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# Delaware's Common Core State Standards for Mathematics Grade 7 Assessment Examples



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## Delaware's Common Core State Standards for 7<sup>th</sup> Grade Mathematics

### Overview

#### **Ratios and Proportional Relationships (RP)**

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

#### **The Number System (NS)**

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

#### **Expressions and Equations (EE)**

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

#### **Geometry (G)**

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

#### **Statistics and Probability (SP)**

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

#### **Mathematical Practices (MP)**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## ***Grade 7 Mathematics – Unpacking the Delaware Common Core State Standards***

This document is designed to help understand the Common Core State Standards (CCSS) in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

### ***What Is the Purpose of This Document?***

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

### ***References***

This document contains explanations and examples that were obtained from State Departments of Education for Utah, Arizona, North Carolina, and Ohio with their permission.

### ***How Do I Send Feedback?***

This document is helpful in understanding the CCSS but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education via [rfry@doe.k12.de.us](mailto:rfry@doe.k12.de.us), and we will use your input to refine this document.

## Ratios and Proportional Relationships (RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard
<p><b>7.RP.1</b> – Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks <math>\frac{1}{2}</math> mile in each <math>\frac{1}{4}</math> hour, compute the unit rate as the complex fraction <math>\frac{1}{2} / \frac{1}{4}</math> miles per hour, equivalently 2 miles per hour.</p> <p><i>Explanation:</i> Students continue to work with unit rates; however, the comparison now includes fractions compared to fractions. For example, if <math>\frac{1}{2}</math> gallon of paint covers <math>\frac{1}{6}</math> of a wall, then the amount of paint needed for the entire wall can be computed by <math>\frac{1}{2} \text{ gallon} \div \frac{1}{6} \text{ wall}</math>. This calculation gives 3 gallons. This standard requires only the use of ratios as fractions. Fractions may be proper or improper.</p> <p><i>Example:</i></p> <ul style="list-style-type: none"> <li>• Travis was attempting to make muffins to take to a neighbor that had just moved in down the street. The recipe that he was working with required <math>\frac{3}{4}</math> cup of sugar and <math>\frac{1}{8}</math> cup of butter.             <ol style="list-style-type: none"> <li>a. Travis accidentally put a whole cup of butter in the mix.                 <ul style="list-style-type: none"> <li>♦ What is the ratio of sugar to butter in the original recipe? What amount of sugar does Travis need to put into the mix to have the same ratio of sugar to butter that the original recipe calls for?</li> <li>♦ If Travis wants to keep the ratios the same as they are in the original recipe, how will the amounts of all the other ingredients for this new mixture compare to the amounts for a single batch of muffins?</li> <li>♦ The original recipe called for <math>\frac{3}{8}</math> cup of blueberries. What is the ratio of blueberries to butter in the recipe? How many cups of blueberries are needed in the new enlarged mixture?</li> </ul> </li> <li>b. This got Travis wondering how he could remedy similar mistakes if he were to dump in a single cup of some of the other ingredients. Assume he wants to keep the ratios the same.                 <ul style="list-style-type: none"> <li>♦ How many cups of sugar are needed if a single cup of blueberries is used in the mix?</li> <li>♦ How many cups of butter are needed if a single cup of sugar is used in the mix?</li> <li>♦ How many cups of blueberries are needed for each cup of sugar?</li> </ul> </li> </ol> </li> </ul>

**Standard**

**7.RP.2** – Recognize and represent proportional relationships between quantities.

- a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
- d. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

*Resources:* Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin  $(0, 0)$  with a constant of proportionality equal to the slope of the line. Proportional relationships are further developed through the analysis of graphs, tables, equations, and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems.

*Explanation:* Students determine if two quantities are in a proportional relationship from a table. For example, the table below gives the price for different number of books. Do the numbers in the table represent a proportional relationship?

*Examples are continued on next page.*

7.RP.2 Examples:

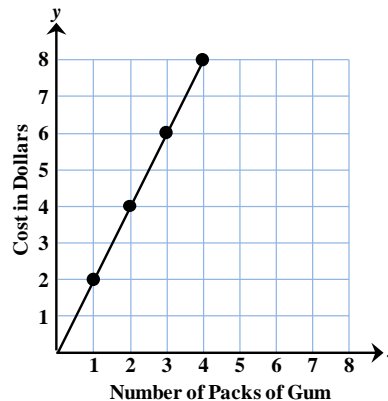
- The graph below represents the cost of books. When graphed, there is not a constant of proportionality.

