# Geometry

## Chapter 12: Circles

### Chapter Focus

Develop and apply the properties of lines and angles that intersect circles.

Analyze the properties of circles in the coordinate plane and use them to solve real-world problems.

### Section 1: Lines and Arcs in Circles

12-1 Lines That Intersect Circles – G.C.2

12-2 Arcs and Chords – G.C.2

12-3 Sector Area and Arc Length – G.C.5

#### Section 2: Angles and Segments in Circles

12-4 Inscribed Angles – G.C.2

12-5 Angle Relationships in Circles – G.C.2

12-6 Segment Relationships in Circles – G.C.2

12-7 Circles in the Coordinate Plane – G.GPE.1

### Previous Knowledge Needed

Used the fundamental vocabulary of circles.

Developed and applied formulas for the area and circumference of circles.

Used circles to solve problems.

### Cadet's Study

Solving problems involving circles.

Finding lengths, angles, measures and areas associated with circles.

Applying circle theorems to solve a wide range of problems.

## Skills Learned

To Use the Pythagorean Theorem to derive the Distance Formula.

Use the distance formula to derive the general form of an equation for a circle.

Find arc length for different arcs in a circle.

Find area of a sector for different angles in a circle.

To calculate distances inside and outside a circle.

# Geometry

## Chapter 12: Circles

### Lesson 1 - Lines That Intersect Circles

**Standards**

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.C.4 – Construct a tangent line from a point outside a given circle to the circle.

**Objectives**

Identify tangents, secants and chords.

Use properties of tangents to solve problems.

**Materials**

Blank paper, compass, Text book, ToolBox

**Vocabulary**

Interior of a circle – points inside a circle

Exterior of a circle – points outside a circle

Chord – segment – endpoints lie on a circle

Secant – line that intersects circle at two points

Tangent – line that intersects circle at one point called a point of tangency.

Point of tangency – The point of intersection of a circle or sphere with a tangent line or plane.

Circles – a set of all points that are a fixed distance from a given point called the center of the circle.

Congruent – radii are equal

Concentric – coplanar with same center

Tangent – coplanar with intersection at one point

Common tangent – a line tangent to two circles

**Formulas**

None

**Symbols**

None

**Postulates**

None

**Theorems**

Theorem – If 2 segments are tangent to a circle from the same external point, then the segments are congruent.

Theorem – If a line is tangent to a circle, then it is perpendicular to the radius drawn to the tangent point.

**Who Uses This?**

Scientists use to solve problems pertaining to the earth.

**Discussion – first day**

Do vocabulary discussing read word and drawing a picture representation of each.

Review example 1 from book. Discuss that there can be 2 identifications for one segment. Example would be a chord that goes through the center is the diameter. Ask what is the relationship between a chord and a diameter? How are they alike and how are they different?

Have students do Check-It-Out #1

Write the first theorem in ToolBox.

Review example 4 from book. Can we conclude that the two segments that are tangent to the circle are congruent? What if they start from the same point? What if they start at different points?

At this point, may need to review solving equations with a variable on both sides.

**Guided Practice – first day**

Check it out #4a and b

**Practice**

None for day 1

**Homework**

None for Day 1

**Discussion – second day**

Review vocabulary.

Questions – What is the relationship between a chord and the diameter?

What do you know about two tangents to a circle that start at the same point?

How many radii are there in a circle?

Review example 2 – find length of radius – if on a coordinate graph – count the squares. If not on a coordinate graph use the points and the distance formula. Review the formula. Find the tangent line and then find the equation of the line. May have to review how to come up with the equation by using the slope-point-formula. How do you know which axis to use – x or y? Discuss how to tell the difference.

Activity – have cadets draw a circle on a piece of plane paper. Make sure the center is identified. Select a point on the circle and draw a radius to that point. Draw a line tangent to the point you selected on the circle. What do you notice about the angle the tangent line makes with the radius? Discuss cadet's thoughts. Have cadets look at other cadets work to see if there is a patterns going on. Should see that the angle is 90 degrees. Draw several examples on the board that are not right angles to show that not all lines intersecting the point on the circle are perpendicular to the radius.

Discuss and write second theorem in ToolBox.

Review Pythagorean Theorem by looking at example 3. Make sure to point out that feet need to be converted to miles. Review how to do that conversion.

