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Unit 1: Quadratics Revisited

Simplify each of the following expressions. https://www.youtube.com/watch?v=KhdZvfH6fGg

1) i^{45} = ***i***

2) i^{72} = ***1***

3) i^{130} = ***-1***

4) i^{453} = ***i***

5) i^{480} = ***1***

6) i^{1241} = ***i***

7) $(2 + 3i) + (9 + 4i)$
11 + 7i

8) $(3 + 4i) + (7 - 2i)$
10 + 2i

9) $(9 + 2i) - (12 - i)$
-3 + i

10) $(5 - 3i)(3 + 7i)$
36 + 26i

11) $(7 - 2i)(3 - 5i)$
11 - 41i

12) $\dfrac{5+6i}{3-5i}$
$\dfrac{-15+43i}{34}$

13) $\dfrac{7-2i}{5+9i}$
$\dfrac{17-73i}{106}$

14) $|2 + 3i|$
$\sqrt{13}$

15) $|5 - 2i|$
$\sqrt{29}$

Solve the following by factoring. https://www.youtube.com/watch?v=fmLM8xt-YIg

16) $x^2 + 8x + 16 = 0$
$(x+4)^2$ x=-4

17) $x^2 - 6x = -9$
$(x-3)^2$ x=3

18) $x^2 + 8x = -12$
$(x+2)(x+6)$ $x = -2$ & -6

19) $x^2 - 7x + 10 = 0$
$(x-5)(x-2)$ $x = 2$ &5

Solve the following by completing the square and the square root method.
https://www.youtube.com/watch?v=xGOQYTo9AKY

20) $x^2 + 4x + 1 = 0$
$-2 \pm \sqrt{3}$

21) $x^2 - 6x + 2 = 0$
$3 \pm \sqrt{7}$

Solve each of the following using the quadratic formula.
https://www.youtube.com/watch?v=EeVqtpuMFOU

22) $x^2 + 3x - 5 = 0$
$\dfrac{-3 \pm \sqrt{29}}{2}$

23) $2x^2 + 5x - 3 = 0$
-3 and ½

Factor and solve the following.

24) $x^2 - 9$
$(x-3)(x+3)$ $x = \pm 3$

25) $x^2 + 9$
$(x-3i)(x+3i)$ $x = \pm 3i$

26) $x^2 + 16$
$(x-4i)(x+4i)$ $x = \pm 4i$

Simplify the following completely. https://www.youtube.com/watch?v=Mm31HKBp1Gc

27) $\sqrt{8x^2}$
$2x\sqrt{2}$

28) $\sqrt{24x^2y^4}$
$2xy^2\sqrt{6}$

29) $(\sqrt{5})(\sqrt[3]{5})$
$5^{\frac{5}{6}}$

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Unit 2: Operations with Polynomials

Find the sum or difference of the following. https://www.youtube.com/watch?v=egPRVWOjfVw

1) $(4y - 2y^2 + 7) - (3y - 2y^2 + 6)$

$y + 1$

2) $(12x - 2x^3 + 4) + (2x - 8x^2)$

$-2x^3 - 8x^2 + 14x + 4$

3) $(4x^2 - 6x - 9) + (-5x^2 + 2x + 11)$

$-x^2 - 4x + 2$

4) $(y - y^2) - (2y - 2y^2)$

$y^2 - y$

Use long division or synthetic division to divide the following polynomials.
https://www.youtube.com/watch?v=bZoMz1Cy1T4&t=46s

5) $(5x^2 - 5x - 18) \div (x - 2)$

$5x + 5 - \frac{8}{x-2}$

6) $\frac{3x^3 - 5x^2 + 2x}{x - 1}$

$3x^2 - 2x$

7) $(8x^4 + 2x - 3) \div (x - 5)$

$8x^3 + 40x^2 + 200x + 1002 - \frac{5007}{x-5}$

8) $(8x^3 - 27) \div (2x - 3)$

$4x^2 + 6x + 9$

Multiply the following expressions. Write your answer in standard form.
https://www.youtube.com/watch?v=wUYa2NAV5t4

9) $(2x + 5)^3$

$8x^3 + 60x^2 + 150x + 125$

10) $(-5x + 7)(6x + 4)$

$-30x^2 + 22x + 28$

Classify the following polynomials by degree and number of terms.