Number of Books	Price
1	3
3	9
4	12
7	18

Students graph relationships to determine if two quantities are in a proportional relationship and interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books cost 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost \$12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs (price per item).

- The graph below represents the cost of gum packs as a unit rate of \$2 dollars for every pack of gum. The unit rate is represented as \$2/pack. Represent the relationship using a table and an equation.

Number of Packs of Gum ( $g$ )	Cost in Dollars ( $d$ )
0	0
1	2
2	4
3	6
4	8



- Equation:  $d = 2g$ , where  $d$  is the cost in dollars and  $g$  is the packs of gum.

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using  $x$  and  $y$ . Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars.” They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost ( $g \times 2 = d$ ).

**7.RP.3** – Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

*Explanations:*

- Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication. Students understand the mathematical foundation for cross-multiplication. A recipe calls for  $\frac{3}{4}$  teaspoon of butter for every 2 cups of milk. If you increase the recipe to use 3 cups of milk, how many teaspoons of butter are needed? Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

$$\frac{\frac{3}{4}}{2} = \frac{x}{3} \qquad \frac{3}{4} \times 3 = 2x$$

- Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related.

*Examples:*

- Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs \$4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say: “The original cost of a gallon of gas is \$4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of \$4.17 is about \$1.04, the projected cost of a gallon of gas should be around \$9.40.”

$$4.17 + 4.17 + (0.24 \times 4.17) = 2.24 \times 4.17$$

100%	100%	24%
\$4.17	\$4.17	?

- A sweater is marked down 33%. Its original price was \$37.50. What is the price of the sweater before sales tax?

\$37.50 Original Price of Sweater	
33% of \$37.50	66% of \$37.50 Sale Price of Sweater

The discount is 33% times \$37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or *Sale Price* =  $0.67 \times$  *Original Price*.

- A shirt is on sale for 40% off. The sale price is \$12. What was the original price? What was the amount of the discount?

Original Price	
Discount 40% of Original Price	Sale Price – \$12 60% of Original Price



## 7.RP.3 Examples (continued)

- Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error.

$$\% \text{ error} = \frac{|\text{your result} - \text{accepted value}|}{\text{accepted value}} \times 100\%$$

For example, you need to purchase a countertop for your kitchen. You measured the countertop as 5 feet. The actual measurement is 4.5 feet. What is the percent error?

$$\% \text{ error} = \frac{|5 \text{ feet} - 4.5 \text{ feet}|}{4.5} \times 100$$

$$\% \text{ error} = \frac{0.5 \text{ feet}}{4.5} \times 100$$

- A salesperson set a goal to earn \$2,000 in May. He receives a base salary of \$500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal?
- After eating at a restaurant, your bill before tax is \$52.60. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pretax amount. How much is the tip you leave for the waiter? How much will the total bill be including tax and tip? Express your solution as a multiple of the bill:  $\text{amount paid} = 0.20 \times \$52.50 + 0.08 \times \$52.50 = 0.28 \times \$52.50$ .

## The Number System (NS)

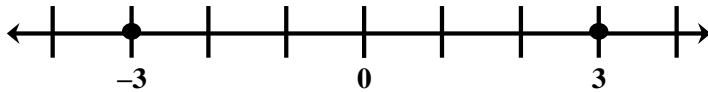
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Standard
<p><b>7.NS.1</b> – Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p> <p>b. Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p> <p><i>Explanation:</i> This cluster builds upon the understandings of rational numbers in grade 6:</p> <ul style="list-style-type: none"><li>• Quantities can be shown using + or – as having opposite directions or values</li><li>• Points on a number line show distance and direction</li><li>• Opposite signs of numbers indicate locations on opposite sides of 0 on the number line</li><li>• The opposite of an opposite is the number itself</li><li>• The absolute value of a rational number is its distance from 0 on the number line</li><li>• The absolute value is the magnitude for a positive or negative quantity</li><li>• Locating and comparing locations on a coordinate grid by using negative and positive numbers</li></ul> <p>Students should explore what happens when negatives and positives are combined. Number lines represent a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as physical and kinesthetic model for adding and subtracting integers. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Visual representations may be helpful as students begin this work—they become less necessary as students become more fluent with the operations.</p>