**Guided Practice – second day**

Check it out #3 – remind cadets to convert feet into miles. This problem is very confusing to cadets as they think that mountains do not extend into the center of the earth. Need to encourage them that they are using the center of the earth to measure the height of the mountain. The sketch is 3D and cadets need to be encouraged to imagine the 3D coming off the paper.

Page 797, # 9 – have cadets work on it and then review.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 11 – 28, 31 – 35, 38 – 40

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets - Problems 11 – 41

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets – Problems 11 – 43

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 14, 15, 16, 20, 26

Page 797 – 13, 16, 18-22, 26, 27, 31, 32, 33 – review each of these after cadets have had time to do them.

**Homework**

Page 73 and 157 in workbook. May be replaced with different work based on needs. Replaced by additional resources listed above.

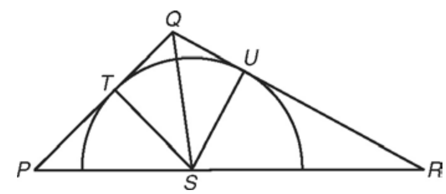
Name ________________________ Date ______________ Class______________

LESSON 12-1

# Challenge

## *Lines That Intersect Circles*

In $\triangle PQR$, in which $PQ = 15$, $QR = 22$, and $PR = 30$, a semicircle is drawn so that its diameter lies on $\overline{PR}$ and it is tangent to $\overline{QP}$ and $\overline{QR}$. If $S$ is the center of the circle, find the measure of $\overline{PS}$ to the nearest hundredth.




**Draw radii $\overline{ST}$ and $\overline{SU}$ to the points of tangency to $\overline{PQ}$ and $\overline{QR}$. Draw $\overline{QS}$.**

| Statements | Reasons |
|---|---|
| 1. ______________________ | 1. All radii have equal measures. |
| 2. $m\angle QTS = m\angle QUS = 90°$ | 2. ______________________ |
| 3. $\overline{QT} \cong \overline{QU}$ | 3. ______________________ |
| 4. $\overline{QS} \cong \overline{QS}$ | 4. ______________________ |
| 5. $\triangle QTS \cong \triangle QUS$ | 5. ______________________ |
| 6. $\angle UQS \cong \angle TQS$ | 6. ______________________ |
| 7. ______________________ | 7. Triangle Angle Bisector Theorem |

Let $PS = x$. Then ______________ and $x =$ ____________.

# Answers for the chapter Circles

## 12-1 LINES THAT INTERSECT CIRCLES

### Practice A

1. E
2. B
3. A
4. C
5. D
6. 2; 1
7. 4; 2
8. $x = 4$
9. tangent
10. congruent
11. perpendicular
12. 5 in.
13. 24
14. 21

### Practice B

1. chords: $\overline{BC}$; secant: $\overleftrightarrow{BC}$; tangent: $\ell$; diam.: $\overline{BC}$; radii: $\overline{AB}$, $\overline{AC}$
2. chords: $\overline{RQ}$, $\overline{ST}$; secant: $\overleftrightarrow{ST}$; tangent: $\overline{UV}$; diam.: $\overline{RQ}$; radii: $\overline{PQ}$, $\overline{PR}$, $\overline{PU}$
3. radius of $\odot D$: 4; radius of $\odot E$: 2; pt. of tangency: (0, –4); eqn. of tangent line: $y = -4$
4. radius of $\odot M$: 1; radius of $\odot N$: 3; pt. of tangency: (–2, –2); eqn. of tangent line: $x = -2$
5. 385,734 km
6. 7.8 m
7. 50 ft

