GSE Algebra I

EOC Review
Units 3 – 6

Unit 3: Quadratic Functions

Key Ideas

Factoring
Vertex and Standard Form
Solving by various methods
Building Functions
Transformations
Characteristics
Analyzing Functions

Solving Quadratics

- Zeros, roots, or x-intercepts are where the graph crosses the x-axis and where the function equals zero.
- Methods:
  - Factoring: $x^2 - 7x + 12 = 0$
  - Completing the Square: $x^2 + 8x + 7 = 0$
  - Taking the Square Root: $3x^2 - 147 = 0$
  - Quadratic Formula: $5x^2 - 6x - 8 = 0$
  - Graphing

Solving Quadratics

- Standard Form: $y = ax^2 + bx + c$
  - Axis of Symmetry: $x = -b/2a$
  - Vertex: $(-b/2a, f(-b/2a))$
- Vertex Form: $y = a(x - h)^2 + k$
  - Vertex: (h, k)
- Ex: Write $f(x) = 2x^2 + 12x + 1$ in vertex form.

Solving Quadratics

- The function $h(t) = -t^2 + 8t + 2$ represents the height, in feet, of a stream of water being squirted out of a fountain after t seconds. What is the maximum height of the water?

Creating Quadratic Equations

- What is the value of r when $S = 0$ for the equation $S = 2\pi r^2 + 2\pi rh$ for r?

- The product of two consecutive positive integers is 132.
  - Write an equation to model the situation.
  - What are the two consecutive integers?

Building Functions

- Annie is framing a photo with a length of 6 inches and a width of 4 inches. The distance from the edge of the photo to the edge of the frame is x inches. The combined area of the photo and frame is 63 square inches.
  - Write a quadratic function to find the distance from the edge of the photo to the edge of the frame.
  - How wide are the photo and frame together?

Transformations

- Parent Function – the basic function from which all the other functions in a family are modeled.

- $y = a(x - h)^2 + k$

- Ex: Compare the graphs of the following functions to f(x).
  - $\frac{1}{2}$ f(x)
  - f(x) – 5
  - f(x – 2) + 1

- Even, Odd, or Neither: $f(x) = 2x^3 + 6x$

Characteristics of Quadratics

- Domain
- Range
- x-intercept, Root, Zero
- y-intercept
- Increasing/Decreasing
- Minimum/Maximum
- End Behavior
- Average ROC

Characteristics of Quadratics

- A ball is thrown into the air from a height of 4 feet at time t = 0. The function that models this situation is $h(t) = -16x^2 + 63t + 4$, where t is measured in seconds and h is the height in feet.
  - What is the height of the ball after 2 seconds?
  - When will the ball reach a height of 50 feet?
  - What is the maximum height of the ball?
  - When will the ball hit the ground?
  - What domain makes sense for the function?

Unit 4: Exponential Functions

Key Ideas

Creating Equations

Transformations

Geometric Sequences

Characteristics

Creating Equations

$y = ab^x$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- Ex: An amount of $1000 is deposited into a bank account that pays 4% interest compounded once a year. If there are no other withdrawals or deposits, what will be the balance of the account after 3 years?

- Ex: The city of Arachna has a spider population that has been doubling every year. If there are 100,000 spiders this year, how many will there be 4 years from now?

Building Functions

$$a_n = a_1(r)^{n-1}$$

- Clara records the number of situps she does over a period of time. Her data for five weeks is 3, 6, 12, 24, 48. Write a sequence to represent her data.

- Growth and Decay

- Ex: The temperature of a large tub of water that is currently at 100 degrees decreases by about 10% each hour.
  - Write an equation to represent the situation.
  - What will the temperature be after 5 hours?