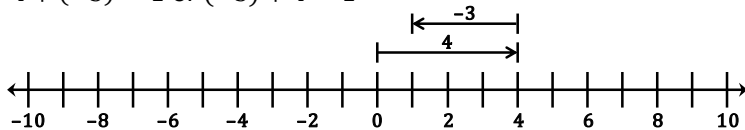
Standard

7.NS.1 Examples

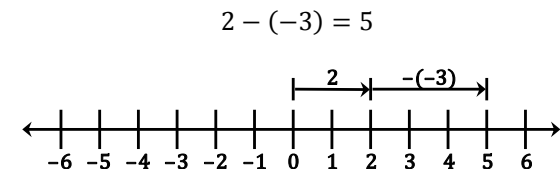
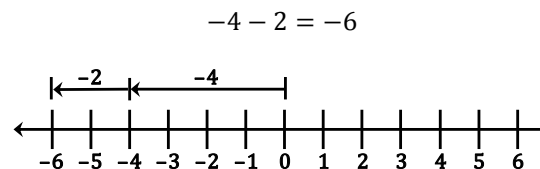
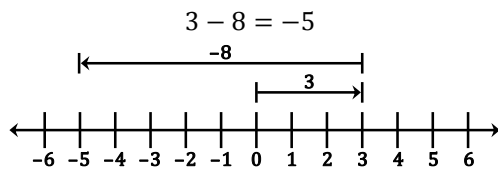
- On a number line,  $-3$  and  $3$  are shown to be opposites because they are equal distance from  $0$  and therefore have the same absolute value and the sum of the number and its opposite is  $0$ .



- Adding on a number line: you have \$4 and you need to pay a friend \$3. What will you have after paying your friend?  
 $4 + (-3) = 1$  or  $(-3) + 4 = 1$



- Subtracting on number line:



- Rules of adding and subtracting integers:

Adding Integers – Mental Rules	
Same Signs	Different Signs
<ul style="list-style-type: none"> <li>Add numbers</li> <li>Carry signs</li> </ul>	<ul style="list-style-type: none"> <li>Subtract numbers</li> <li>Carry sign of larger number</li> </ul>

Subtracting Integers – Mental Rules
$a - b = a + (-b)$
<p>add the opposite      opposite of</p>

**Standard**

**7.NS.2** – Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers, then  $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world contexts.
- c. Apply properties of operations as strategies to multiply and divide rational numbers.
- d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

*Explanation:* Multiplication and division of integers is an extension of multiplication and division of whole numbers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

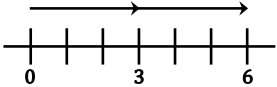

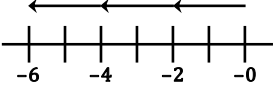
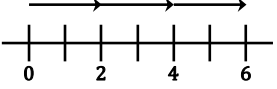
*Examples:*

- Using the language of “the opposite of” helps some students understand the multiplication of negatively signed numbers ( $-4 \times -4 = 16$ , the opposite of 4 groups of  $-4$ ).
- Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

## Standard

## 7.NS.2 Examples (continued)

- Examine the family of equations. What patterns do you see? Create a model and context for each of the products. Write and model the family of equations related to  $3 \times 4 = 12$ .

Equation	Number Line Model	Context
$2 \times 3 = 6$		<ul style="list-style-type: none"> <li>Selling two packages of apples at \$3 per pack</li> </ul>
$2 \times -3 = -6$		<ul style="list-style-type: none"> <li>Spending \$3 each on 2 packages of apples</li> </ul>
$-2 \times 3 = -6$		<ul style="list-style-type: none"> <li>Owing \$2 to each of your 3 friends</li> </ul>
$-2 \times -3 = 6$		<ul style="list-style-type: none"> <li>Forgiving 3 debts of \$2 each</li> </ul>

+	•	+	=	+
-	•	-	=	+
+	•	-	=	-
-	•	+	=	-

- Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language  $[-(p/q) = (-p)/q = p/(-q)]$  is written for the teacher's information and not as an expectation for students.)

