Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade K

(KUD Organizer)

Mathematics

C. Lancour

4/15/2011

Learning Progression (KUD) Organizer

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Course: Math – Grade K
Topic: Counting & Cardinality

Which standards are in this learning progression?

K.NCC.1, K.NCC.2, K.NCC.3, K.NCC.4a-c, K.NCC.5, K.NCC.6, K.NCC.7,

Connections to other domains and/or clusters:

K.MD.1, K.MD.2, K.MD.3, K.G.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Counting strategies can be used to determine the number of objects.

Understand the relationship between numbers and quantities.

KNOW:	DO:
The number-word sequence, combined with the order inherent in the natural numbers, can be used as a foundation for counting. The counting sequence by ones and tens. Ten different digits can be used and sequenced to express any whole number (In K, write numbers 0-20). The last number named when counting said tells the number of objects counted. (cardinality) The number of objects is the same regardless of their arrangement or the order in which they were counted. When comparing two sets of objects or numbers, the one with the largest quantity is more or smallest quantity is less.	**Know number names and the count sequence.** 1. Count to 100 by ones and by tens. **CC.K.CC.1** 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **CC.K.CC.2** 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **CC.K.CC.3** **Count to tell the number of objects.** 4. Understand the relationship between numbers and quantities; connect counting to cardinality. **CC.K.CC.4** a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **CC.K.CC.4a** b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **CC.K.CC.4b** c. Understand that each successive number name refers to a quantity that is one larger. **CC.K.CC.4c** 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **CC.K.CC.5** **Compare numbers.** 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] **CC.K.CC.6** 7. Compare two numbers between 1 and 10 presented as written numerals. **CC.K.CC.7** **Connections to other Domains &/or Clusters:** **Describe and compare measurable attributes.** 8. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **CC.K.MD.1**

[1] Include groups with up to ten objects.

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	2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **CC.K.MD.2** **Classify objects and count the number of objects in each category.** 3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[2] **CC.K.MD.3** **Analyze, compare, create, and compose shapes.** 4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **CC.K.G.4**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Add, Subtract, Number, Count, Compare, Greater than, Less than, Equal to, More, Less**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

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Course: Math – Grade K
Topic: Operations & Algebraic Thinking (Addition & Subtraction)

Which standards are in this learning progression?

K.OA.1, K.OA.2, K.OA.3, K.OA.4, K.OA.5, K.NBT.1

Connections to other domains and/or clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

Understand addition as putting together and adding to.

Understand subtraction as taking apart and taking from.

Numbers can be decomposed into place value parts and represented in multiple ways.

KNOW:	DO:
There are different ways to show addition and subtraction solutions. Objects or drawings can be used to solve addition and subtraction word problems. Record equations to represent addition or subtraction problems. Addition and subtraction facts to 5. Quantities represented by numbers can be composed and decomposed into part-whole relationships (by place value up to 20 in K). (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.)	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** 1. Represent addition and subtraction with objects, fingers, mental images, drawings[3], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **CC.K.OA.1** 2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **CC.K.OA.2** 3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **CC.K.OA.3** 4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **CC.K.OA.4** 5. Fluently add and subtract within 5. **CC.K.OA.5** **Work with numbers 11-19 to gain foundations for place value.** 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **CC.K.NBT.1**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Add, Subtract, Addition, Subtraction, Equation**

[3] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Course: Math – Grade K
Topic: Place Value (NBT)

Which standards are in this learning progression?

K.NBT.1

By the end of this learning progression, students will be able to…

UNDERSTAND:

Teen numbers can be decomposed into place value parts and represented in multiple ways.

KNOW:	DO:
Quantities represented by numbers can be composed and decomposed into part-whole relationships (by place value up to 20 in K). The base ten number system allows for a new place-value unit by grouping ten of the previous place-value units (and this process can be iterated to obtain larger and larger place-value units). The value of a digit in a written numeral depends on its place, or position, in a number. Each composition or decomposition can be recorded by a drawing or equation (e.g., 18 = 10 + 8) (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.)	**Work with numbers 11-19 to gain foundations for place value.** 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **CC.K.NBT.1**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Add, Subtract, Tens, Ones, Eleven, Twelve, Thirteen, Fourteen, Fifteen, Sixteen, Seventeen, Eighteen, Nineteen**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
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Course: Math – Grade K
Topic: Measurement & Data (A strong connection to Number with counting and comparing)

Which standards are in this learning progression?

K.MD.1, K.MD.2, K.MD.3

Connections to other domains and/or clusters:

K.CC.4, K.CC.5, K.CC.6, K.CC.7, K.G.4

By the end of this learning progression, students will be able to…

UNDERSTAND:	
Objects can be sorted/classified by their attributes. Objects can be described and compared by their attributes	
KNOW:	**DO:**
Objects have different attributes that can be measured or compared.	**Describe and compare measurable attributes.** 1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **CC.K.MD.1** 2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **CC.K.MD.2** **Classify objects and count the number of objects in each category.** 3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[4] **CC.K.MD.3** **Connections to other Domains &/or Clusters:** **Count to tell the number of objects.** 4. Understand the relationship between numbers and quantities; connect counting to cardinality. **CC.K.CC.4** a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **CC.K.CC.4a** b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **CC.K.CC.4b** c. Understand that each successive number name refers to a quantity that is one larger. **CC.K.CC.4c** 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

[4] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

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	CC.K.CC.5 **Compare numbers.** 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[5] **CC.K.CC.6** 7. Compare two numbers between 1 and 10 presented as written numerals. **CC.K.CC.7** **Analyze, compare, create, and compose shapes.** 4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **CC.K.G.4**

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Standards for Mathematical Practice:

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4. Model with mathematics
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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[5] Include groups with up to ten objects.

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Course: Math – Grade K
Topic: Geometry (Shapes and Their Attributes)

Which standards are in this learning progression?

K.G.1, K.G.2, K.G.3, K.G.4, K.G.5, K.G.5

Connections to other domains and/or clusters:

K.CC.4, K.CC.5, K.CC.6, K.CC.7, K.MD.1, K.MD.2, K.MD.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

Shapes can be described, compared, and sorted by their attributes.

Shapes can be joined together to make larger shapes.

KNOW:	DO:
A shape has the same name regardless of orientation or size. Shapes have attributes that allow them to be analyzed and compared. Shapes can be combined to form larger shapes. 2-D "lying flat" 3-D "solid"	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** 1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **CC.K.G.1** 2. Correctly name shapes regardless of their orientations or overall size. **CC.K.G.2** 3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **CC.K.G.3** **Analyze, compare, create, and compose shapes.** 4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **CC.K.G.4** 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **CC.K.G.5** 6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"* **CC.K.G.6** **Connections to other Domains &/or Clusters:** **Describe and compare measurable attributes.** 1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **CC.K.MD.1** 2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **CC.K.MD.2** **Classify objects and count the number of objects in each category.**

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	3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[6] **CC.K.MD.3** **Count to tell the number of objects.** 4. Understand the relationship between numbers and quantities; connect counting to cardinality. **CC.K.CC.4** a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **CC.K.CC.4a** b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **CC.K.CC.4b** c. Understand that each successive number name refers to a quantity that is one larger. **CC.K.CC.4c** 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **CC.K.CC.5** **Compare numbers.** 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[7] **CC.K.CC.6** 7. Compare two numbers between 1 and 10 presented as written numerals. **CC.K.CC.7**
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Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

[6] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

[7] Include groups with up to ten objects.

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 1

(KUD Organizer)

Mathematics

4/15/2011

Learning Progression (KUD) Organizer

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Course: Grade 1 - Math
Topic: Operations & Algebraic Thinking (Adding & Subtracting within 20)

Which standards are in this learning progression?

1.OA.1, 1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6, 1.OA.7, 1.OA.8, 1.MD.4

Connections to other domains and/or clusters:

1.MD.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

There are multiple ways to represent and find sums/ differences within 20 (story problems, pictures, equations, computational strategies, manipulatives).

An equation must be balanced and the equal sign represents quantities on each side of the symbol as the same (equal).

The relationship between addition & subtraction can be used to solve problems.

KNOW:	DO:
Addition & subtraction are related operations. Subtraction can be perceived as an unknown addend problem. Addition and subtraction problems can be posed with the missing part being in different positions. The commutative & associative properties of operations can be used to solve problems (but students do not need to know them by name). Symbols can represent an unknown quantity in an equation. Know combinations to 10 fluently. **Strategies**: Counting on, Making Ten, Decomposing, Using Known Facts	**Represent and solve problems involving addition and subtraction.** 4. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1] **CC.1.OA.1** 5. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **CC.1.OA.2** **Understand and apply properties of operations and the relationship between addition and subtraction.** 6. Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **CC.1.OA.3** 7. Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **CC.1.OA.4** **Add and subtract within 20.** 8. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **CC.1.OA.5** 9. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making

[1] See Glossary, Table 1 in the CCSS document.
[2] Students need not use formal terms for these properties.

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ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **CC.1.OA.6**

Work with addition and subtraction equations.

10. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* **CC.1.OA.7**

Footnote to 1.NOA.7: These equations are purposeful in showing students how to determine if an equation is "balanced" (quantity on each side of the equation is the same).

11. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ - 3, 6 + 6 = □. **CC.1.OA.8**

Connections to other Domains &/or Clusters:

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **CC.1.MD.4**

*Footnote for 1.NOA.3: Students need not use formal terms for these properties.

Solve subtraction problems by "adding on".

Students use the counting sequence to solve addition facts and write number sentences efficiently. (+0, +1, +2) Knowing that 18 follows 17 is the same as knowing 17 + 1 = 18.

Footnote to 1.NOA.8: Just as we teach word problems with unknowns in all positions (1.NOA.1) this standard asks teachers to have students practice equations with unknowns in all positions.

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Standards for Mathematical Practice:

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Course: Grade 1 - Math
Topic: NBT - Understanding Place Value

Which standards are in this learning progression?

1.NBT.1, 1.NBT.2, 1.NBT.2a-c, 1.NBT.3, 1.NBT.4, 1.NBT.5

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

The digits of a 2-digit number represent tens and ones.

KNOW:	DO:
The two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: **CC.1.NBT.2** a. 10 can be thought of as a bundle of ten ones — called a "ten." **CC.1.NBT.2a** b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **CC.1.NBT.2b** c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **CC.1.NBT.2c** "10 more" means one more group of tens and "ten less" means one less group of tens. Counting can start with any number (not always with 1). Numbers can be represented in many ways. The placement of the numeral determines its place value meaning (i.e the 5 in 56 means 5 tens or 50. Whereas the 5 in 15 means 5 ones.)	**Extend the counting sequence.** 1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **CC.1.NBT.1** 3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **CC.1.NBT.3** **Use place value understanding and properties of operations to add and subtract.** 5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **CC.1.NBT.5** Compose and decompose teen numbers into tens and ones. Use models (ten frames, hundreds chart, number line, ten train (cubes), to represent any 2-digit number in tens and ones up to 120. Students practice counting around the decades (e.g. 48, 49, 50, 51, 52). **Use place value understanding and properties of operations to add and subtract.** 6. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **CC.1.NBT.4**

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Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Digit, Tens, Ones, Decades, Compose, Decompose, Place Value**

Standards for Mathematical Practice:

9. Make sense of problems & persevere in solving them.
10. Reason abstractly & quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

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Course: Grade 1 - Math
Topic: Number & Operations in Base Ten (Adding & subtracting within 100, including Place Value)

Which standards are in this learning progression?

1.NBT.4, 1.NBT.5, 1.NBT.6

Connections to other domains and/or clusters:

1.MD.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

When adding two-digit numbers, one adds tens and tens, and then the ones and ones.

KNOW:	DO:
Addition & Subtraction are related operations. 10 more or 10 less of any number under 100 mentally. How to add and subtract multiples of 10 to any number (by 20, 30, 70, etc.). Place value strategies can be used to add 2-digit plus 1-digit numbers. Place value strategies for adding & subtracting (counting on, making 10's/100's, breaking apart and putting together, using known facts). Commutative & associative properties of operations can be used to solve problems: (For example students know that if 20+40=60, then 40+20=60 without actually naming the commutative property) Models for adding & subtracting (number line, base ten materials). Methods for recording addition & subtraction strategies using number lines & equations. Symbols can represent an unknown quantity in an equation.	**Use place value understanding and properties of operations to add and subtract.** 4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **CC.1.NBT.4** 5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **CC.1.NBT.5** 6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **CC.1.NBT.6** Students will transfer strategies used for adding to 20 to adding to 100. **Represent and interpret data.** 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **CC.1.MD.4**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Model, Mental Math, Digit, Two-digit, Tens, Ones, Place Value, Sum, Difference**

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Standards for Mathematical Practice:	
1. Make sense of problems & persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly & quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics	8. Look for and express regularity in repeated reasoning.

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Course: Grade 1 - Math
Topic: Measurement (Length & Time)

Which standards are in this learning progression?

1.MD.1, 1.MD.2, 1.MD.3

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

Lengths of objects can be measured and compared using non-standard units.

KNOW:	DO:
Any object can be used as a length unit. The length of two objects can be compared by using the same unit of measure. Analog and digital clocks are used to measure time. Know how to tell time to the hour and half hour (see geometry 1.G.3 as a model for the idea that a half hour is half of a circle.)	**Measure lengths indirectly and by iterating length units.** 7. Order three objects by length; compare the lengths of two objects indirectly by using a third object. **CC.1.MD.1** 8. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **CC.1.MD.2** **Tell and write time.** 9. Tell and write time in hours and half-hours using analog and digital clocks. **CC.1.MD.3**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Digital, Analog, Measure, Length, Compare**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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Course: Grade 1 - Math
Topic: Data (Represent & Interpret)

Which standards are in this learning progression?

1.MD.4

Connections to other domains and/or clusters:

1.OA.1 – 8, and 1.NBT.4 – 6

By the end of this learning progression, students will be able to…

UNDERSTAND:

Organizing, representing, & interpreting data allows for careful analysis and answering questions posed.

Data can be interpreted numerically or categorically.

(Think about these understandings…)

KNOW:	DO:
Organizing data can help with interpreting and answering questions posed. Data can be represented in multiple ways (e.g., line plots, bar graphs/towers of cubes, Venn diagrams, tables) How we interpret data changes depending on the context of the question being asked.	**Represent and interpret data.** 10. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **CC.1.MD.4** Use line plots, bar graphs (towers of cubes), Venn diagrams, and tables to organize and represent data. Interpret data results by counting or using arithmetic strategies. **This KUD connects to addition and subtraction standards in Operations & Algebraic Thinking and Number & Operations in Base Ten.**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Data, Organize, Represent, Interpret**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
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Course: Grade 1 - Math
Topic: Geometry - Reason with Shapes & Their Attributes

Which standards are in this learning progression?

1.G.1, 1.G.2, 1.G.3

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

Shapes have defining attributes that can be compared to other shapes.
Decomposing a shape into more equal shares creates smaller pieces.

KNOW:	DO:
Shapes can be sorted according to their defining geometric attributes such as the number of sides or closed/open figure (not by non-defining attributes such as color, size, orientation, etc.) For example: A triangle is a triangle no matter what color, size or orientation. Shapes can be composed and decomposed into other shapes. Equal partitions of shapes can be described as halves, fourths, as well as half of, fourth of. Distinguishing features of 2D and 3D shapes.	**Reason with shapes and their attributes.** 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. **CC.1.G.1** 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[3] **CC.1.G.2** Use pattern blocks, geoboards, tangrams, etc, to compose composite shapes. 3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **CC.1.G.3** Student's first work with fractional ideas comes from a study of symmetry. Folding circles and rectangles (and squares) into 2 or 4 equals shares defines halves and fourths.

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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[3] Students do not need to learn formal names such as "right rectangular prism."

Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 2

(KUD Organizer)

Mathematics

4/15/2011

Learning Progression (KUD) Organizer

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Course: Grade 2 - Math
Topic: Operations & Algebraic Thinking – Add/Sub within 100 & Foundations for Multiplication

Which standards are in this learning progression?

2.OA.1, 2.OA.2, 2.OA.3, 2.OA.4

Connections to other domains and/or clusters:

NBT.5, 2.NBT.6, 2.NBT.9, 2.MD.5, 2.MD.8, 2.MD.10, 2.G.2

By the end of this learning progression, students will be able to…

UNDERSTAND:

There are multiple ways to represent and find sums / differences within 100 (story problems, pictures, equations, computational strategies, manipulatives, arrays).

KNOW:

Addition and subtraction are related operations.

Subtraction can be perceived as an unknown addend problem.

Addition and subtraction problems can be posed with the missing part being in different positions.

Word problems may require one or two computations to find a solution.

Mental strategies for adding single digit numbers to know combinations to 20 fluently (Doubles +1, Make a Ten, Ten plus…, 9+…)

The objects in an even number set can be paired or broken into two equal groups, and an odd number set of objects cannot.

Methods for recording addition & subtraction strategies using number lines & equations.

Symbols can represent an unknown quantity in an equation.

Rectangular arrays can represent the relationship between repeated addition and the foundations of multiplication.

DO:

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] **CC.2.OA.1**

Add and subtract within 20.

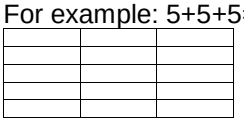
2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers. **CC.2.OA.2**

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **CC.2.OA.3**

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **CC.2.OA.4**

For example: 5+5+5=15 can be shown by a 3 x 5 rectangle.



CONNECTIONS TO OTHER DOMAINS and/or CLUSTERS:

Reason with shapes and their attributes.

2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. CC.2.G.2

Use place value understanding and properties of operations to add and subtract.

[1] See Glossary, Table 1 in the CCSS document.
[2] See standard 1.OA.6 for a list of mental strategies.

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	5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **CC.2.NBT.5** **6.** Add up to four two-digit numbers using strategies based on place value and properties of operations. **CC.2.NBT.6** **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[3] **CC.2.NBT.9** **Relate addition and subtraction to length.** 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **CC.2.MD.5** **Work with time and money.** 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* **CC.2.MD.8** **Represent and interpret data.** 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph. **CC.2.MD.10**
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Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

[3] Explanations may be supported by drawings or objects.

[4] See Glossary, Table 1.

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Course: Grade 2 - Math
Topic: Understanding Place Value - NBT

Which standards are in this learning progression?

2.NBT.1, 2.NBT.2, 2.NBT.3, 2.NBT.4

Connections to other domains and/or clusters:

2.NBT.7, 2.NBT.8, 2.MD.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Three-digit numbers are composed of hundreds, tens, and ones.

KNOW:	DO:
The three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: **CC.2.NBT.1** a. 100 can be thought of as a bundle of ten tens – called a "hundred." **CC.2.NBT.1a** b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **CC.2.NBT.1b** The repeating patterns of the counting sequence up to 1000. The meaning of recording symbols >,=,<.	**Understand place value.** **2.** Count within 1000; skip-count by 5s, 10s, and 100s. **CC.2.NBT.2** **3.** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **CC.2.NBT.3** **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **CC.2.NBT.4** **CONNECTIONS TO OTHER DOMAINS and/or CLUSTERS** **Use place value understanding and properties of operations to add and subtract.** 7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **CC.2.NBT.7** 8. Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. **CC.2.NBT.8** 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[5] **CC.2.NBT.9** **Work with time and money.** 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* **CC.2.MD.8**

[5] Explanations may be supported by drawings or objects.

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Standards for Mathematical Practice:

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Course: Grade 2 - Math
Topic: NBT - Adding & Subtracting Within 1000 including Place Value

Which standards are in this learning progression?

2.NBT.5, 2.NBT.6, 2.NBT.7, 2.NBT.8, 2.NBT.9

Connections to other domains and/or clusters:

2.MD.5, 2.MD.6, 2.MD.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Numbers can be composed & decomposed into place value parts to add and subtract multi-digit numbers efficiently.

KNOW:	DO:
The strategy of mentally adding and subtracting 10 or a 100 to a given number. Addition and subtraction are related operations. Commutative & associative properties of operations can be used to solve problems: For example students know that if 120 + 140 = 260, the 140 + 120 = 260 without actually naming the commutative property. Students know if 2 + 3 + 4 = 9 then they will know that 4 + 3 + 2 = 9 without actually naming the associative property. Place value strategies for adding & subtracting (counting on, making 10's/100's, breaking apart and putting together, using known facts). Models for adding & subtracting (number line, base ten materials). Methods for recording addition & subtraction strategies using number lines & equations. Symbols can represent an unknown quantity in an equation.	**Use place value understanding and properties of operations to add and subtract.** **5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **CC.2.NBT.5** **6.** Add up to four two-digit numbers using strategies based on place value and properties of operations. **CC.2.NBT.6** **7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **CC.2.NBT.7** **8.** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. **CC.2.NBT.8** **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[6] **CC.2.NBT.9** **CONNECTIONS TO OTHER DOMAINS and/or CLUSTERS:** **Relate addition and subtraction to length.** 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **CC.2.MD.5** 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.**CC.2.MD.6** **Work with time and money.** 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes*

[6] Explanations may be supported by drawings or objects.

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	and 3 pennies, how many cents do you have? **CC.2.MD.8** Students will transfer strategies for adding/subtracting within 100 and apply when adding within 1000.

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **mental math, digit, sum, difference, ones, tens, hundreds, thousands, strategy, model**

Standards for Mathematical Practice:

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Course: Grade 2 - Math
Topic: Measurement (Length, Time, Money)

Which standards are in this learning progression?

2.MD.1, 2.MD.2, 2.MD.3, 2.MD.4, 2.MD.5, 2.MD.6, 2.MD.7, 2.MD.8

Connections to other Domains and/or Clusters:

2.MD.9

By the end of this learning progression, students will be able to…

UNDERSTAND:

Tools that measure length, time, & money must have equal intervals between units. (clocks, number lines, coins)

KNOW:	DO:
The appropriate tool and unit of measure should be selected based on the context of the situation. Estimating strategies can be applied to measuring lengths to the closest standard unit of measure. Lengths of an object can be compared by using various units of measure. The value of the measurement of an object will be different depending on the size of the units used to measure it. (See example under #2) When you compare two lengths, you are finding the difference. Strategies used for solving & representing addition/subtraction problems can be utilized to solve and represent measurement word problems. (Word problems involving length, money , & time) Methods for recording addition & subtraction strategies using number lines & equations. Symbols can represent an unknown quantity in an equation. Consecutive whole numbers are equidistant on a number line. (0-10, 10-20, 20-30, etc.)	**Measure and estimate lengths in standard units.** 1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **CC.2.MD.1** 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **CC.2.MD.2** -If you measure an object using larger units, it will take fewer of those units than if you measure the object with a smaller sized unit. Ex: Measure an object using inches and cm. If it measures 12 inches, it takes 12 one-inch units to describe its length. The measurement in cm would be approximately 30cm which would be 30 one-cm units as its length. 3. Estimate lengths using units of inches, feet, centimeters, and meters. **CC.2.MD.3** 4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **CC.2.MD.4** **Relate addition and subtraction to length.** 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **CC.2.MD.5** 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.**CC.2.MD.6** Utilize the number line as model for adding & subtracting within 100. **Work with time and money.** 7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **CC.2.MD.7** 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes*

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The number line can be utilized as a model for adding & subtracting within 100. Time intervals on analog clock skip-count by 5 minutes. The time before 12 noon is a.m., and the time 12 noon and after is p.m.	*and 3 pennies, how many cents do you have?* **CC.2.MD.8** **Connections to other Domains &/or Clusters:** **Represent and interpret data.** 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **CC.2.MD.9**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Measure, estimate, length, standards units, analog clock**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Course: Grade 2 - Math
Topic: Data – Represent & Interpret

Which standards are in this learning progression?

2.MD.9, 2.MD.10

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:	
Data can be organized, represented, and interpreted in multiple ways for a variety of purposes.	
KNOW:	**DO:**
Data can be organized and represented in multiple ways. Data presented in graphs can be interpreted and manipulated to solve problems.	**Represent and interpret data.** 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **CC.2.MD.9** Measure to the nearest in, ft, cm, m. Construct line plots. 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[7] using information presented in a bar graph. **CC.2.MD.10** Interpret and manipulate data to solve problems.
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Measure, line plot, scale, bar graph, picture graph**	
Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

[7] See Glossary, Table 1.

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Course: Grade 2 - Math
Topic: Geometry – Reason with Shapes & Their Attributes

Which standards are in this learning progression?

2.G.1, 2.G.2, 2.G.3

Connections to other domains and/or clusters:

2.OA.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Shapes have defining attributes that can be utilized for comparing and composing/constructing.

Rectangular arrays promote the connection between geometry and the foundations multiplication.

Decomposing shapes into equal size pieces promotes the connection between geometry and fractional concepts.

KNOW:	DO:
Angles and sides are important specified attributes of 2D shapes. (2.G.1) Faces, edges, & vertices are important specified attributes of 3D shapes. Distinguishing features of 2D and 3D shapes. Equal shares of identical wholes do not need to have the same shape. (Ex: ¼ of a square can look different for different equal squares) Rectangular arrays can represent the relationship between repeated addition and the foundations of multiplication.	**Reason with shapes and their attributes.** 1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[8] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **CC.2.G.1** Identify regular shapes Compose shapes given the specified attributes. Distinguish between 2D & 3D shapes 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **CC.2.G.2** 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. **CC.2.G.3** Ex: Students reason that two different shaped halves of identical wholes are each ½ because they are 1 of 2 equal pieces or they may prove that each ½ has the same area. (A sandwich cut on a diagonal vs. down the middle) **Connections to other Domains &/or Clusters:** **Work with equal groups of objects to gain foundations for multiplication.** 4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **CC.2.OA.4** Utilize an array as a model for understanding equal groups. Repeated Addition (beginning Multiplication)

[8] Sizes are compared directly or visually, not compared by measuring.

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
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4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 3

(KUD Organizer)

Mathematics

C. Lancour

4/15/2011

Learning Progression (KUD) Organizer

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Course: Grade 3 - Math
Topic: Operations & Algebraic Thinking – Multiplication & Division

Which standards are in this learning progression?

3.OA.1, 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7, 3.OA.8, 3.OA.9, 3.NBT.3

Connections to other domains and/or clusters:

3.MD.2, 3.MD.3, 3.MD.7a, 3.MD.7b, 3.MD.7c

By the end of this learning progression, students will be able to…

UNDERSTAND:

Multiplication and division situations involve equal-size groups, arrays, and/or area models.

Multiplication and division are inverse operations.

The commutative, associative, and distributive properties can be used to develop efficient strategies to multiply and divide. (Students do not need to know the names of these operations.)

KNOW:

Notation:
- Multiplication and division notation (including different division signs:)
- Methods of recording multiplication strategies using equations and arrays.
- A letter can be used to stand for an unknown quantity.

Strategies:
- Repeated addition
- Skip-counting
- Mental strategies for multiplying single-digit numbers (e.g., using a fact you know to solve a fact that you don't know)
- Partial products for multiplication (partial products can be notated using equations and/or arrays and area)
- Doubling and halving
- Division problems can be solved by thinking of them as unknown factor problems
- Estimation can be used to predict a reasonable answer.

Models/Representations:
- Drawings of equal groups
- Arrays
- Areas

Concepts/Big Ideas:

DO:

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* **CC.3.OA.1**
   - Understand and write story problems for multiplication equations. E.g. write a story problem for 5 x 7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **CC.3.OA.2**
   - Understand and write story problems for division equations. E.g. write a story problem for 56 divided by 8.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] **CC.3.OA.3**
   - Solve multiplication and division word problems that involve equal groups, arrays, and area with products up to 100.
   - Use drawings and equations to represent multiplication and division word problems.
   - Write multiplication and division equations with a symbol for the unknown.
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$. **CC.3.OA.4**
   - Solve multiplication and division problems with the unknown in any position.

Understand properties of multiplication and the relationship between

[1] See Glossary, Table 2 in CCSS document.

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- Division word problems can require finding the unknown number of groups or the unknown group size (grouping problems and sharing problems).
- Multiplication and division are inverse operations.
- Fact families for multiplication and division.

Other:
Be fluent with all products of two one-digit numbers.

multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **CC.3.OA.5**

6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **CC.3.OA.6**

-Use fact families to solve multiplication and division problems.

Multiply and divide by 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **CC.3.OA.7**

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] **CC.3.OA.8**

- Solve two-step word problems that use multiplication and division (as well as addition and subtraction).
- Represent two-step word problems with equations with a letter standing for the unknown quantity (e.g., (4 x a) + 32 =72

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **CC.3.OA.9**

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **CC.3.NBT.3**

CONNECTIONS TO OTHER DOMAINS

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

7. Relate area to the operations of multiplication and addition. **CC.3.MD.7**
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular

[2] Students need not use formal terms for these properties.
[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
[4] A range of algorithms may be used.

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areas in mathematical reasoning. **CC.3.MD.7b**

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. **CC.3.MD.7c**

- Use arrays to model the distributive property of multiplication.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[5] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[6] **CC.3.MD.2**

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **CC.3.MD.3**

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Standards for Mathematical Practice:

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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
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[5] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[6] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2 in CCSS).

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Course: Grade 3 - Math
Topic: Addition and Subtraction (Combined using standards from 3 domains: NBT, OA, and MD

Which standards are in this learning progression?

3.NBT.1, 3.NBT.2, OA.8, OA.9

Connections to other domains and/or clusters:

3.MD.1, 3.MD.2, 3.MD.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

The value of a digit in our number system is determined by its place value position.

Numbers can be decomposed and recomposed into component parts to add and subtract multi-digit numbers efficiently.

KNOW:

Notation:
- Expanded notation for numbers up to 1000.
- How to record addition and subtraction strategies using number lines and/or equations.
- A letter can be used to stand for an unknown quantity.

Strategies (CC.3.NBT.2)**:**
- Partial sums for addition.
- Adding up and subtracting back (in large chunks) for subtraction.
- Creating an equivalent expression (also called compensation for addition and constant difference for subtraction).
- Other place value strategies

Models/Representations:
- Open number line (for thinking about and recording addition and subtraction strategies.)
- Bar graph or picture graph
- Scaled bar graph or picture graph
- Addition table
- Multiplication table

Concepts/Big ideas:
- Rounding is a formal way of estimating. (CC.NBT.1)
- When adding numbers the order of the addends does not matter. E.g., 7 + 10 = 10 + 7. (Commutative property).

DO:

Use place value understanding and properties of operations to perform multi-digit arithmetic.[7]

1. Use place value understanding to round whole numbers to the nearest 10 or 100. **CC.3.NBT.1**
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **CC.3.NBT.2**
   - Break 2 and 3 digit numbers into their place value components.
   - Use strategies such as partial sums for addition, adding up and subtracting back (in large chunks) for subtraction, and/or the regrouping algorithms for addition and subtraction.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[8] **CC.3.OA.8**
   - Write equations to represent two-step word problems, e.g. (4 x 10) + 32 = ___.
   - Represent two-step missing addend problems with a letter for the unknown quantity, e.g. 72 = (4 x 10) + *n*.