### Practice C

1. Possible answer: Draw $\overline{AB}$. A tangent segment is perpendicular to a radius at the point of tangency. So $\angle ACD$ and $\angle BDC$ are right angles. Two segments perpendicular to the same segment are parallel, so $\overline{AC}$ and $\overline{BD}$ are parallel. Because $\overline{AC}$ and $\overline{BD}$ are radii of $\odot A$ and $\odot B$, they are congruent. Therefore *ABDC* is a parallelogram. Opposite sides in a parallelogram are congruent, so $\overline{CD} \cong \overline{AB}$. Similar reasoning will show that $\overline{EF} \cong \overline{AB}$. By the Transitive Property of Congruence, $\overline{CD} \cong \overline{EF}$.
2. Possible answer: It is given that $\overline{RS}$ and $\overline{TU}$ are not parallel, so they must meet at some point. Call this point *X*. $\overline{XR}$ and $\overline{XT}$ are tangent to $\odot P$ and $\odot Q$. Because tangent segments from a common point to a circle are congruent, $XR = XT$ and $XS = XU$. The Segment Addition Postulate shows that $XR = XS + SR$ and $XT = XU + UT$. Thus, by the Transitive Property, $XS + SR = XU + UT$. By the Addition Property of Equality, $SR = UT$, and therefore $\overline{RS} \cong \overline{TU}$.
3. Possible answer: It is given that $\overline{IM}$ and $\overline{JL}$ are tangent segments. They intersect at point *K*. Because tangent segments from a common point to a circle are congruent, $KI = KL$ and $KM = KJ$. By the Addition Property of Equality, $KI + KM = KL + KJ$. The Segment Addition Postulate shows that $IM = KI + KM$ and $JL = KL + KJ$. Thus, by the Transitive Property of Equality, $IM = JL$ and therefore $\overline{IM} \cong \overline{JL}$.
4. 50 m
5. 8.5 ft or 16.5 ft

### Reteach

1. chord: $\overline{FG}$; secant: $\ell$; tangent: *m*; diam.: $\overline{FG}$; radii: $\overline{HF}$ and $\overline{HG}$
2. chord: $\overline{LM}$; secant: $\overleftrightarrow{LM}$; tangent: $\overleftrightarrow{MN}$; radius: $\overline{JK}$
3. $\odot N$: $r = 3$; $\odot P$: $r = 1$; pt. of tangency: (–1, –2); tangent line: $y = -2$
4. $\odot S$: $r = 4$; $\odot T$: $r = 2$; pt. of tangency: (7, 0); tangent line: $x = 7$
5. 6
6. 14
7. 10
8. 19

### Challenge

1. $ST = SU$
2. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency (Theorem 11-1-1).
3. If two segments are tangent to a circle from the same external point, then the

segments are congruent (Theorem 11-1-3).

4. Reflexive Property
5. Hypotenuse-Leg or SSS
6. CPCTC
7. $\frac{PQ}{PS} = \frac{QR}{RS}$
8. $\frac{15}{x} = \frac{22}{30 - x}$; $12.16 = PS$

**Problem Solving**

1. 339 km
2. 27
3. 68 cm
4. 47
5. C
6. H
7. B

**Reading Strategies**

1. interior
2. secant
3. because it intersects the circle at only one point
4. line $z$
5. point $B$
6. line $k$
7. $\overline{XY}$
8. point $R$
9. point $P$

## 12-2 ARCS AND CHORDS

**Practice A**

1. 90°
2. 144°
3. 108°
4. 234°
5. chords
6. chords
7. adjacent
8. perpendicular bisector
9. central angles
10. radius or diameter
11. 150°
12. 240°
13. 125°
14. 11 cm
15. 30 in.
16. 96 m

**Practice B**

1. 115.2°
2. 93.6°
3. 126°
4. 90°
5. 3.6°
6. 241.2°
7. 125°; 227°
8. 67°; 203°
9. 102°
10. 49 cm
11. 76.3 mi
12. 4.9 km