+	÷	+	=	+
-	÷	-	=	+
+	÷	-	=	-
-	÷	+	=	-

- Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.
- Using long division, students understand the difference between terminating and repeating decimals. This understanding is foundational for work with rational and irrational numbers in 8<sup>th</sup> grade. For example, divide fractions to make decimals and identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5).

**Standard**

**7.NS.3** – Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

*Explanation:* Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals. Also include order of operations in the problems.

*Examples:*

- Your cell phone bill is automatically deducting \$32 from your bank account every month. How much will the deductions total for the year?


$$-32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12(-32)$$

- It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?

$$\frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5 \text{ feet per second}$$

## Expressions and Equations (EE)

Use properties of operations to generate equivalent expressions.

Standard
<p><b>7.EE.1</b> – Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p><i>Explanations:</i></p> <ul style="list-style-type: none"> <li>This is a continuation of work on properties and combining like terms. Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions.</li> <li>Properties – Additive Inverse, Multiplicative Identity, Multiplicative Inverse, Associative Property, Commutative Property, Additive Identity, Distributive.</li> <li>Students apply properties of operations and work with rational numbers (integers and positive/negative fractions and decimals) to write equivalent expressions.</li> <li>Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.</li> </ul> <p><i>Examples:</i></p> <ul style="list-style-type: none"> <li>Write an equivalent expression for <math>3(x + 5) - 2</math>.</li> <li>Suzanne thinks the two expressions <math>2(3a - 2) + 4a</math> and <math>10a - 2</math> are equivalent. Is she correct? Explain why or why not.</li> <li>Write equivalent expressions for <math>3a + 12</math>. Possible solutions might include factoring as in <math>3(a + 4)</math> or other expressions such as <math>a + 2a + 7 + 5</math>.</li> <li>A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be <math>w + w + 2w + 2w</math>. Write the expression in two other ways. <ul style="list-style-type: none"> <li>Solution: <math>6w</math> or <math>2(w) + 2(2w)</math>.</li> </ul> <div style="text-align: center;">  </div> </li> <li>An equilateral triangle has a perimeter of <math>6x + 15</math>. What is the length of each of the sides of the triangle? <ul style="list-style-type: none"> <li>Solution: <math>3(2x + 5)</math>, therefore each side is <math>2x + 5</math> units long.</li> </ul> </li> </ul>

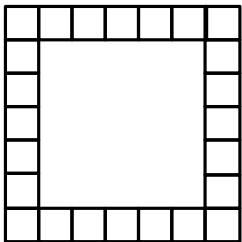
**Standard**

**7.EE.2** – Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example,  $a + 0.05a = 1.05a$  means that "increase by 5%" is the same as "multiply by 1.05."

*Explanation:* Students understand the reason for rewriting an expression in terms of a contextual situation. For example, students understand that a 20% discount is the same as finding 80% of the cost ( $0.80c$ ).

*Examples:*

- Jamie and Ted both get paid an equal hourly wage of \$9 per hour. This week, Ted made an additional \$27 dollars in overtime. Write an expression that represents the weekly wages of both if  $J$  = the number of hours that Jamie worked this week and  $T$  = the number of hours Ted worked this week? Can you write the expression in another way?
  - Students may create several different expressions depending upon how they group the quantities in the problem.
  - One student might say: To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then, I would multiply the number of hours Ted worked by 9. I would add these two values with the \$27 overtime to find the total wages for the week. The student would write the expression:  $(9J) + (9T + 27)$ , and then:  $9J + 9T + 27$ .
  - Another student might say: To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression  $9(J + T) + 27$ .
  - A third student might say: To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the \$27 he earned in overtime. My final step would be to add Jamie's and Ted's wages for the week to find their combined total wages. The student would write the expression  $(9J) + (9T + 27)$ .
- Given a square pool as shown in the picture below, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking.