Transformations

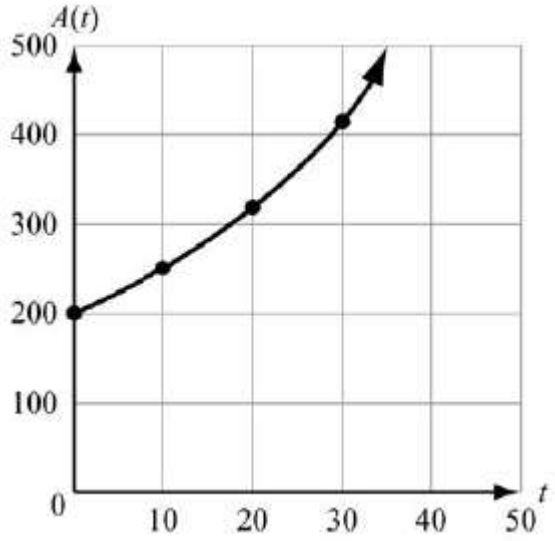
- If $f(x) = 2^x$, how will $g(x) = 3f(x)$, $h(x) = 1/3f(x)$, and $m(x) = -f(x)$ compare?

Function Notation

- A population of bacteria begins with 2 bacteria on the first day and triples every day.
  - Write a function to represent the situation.
  - What is the common ratio of the function?
  - What is the y-intercept of the function?
  - Write a recursive formula for the bacteria growth.
  - What is the population after 10 days?

Characteristics

- The amount accumulated in a bank account over a time period t and based on an initial deposit of $200 is found using the formula $A(t) = 200(1.025)^t$, $t \geq 0$. Time, t, is represented on the horizontal axis. The accumulated amount, A(t), is represented on the vertical axis.
  - What are the intercepts of the function?
  - What is the domain of the function?
  - Why are all the t-values non-negative?
  - What is the range of the function?



Comparing

Two quantities increase at exponential rates. This table shows the value of Quantity A, $f(x)$, after x years.

Quantity A

x	0	1	2	3	4
$f(x)$	100.00	150.00	225.00	337.50	506.25

This function represents the value of Quantity B, $g(x)$, after x years.

$$g(x) = 50(2)^x$$

Which quantity will be greater at the end of the fourth year and by how much?

Unit 5: Comparing and Constrasting Functions

Key Ideas

Construct and Compare Linear, Quadratic, and Exp Models

Interpret Expressions

Transformations

Comparing

- Examine function values carefully.
- Remember that a linear function has a constant rate of change.
- Keep in mind that growth rates are modeled with exponential functions.
- Quadratic functions decrease and increase.
- Look for asymptotes, endpoints, or vertex.

Comparing

This table shows that the value of $f(x) = 5x^2 + 4$ is greater than the value of $g(x) = 2^x$ over the interval [0, 8].

x	$f(x)$	$g(x)$
0	$5(0)^2 + 4 = 4$	$2^0 = 1$
2	$5(2)^2 + 4 = 24$	$2^2 = 4$
4	$5(4)^2 + 4 = 84$	$2^4 = 16$
6	$5(6)^2 + 4 = 184$	$2^6 = 64$
8	$5(8)^2 + 4 = 324$	$2^8 = 256$

As x increases, will the value of $f(x)$ always be greater than the value of $g(x)$? Explain how you know.

Interpreting

- A parameter is the independent variable or variables in a system of equations with more than one dependent variable.

Equation	Parameter(s)
$y = 3x + 5$	coefficient 3, constant 5
$f(x) = \frac{9}{5}x + 32$	coefficient $\frac{9}{5}$, constant 32
$v(t) = v_0 + at$	coefficient a, constant v_0
$y = mx + b$	coefficient m, constant b