	Degree	Number of terms
11) -7	_____	_____
12) $2 - x^2y + x^2$	_____	_____
13) $-3v^5 + 5v^2 - 2v + 3$	_____	_____

Use Pascal's triangle in order to answer #14-15:
https://www.youtube.com/watch?v=C2LDGz5D1Sw

14) $(x + 3)^5$

$x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$

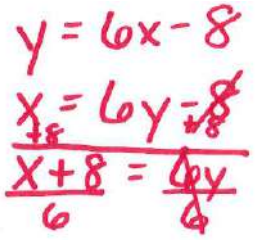
15) $(x - 2)^3$

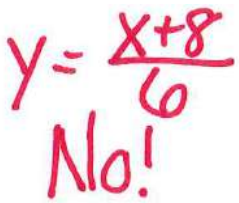
$x^3 - 6x^2 + 12x - 8$

State if the given functions are inverses. Show or explain how you determined the answer.

16) $f(x) = 6x - 8$
$g(x) = \frac{x+6}{8}$

No!





Find the inverse of the function. https://www.youtube.com/watch?v=Ec5YYVxyq44

17) $f(x) = 5x^2 + 7$ **$f^{-1}(x) = \sqrt{\frac{x-7}{5}}$**

18) $y = -4x + 6$ **$f^{-1}(x) = \frac{x-6}{-4}$**

Given the following functions $m(x) = 4x - 3$ *and* $t(x) = -2x^2 + 5$, find:

https://www.youtube.com/watch?v=PCHkzAxiJwU

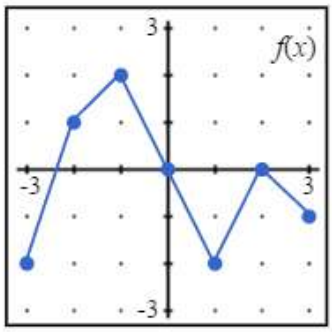
19. $m(t(5)) =$ **-183**

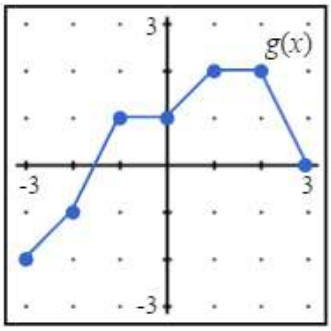
20. $t(m(x)) = -32x^2 + 48x - 13$

21. $m(x) + t(x) = -2x^2 + 4x + 2$

22. $3t(x) - m(x) = -6x^2 - 4x + 18$

Use the graph to answer 23-25.





23. f(g(-1)) = **-3**

24. g(f(-1)) = **2**

25. g(g(0)) = **0**

Unit 3: Polynomial Functions

Find all roots of the following. https://www.youtube.com/watch?v=rP- zFngio

1) $F(x) = x^3 + 3x^2 - 4x - 12$
X = -3, ±2

2) $G(x) = x^3 + 4x^2 + x - 6$
X = -3, -2, 1

3) $H(x) = x^3 + 4x^2 + x + 4$
X = -4, ±*i*

4) $J(x) = x^4 - 12x^2 - 64$
x= ±4, ±2

5)

For the following find each of the roots, classify them and show the factors.

P: (20)
Q: (1)

a. $f(x) = x^3 - 5x^2 - 4x + 20$ **Possible rational roots:** P/Q: ±[1, 2, 4, 5, 10, 20]

Show work for Synthetic Division and Quadratic Formula (or Factoring):

5 | 1 -5 -4 20
↓ 5 0 -20
1 0 -4 [0]

$x^2 - 4 = 0$
$(x+2)(x-2)$

Complete Factorization: $(x-5)(x+2)(x-2)$

Roots and Classification:

Root: 5 — Rational — Irrational — Real — Imaginary

Root: 2 — Rational — Irrational — Real — Imaginary

Root: -2 — Rational — Irrational — Real — Imaginary

6)

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P(8) Q(1)

For the following find each of the roots, classify them and show the factors.