**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.****Standard**

**7.EE.3** – Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional  $\frac{1}{10}$  of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

*Explanations:*

- Students solve contextual problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers.
- To assist students' assessment of the reasonableness of answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. Connections between performing the inverse operation and undoing the operations are appropriate here. It is appropriate to expect students to show the steps in their work. Students should be able to explain their thinking using the correct terminology for the properties and operations.
- Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally, an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs \$12.50 that includes a tax of \$0.75. She only wants to leave a tip based on the cost of the food.* In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost.  $C = (12.50 - 0.75)(1 + T) + 0.75 = 11.75(1 + T) + 0.75$  where  $C$  = cost and  $T$  = tip.
- Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include but are not limited to:
  - Front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts);
  - Clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate);
  - Rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values);
  - Using friendly or compatible numbers such as factors (students seek to fit numbers together—i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000); and
  - Using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

**Standard**

7.EE.3 Example:

- The youth group is going on a trip to the state fair. The trip costs \$52. Included in that price is \$11 for a concert ticket and the cost of 2 passes, one for the rides, and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

$x$	$x$	11
52		

$$2x + \$11.00 = \$52.00$$

$$2x = \$41.00$$

$$x = \$20.50$$

**Standard**

**7.EE.4** – Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- a. Solve word problems leading to equations of the form  $px+q=r$  and  $p(x+q)=r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
- b. Solve word problems leading to inequalities of the form  $px+q > r$  or  $px+q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make and describe the solutions.*

**Explanations:**

- Students solve multistep (at least 2-step) equations and inequalities derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution.
- Students solve and graph 2-step inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context.
- Provide multiple opportunities for students to work with multistep problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

**Examples:**

- Amie had \$26 dollars to spend on school supplies. After buying 10 pens, she had \$14.30 left. How much did each pen cost?
- The sum of three consecutive even numbers is 48. What is the smallest of these numbers?
- Solve:  $\frac{5}{4}n + 5 = 20$
- Florencia has at most \$60 to spend on clothes. She wants to buy a pair of jeans for \$22 dollars and spend the rest on t-shirts. Each T-shirt costs \$8. Write an inequality for the number of t-shirts she can purchase.
- Steven has \$25 dollars. He spent \$10.81, including tax, to buy a new DVD. He needs to set aside \$10 to pay for his lunch next week. If peanuts cost \$0.38 per package, including tax, what is the maximum number of packages that Steven can buy?
- Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution.
- Solve  $\frac{1}{2}x + 3 > 2$  and graph your solution on a number line.

## Geometry (G)

**Draw, construct, and describe geometrical figures and describe the relationships between them.**

### Standard

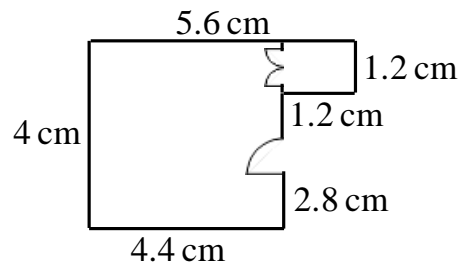
**7.G.1** – Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

*Resources:*

- Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.
- Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations.
- Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.
- Students should move on to drawing scaled figures on grid paper with proper figure labels, scale, and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter? What is the new area? Or, if the scale is 6, what will the new side length look like? Or, suppose the area of one triangle is 16 sq units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?
- Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.

*Example:*

- Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



**Standard**

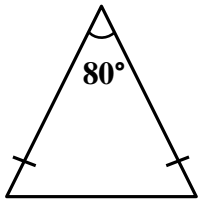
**7.G.2** – Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

*Explanations:*

- Students understand the characteristics of angles that create triangles. For example, can a triangle have more than one obtuse angle? Will three sides of any length create a triangle? Students recognize that the sum of the two smaller sides must be larger than the third side.
- Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles. Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle, or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.
- Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity.

*Examples:*

- Is it possible to draw a triangle with a  $90^\circ$  angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?
- Draw a triangle with angles that are  $60^\circ$ . Is this a unique triangle? Why or why not?
- Draw an isosceles triangle with only one  $80^\circ$  angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?



- Can you draw a triangle with sides that are 13 cm, 5 cm, and 6 cm?
- Draw a quadrilateral with one set of parallel sides and no right angles.