**Practice C**

1. Possible answer: Draw $\overline{AF}$ and $\overline{EF}$. It is given that $\overline{AE}$ is perpendicular to $\overline{FG}$. Therefore $\angle ACF$ and $\angle ECF$ are right angles and are congruent. It is also given that $\overline{AC} \cong \overline{EC}$. $\overline{FC} \cong \overline{FC}$ by the Reflexive Property of Congruence. So $\triangle AFC \cong \triangle EFC$ by SAS. By CPCTC, $\overline{AF} \cong \overline{EF}$. $\overline{AF}$ and $\overline{EF}$ are radii of $\odot A$ and $\odot E$, and circles with congruent radii are congruent circles, so $\odot A \cong \odot E$.
2. Possible answer: Draw $\overline{RU}, \overline{PR}, \overline{PU}, \overline{QR}$, and $\overline{QU}$. Because $\overline{PR}$ and $\overline{PU}$ are radii of $\odot P$, they are congruent and $\triangle PRU$ is isosceles. Similar reasoning shows that $\triangle QRU$ is also isosceles. By the Base Angles Theorem, $\angle PUR \cong \angle PRU$ and $\angle QRU \cong \angle QUR$. So $m\angle PUR$ and $m\angle PRU$ are each equal to $\frac{1}{2}(180 - m\angle RPU)$. Also, $m\angle QRU$ and $m\angle QUR$ are each equal to $\frac{1}{2}(180 - m\angle RQU)$. It is given that $\overparen{RSU} \cong \overparen{RTU}$, so $m\overparen{RSU} = m\overparen{RTU}$. The measure of an arc is equal to the measure of its central angle, so $m\angle RPU = m\angle RQU$. Substitution shows that $m\angle PUR = m\angle PRU = m\angle QRU = m\angle QUR$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. So $\triangle PRU \cong \triangle QRU$ by SAS. By CPCTC, $\overline{PR} \cong \overline{QR}$ and circles with congruent radii are congruent circles, so $\odot P \cong \odot Q$.
3. 60°
4. 19.2°
5. 53.1°
6. 90°
7. 103.5°
8. 180°
9. $0.2r$
10. $0.8r$
11. $1.9r$

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**LESSON 12-1**

# Practice A

## *Lines That Intersect Circles*

**For Exercises 1–5, match the letter of the part of the figure to the names. Use each letter once.**

1. chord ________
2. tangent ________
3. radius ________
4. secant ________
5. diameter ________

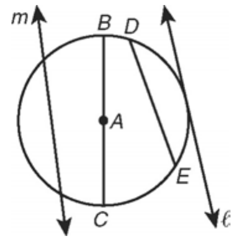
A. $\overline{AB}$

B. $\ell$

C. $m$

D. $\overline{BC}$

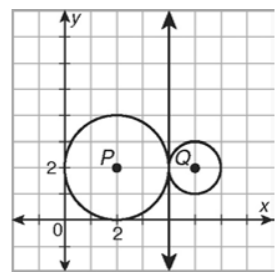
E. $\overline{DE}$




**Use the figure for Exercises 6–8.**

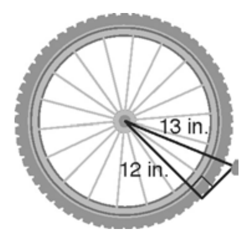
6. radius of $\odot P$ ________ radius of $\odot Q$ ________
7. coordinates of the point of tangency (________, ________)
8. equation of the tangent line at the point of tangency

________________________________




**Fill in the blanks to complete each theorem.**

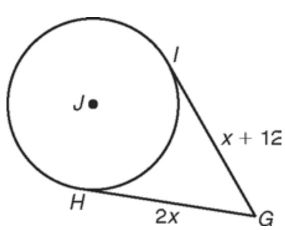
9. If a line is perpendicular to a radius of a circle at a point on the circle, then the line is ____________________ to the circle.
10. If two segments are tangent to a circle from the same external point, then the segments are ____________________.
11. If a line is tangent to a circle, then it is ____________________ to the radius drawn to the point of tangency.
12. Amiko is riding her bike on a wet street. As the bike wheel spins, water drops are sprayed off tangent to the wheel. Amiko's bike wheels have a radius of 12 inches. Use the Pythagorean Theorem to find the distance the water drops have been sprayed when they are 13 inches from the center of the wheel.




________________________________

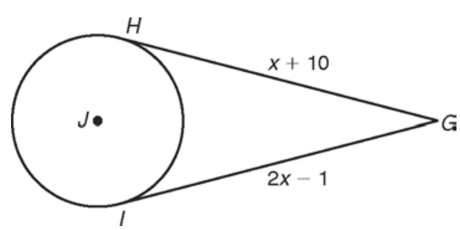
**In Exercises 13 and 14, $\overline{GH}$ and $\overline{GI}$ are tangent to $\odot J$. Find $GH$.**

13.




________________________________

14.