$f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$ **Possible rational roots:** P/Q: ±[1, 2, 4, 8]

Show work for Synthetic Division and Quadratic Formula (or Factoring):

-4 | 1 2 -9 -2 8 → 1 -4 8 4 -8 → $1x^3 - 2x^2 - 1x + 2$

2 | 1 -2 -1 2 → 1 0 -1 0 → $x^2 - 1$ → $(x+1)(x-1)$

Complete Factorization: $(x+4)(x-2)(x+1)(x-1)$

Roots and Classification:

Root: -4	Rational	Irrational	Real	Imaginary
Root: 2	Rational	Irrational	Real	Imaginary
Root: 1	Rational	Irrational	Real	Imaginary
Root: -1	Rational	Irrational	Real	Imaginary

Find the equation, in standard form, for each of the following functions using the given information. https://www.youtube.com/watch?v=HdXjyHnNwKI

7) x-intercepts: (–3, 0), (–1, 0), (1, 0), and (2, 0)
y-intercept: (0, 3)
Find the 4th degree equation $F(x) = x^4 + x^3 - 7x^2 - x + 6 \rightarrow$
$F(x) = \frac{1}{2}x^4 + \frac{1}{2}x^3 - \frac{7}{2}x^2 - \frac{1}{2}x + 3$

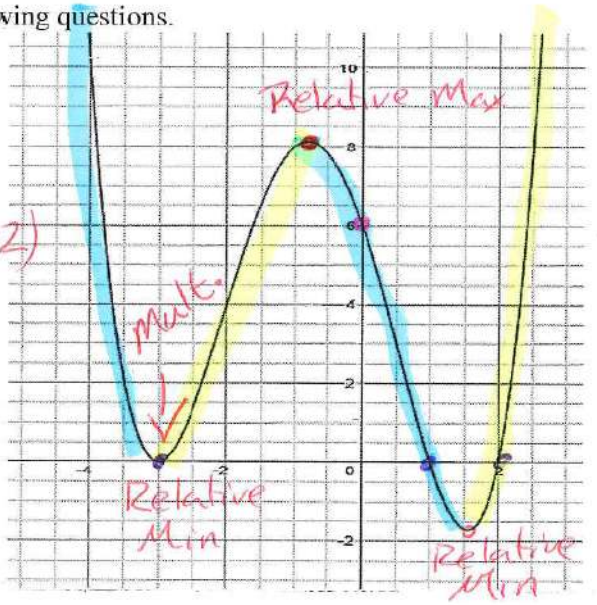
8) x-intercepts: (–5, 0), (–1, 0), and (1, 0)
y-intercept: (0, 5)
Find the 3rd degree equation $F(x) = x^3 + 5x^2 - x - 5 \rightarrow F(x) = -x^3 - 5x^2 + x + 5$

9) x-intercepts: (–2, 0) Mult. 2, (1, 0), and (3, 0)
y-intercept: (0, 12)
Find the 4th degree equation $F(x) = x^4 - 9x^2 - 4x + 12$

10) https://www.youtube.com/watch?v=zE1MlqHmTIQ

10) Using the provided graph, answer all of the following questions.