**Standard**

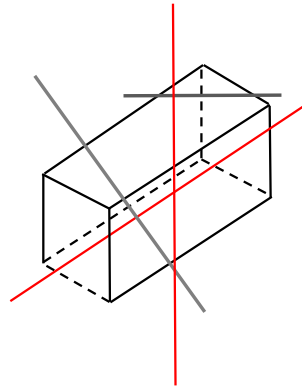
**7.G.3** – Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

*Explanations:*

- Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram; cuts made at an angle through the right rectangular pyramid will also produce a parallelogram.
- Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube,” or “what three-dimensional figure can produce a hexagon slice”? This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices, and resulting two-dimensional figures.

*Example:*

- Using a clay model of a rectangular prism, describe the shapes that are created when planar cuts are made diagonally, perpendicularly, and parallel to the base.



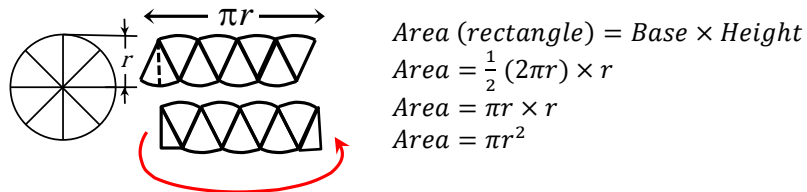
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Standard
<p><b>7.G.4</b> – Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p> <p><i>Explanations:</i></p> <ul style="list-style-type: none"><li>• Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as pi. Building on these understandings, students generate the formulas for circumference and area.</li><li>• This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi, and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters, leads to the realization that a diameter is two times a radius. Given multiple-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi and ultimately deriving a formula for circumference. String or yarn laid over the circle and compared to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.</li><li>• “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of <b>why</b> the formula works and how the formula relates to the measure (area and circumference) and the figure. This understanding should be for <i>all</i> students.</li></ul> <p><i>Examples:</i></p> <ul style="list-style-type: none"><li>• The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?</li><li>• Students measure the circumference and diameter of several circular objects in the room (clock, trashcan, doorknob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures. Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles.</li><li>• Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.</li></ul>

Standard

7.G.4 Examples (continued)

- The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is  $\frac{1}{2}$  the circumference ( $2\pi r$ ). The area of the rectangle (and therefore the circle) is found by the following calculations:



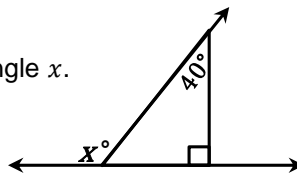
**7.G.5** – Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Explanations:

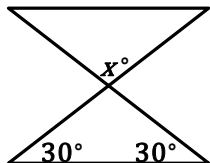
- Students use understandings of angles to write and solve equations.
- In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.
- Angle relationships that can be explored include but are not limited to same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Examples:

- Write and solve an equation to find the measure of angle  $x$ .



- Write and solve an equation to find the measure of angle  $x$ .





**Standard**

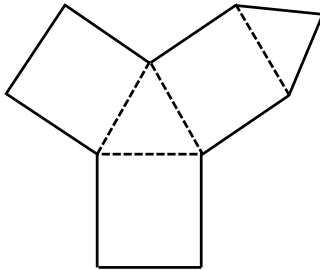
**7.G.6** – Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

*Explanations:*

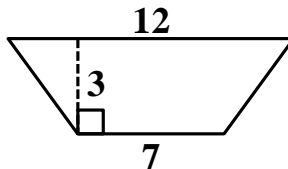
- Students' understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students' understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations.
- Real-world and mathematical multistep problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes, and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios, and various units of measure with same system conversions.

*Examples:*

- Choose one of the figures shown below and write a step-by-step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?



- A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.) Make a poster explaining your work to share with the class.
- Find the area of a triangle with a base length of three units and a height of four units.
- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



## Statistics and Probability (SP)

Use random sampling to draw inferences about a population.

