CCGPS Honors Geometry
Graphing Quadratics
Quadratic Regression

Name ____
Date _________________________ Block ________

Warm – Up

An astronaut on the moon throws a golf ball upward. The height of the golf ball can be modeled by the equation $h(t) = -2.7t^2 + 27t + 5$, where t is measured in seconds and $h(t)$ is measured in feet.

1. What is the initial height of the golf ball? Which intercept, x or y, does this number represent?

2. After how many seconds would it be anticipated that the golf ball would land on the moon? Which intercept, x or y, does this number represent?

Introduction

Data surrounds us in the real world. Every day, people are presented with numbers and are expected to make predictions about future events based upon that given data. A **regression equation** is an equation that best represents a set of data, and it can be used to predict missing data or future data. Different types of equations are suited to different types of data. **Regression** is the mathematical process for determining an equation from a set of given data. Regression is used to make predictions for values of an independent variable. Some data is best represented by linear or exponential equations, as you have seen previously. **Quadratic regression** is the process of finding the equation of a parabola that fits a given set of data. In this lesson, you will work with data sets that are best represented by quadratic equations, and you will learn how to write a **quadratic regression equation**, a regression equation that fits a parabola to data.

Key Concepts

- A linear equation describes a situation where there is a near-constant ______________________________.
- An exponential equation describes a situation where the data changes by a __________________________.

- If you are given a set of data and you are not sure whether the data is best modeled by a linear regression or a quadratic regression, you can look at the _______________ and _______________ differences.

- In a ________________ model, the y-value changes by a constant when the x-value increases by 1. The change in y when x increases by 1 is called a ___________________________. If your ________________________ are all about the same, then a linear model is appropriate.

- In a ___________________ model, the first differences are not the same, but the ________________ in the first differences is constant. The change in successive first differences is called a ________________________.

- A quadratic regression equation fits a________________________ to the data.

Example 1

The following data table shows a car's speed in miles per hour and the car's fuel efficiency in miles per gallon for each speed.

Speed (mph)	18.6	24.9	31.1	37.3	43.5	49.7	55.9	62.1
Fuel efficiency (mpg)	26.1	29.4	31.4	33.1	33.1	31.4	29.4	26.1

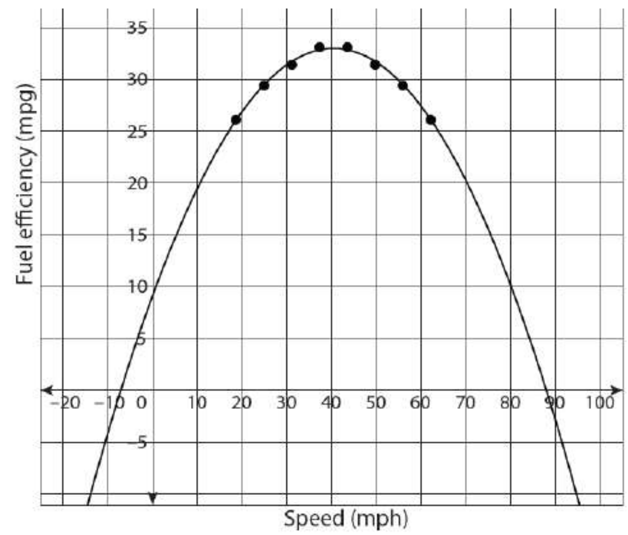
A quadratic regression equation that models this data is given by $m(x) = -0.0146x^2 + 1.1802x + 9.1356$, where x is speed in mph and $m(x)$ is fuel efficiency in mpg. A scatter plot of the data with the graph of this model is shown below. *(this equation can be found using a graphing calculator)*

Use the given regression model to find the car's fuel efficiency in miles per gallon when this car is traveling 31.1 mph.

Compare your answer to the data in the table.

Do these values match?

Then use the graph to estimate the speed(s) that will result in fuel efficiencies of about 25 mpg and 40 mpg. Use the model to check your estimates.



1. Find the x-intercepts of the graph. ________________________

2. Interpret the meaning of the x-intercepts. ____

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3. Find the y-intercept.

4. Interpret the meaning of the y-intercept. ____

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5. Use the equation to predict the fuel efficiency at the speeds of 20 mph, 65 mph, 75 mph, and 90 mph. Determine whether each of these predictions is an *interpolation* or an *extrapolation*, and if any of the predictions seem unreasonable within the context of the problem.

(• ***Interpolation*** is when a regression equation is used to make predictions about a dependent variable that is within the range of the given data. ***Extrapolation*** is when a regression equation is used to make predictions about a dependent variable that is outside the range of the given data.)