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **CC.3.OA.9**

[7] A range of algorithms may be used.
[8] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

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(CC.3.NBT.2)

- Numbers can be decomposed, recomposed, & re-ordered to make adding more efficient. E.g., 8 + 5 = 8 + (2+3)= (8+2) +3= 10+3=13. (Associative property of addition.) (CC.3.NBT.2)
- Addition and subtraction are inverse operations. (CC.3.NBT.2)
- A letter can be used to stand for an unknown quantity. (CC.3.OA.8)

- Students might observe that when counting by 9 (starting at any number) the tens digits increases by 1 and the ones digits decreases by 1. This can be explained by reasoning that adding 9 is equal to adding 10 and subtracting 1.

Connections to other Domains and/or Clusters:

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g. by representing the number on a number line diagram. **CC.3.MD.1**

2. Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. **CC.3.MD.2**

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw bar graph in which each square in the bar graph might represent 5 pets.* **CC.3.MD.3**

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Course: Grade 3 - Math
Topic: Number & Operations - Fractions

Which standards are in this learning progression?

3.G.2, 3.NF.1, 3.NF.2, 3.NF.2a-b, 3.NF.3a-d

Connections to other domains and/or clusters:

3.MD.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Fractions are a special type of numbers.
- They refer to parts of wholes.
- They fall between whole numbers on a number line.

Unit fractions are the building blocks of all other fractions.
- A unit fraction is a quantity.
- Unit fractions refer to "1 out of ___ equal parts". (CC.3.NF.1)
- Non-unit fractions are the sum of unit fractions (e.g. 3/5 = 1/5 + 1/5 + 1/5) (CC.3.NF.1)

(Do we want something about the magnitude of fractions? Knowing that 1/8 is smaller than 1/3)

KNOW:

Notation:
- Fractions are written as *a/b* where the denominator of the fraction indicates the *size of the parts* (the unit fraction it is made of) and the numerator indicates *how many* of those parts are being considered. (CC.3.NF.1)

Models:

Fractions can be represented
- As equal areas of a region.
- As points on a number line.

Concepts/Big ideas:
- The whole on a number line is the interval or space between 0 and 1**.** (CC.NF.2a)
- If the distance on a number line between 0 and 1, is divided into *b* equal intervals, then each interval has a size of 1/*b.* (E.g., if the space on a number line is divided into 4 equal intervals, then each interval represents ¼ of the distance between 0 and 1.) (CC.3.NF.2a)
- When writing fractions on number lines, a fraction a/*b* should be placed a/*b* of the distance from 0 to 1. (E.g., ¼ should be placed on a number line

DO:

** All work with fractions in 3rd grade is limited to fractions with denominators 2, 3, 4, 6, and 8.*

Reason with shapes and their attributes.

1. Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. **CC.3.G.2**

Develop understanding of fractions as numbers.

2. Understand a fraction as a number on number line; represent fractions on a number line diagram **CC.3.NF.2**

a**.** Represent a fraction 1/b on a number line diagram by defining the interval between 0 and 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number on the number line. **CC.NF.2a**
- Correctly place unit fractions on unmarked number lines.

b. Represent a fraction 1/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. **CC.NF.2b**
- Correctly place non-unit fractions between 0 and 1 on unmarked number lines.
- Student should use tick marks to indicate equal intervals.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **CC.3.NF.3.**

b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **CC.3.NF.3b**
- Recognize simple equivalent fractions with denominators 2, 3, 4, 6, and 8.
- Generate simple equivalent fractions with denominators 2, 3, 4, 6,

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at the point that is ¼ of the way from 0 to 1 and 2/3 should be placed at the point that is 2/3 of the distance from 0 to 1.) (CC.3.NF.2a &b)

- Two fractions are equivalent (equal) if they are the same size or the same point on a number line. **CC.3.NF.3a**
- Whole numbers can be written as fractions with a denominator of 1. (3.NF.3c)
- Fractions with the same numerator and denominator are equal to 1. (3.NF.3c)
- The size of a fractional part is relative to the size of the whole. (1/2 of a pizza is bigger than 1/2 of a cookie). (3.NF.3d)
- When comparing the size of two different fractions, one must assume that the wholes are the same size. (3.NF.3d)

and 8.

- Use visual fraction models to prove equivalence.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form of 3 = 3/1; recognize that 6 = 6/1, locate 4/4 and 1 at the same point on a number line diagram.* **CC.3.NF.3c**

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **CC.3.NF.3d**

- Compare fractions with the same numerators by reasoning about their size (e.g. ¾ is greater than $^3/_6$ because ¼ pieces are larger than $^1/_6$ pieces) and justify the answer using visual models.
- Compare fractions with the same denominators by reasoning about their size (e.g.¾ is greater than $^2/_4$ because ¾ refers to more ¼ pieces) and justify the answer visual models.

Connections to other Domains &/or Clusters:

Represent and interpret data.

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units- whole numbers, halves, or quarters. **CC.3.MD.4**
   - Measure lengths with a ruler to the half inch or fourth of an inch.
   - Make line plots with scales marked off in whole units, half units, and quarter units.

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Numerator, Denominator, Whole, Equivalent**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Learning Progression (KUD) Organizer

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Course: Grade 3 - Math
Topic: Measurement and Data (time, liquid volume, mass, & graphing)

Which standards are in this learning progression?

3.MD.1, 3.MD.2, 3.MD.3, 3.MD4

Connections to other domains and/or clusters:

3.OA.1, 3.OA.3, 3.NF.2a-b

By the end of this learning progression, students will be able to…

UNDERSTAND:

Standard units enable people measure data in the same way.

Data can be organized, represented, & interpreted in multiple ways for a variety of purposes.

KNOW:	DO:
Line plots with whole numbers must include all the whole numbers in the range. Line plots with fractions must include all whole numbers and fractions within the range. (3, 3 ¼, 3½, 3 $^{3}/_{4}$ 4,4 ¼,4 ½, etc) It is essential to include the unit when communicating measurement data. One interval on a scaled bar graph represents a larger quantity. One picture on a scaled picture graph represents a larger quantity. Bar graphs, picture graphs, and line plots provide opportunities to make comparisons.	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** 1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **CC.3.MD.1** 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[9] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[10] **CC.3.MD.2** **Represent and interpret data.** 3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **CC.3.MD.3** 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **CC.3.MD.4** Use understanding of fractions on a number line to understand fractions on a ruler. **Connections to other Domains and/or Clusters:** **Represent and solve problems involving multiplication and division.** 5. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total

[9] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[10] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2 in CCSS).

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	number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **CC.3.OA.1** 6. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[11] **CC.3.OA.3** **Develop understanding of fractions as numbers.** 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. **CC.3.NF.2** a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. **CC.3.NF.2a** b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **CC.3.NF.2b**
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Volume, Mass, Kilograms, Grams, Liters, Bar graph, Picture graph, Line plot, Scaled bar graph**	
Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

[11] See Glossary, Table 2 in CCSS document.

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Course: Grade 3 - Math
Topic: Geometric Measurement – Area & Perimeter

Which standards are in this learning progression?

3. MD.5, 3.MD.6, 3.MD.7a-d, 3.MD.8

Connections to other domains and/or clusters:

3.OA.3, 5.OA.5

By the end of this learning progression, students will be able to…

UNDERSTAND:

Area and perimeter are attributes used to describe and measure 2D figures.

5a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **CC.3.MD.5a**

KNOW:	DO:
Area covers: **5b**. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **CC.3.MD.5b** Perimeter is the distance around a figure. Strategies for finding area and perimeter use related to multiplication and addition. Strategies for finding Area: • Counting • Repeated addition • Multiplication of length by width • Decomposing into more than one rectangle Rectangles can have the same perimeter and different areas or the same areas and different perimeter.	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 5. Recognize area as an attribute of plane figures and understand concepts of area measurement. **CC.3.MD.5** 6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **CC.3.MD.6** 7. Relate area to the operations of multiplication and addition. **CC.3.MD.7** 1. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **CC.3.MD.7a** 2. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **CC.3.MD.7b** 3. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. **CC.3.MD.7c** 4. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **CC.3.MD.7d** **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **CC.3.MD.8**

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	• Investigate the relationship of area and perimeter when rectangles have the same perimeter and different areas or the same area and different perimeters. **Connections to other Domains and/or Clusters:** **Represent and solve problems involving multiplication and division.** 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[12] **CC.3.OA.3** **Understand properties of multiplication and the relationship between multiplication and division.** 5. Apply properties of operations as strategies to multiply and divide.[13] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **CC.3.OA.5**

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[12] See Glossary, Table 2 in CCSS document.
[13] Students need not use formal terms for these properties.

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Course: Grade 3 - Math
Topic: Geometry – Reason with Shapes & their Attributes

Which standards are in this learning progression?

3.G.1, 3.G.2

Connections to other domains and/or clusters:

3.NF.1

By the end of this learning progression, students will be able to…

UNDERSTAND:

Shapes in different categories may share attributes and the shared attributes can define a larger category.

KNOW:	DO:
Shapes can be sorted according to their attributes. Quadrilaterals are polygons with four sides. Rectangles, rhombi, and squares are a particular type of quadrilateral (parallelograms).	**Reason with shapes and their attributes.** 1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **CC.3.G.1** • Identify rhombus, rectangle, square, etc as examples of quadrilaterals. • Draw examples of quadrilaterals that do not belong to any subcategory (not rhombi, rectangles, or squares, etc) such as trapezoids and/ or various sizes and shapes of convex and concave quadrilaterals.) 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.* **CC.3.G.2** -This standard should not be taught in isolation, but in conjunction with fractions. **Connections to other Domains &/or Clusters:** **Develop understanding of fractions as numbers.** 1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **CC.3.NF.1**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Polygon, Rhombus, Square, Rectangle, Trapezoid ??, Quadrilateral, Attribute, Partition, Unit fraction**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 4

(KUD Organizer)

Mathematics

C. Lancour

4/15/2011

Learning Progression (KUD) Organizer

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Course: Grade 4 **-** Math
Topic: Operations & Algebraic Thinking - Multiplication

Which standards are in this learning progression?

4.OA.1, 4.OA.2, 4.OA.3, 4.OA.4, 4.OA.5

Connections to other domains and/or clusters:

4.MD.1, 4.MD.2, 4.MD.3, 4.NBT.1, 4.NBT.2, 4.NBT.4, 4.NBT.5, 4.NF.4, 4.NF.5, 4.NF.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

Factors and multiples can be used to determine part-whole relationships.

By utilizing efficient methods of multiplication and division, more complex problem solving is possible.

KNOW:	DO:
Multiplication scenarios can be interpreted differently based on the context of the problem. Ex: A "5 times greater than 7" problem is interpreted differently than "5 groups of 7" but both are derived from 5 x 7. Additive thinking is "how many more". Multiplicative thinking is "how many times more". Problems can be solved by writing the solution pathway in algebraic notation and then solving for the unknown. Estimation in multiplication and division can predict the size of the answer & help to assess the reasonableness of a solution.	**Use the four operations with whole numbers to solve problems.** 1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **CC.4.OA.1** Compare quantities by thinking "N times as large" is necessary to compare units of measure, e.g., when comparing yards to feet, "A yard is 3 times as large as a foot." Compare quantities by thinking "10 times as large" is necessary to compare the place value of the digits, e.g., 70 is 10 times as large as 7. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] **CC.4.OA.2** 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **CC.4.OA.3** EX: "I can write 5 poems every day. I already have a poem in my journal. How many days should I work to have a total 31 poems in my journal?" 5 x N + 1 = 31 **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **CC.4.OA.4**

[1] See Glossary, Table 2 in CCSS document.

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Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **CC.4.OA.5**

Connections to other domains &/or Clusters:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **CC.4.MD.1**

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **CC.4.MD.2**

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **CC.4.MD.3**

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **CC.4.NBT.1**

2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **CC.4.NBT.2**

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. **CC.4.NBT.4**

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **CC.4.NF.4**
   a. Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **CC.4.NF.4a**
   b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **CC.4.NF.4b**
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5*

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	people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? **CC.4.NF.4c** **Understand decimal notation for fractions, and compare decimal fractions.** 5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **CC.4.NF.5** 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **CC.4.NF.7**

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Standards for Mathematical Practice:

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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[2] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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Course: Grade 4 Math
Topic: Number and Operations in Base Ten[3] - Place Value

Which standards are in this learning progression?

4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.NBT.6

Connections to other domains and/or cluster:

4.NF.5, 4.NF.6, 4.NF.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

The number system is a repeated counting pattern based on tens and powers of ten.

Efficient strategies for multi-digit arithmetic are based on applying the properties of operations.

KNOW:	DO:
Expanded notation can be used to show order, values of each digit, and the powers of 10. The Distributive Property of Multiplication can be modeled in an array as well as with expanded notation. Rounding a number to the largest place value can be accomplished by answering: "Is this number closest to N-thousand or N+1 thousand?" Multiplication and division are inverse operations.	**Generalize place value understanding for multi-digit whole numbers.** 1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **CC.4.NBT.1** 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **CC.4.NBT.2** 3. Use place value understanding to round multi-digit whole numbers to any place. **CC.4.NBT.3** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. **CC.4.NBT.4** Use commutative and associative properties to show methods of solving problems. Prove algorithms by using expanded notation. Ex: 400 + 20 + 7 300 + 50 + 2 --- 700 + 70 + 9 = 779 5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **CC.4.NBT.5** Efficient strategies rely on the distributive property of multiplication. Ex: 4327 x 8 = (4000x8) + (300x8) + (20x8) + (7x8) **Or** by decomposing & utilizing the associative property. Ex: 70 x 3 = 7 x 10 x 3 = 7 x 3 x 10 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and

[3] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

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explain the calculation by using equations, rectangular arrays, and/or area models. **CC.4.NBT.6**

Remove groups of 10s, 100s, or multiples of 10s, 100s.

Use knowledge of multiplication to solve division problems.

Connections to other Domains &/or Clusters:

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **CC.4.NF.5**
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **CC.4.NF.6**
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **CC.4.NF.7**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Place Value, Standard Form (1,234), Word Form (One thousand, two hundred thirty-four), Expanded Form (1,000 + 200 + 30 + 4)**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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Course: Grade 4 Math
Topic: Number and Operations – Fractions[5] - Equivalence, Comparing Fractions & Decimals

Which standards are in this learning progression?

4.NF.1, 4.NF.2, 4.NF.4, 4.NF.6, 4.NF.7

Connections to other domains and/or clusters:

4.MD.1, 4.MD.2

By the end of this unit, students will be able to…

<table>
<tr><th colspan="2">UNDERSTAND:</th></tr>
<tr><td colspan="2">Equivalent fractions or decimal fractions represent the same quantity in multiple ways.
Using visual models and place value is helpful in comparing fractions and decimals.</td></tr>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>Multiplying a fraction by one always results in an equivalent fraction. Ex: ¼ x 3/3 = 3/12

Equivalent fractions can be generated using area models, ratio models, number lines and fractions bars.

Compare fractions using common denominator, common numerator, comparison to benchmark and distance to benchmark; as well as determining when each strategy is appropriate.

Compare decimal fractions using 10x10 grid, a number line, and measurement such as metric system, money.</td><td>Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. CC.4.NF.1

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. CC.4.NF.2

• Models that can show equivalence include area models, ratio model, number line, and fraction bars.
• 3/6=4/8 because both are equal to ½.
• ½=3/6 because numerator and denominator are multiplied by the same number (1/2 x 3/3 = 3/6).

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[6] For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. CC.4.NF.5</td></tr>
</table>

[5] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
[6] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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	6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **CC.4.NF.6** 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **CC.4.NF.7** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **CC.4.MD.1** 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **CC.4.MD.2**
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Numerators, Denominators, Equivalent fractions, Benchmark fractions, Tenth, Hundredth**	
Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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Course: Grade 4 Math
Topic: Number and Operations – Fractions[7] - Operations

Which standards are in this learning progression?