________________________________

# Answers for the chapter Circles

## 12-1 LINES THAT INTERSECT CIRCLES

### Practice A

1. E
2. B
3. A
4. C
5. D
6. 2; 1
7. 4; 2
8. $x = 4$
9. tangent
10. congruent
11. perpendicular
12. 5 in.
13. 24
14. 21

### Practice B

1. chords: $\overline{BC}$; secant: $\overleftrightarrow{BC}$; tangent: $\ell$; diam.: $\overline{BC}$; radii: $\overline{AB}$, $\overline{AC}$
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5. 385,734 km
6. 7.8 m
7. 50 ft

### Practice C

1. Possible answer: Draw $\overline{AB}$. A tangent segment is perpendicular to a radius at the point of tangency. So $\angle ACD$ and $\angle BDC$ are right angles. Two segments perpendicular to the same segment are parallel, so $\overline{AC}$ and $\overline{BD}$ are parallel. Because $\overline{AC}$ and $\overline{BD}$ are radii of $\odot A$ and $\odot B$, they are congruent. Therefore *ABDC* is a parallelogram. Opposite sides in a parallelogram are congruent, so $\overline{CD} \cong \overline{AB}$. Similar reasoning will show that $\overline{EF} \cong \overline{AB}$. By the Transitive Property of Congruence, $\overline{CD} \cong \overline{EF}$.
2. Possible answer: It is given that $\overline{RS}$ and $\overline{TU}$ are not parallel, so they must meet at some point. Call this point *X*. $\overline{XR}$ and $\overline{XT}$ are tangent to $\odot P$ and $\odot Q$. Because tangent segments from a common point to a circle are congruent, $XR = XT$ and $XS = XU$. The Segment Addition Postulate shows that $XR = XS + SR$ and $XT = XU + UT$. Thus, by the Transitive Property, $XS + SR = XU + UT$. By the Addition Property of Equality, $SR = UT$, and therefore $\overline{RS} \cong \overline{TU}$.
3. Possible answer: It is given that $\overline{IM}$ and $\overline{JL}$ are tangent segments. They intersect at point *K*. Because tangent segments from a common point to a circle are congruent, $KI = KL$ and $KM = KJ$. By the Addition Property of Equality, $KI + KM = KL + KJ$. The Segment Addition Postulate shows that $IM = KI + KM$ and $JL = KL + KJ$. Thus, by the Transitive Property of Equality, $IM = JL$ and therefore $\overline{IM} \cong \overline{JL}$.
4. 50 m
5. 8.5 ft or 16.5 ft

### Reteach

1. chord: $\overline{FG}$; secant: $\ell$; tangent: *m*; diam.: $\overline{FG}$; radii: $\overline{HF}$ and $\overline{HG}$
2. chord: $\overline{LM}$; secant: $\overleftrightarrow{LM}$; tangent: $\overleftrightarrow{MN}$; radius: $\overline{JK}$
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5. 6
6. 14
7. 10
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### Challenge

1. $ST = SU$
2. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency (Theorem 11-1-1).
3. If two segments are tangent to a circle from the same external point, then the

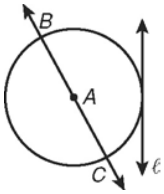
Name ________________________ Date ______________ Class ______________

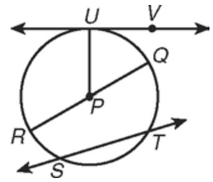
LESSON 12-1

# Practice B

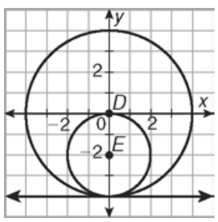
## *Lines That Intersect Circles*

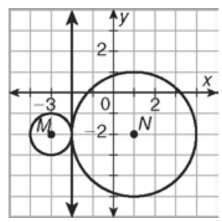
**Identify each line or segment that intersects each circle.**

1. 

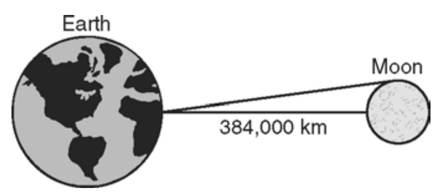

2. 


**Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.**

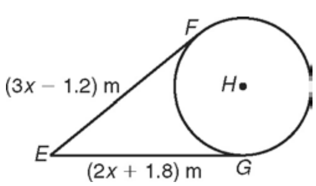
3. 