A) x-intercepts: -3, 1, 2
B) y-intercept: 6
C) 3rd degree equation: $x^3 - 7x + 6$
D) Relative Maximum(s): (-.7, 8)
E) Relative Minimum(s): (-3, 0) & (1.5, -2)
E) Absolute Maximum: None
F) Absolute Minimum(s): (1.5, -2)
F) Domain: ℝ
G) Range: $-2 \leq y < \infty$
H) Odd, even, or neither: Neither
I) Intervals of increase:
J) Intervals of decrease:
J) Multiplicities: -3



Unit 4: Rational and Radical Relationships

https://www.youtube.com/watch?v=ZWTZm6Aveqg

Solve, show all work:

1) $\frac{7}{4x} - \frac{3}{x^2} = \frac{1}{2x^2}$

Needs: x 4 2

$7x - 12 = 2$

$7x - 12 = 2$

$7x = 14$

$x = 2$

2) $\frac{3x}{2} - \frac{x}{3} = 7$

Needs: 3 2 6

$9x - 2x = 42$

$9x - 2x = 42$

$7x = 42$

$x = 6$

Simplify: https://www.youtube.com/watch?v=x6XyP2_RfA4

3) $\frac{x-5}{x^2-13x+40}$

$\frac{x-5}{(x-5)(x-8)} = \frac{1}{x-8}$ $x \neq 5$

4) $\frac{4}{x+1} - \frac{2}{x+2}$

$\frac{4(x+2)}{(x+1)(x+2)} - \frac{2(x+1)}{(x+1)(x+2)} = \frac{2x+6}{(x+1)(x+2)}$

5) $\frac{2}{x^2-9} + \frac{4}{x+3}$

$\frac{2}{(x+3)(x-3)} + \frac{4(x-3)}{(x+3)(x-3)} = \frac{4x-10}{x^2-9}$

6) $\frac{12x(x-7)}{(x-7)(x+3)} \div \frac{(x-6)(x+8)}{x+3}$

$\frac{12x(x-7)}{(x-7)(x+3)} \cdot \frac{(x+3)}{(x-6)(x+8)} = \frac{12x}{(x-6)(x+8)}$ $x \neq 7, -3$

7) $\frac{10}{x-3} \cdot \frac{2x+10}{x+5}$

$\frac{10}{x-3} \cdot \frac{2(x+5)}{(x+5)} = \frac{20}{x-3}$ $x \neq -5$

8) $\frac{x+7}{x^2+20x-12} \div \frac{x+9}{6}$

$\frac{6(x+7)}{(x^2+20x-12)(x+9)}$

Solve (Remember to check your answers!): https://www.youtube.com/watch?v=3MoY3kEVY8o

9) $\sqrt{18-3x} = (x-6)^2$

$18 - 3x = x^2 - 12x + 36$

$0 = x^2 - 9x + 18$

$(x-6)(x-3)$

$x = 6, 3$

$\sqrt{18-3(6)} = 6 - 6$

$0 = 0$ ✓

$\sqrt{18-3(3)} = 3 - 6$

$-3 = -3$ ✓

10) $\sqrt[3]{x-5} = \sqrt[3]{x+1}$

$x - 5 = x + 1$

$0 = 6$

No Solution

11) Nancy takes 2 hours to spread mulch. Jessica takes 1 hour and 30 minutes to complete the same job. If they work together, how long, in minutes, will it take them?