4.NF.3a-d, 4.NF.4a-c

Connections to other domains and/or clusters:

4.MD2, 4.MD.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Fractions are built from unit fractions through the process of addition and multiplication.

Visual fraction models and equations are tools for adding fractions, subtracting fractions, and multiplying a fraction by a whole number

KNOW:	DO:
A fraction $^{a}/_{b}$ is a multiple of $^{1}/_{b}$ (i.e., *a* groups of $^{1}/_{b}$ = *a* x $^{1}/_{b}$. For example: $^{5}/_{4}$ is the same as 5 sets of ¼ or 5 x ¼) A mixed number is the sum of its decomposed fractional parts. For example: 2 ¼ = $^{4}/_{4}$ + $^{4}/_{4}$ + ¼ Decomposing ¾ into ¼ + ¼ + ¼ allows for adding or subtracting fourths. Either factor can be the multiplier when multiplying a fraction by a whole number: ½ x 6 or 6 x ½ Visual Fraction models: Area model, array, number line, fraction bars, clock model	*Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.* **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** 3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **CC.4.NF.3** a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **CC.4.NF.3a** b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 $^{1}/_{8}$ = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **CC.4.NF.3b** c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **CC.4.NF.3c** Ex: Us the associative property to solve problems. 2 ¼ + 3 ¾ = 2 + 3 + ¼ + ¾ d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **CC.4.NF.3d** 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **CC.4.NF.4** a. Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **CC.4.NF.4a** b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **CC.4.NF.4b** *c.* Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For*

[7] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? **CC.4.NF.4c**

Connections to other Domains &/or Clusters:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **CC.4.MD.2**

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* **CC.4.MD.4**

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Decompose, Justify, Mixed Numbers, Equivalence, Equivalent Fractions**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Course: Grade 4 Math
Topic: Measurement and Data – Measurement Systems, Area, Perimeter, Data

Which standards are in this learning progression?

4.MD.1, 4.MD.2, 4.MD.3

Connections to other domains and/or clusters:

4.NF.3d, 4.NF.6, 4.NF.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

Within a single system of measurement larger units are made from smaller units. (1 km=1,000 meters) Smaller units are divisions of larger unit (1 cm = 1/100 of a meter)

Formulas are an efficient way to solve for area and perimeter.

Line plots can be used to represent data.

KNOW:

Relative sizes of measurement units (km, cm, kg, ,g, lb, oz., liter, ml, min. sec. Hour)

Equivalent measurements within a measurement system can be used to solve problems. Ex: 4m = 400cm, and 24in = 2 ft.

An array model can justify the formulas: A=LxW and P=2L+2W

Line plots with whole numbers must include all the whole numbers in the range.

Line plots with fractions must include all whole numbers and fractions within the range. (3, 3 ½, 4, ½,)

Consistent increments

DO:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **CC.4.MD.1**
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **CC.4.MD.2**
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **CC.4.MD.3**

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* **CC.4.MD.4**

Connections to other Domains and/or Clusters:

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

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	3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **CC.4.NF.3** d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **CC.4.NF.3d** **Understand decimal notation for fractions, and compare decimal fractions.** 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **CC.4.NF.6** 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **CC.4.NF.7**

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Standards for Mathematical Practice:

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Course: Grade 4 Math
Topic: Geometry & Angle Measurement

Which standards are in this learning progression?

4.MD.5a-b, 4.MD.6, 4.MD.7, 4.G.1, 4.G.2, 4.G.3

Connections to other domains and/or clusters:

4.NF.4a

By the end of this learning progression, students will be able to…

UNDERSTAND:

An angle is measured with reference to a circle and a circle is measured in terms of 360 degrees (full circle= 360 degrees)

Two-dimensional shapes can be classified based on properties of their angles (right, acute, obtuse,) and/or properties of their line segments (parallel, perpendicular).

KNOW:	DO:
An angle is a turn Angles are measured in degrees.(1 full turn is 360 degrees, ½ turn =180 degrees, ¼ turn=90 degrees) A larger angle can be decomposed into smaller angles Two or more angles can be combined to make a larger angle 2D shapes have angles at every vertex. Perpendicular lines intersect at a 90 degree angle. Parallel lines never intersect. A 2D figure has line symmetry if it can be folded along the line into matching parts.	**Geometric measurement: understand concepts of angle and measure angles.** 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **CC.4.MD.5** a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **CC.4.MD.5a** b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **CC.4.MD.5b** 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **CC.4.MD.6** 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **CC.4.MD.7** **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **CC.4.G.1** 2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **CC.4.G.2** -Distinguish between parallel & perpendicular lines.

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	3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. **CC.4.G.3** **Connections to other Domains &/or Clusters:** 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **CC.4.NF.4** a. Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **CC.4.NF.4a**

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 5

(KUD Organizer)

Mathematics

C. Lancour

4/15/2011

Learning Progression (KUD) Organizer

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Course: Grade 5 - Math
Topic: Operations & Algebraic Thinking (Expressions & Relationships)

Which standards are in this learning progression?

5.OA.1, 5.OA.2, 5.OA.3

Connections to other domains and/or clusters:

5.G.1, 5.G.2

By the end of this learning progression, students will be able to…

UNDERSTAND:

Mathematical rules and expressions depict mathematical relationships.

KNOW:	DO:
Order of Operations – Mathematical computations are performed following a given order: the order of operations. Functions of mathematical symbols Numerical patterns can be generated based on a rule. Ordered pairs form a relationship that generate a pattern and can be represented in multiple ways (tables, graphs, etc.) Numerical patterns (i.e. 0, 3, 6, 9, etc.) Parts of ordered pairs	**Write and interpret numerical expressions.** 1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **CC.5.OA.1** 2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **CC.5.OA.2** **Analyze patterns and relationships.** 3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **CC.5.OA.3** **Connections to other Domains &/or Clusters:** **Graph points on the coordinate plane to solve real-world and mathematical problems.** 4. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **CC.5.G.1** 5. Represent real world and mathematical problems by graphing points

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	in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **CC.5.G.2**

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Standards for Mathematical Practice:

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Course: Grade 5 - Math
Topic: Number & Operations in Base Ten – Understanding Place Value

Which standards are in this learning progression?

5.NBT.1, 5.NBT.2, 5.NBT.3a-b, 5.NBT.4, 5.NBT.5, 5.NBT.6, 5.NBT.7

Connections to other domains and/or clusters:

5.MD.4, 5.MD.5

By the end of this learning progression, students will be able to…

UNDERSTAND:

The value of a digit in our number system is determined by its place value position.

Place value patterns are continued in decimal numbers.

Computational strategies with whole numbers can be applied to decimals.

KNOW:	DO:
Place value is based on multiples of ten. A digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. When multiplying/dividing whole numbers & decimals by powers of 10, the digits move based on place value, not the decimal point. Multiplying & dividing whole numbers and decimals by 10 results in a pattern of zeros. (See example for standard 5.NBT.2). Decimal numbers fall between whole numbers. Proximity of a decimal to the nearest whole number. Rounding is a formal way of estimating. Process of standard algorithm for multiplication (There are multiple standard algorithms: partial products/distributed multiplication, traditional). Strategies for dividing whole	**Understand the place value system.** 1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **CC.5.NBT.1** 2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **CC.5.NBT.2** Ex: 247 x 100 = (200 x 100) + (40 x 100) + (7 x 100) = 20,000 + 4,000 + 700 = 24,700 247/10 = (200/10) + (40/10) + (7/10) = 20 + 4 + .7 = 24.7 24.7 x 10 = (20 x 10) + (4 x 10) + (.7 x 10) = 200 + 40 + 7 = 247 3. Read, write, and compare decimals to thousandths. **CC.5.NBT.3** a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **CC.5.NBT.3a** b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **CC.5.NBT.3b** 4. Use place value understanding to round decimals to any place. **CC.5.NBT.4** **Perform operations with multi-digit whole numbers and with decimals to hundredths.** 5. Fluently multiply multi-digit whole numbers using the standard algorithm. **CC.5.NBT.5** 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication

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numbers (See standard 5.NBT.6 - Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models) Strategies to perform all operations. Properties of operations (i.e. distributive property) Addition & subtraction are inverse operations. Multiplication & division are inverse operations.	and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **CC.5.NBT.6** 7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **CC.5.NBT.7** **Connections to other Domains &/or Clusters:** **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 4. unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **CC.5.MD.4** 5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **CC.5.MD.5** a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **CC.5.MD.5a** b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **CC.5.MD.5b** c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **CC.5.MD.5c**

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Standards for Mathematical Practice:

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Course: Grade 5 - Math
Topic: Number & Operations: Fractions – Adding & Subtracting

Which standards are in this learning progression?

5.NF.1, 5.NF.2

Connections to other domains and/or clusters:

5.MD.2

By the end of this learning progression, students will be able to…

UNDERSTAND:

Equivalent fractions are a powerful strategy for adding and subtracting fractions.

Multiple strategies & models can be utilized to solve a variety of problems involving fractional concepts.

KNOW:	DO:
Fractions can be added & subtracted using area models, ratio models, number lines, fraction bars, and finding common denominators. Relationship between numbers and their multiples are used to find equivalent fractions. Benchmark fractions and fraction number sense can be used to estimate fraction sums and differences and assess the reasonableness of solutions. Methods for recording strategies for adding & subtracting fractions using models or equations.	**Use equivalent fractions as a strategy to add and subtract fractions.** 1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **CC.5.NF.1** -Add and subtract fractions with like denominators - Add and subtract fractions with unlike denominators using models. - Find common denominators. 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **CC.5.NF.2** -Use benchmark fractions and fraction number sense to estimate fraction sums and differences. -Check the reasonableness of an answer within the context of the problem. **Connections to other Domains &/or Clusters:** **Represent and interpret data.** 2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **CC.5.MD.2**

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Course: Grade 5 - Math
Topic: Number and Operations – Fractions (Multiplying & Dividing)

Which standards are in this learning progression?

5.NF.3, 5.NF.4a-b, 5.NF.5a-b, 5.NF.6, 5.NF.7a-c

Connections to other domains and/or clusters:

5.NBT.6, 5.NBT.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

Extending previous understandings of multiplication & division can help you solve problems involving multiplying & dividing fractions.

KNOW:	DO:
Fractions can be perceived and utilized as division of the numerator by the denominator. Multiplying a whole number by a number greater than 1 results in a product greater than the given number. (Ex: $3^1/_2$ x 5 will result in a number more than $3^1/_2$) Multiplying a whole number by a number smaller than 1 results in a product less than the given number. (Ex:$3^1/_2$ x $^1/_4$ will result in a number less than $3^1/_2$) Multiplying a whole number by a number/ fraction equal to 1 results in a number that represents the same quantity. When multiplying fractions, either factor can be the multiplier. (Ex:$^2/_3$ x 4 can be interpreted as $^2/_3$ of 4 or 4 groups of $^2/_3$.) Visual fraction models can be used to solve problems like $^1/_3$ divided by 4 or 4 divided by $^1/_3$. An array model can justify the formula: A=L*x*W	Decompose & recompose fractions to solve problems involving multiplication and division of fractions. **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **CC.5.NF.3** Interpret a fraction as division of the numerator by the denominator. Use visual fraction models or equations to solve problems involving division of whole numbers in which the answers are fractions or mixed numbers. 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **CC.5.NF.4** a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* **CC.5.NF.4a** Use visual models to show the multiplication of two fractions (Ex: $^1/_2$ of $^1/_4$) Multiply a fraction by a whole number. (EX: $^2/_3$ x 4 is 4 sets of $^2/_3$ or 8 sets of $^1/_3$) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side

lengths to find areas of rectangles, and represent fraction products as rectangular areas. **CC.5.NF.4b**

5. Interpret multiplication as scaling (resizing), by: **CC.5.NF.5**
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **CC.5.NF.5a**
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **CC.5.NF.5b**

Reason whether multiplying by a fraction greater or less than 1 will result in a product greater or less than 1.

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **CC.5.NF.6**

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] **CC.5.NF.7**
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* **CC.5.NF.7a**
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. F*or example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* **CC.5.NF.7b**
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* **CC.5.NF.7c**

Connect division of a whole number by a fraction (2 divided by $^1/_3$) to a context such as "How many $^1/_3$ cup servings are in 2 cups of raisins?"

Connect division of a fraction by a whole number ($^1/_2$ divided by 3) to a context such as "How much chocolate will each person get if 3 people share $^1/_2$ a pound of chocolate equally?"

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

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	Connections to other Domains &/or Clusters: 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **CC.5.NBT.6** 7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **CC.5.NBT.7**

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Course: Grade 5 - Math
Topic: Measurement & Data (Measurement system, line plots, volume)

Which standards are in this learning progression?

5.MD.1, 5.MD.2, 5.MD.3, 5.MD.4, 5.MD.5

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

There are a multiple ways to organize, recognize, and interpret data for a variety of purposes.

The concepts of volume are related to area, multiplication and division.

KNOW:	DO:
Standard Measurement Units can be used interchangeably. Data can be organized, represented, & interpreted in multiple ways. Volume is an attribute of solid figures relating length, width, and height (depth). Volume is "filling" the inside space if a 3D shape. Volume is additive: The volumes of two non-overlapping rectangular prisms can be added to find a total volume. The formula V=B•h relates the total volume as multiple layers of the Base (area). The area of a rectangular base can be utilized when calculating the volume.	**Convert like measurement units within a given measurement system.** 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **CC.5.MD.1** **Represent and interpret data.** 2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **CC.5.MD.2** Connect to operations with fractions to solve problems in measurement and data contexts. **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **CC.5.MD.3** a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **CC.5.MD.3a** b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **CC.5.MD.3b** 4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **CC.5.MD.4** 5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **CC.5.MD.5** a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative

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	property of multiplication. **CC.5.MD.5a** b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **CC.5.MD.5b** c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **CC.5.MD.5c** Clarification of 5.MD.2 can be interpreted as students displaying a data set that includes fractions, using the operations of fractions to interpret the data (i.e. finding the mean (average) of fractional data.) Decompose a rectangular prism into multiple layers of the area of the base layer of the figure and relate that back their understanding of volume. Connect to multiplication concepts in Number & Operations in Base Ten to solve problems in geometric measurement contexts.

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Course: Grade 5 - Math
Topic: Geometry – Graphing & Properties of 2D Figures

Which standards are in this learning progression?

5.G.1, 5.G.2, 5.G.3, 5.G.4

Connections to other domains and/or clusters:

5.OA.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

Coordinate systems can be used to describe locations precisely.

2-D shapes can be identified, classified and analyzed by their properties.

KNOW:	DO:
A pair of perpendicular lines form a coordinated system, with the intersection of the lines (the origin) is coordinated to form the point (0,0). The first number in an ordered pair tells how far to travel from the origin on the x-axis, and second number says how far to travel on the y-axis. Points that lie on a graphed (linear) line express equivalent ratios. Points graphed on a coordinate plan can be interpreted to solve problems. Two-dimensional shapes are classified by their attributes (i.e. # of sides, # of angles, types of angles, regular vs. irregular polygons, etc.).	**Graph points on the coordinate plane to solve real-world and mathematical problems.** 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **CC.5.G.1** 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **CC.5.G.2** **Classify two-dimensional figures into categories based on their properties.** 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **CC.5.G.3** 4. Classify two-dimensional figures in a hierarchy based on properties. **CC.5.G.4** **Connections to other Domains &/or Clusters:** **Analyze patterns and relationships.** 3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice*

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	the corresponding terms in the other sequence. Explain informally why this is so. **CC.5.OA.3**

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS – Grade 6

(KUD Organizer)

Mathematics

C. Lancour

5/10/2011

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Course: Grade 6 Math
Topic: Rates & Ratios

Which standards are in this learning progression?

