

# Dilations and Scale Factor Notes

## Vocabulary

**Transformation** – operations that alter the form of a figure (such as changing size or direction)

**Dilation** – a type of transformation that changes the size of the image to create a similar image

**Scale Factor** – the ratio of the lengths of corresponding sides of similar figures

**Enlargement** – when the image (or the new figure) is bigger than the original – happens when the scale factor is  $> 1$

**Reduction** – when the image (or the new figure) is smaller than the original – happens when the scale factor is  $< 1$  and  $> 0$ .

## Dilations

When a figure is dilated, its size is changed by multiplying the length of each side by a scale factor. All angles remain the same and so the new shape (or image) is similar to the original.

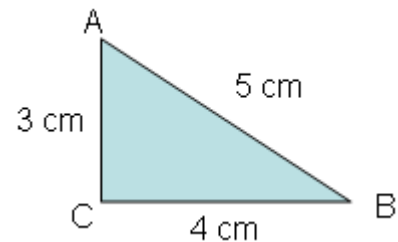
Example 1:

Use the scale factor 3 to dilate triangle ABC to triangle A'B'C'. Give the length of the sides of triangle A'B'C'.

First, find the lengths of the sides of the triangle ABC.

The lengths are:

$$\begin{aligned} AB &= 5 \text{ cm} \\ BC &= 4 \text{ cm} \\ CA &= 3 \text{ cm} \end{aligned}$$



Then use the given scale factor, 3, to enlarge the triangle's side lengths by multiplying each of the original lengths by the scale factor.

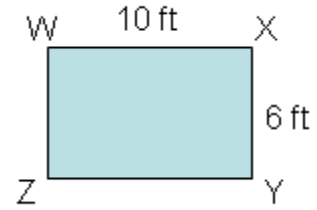
$$\begin{aligned} AB \text{ to } A'B' & \quad 5\text{cm} \times 3 = 15 \text{ cm} \\ BC \text{ to } B'C' & \quad 4\text{cm} \times 3 = 12 \text{ cm} \\ CA \text{ to } C'A' & \quad 3\text{cm} \times 3 = 9 \text{ cm} \end{aligned}$$

So the lengths of the sides of triangle A'B'C' are 15 cm, 12 cm, and 9 cm.

Example 2:

Given the scale factor 2.5, and rectangle WXYZ, find the lengths of the sides of the dilated image rectangle W'X'Y'Z'.

Sides:      WX = 10 ft  
              XY = 6 ft  
              YZ = 10 ft  
              ZW = 6 ft



WX to W'X' and YZ to Y'Z'

$$10 \text{ ft} \times 2.5 = 25 \text{ ft}$$

XY to X'Y' and ZW to Z'W'

$$6 \text{ ft} \times 2.5 = 15 \text{ ft}$$

So the lengths of the sides of W'X'Y'Z' are 25 ft and 15 ft.

Example 3:

Remember that if you multiply something by a fraction less than one, you get a smaller number? The same is true if your scale factor is less than one.

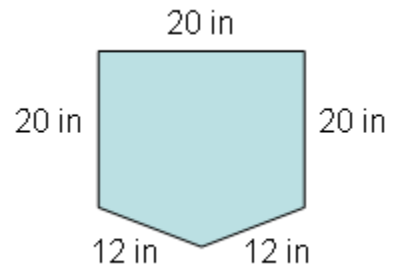
SF =  $\frac{3}{4}$       Find the lengths of the sides of the shape given the scale factor.

There are two side lengths, 20 and 12.

Taking the lengths and multiplying them by the scale factor...

$$20 \times \frac{3}{4} = 15$$

$$12 \times \frac{3}{4} = 9$$



So the lengths of the sides of the new shape would be 15 and 9.

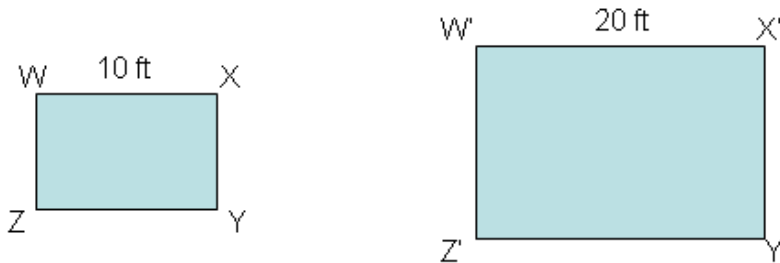
## Finding Scale Factor Given the Shapes

If you are given two shapes and need to find the scale factor, you must know which one was the original and which one is the image or the new shape. Then you need to know the length of corresponding sides and set them up in a ratio like so:

$$\frac{\text{IMAGE}}{\text{ORIGINAL}} = \text{SCALE FACTOR}$$

Example:

What scale factor was used to dilate rectangle WXYZ to rectangle W'X'Y'Z'?



First, determine which shape is the original and which is the image.

Since the smaller one does not have the “prime” marks (the little apostrophes) it must be the original one and the one with the marks is the image.

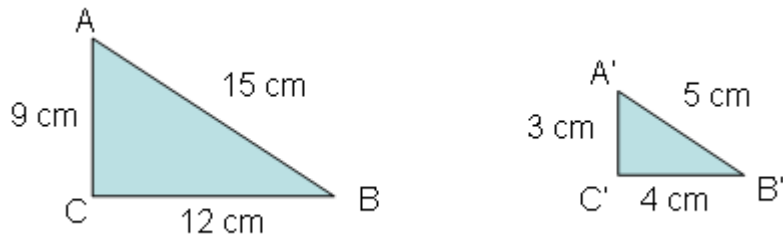
So we set up our ratio like the one above using the lengths given.

$$\frac{\text{IMAGE}}{\text{ORIGINAL}} = \frac{20 \text{ ft}}{10 \text{ ft}} = 2$$

So the scale factor is 2.

Example:

What scale factor was used to dilate triangle ABC to triangle A'B'C'?



First, A'B'C' is the new shape or the image and ABC is the old shape or the original.

$$\frac{\text{IMAGE}}{\text{ORIGINAL}} = \frac{5 \text{ cm}}{15 \text{ cm}} = \frac{1}{3}$$

So the scale factor is  $\frac{1}{3}$ .

Note: the new image is smaller than the original and that the scale factor is less than 1.