2 hours = 120minutes 1hr & 30 min = 90 minutes

$\frac{1}{120} + \frac{1}{90} = \frac{1}{x}$

Needs: 3x 4x 360

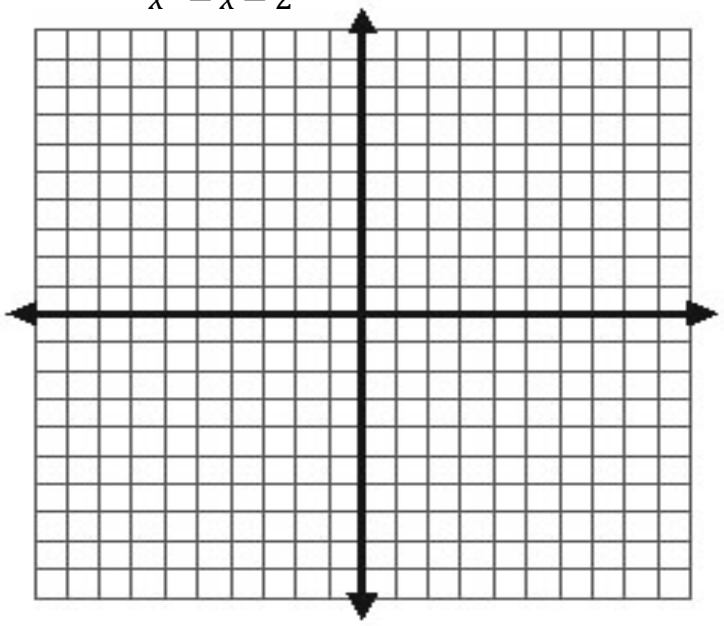
$3x + 4x = 360$

$7x = 360$

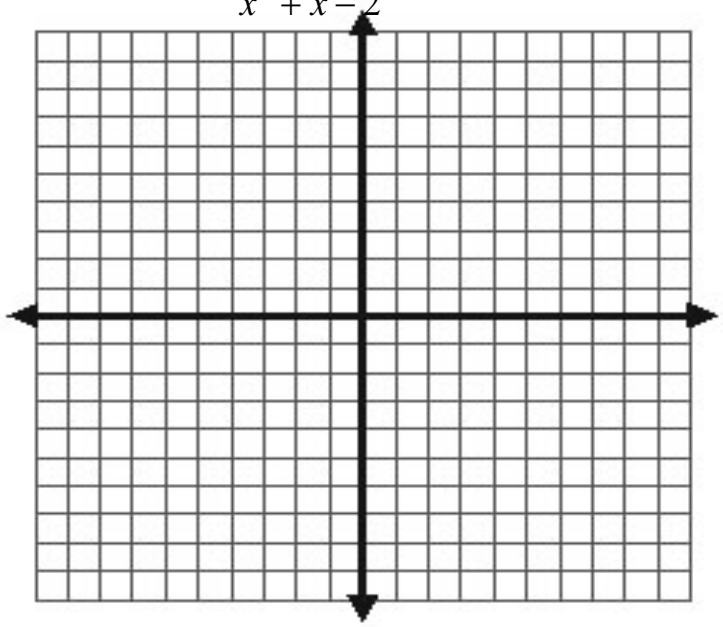
51.4 min

Identify all aspects of the graph that we have discussed in this unit. (Vertical asymptotes, horizontal asymptotes, holes in the graph, solutions, domain, and range.) Then graph.
https://www.youtube.com/watch?v=hWjMovgqvi4

12) $y = \dfrac{x^2 + 2x - 8}{x^2 - x - 2}$



13) $y = \dfrac{x^2 + 2x - 3}{x^2 + x - 2}$



20) Identify all aspects of the graph of $y = \dfrac{x^2 + 2x - 3}{x^2 + x - 2}$ that we have discussed in this unit. (Vertical asymptotes, horizontal asymptotes, holes in the graph, solutions, domain, and range.) Then graph.