6.RP.1, 6.RP.2, 6.RP.3a-d,

Connections to other Domains and/or Clusters:

6.NS.6a-c, 6.7.a-d

By the end of this learning progression, students will be able to…

UNDERSTAND:

Reasoning about multiplication & division is critical to the understanding of ratio concepts & their application to solving problems.

A rate is a set of infinitely many equivalent ratios.

KNOW:

A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

Ratios & rates are connected to multiplication and division.

Various ways for representing ratios (e.g., in words, with a colon, in fraction notation).

Ratios are connected to fractions:
- Ratios are often expressed in fraction notation, although ratios & fractions do not have identical meaning.
- Ratios are often used to make "part-part" comparisons, but fractions are not.
- Ratios and fractions can be thought of as overlapping sets.
- Ratios can be meaningfully re-interpreted as fractions.

(source: *Essential Understanding of Ratios, Proportions, & Proportional Reasoning gr 6-8* from NCTM 2010)

When representing a ratio, quantities must correspond to the context of the situation.

Idea of unit rate is comparing quantity per one.

A variety of representations (ratio table,

DO:

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **CC.6.RP.1**
2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] **CC.6.RP.2**
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **CC.6.RP.3**
   Connection to 6.NS.6a-c & 6.NS.7a-d when comparing, ordering, & understanding equivalence of rational numbers.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **CC.6.RP.3a**
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **CC.6.RP.3b**
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **CC.6.RP.3c**
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **CC.6.RP.3d**

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

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graph, double number lines, scale drawings, etc) model proportionality. Flexibility between representations of ratios & proportions (listed above) empowers problem solving. Scaling up/down a ratio proportionally maintains equivalency. Percent of a quantity is per 100. Flexibility between representations of numbers (fractions, decimals, percent) empowers problem solving. Equivalent measurements within measurement systems can be used to solve problems.	

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Course: Grade 6 Math
Topic: The Number System – Operations w/ Rational Numbers (excluding integers)

Which standards are in this learning progression?

6.NS.1, 6.NS.2, 6.NS.3, 6.NS.4

Connections to other Domains and/or Clusters:

By the end of this learning progression, students will be able to…

UNDERSTAND:

Multiple strategies & models can be utilized to solve a variety of problems involving fractions and decimals.

Relative magnitude of a solution (quotient) depends on the size of the divisor.

KNOW:	DO:
Computation with rational numbers is an extension of computation with whole numbers, but introduces some new ideas and processes. • Strategies utilized for multiplying & dividing whole numbers (and whole numbers by fractions) extend to fractions/decimals. Ex: The same reasoning used to solve **6 ÷ 2** (*How many 2s are in 6?*) can be used to solve problems involving fractions such as **8 ÷ ½** (*How many one-halves are in 8?*) and **¾ ÷ ¼** (*How many one-fourths are in ¾?*) • When dividing by a fraction, the solution may be larger than what you started with (the original value). • Multiplying by a number greater than 1 results in a product greater than the given number. (Ex: $3^1/_2$ x 5 will result in a number more than $3^1/_2$) • Multiplying by a number smaller than 1 results in a product less than the given number. (Ex:$3^1/_2$ x $^1/_4$ will result in a number less than $3^1/_2$) **Models**: Number line, ratio table, area model, arrays, bar model, fraction circles, picture/visual Estimation as a means for predicting & assessing the reasonableness of a solution. Computational fluency is built upon understandings of models and decomposing & recomposing numbers. Fluency with mental math and estimation facilitates efficient problem solving. Flexibility with the equivalent forms of the	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* **CC.6.NS.1** **Compute fluently with multi-digit numbers and find common factors and multiples.** 2. Fluently divide multi-digit numbers using the standard algorithm. **CC.6.NS.2** There are multiple ways to develop a conceptual understanding of division and division algorithms. 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **CC.6.NS.3** Conceptual understanding of operational algorithms is built in a variety of ways and extends upon the strategies utilized in lower grades. 4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* **CC.6.NS.4** Solve problems efficiently by converting between equivalent forms of the distributive property (expanded form and factored form) through the use of other concepts such as GCF.

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distributive property (expanded form and factored form) allows for efficient problem solving.	

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Standards for Mathematical Practice:

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Course: Grade 6 Math
Topic: Expanding Rational Numbers

Which standards are in this learning progression?

6.NS.5, 6.NS.6a-c, 6.NS.7a-d, 6.NS.8

Connections to other Domains and/or Clusters:

6.G.3, 6.RP.3a-d

By the end of this learning progression, students will be able to…

UNDERSTAND:

Rational numbers are a subset of the number system including & beyond whole numbers.

Although every whole number corresponds to a single numeral, every rational number can be written in many different ways.

KNOW:	DO:
Rational numbers are a set of numbers that includes the whole numbers, integers, as well as numbers that can be written as quotient of two integers, a ÷ b, where b≠0. Much of the space between adjacent integers on a number line is taken by rational numbers that are not integers. Between any two rational numbers, there are infinitely many rational numbers. Rational numbers can be represented as fractions, decimals, & percents in infinitely many equivalent forms. Rational numbers have multiple interpretations, and making sense of them depends on identifying the unit. • The concept of *unit* is fundamental to the interpretation of rational numbers • Rational numbers can be interpreted as: o A part-whole relationship o As a measure o As a quotient o As a ratio o As an operator	**Apply and extend previous understandings of numbers to the system of rational numbers.** 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **CC.6.NS.5** 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **CC.6.NS.6** -Compare & order fractions, decimals, & percents efficiently and find their approximate locations on a number line. -This connects to Ratios & Proportional Relationships in which students begin to understand percents and represent quantities with models: 6.G.3a-d -Develop a meaning for integers and represent & compare quantities with them. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. **CC.6.NS.6a** b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by **reflections** across one or both axes. **CC.6.NS.6b** -In grade 4, students are exposed to line symmetry. Here in grade 6 is an informal way of introducing **reflections** in terms of understanding locations on a coordinate plane. Reflections become more formalized in grade 8. -This connects to Geometry where students must draw polygons in the coordinate plane: 6.G.3 c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **CC.6.NS.6c** 7. Understand ordering and absolute value of rational numbers. **CC.6.NS.7**

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(Source: *Essential understanding of Rational Numbers* by NCTM 2010) Estimation and mental math are more complex with rational numbers than with whole numbers.	a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* **CC.6.NS.7a** b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^oC > -7^oC$ to express the fact that -3^oC is warmer than -7^oC.* **CC.6.NS.7b** c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* **CC.6.NS.7c** d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* **CC.6.NS.7d** 8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **CC.6.NS.8** Ex: If the context includes finding the distance between (2, 4) and (2, -7), students must find the number of units from 0 to 4 and from 0 to -7 to find the total distance. They could use absolute value to find the total distance of 11. This standard connects to the Geometry standard for drawing polygons in the coordinate plane: 6.G.3

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Course: Grade 6 Math
Topic: Algebraic Reasoning – Expressions

Which standards are in this learning progression?

6.EE.1, 6.EE.2a-c, 6.EE.3, 6.EE.4

Connections to other Domains and/or Clusters:

6.G.1, 6.G.2, 6.EE.5 - 9

By the end of this learning progression, students will be able to…

UNDERSTAND:

Mathematical expressions can be used to represent and solve real-world and mathematical problems.

Flexibility in manipulating expressions to suit a particular purpose (rewriting an expression to represent a quantity in a different way to make it more compact or to feature different information) helps with solving problems efficiently.

KNOW:	DO:
Variables can be used to represent numbers whose exact values are not yet specified. Expressions can be manipulated to generate equivalent expressions to simplify the problem. Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions. The equals sign as a symbol of equivalence.	**Apply and extend previous understandings of arithmetic to algebraic expressions.** 1. Write and evaluate numerical expressions involving whole-number exponents. **CC.6.EE.1** 2. Write, read, and evaluate expressions in which letters stand for numbers. **CC.6.EE.2** a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* **CC.6.EE.2a** b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **CC.6.EE.2b** Decompose & recompose expressions in meaningful ways. *c.* Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* **CC.6.EE.2c** Note: This is a connection to the geometry standards for students to be solving problems in the context of area, SA, & volume. (6.G.1, 6.G.2) 3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **CC.6.EE.3** An example of the distributive property is in the context of area: 3(x+7) = 3x + 21 X + 7 [ ][ ] 3

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	Create, write, interpret, and use equivalent expressions. 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* **CC.6.EE.4** How can this standard be unpacked to encourage higher order thinking?

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Course: Grade 6 Math
Topic: Algebraic Reasoning – Equations & Inequalities

Which standards are in this learning progression?

6.EE.5, 6.EE.6, 6.EE.7, 6.EE.8

Connections to other Domains and/or Clusters:

6.EE.1 – 4, 6.EE.9, 6.G.1, 6.G.2

By the end of this learning progression, students will be able to…

UNDERSTAND:

Mathematical expressions, equations, and inequalities are used to represent and solve real-world and mathematical problems.

KNOW:	DO:
Variables can be used to represent numbers whose exact values are not yet specified. Inverse operations are used to solve equations and inequalities. Solutions to an equation/ inequality are the values of the variables that make the equation/ inequality true. There are some inequalities that have infinitely many solutions (those in the form of $x>c$ or $x<c$). Solutions to an inequality are represented symbolically or using a number line. The equals sign as a symbol of equivalence.	**Reason about and solve one-variable equations and inequalities.** 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **CC.6.EE.5** This is connected to 6.EE.9 in which students are to represent relationships between variables (quantities). 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **CC.6.EE.6** 7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **CC.6.EE.7** This is connected to: 6.EE.9 in which students are to represent relationships between variables (quantities). 6.RP.3a-b in which students utilize rates to solve problems. 8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **CC.6.EE.8** This is also connected to: 6.NS.6a-c 7 6.NS.7a-d in which students compare and order rational numbers utilizing the number line.

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Course: Grade 6 Math
Topic: Algebraic Reasoning – Quantitative Relationships

Which standards are in this learning progression?

6.EE.9

Connections to other Domains and/or Clusters:

6.RP.3a-b, 6.EE.3 - 8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Quantitative relationships between dependent and independent variables can be represented in multiple ways including algebraic (equation), graphical, verbal (scenario), and tabular.

KNOW:	DO:
Quantities that change in relationship to one another can be represented using variables. There is a relationship between independent and dependent variables. Different representations of the relationship provide varied opportunities to analyze changes in quantities (e.g., as in linear relationships). Various representations of quantitative relationships including: scenario (context) table, graph, and equation.	**Represent and analyze quantitative relationships between dependent and independent variables.** 9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **CC.6.EE.9** Use graphs to analyze the nature of changes in quantities in linear relationships. This connects to: 6.RP.3a-b in which students utilize rates to solve problems. 6.EE.5-8 in which students are writing and solving equations and inequalities for contextual and mathematical situations.

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Course: Grade 6 Math
Topic: Geometry – Area, Surface Area, Volume

Which standards are in this learning progression?

6.G.1, 6.G.2, 6.G.3, 6.G.4

Connections to other Domains and/or Clusters:

6.NS.8, 6.EE.6, 6.EE.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

By decomposing two- and three-dimensional shapes into smaller component shapes, the concept of surface area and the formulas for area and volume can be developed and justified.

KNOW:	DO:
Areas of polygons can be found by decomposing them into triangles and/or other shapes, rearranging or removing shapes, and relating the shapes to rectangles. (Formulas for the areas of triangles and parallelograms can be developed and justified with these methods.) Linear, square, and cubic measurements are described with different units (connect the units to what is being measured). Area is 'covering', and is measured in square units. Surface area is 'covering' all faces of a 3D shape with square units. Volume is 'filling' and is measured in cubic units. The volume of a rectangular prism can be decomposed into multiple layers. Volume can be thought of as multiple layers (h) of the number of cubes needed for the base (B). ($V=B*h$)	**Solve real-world and mathematical problems involving area, surface area, and volume.** 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **CC.6.G.1** Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. (Source: Narrative of CCSS for Grade 6) 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **CC.6.G.2** 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **CC.6.G.3** This connects to **6.NS.8** in which students are to solve problems by graphing, and find distances on the coordinate plane when one value in the coordinates (x or y) stays the same. 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **CC.6.G.4** Use 2D representations of 3D objects to visualize and solve problems such as those involving surface area and volume. (Source: *NCTM Focal Points*)

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Course: Grade 6 - Math
Topic: Statistics – Describing Data (Measures of Center & Variability)

Which standards are in this learning progression?

6.SP.1, 6.SP.2, 6.SP.3

Connections to other Domains and/or Clusters:

6.NS (Number operations, comparing/ordering numbers)

By the end of this learning progression, students will be able to…

UNDERSTAND:

Descriptive statistics (mean, median, mode, range, inter-quartile range) and the various graphical representations allow you to summarize and compare data sets.

KNOW:	DO:
Numerical data can be summarized with one or more numbers. Descriptive statistics (mean, median, mode, range, inter-quartile range) allow you to summarize and compare data sets. A single measure of center will not thoroughly describe a data set because very different data sets can share the same measure of center. Different measures of center may be selected to describe data for different purposes or contexts. Different representations of data may be appropriate based on the context or purpose for displaying the data.	**Develop understanding of statistical variability.** 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **CC.6.SP.1** 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **CC.6.SP.2** Describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. (Source: Narrative of CCSS for Grade 6) 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **CC.6.SP.3** Find, use, and interpret measures of center and spread, including mean, median, mode, range, inter-quartile range. 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **CC.6.SP.4** Organize and display data to pose and answer questions. Select, create and use appropriate graphical representations of data. Compare and discuss how different representations correspond to the data sets, and evaluate how each representation shows important aspects of the data. 5. Summarize numerical data sets in relation to their context, such as by: **CC.6.SP.5** a. Reporting the number of observations. **CC.6.SP.5a** b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **CC.6.SP.5b** c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **CC.6.SP.5c** -Use measures of center and spread, including mean, median, mode, range, inter-quartile range, to summarize & compare data sets. -Compare the information that is provided by the mean, median, & mode. -Explore how changes in the data values will affect these measures of center.

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	d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. **CC.6.SP.5d** The standards in this progression are connected to the **Number System** (NS) domain in which students operate on, compare, and order numbers.

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grade 7

(KUD Organizer)

Mathematics

C. Lancour

5/10/2011

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Course: Mathematics Grade 7
Topic: Ratios and Proportional Relationships

Which standards are in this learning progression?

CC.7.RP.1, CC.7.RP.2a-d, CC.7.RP.3

Connections to other Domains and/or Clusters:

7.G.1, 7.EE, 7.NS

By the end of this learning progression, students will be able to…

UNDERSTAND:

Extending an understanding of ratios develops a deeper understanding of proportionality builds the knowledge and skill levels needed to solve single- and multi-step problems.

KNOW:	DO:
Multiple strategies for finding unit rates. There are various strategies for deciding if a relationship is proportional (for example – equivalent ratios in a table, observing points graphed on a coordinate plane, analyzing ratios for equivalence, etc). Ratio (rate) tables are used to build equivalent ratios/ rates. Proportional relationships can be represented symbolically (equation), graphically (coordinate plane), in a table, in diagrams, and verbal descriptions. The coordinates representing a proportional linear context can be interpreted in terms of the context. Special attention should be spent on analyzing the points (0, 0) and 1, *r*) where *r* is the unit rate. Ratio and proportional reasoning strategies can be extended and applied to multi-step ratio and percent problems.	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** 1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* **CC.7.RP.1** 2. Recognize and represent proportional relationships between quantities. **CC.7.RP.2** a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **CC.7.RP.2a** Distinguish proportional relationships from other relationships. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **CC.7.RP.2b** c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. **CC.7.RP.2c** d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **CC.7.RP.2d** Interpret the coordinate pairs of a proportional linear situation in terms of the context. For example, if the context is about earning $30 per day for babysitting for six hours, the unit rate of $5/hour can be interpreted when graphed as the coordinate (1, 5) where the person earns $5 for every hour of babysitting. 3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **CC.7.RP.3** Use knowledge of models such as ratio (rate) tables, double number lines, graphs, scale drawings, equations, etc to solve multi-step ratio and percent problems. This progression connects to standards in other domains such as Expressions and

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	Equations (7.EE), The Number System (7.NS), and Geometry (7.G.1 – scale drawings).