6. Predict the fuel efficiency at 20 mph and determine if the prediction seems to be reasonable.

7. Predict the fuel efficiency at 65 mph and determine if the prediction seems to be reasonable.

8. Predict the fuel efficiency at 75 mph and determine if the prediction seems to be reasonable.

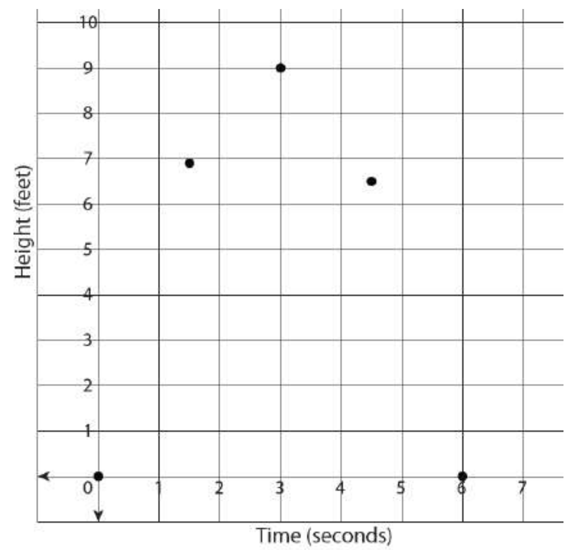
9. Predict the fuel efficiency at 90 mph and determine if the prediction seems to be reasonable.

Example 2

The table below shows the height in feet of a children's roller coaster at different times throughout the ride.

Time (seconds)	0	1.5	3	4.5	6
Height (feet)	0	6.9	9	6.5	0

Create a scatter plot of the data. Should a quadratic regression model be used to model the height of the roller coaster? If so, find a quadratic equation that fits this data.



The scatter plot shows that the data is parabolic. The data rises, then falls. A quadratic regression model is appropriate.

1. Estimate the vertex of the parabola.

3. Find the equation of the parabola.

Example 3

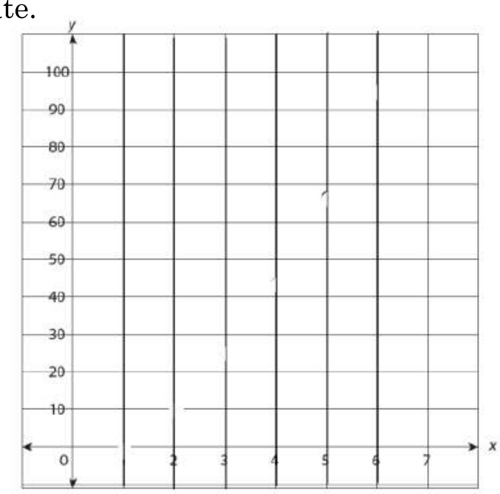
Look at the data given in the table that follows. What is the most appropriate regression for the data: linear, exponential, or quadratic? Create a scatter plot of the data and confirm the appropriateness of the model chosen.

x	1	2	3	4	5	6
y	–1	10	25	44	67	94

1. Find the first differences by finding differences between consecutive y-values.

2. Find the second differences by finding the differences between the first differences.
3. Which differences are the same?

4. This means a ________________________ model is appropriate.



5. Make a scatterplot of the data.

6. Does this look like an exponential graph?

7. How do you know if it is?

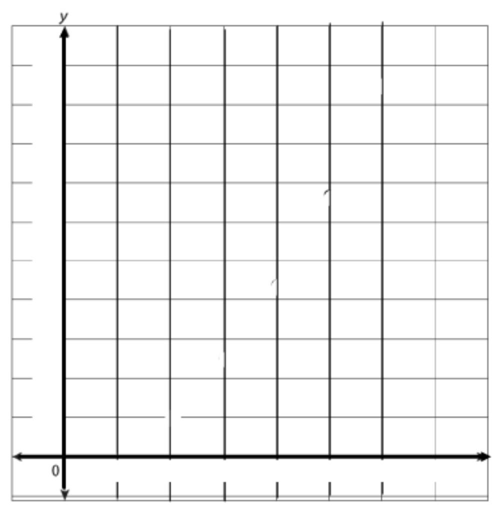
8. Use a calculator to find the regression equation.

Practice

The amount of medication in a patient's bloodstream varies over time. The table below shows the concentration of a certain medication in milligrams per liter at various time intervals after being administered.

Time (minutes)	0	30	60	90	120	150
Concentration (mg/L)	0	39.02	49.93	42.34	25.06	7.78

1. Create a scatter plot of the data. Is a quadratic regression model appropriate? Explain.



2. Use the scatter plot to estimate the time(s) at which the concentration is 30 mg/L.

3. A quadratic regression model for the data is
$\boldsymbol{y = -0.0078x^2 + 1.1633x + 4.6421}$.
Use this model to check your estimate from problem 2.

4. Using the model given in problem 3, find and interpret the x-intercepts.

5. Using the model given in problem 3, find and interpret the y-intercept.

6. Use the model given in problem 3 to estimate the concentration of the medication 12 hours, or 720 minutes, after the medication is administered. Is your answer reasonable? Why or why not?