$\dfrac{(x+3)(x-1)}{(x+2)(x-1)}$

H.A = 1

V.A = -2

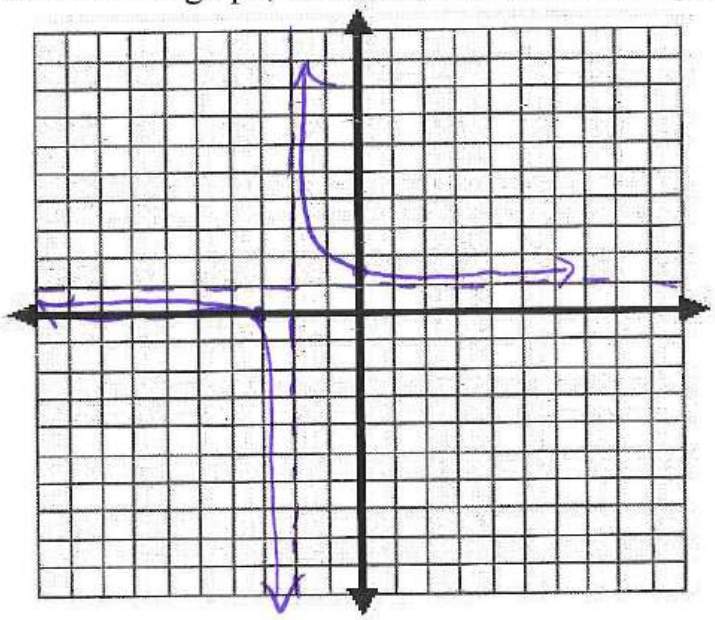
Holes = $(1, \frac{4}{3})$

Domain = $\mathbb{R}$ $x \neq 1$

Range = $\mathbb{R}$ $y \neq -2$

Y-int = $\frac{3}{2}$

X-int = -3



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Unit 5: Exponential & Logarithmic Functions

A new solar system was discovered far from the Milky Way in 1999. After much preparation, NASA decided to send a group of astronauts to explore Exponentia, one of the planets in the system. Upon landing on the planet, the astronauts discovered life on the planet. Scientists named the creatures Viètians (vee-et-ee-ans), after the French mathematician François Viète who led the way in developing our present system of notating exponents. After observing the species for a number of years, NASA biologists determined that the population was growing by 10% each year.

1. The estimated number of Viètians was 1 million in 1999 and their population increases 10% a year. Complete the table to show the population for the next 4 years after 1999.

Years since 1999	0	1	2	3	4
Population in millions	1	1 + (.1)1 = 1.1 1.1	1.1+(.1)(1.1) =1.1(1+.1) =1.1(1.1) $=(1.1)^2$ =1.21 million	$(1.1)^2+.1(1.1)^2 =(1.1)^2(1+.1)$ $=(1.1)^2(1.1)$ $=(1.1)^3$ =1.331 million	1.4641 million

2. Write an equation for the population of Exponentia, *P*, as a function of the number of years, *t*, since 1999. How can you express the population as an expression in the table rather than as a computed value to help you see patterns to create the function?

Solution: $P(t) = 1.1^t$, where population is measured in millions

3. What was the population in 2005? What will the population be in 2015 if the population growth rate remains the same?

***2005 is 6 years after 1999. Thus, we find P(6) = 1.771561million = 1,771,561* Viètians.**

2015 *is 16 years after 1999. We find P(16) =* 1.1^{16} ***= 4.594972986 million = 4,594,972* Viètians.**

4. Suppose you want to know when the population reached 2 million. Write an equation that could be solved to answer this question. Determine the answer graphically and algebraically.

1.1^x=2. Graphically, we could see where the line y = 2 intersects the graph in part (d). The point of intersection is (7.2725, 2) so it took approximately 7.3 years after 1999 for the population to reach 2 million. Algebraically, we have lll1.12=x so lll 2lll 1.1 or approx 7.3 years.

Solve the following. https://www.youtube.com/watch?v=ZIwmZ9m0byI

7. $12^x = 15^3$

$\log_{12} 3375 = x$

X = 3.27

8. $\log_x 8 = 3$

$x^3 = 8$

x = 2

9. $7^{2t} = 1955$

$\log_{17} 1955 = 2t$

2t = 2.675 t=1.337

10. $e^{3t} = 52$

$\ln 52 = 3t$

t = 1.317

11. $\log_3 X = 5$

$3^5 = x$

x = 243

12. $\log_{19} X = 0$

$19^0 = x$

x = 1

13. $\log(n^2 + 3) = \log(5n - 1)$

$n^2 + 3 = 5n - 1$

n = 4 and 1

14. $\log_4(3x - 2) - \log_4(4x + 1) = 2$

$\log_4 \frac{3x-2}{4x+1} = 2$ $4^2 = \frac{3x-2}{4x+1}$

$x = -\frac{18}{61}$

15. $\ln(6x - 5) = 3$

$e^3 = 6x - 5$

X = 4.18

16. $\log_3(x - 6x) = 3$

$3^3 = x - 6x$

x = -5.4

17. $\log_8 x + \log_8(x + 6) = \log_8(5x + 12)$

$x(x + 6) = 5x + 12$

X = 3 & -4

18. $\log_6 x + \log_6(x - 9) = 2$

$6^2 = x(x - 9)$

x = 12 and -3

Unit 6: Mathematical Modeling

Write the following series in summation notation and find the sum of the first 10 terms.
https://www.youtube.com/watch?v=hEPk36Yncxg