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Proportional Relationship, Equivalent, Coordinate Plane, Origin, Unit Rate, Ratio, Rate, Simple Interest, Tax, Tip, Percent increase/decrease, Commission, Percent Error**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Course: Math – Grade 7
Topic: The Number System – Operations w/ Rational Numbers

Which standards are in this learning progression?

7.NS.1a-d, 7.NS.2a-d, 3

Connections to other Domains and/or Clusters:

7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-b, 7.RP.1, 7.RP.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

Rational numbers are a subset of the number system including & beyond whole numbers.

Properties of whole number operations can be applied to solving real world and mathematical problems involving rational numbers, including integers.

KNOW:

Properties of whole number operations can be applied when solving problems involving operations with rational numbers (Distributive Property, Commutative, Associative, and Identity Properties of Addition and Multiplication, Additive Inverse Property).

Strategies to represent and solve problems involving operations with rational numbers (including decimals, fractions, integers).

Strategies for converting a rational number into a decimal.

The decimal form of a rational number terminates in zeros or eventually repeats.

Opposites and absolute value of rational numbers.

A negative number can also be interpreted as the opposite of the positive number. (Ex: -5 can be interpreted as the opposite of 5.)

Computation with integers is an extension of computation with fractions and decimals:

- Strategies utilized for multiplying & dividing fractions and decimals numbers extend to integers. Ex: The same reasoning used to solve **6x2** (*What is 6 groups of 2?)* can be used to solve problems involving integers such as **6 x -2** (*What is 6 groups of -2?*)

Models: Number line, chip model, area model, arrays, bar model, fraction circles, picture/visual

Estimation as a means for predicting & assessing the reasonableness of a solution.

DO:

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **CC.7.NS.1**

   Use number lines, chip models, or other models to represent addition and subtraction of integers.

   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **CC.7.NS.1a**

   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **CC.7.NS.1b**

   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **CC.7.NS.1c**

   Find distance between rational numbers (including integers) on a number line.

   d. Apply properties of operations as strategies to add and subtract rational numbers. **CC.7.NS.1d**

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **CC.7.NS.2**

   Use strategies such as repeated addition/subtraction and/or patterning to develop conceptual understanding of multiplication and division with integers. For example:
   -4 x 3 = (-4) + (-4) + (-4) = -12

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Computational fluency is built upon understandings of models and decomposing & recomposing numbers.

Fluency with mental math and estimation facilitates efficient problem solving.

Flexibility with the equivalent forms of the distributive property (expanded form and factored form) allows for efficient problem solving.

Rational numbers are a set of numbers that includes the whole numbers, integers, as well as numbers that can be written as quotient of two integers, a ÷ b, where b≠0.

Between any two rational numbers, there are infinitely many rational numbers.

Rational numbers can be represented as fractions, decimals, & percents in infinitely many equivalent forms.

Rational numbers have multiple interpretations, and making sense of them depends on identifying the unit.

- The concept of *unit* is fundamental to the interpretation of rational numbers
- Rational numbers can be interpreted as:
  - A part-whole relationship
  - As a measure
  - As a quotient
  - As a ratio
  - As an operator

(Source: *Essential understanding of Rational Numbers* by NCTM 2010)

Estimation and mental math are more complex with rational numbers than with whole numbers.

And:

4 x 3 = 12	-4 x 3 = -12
4 x 2 = 8	-4 x 2= -8
4 x 1 = 4	-4 x 1= -4
4 x 0 = 0	-4 x 0 = 0
4 x -1 = -4	-4 x -1= 4
4 x -2 = -8	-4 x -2 = 8

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **CC.7.NS.2a**

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real- world contexts. **CC.7.NS.2b**

c. Apply properties of operations as strategies to multiply and divide rational numbers. **CC.7.NS.2c**

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **CC.7.NS.2d**

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1] **CC.7.NS.3**

Understand the meaning and **effect** of arithmetic operations with fractions, decimals, and integers.

This cluster is connected to 7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-b in which students must utilize their knowledge of rational numbers in the context of algebraic reasoning.

It also connects to the Ratios and Proportional Reasoning domain in which students are utilizing operations with rational numbers to compute unit rates and solve multi-step problems (7.RP.1 7.RP.3).

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[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

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Course: Math – Grade 7
Topic: Problem Solving with Expressions & Equations

Which standards are in this learning progression?

7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-b

Connections to other Domains and/or Clusters:

7.NS.1a-d, 7.NS.2a-d, 7.NS.3,

By the end of this learning progression, students will be able to…

UNDERSTAND:

Expressions can be manipulated to suit a particular purpose and solving problems efficiently.

Mathematical expressions, equations, and inequalities are used to represent and solve real-world and mathematical problems.

KNOW:	DO:
Variables can be used to represent numbers whose exact values are not yet specified. Expressions can be manipulated to generate equivalent expressions to simplify the problem. Expressions can be decomposed and recomposed in different ways to generate equivalent forms. Flexibility with the equivalent forms of an expression (expanded form, factored form, etc) allows for efficient problem solving. Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions. The equals sign demonstrates equivalence. Ex: 2x + x = 3x (equivalent expressions) 2x + x= 3x + 4 = 3x + 4 (not equivalent expressions) Rational numbers can be represented in equivalent forms to solve problems efficiently (25% can be represented as ¼ or 0.25). Estimation as a means for predicting & assessing the reasonableness of a solution.	**Use properties of operations to generate equivalent expressions.** 1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **CC.7.EE.1** Decompose and recompose expressions using properties of operations to generate equivalent forms. This is connected to 7.NS.1a-d, 7.NS.2a-d, 3. Students will utilize their knowledge of rational numbers, operations, and the number system in other contexts such as algebraic reasoning. 2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **CC.7.EE.2** Rewrite expressions to represent a quantity in a different way to make it more compact or to feature different information. **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **CC.7.EE.3** This is connected to 7.NS.1a-d, 7.NS.2a-d, 3. Students will utilize their knowledge of rational numbers, operations, and the number system in other contexts such as algebraic reasoning. This is connected to 7.EE.1 and 7.EE.2 in which students are working with

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Fluency with mental math and estimation facilitates efficient problem solving. Inverse operations are used to solve equations and inequalities. Solutions to an equation/ inequality are the values of the variables that make the equation/ inequality true. There are some inequalities that have infinitely many solutions (those in the form of $x>c$ or $x<c$). Solutions to an inequality are represented symbolically or using a number line.	expressions. 4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **CC.7.EE.4** a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **CC.7.EE.4a** *b.* Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* **CC.7.EE.4b** Graph the solution set on a number line. For example, x > 6 can be represented on a line with an open circle on the number 6 and shading to the right of the 6.
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Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

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Course: Math – Grade 7
Topic: Geometry – Relationships in 2D Geometric Figures

Which standards are in this learning progression?

7.G.1, 7.G.2, 7.G.3

Connections to other Domains and/or Clusters:

7. RP. 1. 7.RP.2a

By the end of this learning progression, students will be able to…

UNDERSTAND:

Two-dimensional geometric figures are representations of our three-dimensional world.

Experimenting with and investigating the relationships between 2D and 3D geometric figures connects and integrates these concepts for problem solving.

KNOW:	DO:
Scaling up/down is an application of proportional reasoning. The relationship between dimensions of a scale drawing and the original figure is proportional. Attributes of triangles and angles. Depending on the attributes given, a unique triangle, more than one triangle, or no triangle can be the result. There are certain given conditions that will produce only one, unique triangle. Some given conditions may produce more than one triangle or no triangle at all. Slicing/ cross-sectioning 3D shapes (including but not limited to right rectangular prisms and right rectangular pyramids) will result in 2D shapes.	**Draw, construct, and describe geometrical figures and describe the relationships between them.** 1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **CC.7.G.1** Similarity is not formally introduced until grade 8, but scale drawings are used as a context for proportional reasoning, in geometry. Similarity is introduced informally in grade 6 & 7 through proportional reasoning and geometry. 2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **CC.7.G.2** This can be an **informal** exploration the relationships of sides and angles of triangles. This is the **exploration** phase done prior to more formalized work in high school around the relationships between shapes and their angles and sides. For example, students may explore the Triangle Sum Theorem, SAS, SSS, etc. 3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **CC.7.G.3** Visualize the cross-sections of rectangular prisms and rectangular pyramids. While in elementary grades we analyze the 2D faces of 3D figures. Now students will investigate the 2D shapes found inside 3D figures. Investigate and compare 2D cross-sections parallel to the base of a 3D figure. For example the cross-sections parallel to the base of a rectangular prism are all congruent rectangles. The cross-sections parallel to the base of a rectangular prism are all rectangles, but they get smaller as they are sliced closer to the vertex

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Course: Math – Grade 7
Topic: Geometry – Problem Solving with Angles, Area, SA, Volume

Which standards are in this learning progression?

7.G.4, 7.G.5, 7.G.6

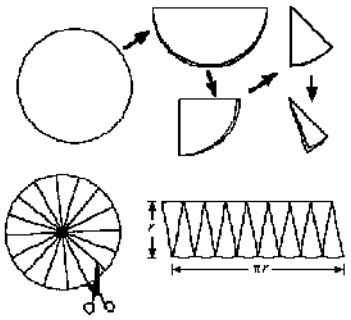
Connections to other Domains and/or Clusters:

7.RP

By the end of this learning progression, students will be able to…

UNDERSTAND:

Relationships between geometric figures are useful for building new knowledge and solving real world and mathematical problems accurately.

KNOW:	DO:
There is a relationship between the circumference and the diameter of a circle. The ratio of the circumference to the diameter of a circle is pi (π). There is a proportional relationship between the circumference and area of a circle. (This is informal) [The area of a circle can be found by multiplying half the circumference by the radius (A=1/2 * C * *r*) or multiplying one-fourth the circumference by the diameter (A=1/4 *C * *d*). Relate this to the formula for finding the area of a rectangle (A=*l***w*)] – See picture to right. Circumference of a circle: C= 2π*r* or π*d* Area of a circle: A= πr^2 Supplementary angles sum to 180º Complementary angles sum to 90º Vertical angles are created by intersecting lines and are congruent. Adjacent angles in parallelogram are supplementary (sum to 180º). Previous knowledge of area and volume to solve problems involving area, volume, and SA of additional 2D	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **CC.7.G.4** Develop and utilize formulas for the area and circumference of circles. Explore the relationship between the circumference and area of a circle by cutting the circle into segments and laying them side-by-side in a shape that is close to the rectangle. How can you use the circumference and the radius or diameter to fin the area? How does this relate to the area of a rectangle?  5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **CC.7.G.5** Explore the relationships between the angles formed by intersecting lines. Use these relationships to solve problems. 6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **CC.7.G.6** Decompose shapes into smaller component shapes (described in standard 6, above) to solve problems involving area, volume, and surface area.

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and 3D figures.	

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Course: Math – Grade 7
Topic: Statistics

Which standards are in this learning progression?

7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4

Connections to other Domains and/or Clusters:

7.RP, 7.SP.5-8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Formulating questions, designing studies, and collecting data about a population through random sampling allow us to make inferences and compare data.

KNOW:	DO:
Random sampling tends to produce representative samples. Finding a valid, representative sample will enable valid inferences to be made about a population. What it means to have a valid, random sample representative of a population(s). Inferences about a population are only valid if the sample is random and representative. Proportional reasoning is used to make estimates or predictions about a population. Having multiple samples for the same population allows for gauging the variation of estimates or predictions. Measures of center can be used to compare data and measure variability between data sets. Data displays are used to visually compare data sets and draw informal comparative inferences. Box plots are way to show measures of variability such as the range (Other data displays may highlight other measures of variability).	**This set of standards is connected to Ratios & Proportional Reasoning and Probability.** **Use random sampling to draw inferences about a population.** 1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **CC.7.SP.1** Formulate questions, design studies, and collect data about a characteristic shared by two populations, or different characteristics within one population. 2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **CC.7.SP.2** Use proportional reasoning to make estimates relating to a population on the basis of a sample. Generate multiple samples for making estimates and predictions. **Draw informal comparative inferences about two populations.** 3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **CC.7.SP.3** Select, create, and use appropriate graphical representation of data including histograms, box plots, and scatter plots. (This was learned in grade 6). Use the mean (or other measures of center) to find the difference between the centers as a measure of variability.

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Mean absolute deviation is an element of a data set that is the absolute difference between that element and a given point. Typically the point from which the deviation is measured is a measure of central tendency, most often the median or sometimes the mean of the data set.	Find the mean absolute deviation of the values in a data set. Use observations to about mean differences between two or more samples to make conjectures about the populations from which the samples were taken. 4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **CC.7.SP.4** Make conjectures about data by selecting and using appropriate measures of center and variability.

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Standards for Mathematical Practice:

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Course: Math – Grade 7
Topic: Probability

Which standards are in this learning progression?

7.SP.5, 7.SP.6, 7.SP.7a-b, 7.SP.8a-c

Connections to other Domains and/or Clusters:

7.RP, 7.NS.2

By the end of this learning progression, students will be able to…

UNDERSTAND:

Probabilities are fractions derived from modeling real world experiments and simulations of chance.

KNOW:	DO:
Probability is the likelihood of an event occurring. The likelihood of a chance event is a number between 0 and 1. Larger numbers (closer to 1) indicate greater likelihood of an event occurring. The benchmark of ½ can be used to determine if an event is more likely or less likely to occur. Theoretical probability is the likelihood of an happening based on all possible outcomes. Experimental probability of an event occurring after an experiment was conducted. Theoretical and experimental probabilities and proportional reasoning are used to make predictions. Equivalent fractions (and prior fraction knowledge) are used for making predictions. Probability in a **uniform** probability model is the number of favorable outcomes as 1 (numerator) out of the number of all possible outcomes (denominator). For example – probability of rolling a 5 on a regular die is 1/6. Probabilities of compound events	This set of standards is connected to Ratios and Proportional Reasoning. **Investigate chance processes and develop, use, and evaluate probability models.** 5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **CC.7.SP.5** 6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **CC.7.SP.6** Use Theoretical and Experimental probability and proportional reasoning to make approximate predictions. 7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **CC.7.SP.7** Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **CC.7.SP.7a** Determine the probabilities of events with equally likely outcomes. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **CC.7.SP.7b** Determine the probabilities of events with outcomes that may or may not equally likely.

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are found by first finding the probabilities of each independent event. Then multiply the probabilities of the independent events. Multiplication of fractions can be used to find probabilities of compound events. Sample space represents all possible outcomes Models can be used to represent the sample space of a compound event in order to connect and build understanding of the probability calculations. Models of probability – area model, tree diagrams, organized lists, table	8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **CC.7.SP.8** a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **CC.7.SP.8a** b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. **CC.7.SP.8b** Used lists, tables, area models, and tree diagrams to represent sample spaces and finding the probabilities of compound events. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* **CC.7.SP.8c** Use models to represent the sample space and connect to the calculation of the probability.

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Standards for Mathematical Practice:

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grade 8

(KUD Organizer)

Mathematics

C. Lancour, D. Roscoe, J. Dick, and E. Shane

2/1/2012

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Course: Math – Grade 8
Topic: The Number System – Rational and Irrational Numbers (This is not a stand-alone unit, but topics that are embedded within other units.)

Which standards are in this learning progression?