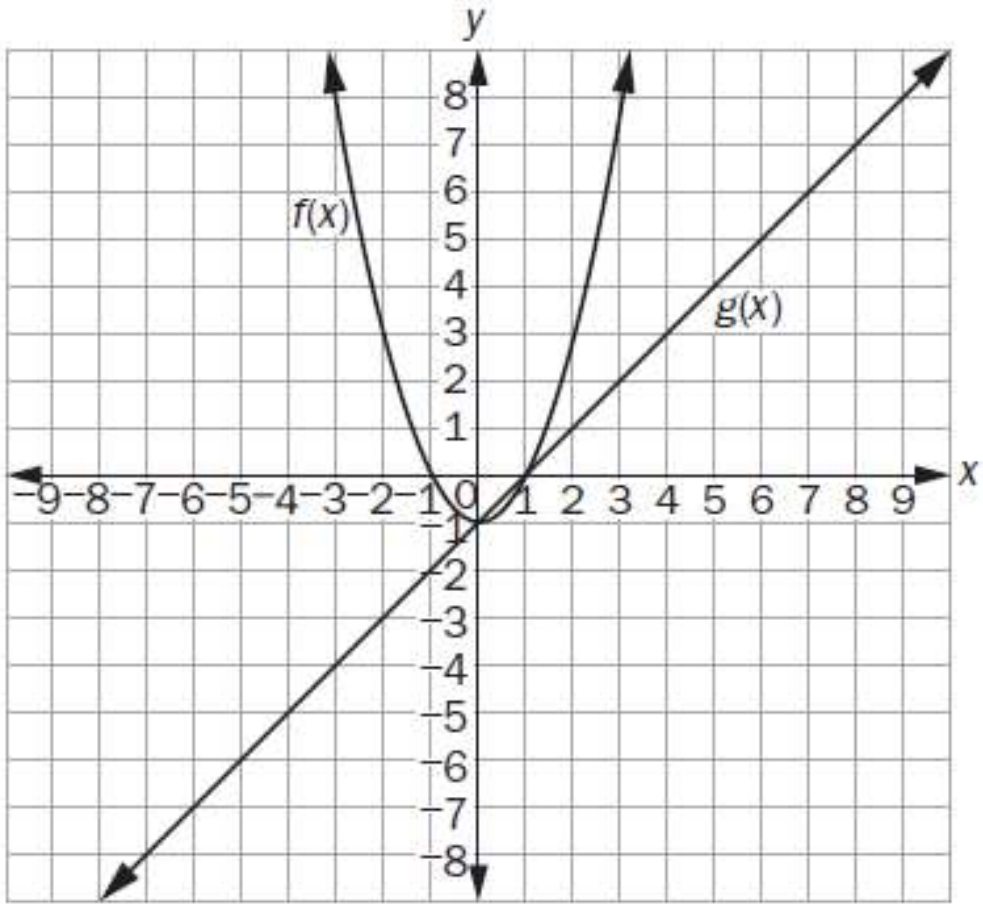
Interpreting

- Katherine has heard that you can estimate the outside temperature from the number of times a cricket chirps. It turns out that the warmer it is outside, the more a cricket will chirp. She has these three pieces of information:
  - A cricket chirps 76 times a minute at 56 degrees (76, 56).
  - A cricket chirps 212 times per minute at 90 degrees (212, 90).
  - The relationship is linear.

  Estimate the function.

Transformations

Look at the graphs of the function $f(x) = x^2 + 1$ and $g(x) = x - 1$.



What transformation makes $g(x) \geq f(x)$ only for the interval $-2 \leq x \leq 3$?

Unit 6: Describing Data

Key Ideas

One Variable Stats

Bivariate Stats

Linear Models

One Variable Statistics

- Measures of Central Tendency:
  - Mean
  - Median
- First Quartile
- Third Quartile
- Interquartile Range
- Box Plot
- Histogram
- Outliers
- Mean Absolute Deviation
- Skewness

Bivariate Data

- Two variable statistics
- Categorical (color, gender, ethnicity) and Quantitative (age, years of schooling, height)
- Bivariate Chart = two-way frequency chart
- Joint Frequencies
- Marginal Frequencies
- Conditional Frequencies
- Scatter Plot
- Line of Best Fit
- Regression
- Correlation Coefficient