Standard
<p><b>7.SP.1</b> – Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p><i>Explanation:</i> Students recognize that it is difficult to gather statistics on an entire population. Instead, a random sample can be representative of the total population and will generate valid results. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.</p> <p><i>Example:</i></p> <ul style="list-style-type: none"> <li>• The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students’ preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why?             <ol style="list-style-type: none"> <li>a. Write all of the students’ names on cards and pull them out in a draw to determine who will complete the survey.</li> <li>b. Survey the first 20 students that enter the lunchroom.</li> </ol> </li> </ul>
<p><b>7.SP.2</b> – Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. <i>For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</i></p> <p><i>Resources:</i></p> <ul style="list-style-type: none"> <li>• Students collect and use multiple samples of data to answer question(s) about a population. Issues of variation in the samples should be addressed.</li> <li>• Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. Have students compare the random sample to population, asking questions like “Are all the elements of the entire population represented in the sample?” and “Are the elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.</li> <li>• Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.</li> </ul>

**Standard**

7.SP.2 Example:

- Below is the data collected from two random samples of 100 students regarding student's school lunch preference. Make at least two inferences based on the results.

Lunch Preferences				
Student Sample	Hamburgers	Tacos	Pizza	Total
#1	12	14	74	100
#2	12	11	77	100

**Draw informal comparative inferences about two populations.**

**Standard**

**7.SP.3** – Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

*Resources:* Students can readily find data as described in the example on sports team or college websites. Other sources for data include [American Fact Finder](#) (Census Bureau), [Fed Stats](#), [USGS](#), or [CIA World Factbook](#). Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistical functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

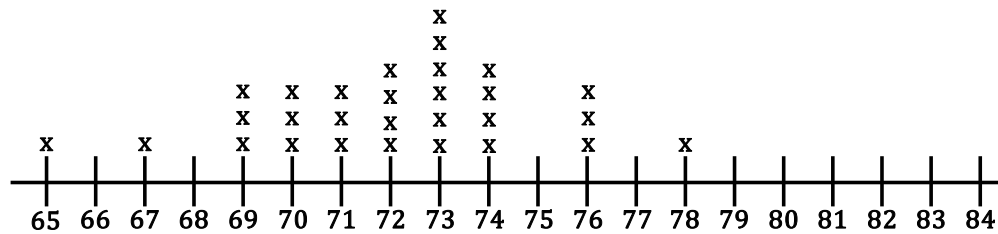
*Example:*

- Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but does not know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists.
    - Basketball team – Height of players in inches for 2010-2011 season:  
75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84
    - Soccer team – Height of Players in inches for 2010  
73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69
- To compare the data sets, Jason creates two dot plots on the same scale. The shortest player is 65 inches, and the tallest players are 84 inches.

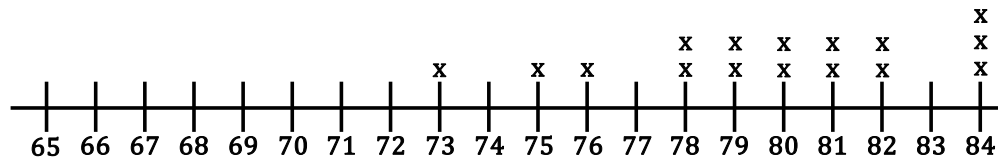
Standard

7.SP.3 Example (continued)

Height of Soccer Players (inches)



Height of Basketball Players (inches)



In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation (MAD) to compare the data sets. Jason sets up a table for each data set to help him with the calculations—see data set tables on next page.

The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches.

The MAD is calculated by taking the mean of the absolute deviations for each data point. The difference between each data point and the mean is recorded in the second column of the table. Jason used rounded values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set.

The MAD is 2.14 inches for the basketball players and 2.53 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams is approximately 3 times the variability of the data sets ( $7.68 + 2.53 = 3.04$ ).

## Standard

7.SP.3 Example (continued)

