Dilations



MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- a) A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b) The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Dilation is where the polygon grows or shrinks but keeps the same overall shape. It's a little like zooming in or out on a camera. In the figure above, the polygon is a rectangle ABCD. As you adjust the slider on the right, the transformed rectangle A'B'C'D gets bigger and smaller, but remains the same shape. (http://www.mathopenref.com/dilate.html)

In fact, a great place to start would be to reference the site above because it has an awesome applet that lets you visualize dilation of a square at zero or any other point.

In short, dilation involves drawing points from the vertices of the dilated images through the point of origin which is often times the origin (sometimes it is not) and multiplying the values of x and y by a scale factor, k, that can be less than zero or greater than zero.

1. Let's begin with a simple experience with dilations. We're going to plot a line segment and then it's dilations with a scale factor, k, where k < 1 (reduction) and k > 1 (enlargement).



In number 1, you created a dilation of the initial image (the line segment to the far left). The scale factor of that image to the result on the far right can be found using a formula. Please note that A and B are the endpoints of the initial image. A' and B' are the endpoints of the resulting image. Also, let A'B' and AB represent the lengths of each segment.

$$k = \frac{A'B'}{AB}$$

2. What is the scale factor that was used in #1?

3. What would be the endpoints of the resulting image if the scale factor was 2?

4. What would be the endpoints of the resulting image if the scale factor was one-fourth?

Let's now begin working on more complex images like triangles, quadrilaterals, and other polygons. I would like to state that dilations are not just limited to geometry as you can see with the image at the top of page 1.

5. Consider the points (-6, -9), (-9, -12), and (-3, -12). What type of shape do these points create? *Be specific*.



6. What would be the coordinates of a triangle is a dilation of it with a scale factor of one-third?

7. How could you prove using some algebra that your coordinates are correct?

8. Plot the points (-6, 6), (-6, 2), (-2, 6), and (-2, 2). Draw your lines that pass through the origin like those in #1. Find the coordinates of a dilation that would exist in quadrant four. Use the graph below to illustrate your work.



9. What is the scale factor of the dilation from the initial image to the resulting image? Prove it.

Let's now look at dilations from points not at the origin. To make things simple, let's use (1, 1) as our center. Please note that the rules regarding the drawn lines and scale factor are still the same.



- 10. Plot the points (3, 5), (5, 3), and (5, 5). Find the other points of dilation with a scale factor of two-fifths.
- 11. How could you prove that you are correct?

Additional Resources

http://youtu.be/mWrvZdxQ0 A - Video #1

http://youtu.be/8GQVZRPof5o - Video #2

http://youtu.be/Y101xMYcnT4 - Video #3 (Negative Scale Factor)

ML Geometry 8-7 Dilations.pdf

http://www.nexuslearning.net/books/ml-geometry/Chapter8/ML%20Geometry%208-7%20Dilations.pdf

C2 OnlineLesson 1.pdf

http://www.glencoe.com/sites/north_carolina/support_student/additional_lessons/Course_2/C2_OnlineLesso n_1.pdf

dilations Ocenter 3vertices 4scale 5point all.pdf

 $http://math.uttyler.edu/ut3mc/resources/problem_based_challenges/transformations/dilations_0center_3vertices_4scale_5point_all.pdf$

Homework

- 1. Take pictures that illustrate dilations in your home, neighborhood, or place of work. If you can, email them to your teacher. If not, show your teacher the pictures of your dilations an initial image and a reduction or an enlargement (you don't need both).
- 2. Come up with three figures and their corresponding points a triangle, a quadrilateral, and a circle. For each image, you must show an enlargement and a reduction. You must identify your scale factor in each case and prove that the scale factor is correct. *Considering your level of neatness, you can draw both requirements for each shape on the same graph/coordinate plane.*
- 3. What does it mean if the scale factor is one?