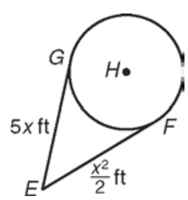

4. 


5. The Moon's orbit is not exactly circular, but the average distance from its surface to Earth's surface is 384,000 kilometers. The diameter of the Moon is 3476 kilometers. Find the distance from the surface of Earth to the visible edge of the Moon if the Moon is directly above the observer. Round to the nearest kilometer. (*Note:* The figure is not drawn to scale.)




**In Exercises 6 and 7, $\overline{EF}$ and $\overline{EG}$ are tangent to ⊙*H*. Find *EF*.**

6. 


7. 

# Answers for the chapter Circles

## 12-1 LINES THAT INTERSECT CIRCLES

### Practice A

1. E
2. B
3. A
4. C
5. D
6. 2; 1
7. 4; 2
8. $x = 4$
9. tangent
10. congruent
11. perpendicular
12. 5 in.
13. 24
14. 21

### Practice B

1. chords: $\overline{BC}$; secant: $\overleftrightarrow{BC}$; tangent: $\ell$; diam.: $\overline{BC}$; radii: $\overline{AB}$, $\overline{AC}$
2. chords: $\overline{RQ}$, $\overline{ST}$; secant: $\overleftrightarrow{ST}$; tangent: $\overline{UV}$; diam.: $\overline{RQ}$; radii: $\overline{PQ}$, $\overline{PR}$, $\overline{PU}$
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### Practice C

1. Possible answer: Draw $\overline{AB}$. A tangent segment is perpendicular to a radius at the point of tangency. So $\angle ACD$ and $\angle BDC$ are right angles. Two segments perpendicular to the same segment are parallel, so $\overline{AC}$ and $\overline{BD}$ are parallel. Because $\overline{AC}$ and $\overline{BD}$ are radii of $\odot A$ and $\odot B$, they are congruent. Therefore $ABDC$ is a parallelogram. Opposite sides in a parallelogram are congruent, so $\overline{CD} \cong \overline{AB}$. Similar reasoning will show that $\overline{EF} \cong \overline{AB}$. By the Transitive Property of Congruence, $\overline{CD} \cong \overline{EF}$.
2. Possible answer: It is given that $\overline{RS}$ and $\overline{TU}$ are not parallel, so they must meet at some point. Call this point $X$. $\overline{XR}$ and $\overline{XT}$ are tangent to $\odot P$ and $\odot Q$. Because tangent segments from a common point to a circle are congruent, $XR = XT$ and $XS = XU$. The Segment Addition Postulate shows that $XR = XS + SR$ and $XT = XU + UT$. Thus, by the Transitive Property, $XS + SR = XU + UT$. By the Addition Property of Equality, $SR = UT$, and therefore $\overline{RS} \cong \overline{TU}$.
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4. 50 m
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### Reteach

1. chord: $\overline{FG}$; secant: $\ell$; tangent: $m$; diam.: $\overline{FG}$; radii: $\overline{HF}$ and $\overline{HG}$
2. chord: $\overline{LM}$; secant: $\overleftrightarrow{LM}$; tangent: $\overleftrightarrow{MN}$; radius: $\overline{JK}$
3. $\odot N$: $r = 3$; $\odot P$: $r = 1$; pt. of tangency: (–1, –2); tangent line: $y = -2$
4. $\odot S$: $r = 4$; $\odot T$: $r = 2$; pt. of tangency: (7, 0); tangent line: $x = 7$
5. 6
6. 14
7. 10
8. 19

### Challenge

1. $ST = SU$
2. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency (Theorem 11-1-1).
3. If two segments are tangent to a circle from the same external point, then the