Basketball Players (n=16)			Soccer Players (n=29)		
Height	Deviation from Mean (inches)	Absolute Deviation (inches)	Height	Deviation from Mean (inches)	Absolute Deviation (inches)
73	-7	7	65	-7	7
75	-5	5	67	-5	5
76	-4	4	69	-3	3
78	-2	2	69	-3	3
78	-2	2	69	-3	3
79	-1	1	70	-2	2
79	-1	1	70	-2	2
80	0	0	70	-2	2
80	0	0	71	-1	1
81	+1	1	71	-1	1
81	+1	1	71	-1	1
82	+2	2	72	0	0
82	+2	2	72	0	0
84	+4	4	72	0	0
84	+4	4	72	0	0
84	+4	4	73	+1	1
			73	+1	1
			73	+1	1
			73	+1	1
			73	+1	1
			73	+1	1
			74	+2	2
			74	+2	2
			74	+2	2
			74	+2	2
			76	+4	4
			76	+4	4
			76	+4	4
			78	+6	6
$\Sigma = 1276$		$\Sigma = 40$	$\Sigma = 2090$		$\Sigma = 62$
$Mean = 1276 \div 16 = 80$ inches $MAD = 40 \div 16 = 2.5$ inches			$Mean = 2090 \div 29 = 72$ inches $MAD = 62 \div 29 = 2.13$ inches		

**Standard**

**7.SP.4** – Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

*Explanation:* Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.

*Example:*

- The two data sets below depict random samples of the housing prices sold in the King River and Toby Ranch areas of Arizona. Based on the prices below, which measure of center will provide the most accurate estimation of housing prices in Arizona? Explain your reasoning.
  - King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}
  - Toby Ranch homes {5 million, 154000, 250000, 250000, 200000, 160000, 190000}

**Investigate chance processes and develop, use, and evaluate probability models.**

**Standard**

**7.SP.5** – Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around  $\frac{1}{2}$  indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

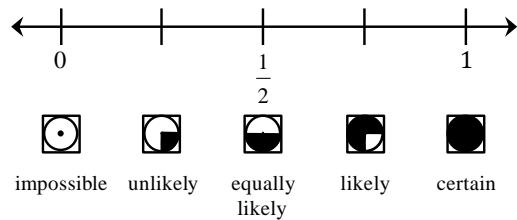
*Explanation:* This is students' first formal introduction to probability. Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line. The sum of all possible outcomes is 1.

*Example:*

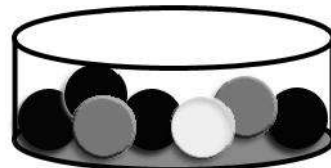
- There are three choices of jellybeans—grape, cherry, and orange. If the probability of getting a grape is  $\frac{3}{10}$  and the probability of getting cherry is  $\frac{1}{5}$ , what is the probability of getting orange? The probability of any single event can be recognized as a fraction. The closer the fraction is to 1, the greater the probability the event will occur. Larger numbers indicate greater likelihood. For example, if you have 10 oranges and 3 apples, you have a greater likelihood of getting an orange.

- Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

- Marble Mania: <http://sciencenetlinks.com/interactives/marble/marblemania.html>
- Random Drawing Tool: <http://illuminations.nctm.org/ActivityDetail.aspx?ID=67>



- The container below contains 2 gray, 1 white, and 4 black disks. Without looking, if you choose a disk from the container, will the probability be closer to 0 or to 1 that you will select a white disk? A gray disk? A black disk? Justify each of your predictions.



**Standard**

**7.SP.6** – Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times but probably not exactly 200 times.*

*Explanations:* Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability. The focus of this standard is relative frequency. The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful events.

*Resources:* Students can collect data using physical objects, a graphing calculator, or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.

*Example:*

- Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn, and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (how many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?).
- Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, and 3 blue marbles.)
- Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.



**Standard**

**7.SP.7** – Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

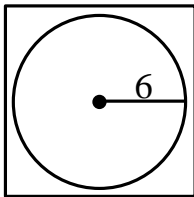
- a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

*Explanations:*

- Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.
- Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects, a graphing calculator, or web-based simulations. Students can also develop models for geometric probability (i.e., a target).

*Example:*

- If you choose a point in the square, what is the probability that it **is not** in the circle?



**Standard**

**7.SP.8** – Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

*Resources:* Students use tree diagrams, frequency tables, organized lists, and simulations to determine the probability of compound events.

*Explanation:* Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

*Examples:*

- Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles, and two purple marbles. Students will draw one marble without replacement and then draw another. What is the sample space for this situation? Explain how you determined the sample space and how you will use it to find the probability of drawing one blue marble followed by another blue marble.
- Show all possible arrangements of the letters in the word FRED using a tree diagram. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your "word" will have an F as the first letter?