1) $\frac{1}{2} + 3 + 18 + \ldots$ $\sum_{x=12}^{10} \frac{1}{12} \cdot 6^{x-1}$ **Sum: 6,046,617.5**

2) $-4 + 12 - 36 + 108 + \ldots$ $\sum_{x=1}^{10} -4 \cdot -3^{x-1}$ **Sum: 59,048**

Isolate the designated variable. https://www.youtube.com/watch?v=pb7iiaTryQg

2) $E = mc^2$ for c $C = \sqrt{\frac{E}{M}}$ 3) $E = \frac{F}{q}$ for q $Q = \frac{F}{E}$

https://www.youtube.com/watch?v=B3IdfBcXrLA

4) Ten years ago, Josh put money into an account paying 5.5% compounded quarterly. If the account has $12,000 now, how much money did he deposit?

$$12{,}000 = p\left(1 + \frac{.055}{4}\right)^{4 \cdot 10}$$

12,000 = p(1.72677)

P = $6,949.39

5) Your favorite neighbor, Mr. Nelson, needs your advice. He has $50,000 that he received as an inheritance from his late father. He would like to invest it into a savings account. He has the following two options:

Option A: This is a continuously compounding account that has a 4% interest rate.

Option B: This a monthly compounding account that bears a 4.25% interest rate.

If Mr. Nelson is planning to invest for 5 years, which account would you suggest?

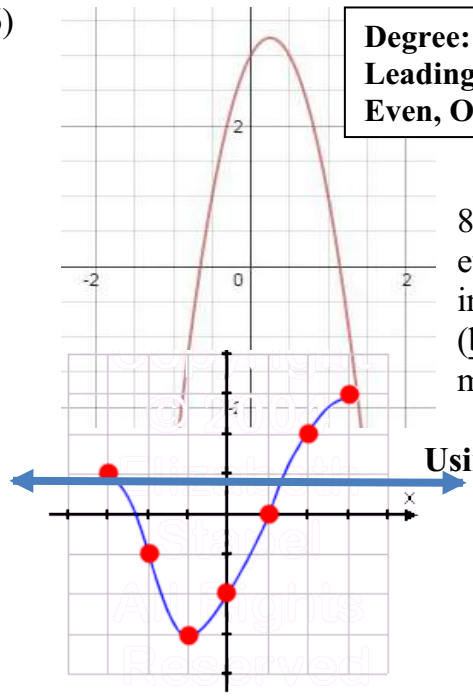
Option A: $50{,}000e^{.04 \cdot 5}$ **$61, 070.14**

Option B: $50{,}000\left(1 + \frac{.0425}{12}\right)^{12 \cdot 5}$ **$61,815.09**

OPTION B!

Determine the degree type, leading coefficient, and function type (i.e. even, odd, or neither) for each of the following.

6)



Degree: Quadratic
Leading coefficient: -
Even, Odd, or Neither: Neither

7) $x^3 + x$

Degree: Cubic
Leading coefficient: +1
Even, Odd, or Neither: Odd

8) You and your family are away on vacation. Instead of eating out every night, it is decided that you will buy groceries and store them in the fridge in the condo. You buy $237.44 worth of groceries (before tax). If the sales tax in your current location is 6%, how much were your groceries after tax?

$224

Using the graph to the left find each of the following.

9) f(3) = 3

10) f(0) = -2

11) f(-2) = -1

12) Is f(x) a function? **YES!**

13) f(g(3)) = 0