

PC Statistics – Day 06 – Expected Value Worksheet

1. A \$20 bill, two \$10 bills, three \$5 bills and four \$1 bills are placed in a bag. If a bill is chosen at random, what is the expected value for the amount chosen?

X	f	P(x)
20	1	.1
10	2	.2
5	3	.3
1	4	.4

$$E(X) = 20(.1) + 10(.2) + 5(.3) + 1(.4)$$

$$E(X) = \$5.90$$

2. In a game you flip a coin twice, and record the number of heads that occur. You get 10 points for 2 heads, zero points for 1 head, and 5 points for no heads. What is the expected value for the number of points you'll win per turn?

Points	f/P
10	1/4
0	2/4
5	1/4

$$E(X) = 10\left(\frac{1}{4}\right) + 0\left(\frac{2}{4}\right) + 5\left(\frac{1}{4}\right)$$

$$= 2.5 + 0 + 1.25$$

$$E(X) = 3.75$$

3. There is an equally likely chance that a falling dart will land anywhere on the rug below. The following system is used to find the number of points the player wins. What is the expected value for the number of points won?

Black = 40 points
 Gray = 20 points
 White = 0 points



Points	P(x)
40	6/15
20	6/15
0	3/15

$$E(X) = 40\left(\frac{6}{15}\right) + 20\left(\frac{6}{15}\right) + 0\left(\frac{3}{15}\right)$$

$$E(X) = 24$$



4. A mysterious card-playing squirrel (pictured) offers you the opportunity to join in his game. The rules are:

- To play you must pay him \$2.
- If you pick a spade from a shuffled pack, you win \$9.

Find the expected value you win (or lose) per game.

X	P(x)
-2	39/52
7	13/52

$$E(X) = -2\left(\frac{39}{52}\right) + 7\left(\frac{13}{52}\right) = \$.25$$

5. A dice game involves rolling 2 dice. If you roll a 2, 3, 4, 10, 11, or a 12 you win \$5. If you roll a 5, 6, 7, 8, or 9 you lose \$5. Find the expected value you win (or lose) per game.

Sum	P(x)
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

$$E(X) = 5\left(\frac{12}{36}\right) + -5\left(\frac{24}{36}\right) = \$.167$$

6. Consider a hat with pieces of paper inside. The papers are numbered as follows: 5 pieces with the number "1", 6 pieces with the number "7", and 9 pieces with the number "50." Find the expected value for drawing a ticket from this hat.

X	f
1	5
7	6
50	9

$$E(X) = 1\left(\frac{5}{20}\right) + 7\left(\frac{6}{20}\right) + 50\left(\frac{9}{20}\right) = 24.85$$

7. Someone is using a loaded die. The probability of rolling a "1" is $\frac{1}{2}$ and the rest of the values have equal probabilities. Find the expected value for rolling this die.

1	$\frac{1}{2}$
2	$\cdot 1$
3	$\cdot 1$
4	$\cdot 1$
5	$\cdot 1$
6	$\cdot 1$

$$E(X) = 1\left(\frac{1}{2}\right) + 2(\cdot 1) + 3(\cdot 1) + 4(\cdot 1) + 5(\cdot 1) + 6(\cdot 1) = 2.5$$

8. Construct a counting tree to list the possible outcomes of a 3-child family. Verify that the expected value (of having 0, 1, 2 or 3 boys) is 1.5.

BBB	GBB	GGB
BBG	BGG	GGG
BGB	GBG	

$$E(X) = 3\left(\frac{1}{8}\right) + 2\left(\frac{3}{8}\right) + 1\left(\frac{3}{8}\right) + 0\left(\frac{1}{8}\right) = 1.5$$

9. "Wheel of Fortune" just got a new wheel! On it there are 6 slots worth \$200, 15 slots worth \$400, 2 slots worth \$600, 6 slots with no money, 1 slot with \$1000 and 1 slot with a car worth \$20,000.

a. What is the expected winnings on one turn (cash and prizes)?

$$E(X) = 200\left(\frac{6}{31}\right) + 400\left(\frac{15}{31}\right) + 600\left(\frac{2}{31}\right) + 1000\left(\frac{1}{31}\right) + 20000\left(\frac{1}{31}\right) + 0\left(\frac{6}{31}\right)$$

X	f
200	6
400	15
600	2
1000	1
20000	1
0	6
31	

b. If you get 20 spins, what are your expected winnings?

$$20(948.39) = \$18967.74$$

10. A "fair game" is any game with an expected value of 0. Are these games fair? Show work for all expected values.

a. We flip 3 coins. You win \$1 if there are exactly two heads or all tails; otherwise, you lose \$1.

HHH HHT HTH HTT TTT
TTH THT TTT

X	P(X)
1	$\frac{4}{8}$
-1	$\frac{4}{8}$

$$1\left(\frac{4}{8}\right) + (-1)\left(\frac{4}{8}\right) = 0$$

Yes fair game.

b. We draw cards. You win \$1 for a red card under a jack (ace is low). You win \$5 for any face card. Otherwise, I win \$4.

$$E(X) = 1\left(\frac{20}{52}\right) + 5\left(\frac{12}{52}\right) - 4\left(\frac{20}{52}\right) = 0$$

Yes its fair.

X	P(X)
1	20
5	12
-4	20

c. You enter a lottery for \$1.50 with one prize of \$2000. 15007 tickets are sold.

$$-1.50\left(\frac{15006}{15007}\right) + 2000\left(\frac{1}{15007}\right) = -1.37$$

11. You walk into a car dealership to buy a car and the salesperson offers you a deal. You roll a pair of 6-sided dice, and, using the following chart, you find what the \$18,000 car will cost you. Find the expected value, and decide if it makes sense to play the game.

Sum of dice	Amount	
2	Pay nothing	1
3	\$10000	2
4	\$16000	3
5	\$18000	4
6	\$20000	5
7	\$24000	6
8	\$20000	5
9	\$18000	4
10	\$16000	3
11	\$10000	2
12	Pay nothing	1

$$E(X) = 0\left(\frac{2}{36}\right) + 10000\left(\frac{4}{36}\right) + 16000\left(\frac{6}{36}\right) + 18000\left(\frac{8}{36}\right) + 20000\left(\frac{10}{36}\right) + 24000\left(\frac{10}{36}\right)$$

$$E(X) = \$17333.33$$

12. In a game, you roll a die. If you get a 1 or a 5, you would win \$5. If you roll a 4 you win \$15 and if you roll a 2, 3, or 6 you lose \$10. What is the expected value of one roll of the die?

X	P(X)
5	$\frac{2}{6}$
15	$\frac{1}{6}$
-10	$\frac{3}{6}$

$$E(X) = 5\left(\frac{2}{6}\right) + 15\left(\frac{1}{6}\right) - 10\left(\frac{3}{6}\right) = -\$1.83$$