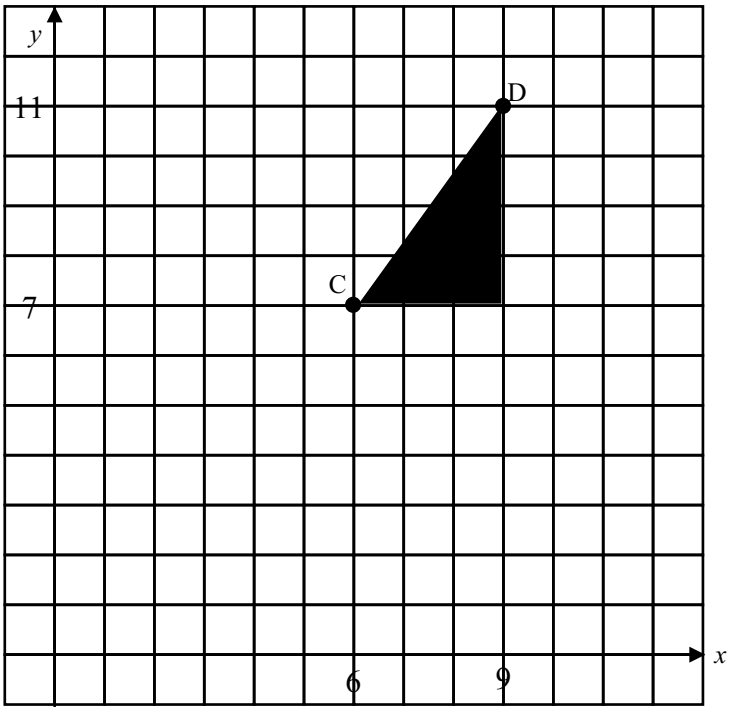
Discovering the Distance Formula

1. Consider the picture of two points, C(6, 7) and D(9, 11). We are interested in the distance between them.



a) Draw in line segment $\overline{CD}$ using a straightedge.

b) Draw in and label point E, which is located 3 units right of point C, and 4 units down from point D.

c) Draw in the 2 legs of right triangle ΔCDE.

d) Label $\overline{CE}$ with its length.

e) Label $\overline{DE}$ with its length.

f) Label point D with its coordinates $(9,11)$.

g) Using the most important theorem in all of Geometry, use the right triangle to determine the length of the hypotenuse.

h) So, what is the distance between points C and D?

i) Label point E with its coordinates $(9,7)$.

j) Our goal now is to see how to combine the coordinates of endpoint $C(6,7)$ with endpoint $D(9,11)$ to come up with the distance of 5.

k) Write down only the x-coordinates of points C and D:

Point C x-coordinate:

Point D x-coordinate:

l) Notice that the horizontal leg $\overline{CE}$ is 3 units long: How could you combine the two numbers from (k) to get a result of 3 on your calculator?

m) Write a mathematical expression to represent the idea from (l). Your expression should have the numerals 9 and 6 in it:

(Put parentheses around it.)

n) Write down only the y-coordinates of point C and D:

Point C y-coordinate:

Point D y-coordinate:

o) Notice that the vertical leg $\overline{DE}$ is 4 units long: How could you combine the two numbers from (n) to get a result of 4 on your calculator?

p) Write a mathematical expression to represent the idea from (o). Your expression should have the numerals 11 and 7 in it:

(Put parentheses around it.)

q) Looking at Pythagorean Theorem work from part (g), we see the basic equation:

$3^2 + 4^2 = 5^2$:

Rewrite this equation by replacing the 3 with $(9-6)$ (don't simplify it!), and replace the 4 with $(11-7)$ (again, don't simplify).

r) Let's relabel the numbers for points C and D. Instead of numerals, let's use symbols. We'll call point C "Point 1," and label it accordingly:

[ ] [ ]

C (6 , 7)

And we'll label point D as "Point 2":

[ ] [ ]

D (9 , 11)

s) Now, let's rewrite your answer from (q):

- Replace the 6, 7, 9, and 11 with $x_1, y_1, x_2, and\ y_2$ (Make sure you match them up as specified in part (r);
- Replace the 5 with a "*d*" to represent the distance:

2. You should now be looking at the equation $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$:

a) Rewrite it so the d^2 term is on the left side, and the radical part is on the right side:	b) Take the square root of both sides:	c) Simplify. Your equation should begin: $d =$

3. Let's summarize: the distance formula for the distance *d* between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

a) Let's apply the formula to figure out the distance between $(2,8)$ and $(7,20)$: First, label the points (x_1, y_1) and (x_2, y_2): (Put the little labels over them): $(2,8)$ and $(7,20)$	b) Copy the formula:
c) Substitute:	d) Simplify. Round to the nearest hundredth if necessary.

4. Now, let's apply the formula to figure out the distance between the points (4, –6) and (9, –14):

a) Label the points by putting the little labels over them: (4, –6) and (9, 14)	b) Copy the formula:
c) Substitute:	d) This problem is slightly different from before: What's different this time?
e) Simplify, being very careful with the negatives:	f) Simplify, round to the nearest hundredth if necessary:

5. Use the distance formula to determine the distance between each pair of points. Simplify to nearest hundredth if necessary.

a) (3, 10) and (–4, 31)	b) (–2, –12) and (4, –9)	c) (3.2, 4.9) and (6.3, –9.2) . Round to the nearest hundredth:

6. Write out each of the following the long way, and then simplify:

a) x^2x^5	b) $(x^2)^5$	c) $\frac{x^5}{x^2}$	d) $\frac{x^2}{x^5}$ This is a different problem from (c), so it should have a different answer!	e) $x^5 - x^2$ This is a trick question. The answer is NOT x^3

7. Consider the equation for the area of a trapezoid: $A = \frac{(b_1 + b_2)h}{2}$. Suppose we know the area of a trapezoid is 98 square centimeters, the height is 7 cm, and one base is 12 cm:

a) Substitute all of the known information into the equation:	b) Make an order of operations ladder for the unknown variable:	c) Solve the equation for the unknown variable:
d) Make a sketch of this trapezoid, clearly labeling its height and two bases.	e) Show that its area is, in fact, equal to 98 square centimeters.	f) If you added two centimeters to each base, and to the height, what would be the new area? Show all the work.

8. **Test Preparation**: Simplify: $-3(2x-5)$

9. Simplify: $(2x-3)-(x+7)$

The answer is NOT $x+4$! A negative times a positive is a negative!

10. Write the equation for the line with slope –3 that passes through (4, 7). Show a check.

11. Solve for *x*, then check your work: $8x - 24 = 3(x+12)$