8.NS.1, 8.NS.2

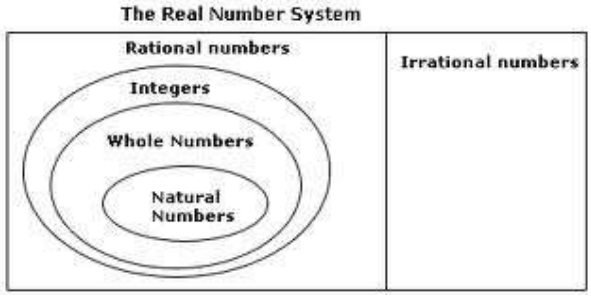
Connections to other domains and/or clusters:

8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4, 8.G.6, 8.G.7, 8.G.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

In the real number system, numbers can be defined by their decimal representations.

KNOW:	DO:
There are numbers that are not rational called "irrational". Irrational numbers are a subset of the Real Number System.  Every number has a decimal representation: • Irrational decimals are non-repeating and non-terminating • Rational number decimals eventually terminate or repeat. Irrational numbers can be approximated for comparing and ordering them.	**Know that there are numbers that are not rational, and approximate them by rational numbers.** 1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **CC.8.NS.1** 2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **CC.8.NS.2** This standard is connected to 8.G.6, 8.G.7, and 8.G.8 in which estimates of irrational numbers are used when developing a conceptual understanding of the Pythagorean Theorem and applying it to find distances on a coordinate plane.

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Course: Math – Grade 8
Topic: Expressions & Equations – Radicals & Integer Exponents

Which standards are in this learning progression?

8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4

Connections to other domains and/or clusters:

8.NS.1, 8.NS.2, 8.G.6, 8.G.7, 8.G.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

The properties of number systems and their relationships remain consistent when applied to integer exponents.

KNOW:	DO:
Properties of integer exponents. A perfect square is a number in which the square root is an integer. A perfect cube is a number in which the cube root is a whole number. Perfect squares and perfect cube numbers up to 100. The √2 is irrational. The base ten number system can be applied to represent very large and very small numbers using powers of 10. Flexibility with the equivalent forms of an expression allows for efficient problem solving. Strategies for computing with numbers expressed in scientific notation. Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions. Estimation as a means for predicting & assessing the reasonableness of a solution.	**Work with radicals and integer exponents.** 1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **CC.8.EE.1** Build understanding of the properties of integer exponents through analyzing number patterns and relationships. 2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. **CC.8.EE.2** This standard is connected to the Geometry standards in which the understanding of square roots is embedded in the development of the Pythagorean Theorem, and representations of cube numbers are embedded within applications of volume. 3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* **CC.8.EE.3** Use powers of 10 and knowledge of the base 10 number system to decompose and recompose numbers in a different representation for estimating and approximating mental computations. 4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **CC.8.EE.4**

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Course: Math – Grade 8
Topic: Expressions & Equations – Proportional Linear Relationships

Which standards are in this learning progression?

8.EE.5, 8.EE.6

Connections to other domains and/or clusters:

8.EE.7, 8.EE.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

The connection between linearity and proportionality (as a special case of linearity) is based on an understanding of slope as the constant rate of change and the y-intercept.

KNOW:	DO:
Proportional relationships can be represented symbolically (equation), graphically (coordinate plane), in a table, in diagrams, and verbal descriptions. The coordinates representing a proportional linear situation can be interpreted in terms of the context. In a proportional linear relationship, the point (0, 0) is the y-intercept and (1, *r*) is the slope, where *r* is the unit rate. Slope of a line is a constant rate of change. The y-intercept is the point at which a line intersects the vertical axis (y-axis). One form of an equation for a line is *y=mx* + b, where *m* is the slope and b is the y-intercept. A special case of linear equations (proportional relationships) are in the form of *y/x* = *m* and *y* = *mx*.	**Understand the connections between proportional relationships, lines, and linear equations.** 5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **CC.8.EE.5** Represent proportional linear relationships in tables, graphs, equations, and scenarios. Interpret the slope in terms of the context of the situation. Compare the different representations to determine which representation best highlights the information. Analyze how the slope and y-intercept appear in different representations. 6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*. **CC.8.EE.6** Interpret slope and y-intercept in terms of the context of the situation. Relate the similar "slope" triangles to the slope of the line.

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Course: Math – Grade 8
Topic: Expressions & Equations – Systems of Equations

Which standards are in this learning progression?

8.EE.7, 8.EE.8

Connections to other domains and/or clusters:

8.EE.5, 8.EE.6

By the end of this learning progression, students will be able to…

UNDERSTAND:

Linear equations, systems of equations, linear functions, and their understanding of slope of a line can be used to analyze situations and solve problems.

KNOW:	DO:
Inverse operations are used to solve equations. Linear equations in one variable can have one solution, infinitely many solutions, or no solution. Solutions to a system of equations are the values of the variables that make both equations true (one point of intersection). Systems of equations that have infinitely many solutions are equivalent forms of the same equation, and represent the same line in a plane (all points intersect because they are the same line). Systems of equations that have no solution are parallel lines having equivalent slopes (no point of intersection).	**Analyze and solve linear equations and pairs of simultaneous linear equations.** 7. Solve linear equations in one variable. **CC.8.EE.7** a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). **CC.8.EE.7a** Examples: 2x + 8= 14 (x=a) 2x + 8= ½(4x + 16) (a=a) 4x + 16= 4x + 28 (a=b) b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **CC.8.EE.8b** 8. Analyze and solve pairs of simultaneous linear equations. **CC.8.EE.8** a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **CC.8.EE.8a** b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* **CC.8.EE.8b** Elimination, substitution, and graphing incorporating technology. c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **CC.8.EE.8c**

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Course: Math – Grade 8
Topic: Functions – Linear & Nonlinear

Which standards are in this learning progression?

8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5

Connections to other domains and/or clusters:

8.EE.5 - 8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Functions can be classified into different families of functions (linear and nonlinear) that can be used to model different real-world phenomena.

KNOW:	DO:
Input/output tables can be used as a tool to generate a function rule. Functions can be represented algebraically, graphically, numerically in tables (ordered pairs), or by verbal descriptions. Changing the way a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations may be more useful than others and may highlight different characteristics. Some representations of functions may show only part of the function. Functions are used to model real-world phenomena.	Exploring patterns, generalizing patterns and developing informal notions of a variable as a quantity that changes leads to the informal study of functions as special relationships between variables. **Define, evaluate, and compare functions.** 1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] **CC.8.F.1** 2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **CC.8.F.2** 3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.* **CC.8.F.3** Distinguish between linear and nonlinear functions. **Use functions to model relationships between quantities.** 4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **CC.8.F.4** 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **CC.8.F.5**

[1] Function notation is not required in Grade 8.

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Course: Math – Grade 8
Topic: Geometry – Transformations & Angle Relationships

Which standards are in this learning progression?

8.G.1, 8.G.2, 8.G.3, 8.G.4, 8.G.5

Connections to other domains and/or clusters:

8.G.6 – 8, 8.EE.6

By the end of this learning progression, students will be able to…

UNDERSTAND:

Use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarities can describe and analyze two-dimensional figures and solve problems.

KNOW:	DO:
Reflections, rotations, and translations are rigid transformations and maintain congruence (do not change the size and shape of the object being transformed). Congruent figures have the same size and shape. Dilations may change the size of the object being transformed, but not the shape. Similar figures have the same shape; corresponding angle measures remain congruent, with a scale factor relating corresponding sides (corresponding sides are proportional). Scale factors larger than 1 enlarge a figure. Scale factors less than 1 shrink a figure. A scale factor of exactly 1 maintains congruence. Two triangles are similar if at least two pairs corresponding angles are congruent (AA postulate for similarity) Symbols for congruency and similarity (≅ and ~)	**Understand congruence and similarity using physical models, transparencies, or geometry software.** 1. Verify experimentally the properties of rotations, reflections, and translations: **CC.8.G.1** a. Lines are taken to lines, and line segments to line segments of the same length. **CC.8.G.1a** b. Angles are taken to angles of the same measure. **CC.8.G.1b** c. Parallel lines are taken to parallel lines. **CC.8.G.1c** Identify corresponding parts of figures. 2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **CC.8.G.2** Examine the congruence, similarity, and line or rotational symmetry of objects using transformations. 3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **CC.8.G.3** Recognize the relationship between the coordinates of the pre-image and image and the scale factor following a dilation from the origin. 4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **CC.8.G.4** Examine the congruence, similarity, and line or rotational symmetry of objects using transformations. 5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the*

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The sum of the angles of a triangle is 180°. The sum of interior angles of a polygon is related to the number of triangles that the polygon is composed of, and that relates to a generalization (formula).	*three angles appears to form a line, and give an argument in terms of transversals why this is so.* **CC.8.G.5** Construct various triangles and find measures on interior and exterior angles. Make conjectures about the relationship between the measure of an exterior angle and the opposite interior angles, and the sum of exterior angles. Use reasoning to find missing angle measures of various figures. Use parallel lines cut by a transversal to recognize angle pairs learned in grade 7 and build on this knowledge of angle relationships to identify and analyze additional pairs of congruent angles (Interior Angles, Exterior Angles, Alternate Interior Angles, Alternate Exterior Angles, and Vertical Angles).

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Course: Math – Grade 8
Topic: Geometry – Pythagorean Theorem

Which standards are in this learning progression?

8.G.6, 8.G.7, 8.G.8

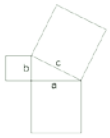
Connections to other domains and/or clusters:

8.G.1 – 5, 8.EE.2, 8.NS.1 - 2

By the end of this learning progression, students will be able to…

UNDERSTAND:

The Pythagorean Theorem, the special relationship between side lengths of a right triangle, can be used to find and describe unknown length.

KNOW:	DO:
The square root of the area of a square represents the side length of the square (Ex: A square with an area of 9 cm^2 has a side length of $\sqrt{9}$ = 3cm). There is a special relationship between the side lengths of a right triangle that states that the sums of the squares of the legs equal the square of the hypotenuse. This relationship is called the Pythagorean Theorem. Numbers that have two identical factors are called perfect squares. (Ex: 16 is a square number because 4 * 4 = 16), so the square root (as a side length) of these numbers are whole numbers ($\sqrt{16}$ = 4). Estimating can help to assess the reasonableness of a square root calculation (Ex: $\sqrt{15}$ is between 3 and 4, but closer to 4 because $\sqrt{9}$=3 and $\sqrt{16}$=4 and 15 is between 9 and 16, but closer to 16).	**Understand and apply the Pythagorean Theorem.** 6. Explain a proof of the Pythagorean Theorem and its converse. **CC.8.G.6** Relate the area of a square to its side length. Estimate the values of square roots of whole numbers (8.EE.2). Calculate the areas of the squares for the sides of a right triangle to prove the relationship between the three sides of a right triangle.  Distinguish right triangles from non-right triangles using the relationship among the side lengths (Pythagorean Theorem). 7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **CC.8.G.7** 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **CC.8.G.8** Students build on their work from 6th grade (finding vertical and horizontal distances on the coordinate plane 6.G.3 and 6.NS.8) to explore the Pythagorean Theorem and make conjectures about areas of squares and their side lengths. Determine the lengths of sides of right triangles in a coordinate plane.

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Learning Progression (KUD) Organizer

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Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
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Course: Math – Grade 8
Topic: Geometry – Volume

Which standards are in this learning progression?

8.G.9

Connections to other domains and/or clusters:

7.G.3-6

By the end of this learning progression, students will be able to…

UNDERSTAND:

The volume of any three dimensional shape is dependent on the area of its base, height of the shape and the number of parallel bases (layers) that the shape has.

KNOW:	DO:
Volume is measured in cubic units. There is a relationship between the volumes of cylinders, cones, and spheres. The volume of a cone is 1/3 of the volume of a cylinder (with same radius and height). The volume of a sphere is 2/3 the volume of a cylinder (with same radius and height).	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** 9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **CC.8.G.9** Students build on their knowledge of area of circles and volume in grade 7 (7.G.4 and 7.G.6) to extend their understanding of volume to include cones, cylinders, and spheres. Explore the relationship between the volumes of cones, cylinders, and spheres. Solve real-world problems that involve realistic applications for the volume of these figures.

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Course: Math – Grade 8
Topic: Statistics & Probability – Scatter Plots, Linear vs. Non-linear data associations & Linear Regression

Which standards are in this learning progression?

8.SP1, 8.SP.2, 8.SP.3, 8.SP.4

Connections to other domains and/or clusters:

8.F.1-5, 8.EE.5, 8.EE.6

By the end of this learning progression, students will be able to…

UNDERSTAND:

The same characteristics used to describe linear relationships allow us to describe, classify, and analyze the association of bivariate measurement data.

KNOW:	DO:
The characteristics of a linear relationship, such as: -Can be written in the form y=mx + b in which x is the independent variable, y is the dependent variable, m is the slope, and b is the y-intercept. -Appears to be a straight line in a xy coordinate graph. -When the constant rate is positive, the line will extend northeast and southwest. -When the constant rate is negative, the line will extend northwest and southeast. Strategies for modeling a line of best fit given a set of data. -Line drawn with the least total deviation from the actual data points. -Strategies include using a graphing calculator, strand of spaghetti, ruler, etc… Linear trends can be identified as positive or negative, while some trends have no correlation.	**Investigate patterns of association in bivariate data.** 1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **CC.8.SP.1** 2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **CC.8.SP.2** 3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **CC.8.SP.3** After informally making a line of best fit, students will make sense of slope and y-intercept.

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Identifying outliers and other data characteristics allows for meaningful data interpretation and analysis.

The line of best fit represents the data set as a whole, fitted through the majority of points.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **CC.8.SP.4**

Example:

Table Schemes for Comparing Frequencies and Row Proportions

(Favorite Music Type of 6th and 8th Graders)

	Rock	Rap	Country	Total
6th graders	a	b	c	d
8th graders	e	f	g	h

	Rock	Rap	Country	Total
6th graders	a/d	b/d	c/d	d
8th graders	e/h	f/h	g/h	h

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grades 9 – 12
Algebra Strand

(KUD Organizer)

Mathematics
D. Roscoe
4/26/2011

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Course: Math Grades 9-12
Topic: Algebra: Seeing Structure in Expressions
Team: H. Sutton, J. McCarthy, D. Roscoe, V. Maxwell, J. Riser, K. Clifton

Which standards are in this learning progression?

9-12.A.SSE.1a-b, 9-12.A.SSE.2, 9-12.A.SSE.3a-c, 9-12.A.SSE.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

KNOW:	DO:
Number properties that facilitate the writing of the equivalent forms	**9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.* **9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients. **9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.* **9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* **9-12.A.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines. **9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **9-12.A.SSE.3*c*** Use the properties of exponents to transform expressions for exponential functions*. For example the expression* 1.15^t *can be rewritten as* $(1.1^{51/12)12t} \approx 1.01^{212t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **9-12.A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.** Interpret and model a given context using expressions Decompose and recompose algebraic expressions using number properties in the context of solving problems Demonstrate flexible and strategic use of equivalent expressions to model and solve world problems.

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Course: Math Grades 9-12
Topic: Algebra – Arithmetic with Polynomials and Rational Expressions
Team: High School Team

Which standards are in this learning progression?

9-12.A.APR1, 9-12.A.APR2, 9-12.A.APR3, 9-12.A.APR4, 9-12.A.APR6

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand:

How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation.

KNOW:	DO:
Know that polynomials form a system analogous to the integers, namely, they are closed under the operation of addition, subtraction, and multiplication. Know that polynomials can be decomposed and recomposed.	**9-12.A.APR.1** Add, subtract, and multiply polynomials. **9-12.A.APR.2** Apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x-a is p(a), so p(a) = 0 if and only if (x-a) is a factor of p(x). **9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are defined by the polynomial. **9-12.A.APR.4** Prove polynomial identities and use them to describe numerical relationships. **9-12.A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

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Course: Math Grades 9-12
Topic: Algebra: Creating Equations
Team: High School Team

Which standards are in this learning progression?