Name ______________________ Date ______________ Class______________

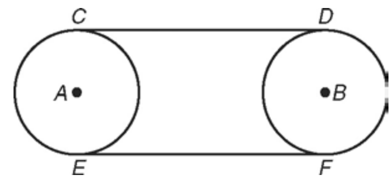
LESSON 12-1

# Practice C

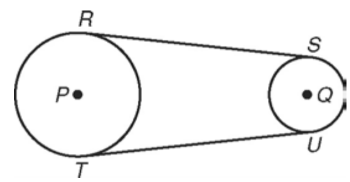
## *Lines That Intersect Circles*

**Write paragraph proofs for Exercises 1–3.**

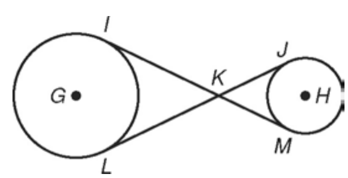
1. **Given:** $\odot A$ and $\odot B$ with congruent radii. $\overline{CD}$ and $\overline{EF}$ are common tangent segments.
**Prove:** $\overline{CD} \cong \overline{EF}$ (*Hint:* Draw $\overline{AB}$. Use properties of quadrilaterals to show that $CD = AB = EF$.)



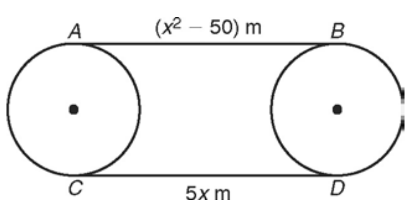

2. **Given:** $\odot P$ and $\odot Q$ with different radii. $\overline{RS}$ and $\overline{TU}$ are common tangent segments. $\overline{RS}$ and $\overline{TU}$ are not parallel.
**Prove:** $\overline{RS} \cong \overline{TU}$



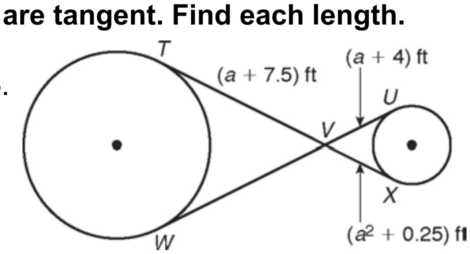

3. **Given:** $\odot G$ and $\odot H$. $\overline{IM}$ and $\overline{JL}$ are common interior tangent segments.
**Prove:** $\overline{IM} \cong \overline{JL}$




**Assume the segments that appear to be tangent are tangent. Find each length.**

4. 


$CD$ ______________________

5. 


$UW$ ______________________

# Answers for the chapter Circles

## 12-1 LINES THAT INTERSECT CIRCLES

### Practice A

1. E
2. B
3. A
4. C
5. D
6. 2; 1
7. 4; 2
8. $x = 4$
9. tangent
10. congruent
11. perpendicular
12. 5 in.
13. 24
14. 21

### Practice B

1. chords: $\overline{BC}$; secant: $\overleftrightarrow{BC}$; tangent: $\ell$; diam.: $\overline{BC}$; radii: $\overline{AB}$, $\overline{AC}$
2. chords: $\overline{RQ}$, $\overline{ST}$; secant: $\overleftrightarrow{ST}$; tangent: $\overleftrightarrow{UV}$; diam.: $\overline{RQ}$; radii: $\overline{PQ}$, $\overline{PR}$, $\overline{PU}$
3. radius of $\odot D$: 4; radius of $\odot E$: 2; pt. of tangency: (0, –4); eqn. of tangent line: $y = -4$
4. radius of $\odot M$: 1; radius of $\odot N$: 3; pt. of tangency: (–2, –2); eqn. of tangent line: $x = -2$
5. 385,734 km
6. 7.8 m
7. 50 ft

### Practice C

1. Possible answer: Draw $\overline{AB}$. A tangent segment is perpendicular to a radius at the point of tangency. So $\angle ACD$ and $\angle BDC$ are right angles. Two segments perpendicular to the same segment are parallel, so $\overline{AC}$ and $\overline{BD}$ are parallel. Because $\overline{AC}$ and $\overline{BD}$ are radii of $\odot A$ and $\odot B$, they are congruent. Therefore *ABDC* is a parallelogram. Opposite sides in a parallelogram are congruent, so $\overline{CD} \cong \overline{AB}$. Similar reasoning will show that $\overline{EF} \cong \overline{AB}$. By the Transitive Property of Congruence, $\overline{CD} \cong \overline{EF}$.
2. Possible answer: It is given that $\overline{RS}$ and $\overline{TU}$ are not parallel, so they must meet at some point. Call this point *X*. $\overline{XR}$ and $\overline{XT}$ are tangent to $\odot P$ and $\odot Q$. Because tangent segments from a common point to a circle are congruent, $XR = XT$ and $XS = XU$. The Segment Addition Postulate shows that $XR = XS + SR$ and $XT = XU + UT$. Thus, by the Transitive Property, $XS + SR = XU + UT$. By the Addition Property of Equality, $SR = UT$, and therefore $\overline{RS} \cong \overline{TU}$.
3. Possible answer: It is given that $\overline{IM}$ and $\overline{JL}$ are tangent segments. They intersect at point *K*. Because tangent segments from a common point to a circle are congruent, $KI = KL$ and $KM = KJ$. By the Addition Property of Equality, $KI + KM = KL + KJ$. The Segment Addition Postulate shows that $IM = KI + KM$ and $JL = KL + KJ$. Thus, by the Transitive Property of Equality, $IM = JL$ and therefore $\overline{IM} \cong \overline{JL}$.
4. 50 m
5. 8.5 ft or 16.5 ft