9-12.A.CED.1, 9-12.A.CED.2, 9-12.A.CED.3, 9-12.A.CED.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function.

Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems.

KNOW:	DO:
The relationship of two or more variables can be represented as an equation or inequality The relationship of two or more variables can be represented graphically Knows how to interpret solutions in an equation or inequality- is the solution viable or non-viable in the context of a problem Know what a constraint is and how to recognize it in the context of a problem Know what number properties can be applied to symbols in order to rearrange formulas.	**9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. **9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities: graph equations on coordinate axes with labels and scales **9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems and/or inequalities **9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

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Course: Math Grades 9-12
Topic: Algebra – Reasoning with Equations and Inequalities
Team: High School Team

Which standards are in this learning progression?

9-12.A.REI.1, 9-12.A.REI.2, 9-12.A.REI.3, 9-12.A.REI.4a-b, 9-12.A.REI.5, 9-12.A.REI.6 9-12.A.REI.7 9-12.A.REI.10, 9-12.A.REI.11, 9-12.A.REI.12

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.

KNOW:	DO:
Know algebraic properties and inverse operations used to justify the steps in solving equations. Know the conditions under which to use factoring, completing the square, and the quadratic formula to solve problems. **9-12.A.REI.10** Knows that a graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Know that problems can be solved graphically and/or symbolically.	**9-12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **9-12.A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **9-12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **9-12.A.REI.4** Solve quadratic equations in one variable. a) Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form (*x*-*p*)2 = *q* that has the same solutions Derive the quadratic formula from this form. b) Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a+- bi for real numbers a and b. **9-12.A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **9-12.A.REI.6** Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. **9-12.A.REI.7** Solve a simple system consisting of a linear equation and quadratic equation in two variables algebraically and graphically. **9-12.A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations

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	$y=f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) =g(x)$; find the solutions approximately. **9-12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

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Learning Progressions

CCSS: Grades 9 – 12
Functions Strand

(KUD Organizer)

Mathematics

D. Roscoe

4/26/2011

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Course: Math Grades 9-12
Topic: Functions - Interpreting Functions Domain
Team: High School Team

Which standards are in this learning progression?

9-12.F.IF.1, 9-12.F.IF.2, 9-12.F.IF.3, 9-12.F.IF.4, 9-12.F.IF.5, 9-12.F.IF.6, 9-12.F.IF.7a-c,e, 9-12.F.IF.8, 9-12.F.IF.8a-b, 9-12.F.IF.9

By the end of this learning progression, students will be able to…

UNDERSTAND:

Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.

KNOW:	DO:
9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). Recognizing characteristics of graphs, tables, and equations that model families of functions. Strategies for interpreting key features of representations.	**9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **9-12.F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* **9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** **9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **9-12.F.IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima. **9-12.F.IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.9-12.F.IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

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	9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **9-12.F.IF.8b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

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Course: Math Grades 9-12
Topic: Functions - Building Functions Domain
Team: High School Team

Which standards are in this learning progression?

9-12.F.BF.4a, 9-12.F.BF.1a, 9-12.F.BF.1b, 9-12.F.BF.2, 9-12.F.BF.3, 9-12.F.BF.4, 9-12.F.BF.1

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

KNOW:	DO:
Know the difference between a recursive rule and an explicit expression for a function. How a recursive rule can be used to generate an explicit expression. Knows that manipulating the parameters of the symbolic rule will result in a predictable transformation of the graph. Know a number of strategies for finding and verifying whether an inverse of a function is itself a function (strategies using tables, graphs or symbolic transformations). Know that Logarithmic Functions are inverse Exponential Functions.	**9-12.F.BF.1** Write a function that describes a relationship between two quantities.* **9-12.F.BF.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context. **9-12.F.BF.1b** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **9-12.F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* **9-12.F.BF.3** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f*(*x*), *f*(*kx*), and *f*(*x + k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **9-12.F.BF.4** Find inverse functions. **9-12.F.BF.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for x≠1.*

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Recursive, Explicit, Inverse Functions, Arithmetic, Geometric, Sequence, Exponential Function, Linear Function**

Standards for Mathematical Practice:

1. Make sense of problems & persevere in solving them.
2. Reason abstractly & quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Course: Math Grades 9-12
Topic: Functions - Linear, Quadratic, and Exponential Models Domain
Team: High School Team

Which standards are in this learning progression?

9-12.F.LE.1, 9-12.F.LE.1a-c, 9-12.F.LE.2, 9-12.F.LE.3, 9-12.F.LE.4, 9-12.F.LE.5

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate.

KNOW:	DO:
Characteristic graph, table, and equation formats for linear, exponential, and quadratic functions. Linear functions with a constant term of zero describe proportional relationships.	**9-12.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **9-12.F.LE.1a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **9-12.F.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **9-12.F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **9-12.F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic ally, or (more generally) as a polynomial function. **9-12.F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **9-12.F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.

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Course: Math Grades 9-12
Topic: Functions - Trigonometric Functions Domain
Team: High School Team

Which standards are in this learning progression?

9-12.F.TF.1, 9-12.F.TF.2, 9-12.F.TF.5, 9-12.F.TF.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand:

The connection between extending the domain of trigonometric functions using the unit circle and graphing trigonometric functions in the Cartesian coordinate system to model periodic phenomena across the extended domain.

That the graph of a function is a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties (amplitude, frequency, and midline).

KNOW:	DO:
9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	**9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* **9-12.F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

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Standards for Mathematical Practice:

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grades 9 – 12
Geometry Strand

(KUD Organizer)

Mathematics

D. Roscoe

4/26/2011

Learning Progression (KUD) Organizer

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Course: Math Grades 9-12
Topic: Geometry: Congruence
Team: High School Team

Which standards are in this learning progression?

9-12.G.CO.1, 9-12.G.CO.2, 9-12.G.CO.3, 9-12.G.CO.4, 9-12.G.CO.5, 9-12.G.CO.6, 9-12.G.CO.7, 9-12.G.CO.8, 9-12.G.CO.9, 9-12.G.CO.10, 9-12.G.CO.11, 9-12.G.CO.12, 9-12.G.CO.13

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students understand that:

The concept of congruence and symmetry can be understood from the perspective of geometric transformation.

Once the triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures

Construction is another way to visualize and create a strategic pathway to proof.

KNOW:	DO:
9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. Know that congruence will be maintained when a shape is rotated, reflected and translated. Know how to use both verbal and symbolic language to develop arguments related to location, transformation and congruence. Know what it means to prove or disprove a conjecture. Make sense of and know why point, line, distance along a line and distance around a circular arc are undefined.	**9-12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **9-12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **9-12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **9-12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **9-12.G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **9-12.G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **9-12.G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **9-12.G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

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	9-12.G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **9-12.G.CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **9-12.G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

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Standards for Mathematical Practice:

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Course: Math Grades 9-12
Topic: Geometry: Similarity, Right Triangles, and Trigonometry
Team: High School Team

Which standards are in this learning progression?

9-12.G.SRT.1a-b, 9-12.G.SRT.2, 9-12.G.SRT.3, 9-12.G.SRT.4, 9-12.G.SRT.5, 9-12.G.SRT.6, 9-12.G.SRT.7, 9-12.G.SRT.8

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students understand that:

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.

KNOW:	DO:
9-12.G.SRT.1a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **9-12.G.SRT.1b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. Know the trigonometric ratios, Sine, Cosine, and Tangent.	**9-12.G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor: **9-12.G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **9-12.G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **9-12.G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **9-12.G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **9-12.G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **9-12.G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **9-12.G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

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Course: Math Grades 9-12
Topic: Geometry - Circles
Team: High School Team

Which standards are in this learning progression?

9-12.G.C.1, 9-12.G.C.2, 9-12.G.C.3, 9-12.G.C.5

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Properties of Circles can be described by theorems that integrate algebraic and geometric understanding, modeling, and proof.

Properties of Circles can be used to derive an understanding of the radian measure of an angle.

KNOW:	DO:
Know that all circles are similar. Know strategies for geometric constructions. Know how the application of proportional reasoning is used to develop the concept of radian measure.	**9-12.G.C.1** Prove that all circles are similar. **9-12.G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **9-12.G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **9-12.G.C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

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Course: Math Grades 9-12
Topic: Geometry: Expressing Geometric Properties with Equations
Team: High School Team

Which standards are in this learning progression?

9-12.G.GPE.1, 9-12.G.GPE.2, 9-12.G.GPE.4, 9-12.G.GPE.5, 9-12.G.GPE.6, 9-12.G.GPE.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions.

This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa

Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

KNOW:	DO:
Know the equation of a circle. Know that the distance formula is an application of the Pythagorean Theorem. Know strategies for using the coordinate plane to justify geometric relationships.	**9-12.G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **9-12.G.GPE.2** Derive the equation of a parabola given a focus and directrix. **9-12.G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **9-12.G.GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **9-12.G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **9-12.G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

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Course: Math Grades 9-12
Topic: Geometry: Geometric Measurement and Dimension
Team: High School Team

Which standards are in this learning progression?

9-12.G.GMD.1, 9-12.G.GMD.3, 9-12.G.GMD.4

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:
Perimeter, Area, and Volume of Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.
Geometric transformations of shape (composing, decomposing or slicing) correspond to algebraic changes in their equations.

KNOW:	DO:
Know strategies for dissection and partitioning that support the visualizations necessary to build informal arguments.	**9-12.G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **9-12.G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* **9-12.G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Cross-section, Cylinder, Pyramid, Cone, Circumference, Area**

Standards for Mathematical Practice:

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Course: Math Grades 9-12
Topic: Geometry: Modeling with Geometry
Team: High School Team

Which standards are in this learning progression?

9-12.G.MG.1, 9-12.G.MG.2, 9-12.G.MG.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Real-world situations are not organized and labeled for analysis; formulating flexible geometric models, representing such models, and analyzing them is a creative process.

The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them.

KNOW:	DO:
Know that modeling is the process of choosing and using appropriate mathematics to analyze and understand geometric situations. Know that when making mathematical models, technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.	**9-12.G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* **9-12.G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* **9-12.G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: Geometric Modeling, Approximation, Density, Design Problem

Standards for Mathematical Practice:

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Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grades 9 – 12
Number & Quantity Strand

(KUD Organizer)

Mathematics
D. Roscoe
4/26/2011

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Course: Math Grades 9-12
Topic: Number and Quantity: The Real Number System Domain
Team: High School Team

Which standards are in this learning progression?

9-12.N.RN.1, 9-12.N.RN.2, 9-12.N.RN.3

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

How properties of rational exponents, rational number, and irrational number are defined using characteristic patterns of equivalency and computation, to build a comprehensive knowledge of the structure and order of the real number system.

KNOW:	DO:
Know the patterns of equivalency and computation that determine the laws of rational exponents. Know equivalent expressions for real numbers to include radicals and numbers in exponential form. *Number Systems graphic organizer (know and/or create it).	**9-12.N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. Compare and order numbers in the Real Number System by size and/or position on a number line (to include demonstrating an ability to identify equivalent terms). **9-12.N.RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are: **Equivalency, Real Number System, Rational Numbers, Irrational Numbers, Integers, Radical.**

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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Learning Progression (KUD) Organizer

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Course: Math Grades 9-12
Topic: Number and Quantity: Quantities Domain
Team: High School Team

Which standards are in this learning progression?

9-12.N.Q.1, 9-12.N.Q.2, 9-12.N.Q.3

By the end of this learning progression, students will be able to…

UNDERSTAND:	
Students will understand: How unit and scale can be used as a tool to effectively model context and solve problems.	
KNOW:	**DO:**
Strategies that can be used to identify an appropriate scale in the context of the representation, model, or problem. Standard unit conversions such as 5280ft = 1mile, 3ft = 1yd, 12in = 1ft, metric place value fluency.	**9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. **9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
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Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

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Course: Math
Topic: Number and Quantity- Complex Numbers
Team: High School Team

Which standards are in this learning progression?

9-12.N.CN.1; 9-12.N.CN.2; 9-12.N.CN.7

By the end of this learning progression, students will be able to…

UNDERSTAND:	
Students will understand: How knowledge of number properties in the Real Number System can be use to develop and apply properties of the Complex Number System.	
KNOW:	**DO:**
Recognize the form of a complex number.	**Perform arithmetic operations with complex numbers.** **9-12.N.CN.1** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
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Standards for Mathematical Practice: 1. Make sense of problems & persevere in solving them. 2. Reason abstractly & quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

Delaware Department of Education

Teaching and Learning Branch

Learning Progressions

CCSS: Grades 9 – 12
Statistics & Probability Strand

(KUD Organizer)

Mathematics
D. Roscoe

4/26/2011

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Course: Math Grades 9-12
Topic: Statistics and Probability: Interpreting Categorical and Quantitative Data
Team: High School Team

Which standards are in this learning progression?

9-12.S.ID.1, 9-12.S.ID.2, 9-12.S.ID.3, 9-12.S.ID.4, 9-12.S.ID.5, 9-12.S.ID.6, 9-12.S.ID.6a, 9-12.S.ID.6b, 9-12.S.ID.6c, 9-12.S.ID.7, 9-12.S.ID.8, 9-12.S.ID.9

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns.

Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data.

Causation implies correlation yet correlation does not imply causation.

KNOW:	DO:
Know that Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. Know that the shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Know strategies for fitting a function to a data display and informally assessing the fit. Know what the slope and intercept of the linear model mean within the context of the data.	**9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots). **9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **9-12.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **9-12.S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **9-12.S.ID.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* **9-12.S.ID.6b** Informally assess the fit of a function by plotting and analyzing residuals. **9-12.S.ID.6c** Fit a linear function for a scatter plot that suggests a linear association. **9-12.S.ID.7** Interpret the slope (rate of change) and the intercept (constant term)

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	of a linear model in the context of the data. **9-12.S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit. **9-12.S.ID.9** Distinguish between correlation and causation

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Course: Math Grades 9-12
Topic: Statistics and Probability: Making Inferences and Justifying Conclusions
Team: High School Team

Which standards are in this learning progression?

9-12.S.IC.1, 9-12.S.IC.2, 9-12.S.IC.3, 9-12.S.IC.4, 9-12.S.IC.5, 9-12.S.IC.6

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account.

Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.

KNOW:	DO:
9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies.	**9-12.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **9-12.S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **9-12.S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; ***explain how randomization relates to each.*** **9-12.S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **9-12.S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **9-12.S.IC.6** Evaluate reports based on data.

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Course: Math Grades 9-12
Topic: Statistics and Probability: Conditional Probability and the Rules of Probability
Team: High School Team

Which standards are in this learning progression?

9-12.S.CP.1, 9-12.S.CP.2, 9-12.S.CP.3, 9-12.S.CP.4, 9-12.S.CP.5, 9-12.S.CP.6, 9-12.S.CP.7

By the end of this learning progression, students will be able to…

UNDERSTAND:

Students will understand that:

In a probability model, sample points represent outcomes and combine to make up events.

The probabilities of the events can be computed by applying the Addition and Multiplication Rules.

Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

KNOW:	DO:
9-12.S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities	**9-12.S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
9-12.S.CP.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	**9-12.S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, ***and use this characterization to determine if they are independent.***
Construct a two-way table as a sample space.	**9-12.S.CP.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
Recognize the concepts of conditional probability and independence in everyday language and everyday situations.	**9-12.S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
Know the characteristics of any context used to determine when it is appropriate to apply the Addition and Multiplication Rules.	**9-12.S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
	9-12.S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.
	9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.

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