### Reteach

1. chord: $\overline{FG}$; secant: $\ell$; tangent: *m*; diam.: $\overline{FG}$; radii: $\overline{HF}$ and $\overline{HG}$
2. chord: $\overline{LM}$; secant: $\overleftrightarrow{LM}$; tangent: $\overleftrightarrow{MN}$; radius: $\overline{JK}$
3. $\odot N$: $r = 3$; $\odot P$: $r = 1$; pt. of tangency: (–1, –2); tangent line: $y = -2$
4. $\odot S$: $r = 4$; $\odot T$: $r = 2$; pt. of tangency: (7, 0); tangent line: $x = 7$
5. 6
6. 14
7. 10
8. 19

### Challenge

1. $ST = SU$
2. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency (Theorem 11-1-1).
3. If two segments are tangent to a circle from the same external point, then the

Name ________________________________________ Date ___________________ Class___________________

LESSON 12-1

# Reading Strategies

## *Focus on Vocabulary*

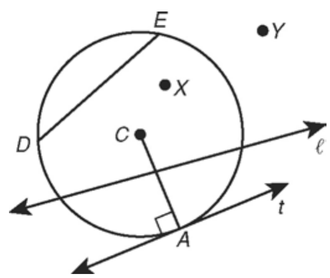
The diagram below describes vocabulary words that are used with circles.

A **secant** is a line that intersects a circle at two points. Line $\ell$ is a secant.

A **chord** is a segment whose endpoints lie on a circle. $\overline{DE}$ is a chord.

A **tangent** is a line in the same plane as the circle that intersects it at exactly one point. Line *t* is a tangent.

The **interior** of a circle is the set of all points inside the circle. Point *X* is in the interior of the circle.




The **point of tangency** is the point where the tangent and the circle intersect. Point *A* is a point of tangency.

The **exterior** of a circle is the set of all points outside the circle. Point *Y* is in the exterior of the circle.

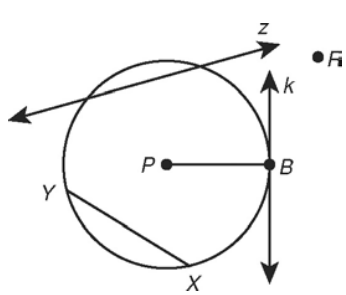
A tangent line is **perpendicular** to the **radius** of a circle drawn to the point of tangency. $\overline{CA} \perp$ line *t*
A line that is perpendicular to the radius of a circle at a point on the circle is a tangent line to the circle.

**Answer the following.**

1. The ___________________ of a circle is the set of all points inside the circle.
2. A ___________________ is a line that intersects a circle at two points.
3. Look at circle *C* above. Why is line *t* not a secant?

_____________________________________________________________________________

**Use circle *P* to identify each line, segment, or point.**




4. secant line ___________________
5. point of tangency ___________________
6. tangent line ___________________
7. chord ___________________
8. a point in the exterior of the circle ___________________
9. a point in the interior of the circle ___________________

segments are congruent (Theorem 11-1-3).

4. Reflexive Property
5. Hypotenuse-Leg or SSS
6. CPCTC
7. $\frac{PQ}{PS} = \frac{QR}{RS}$
8. $\frac{15}{x} = \frac{22}{30 - x}$; $12.16 = PS$

**Problem Solving**

1. 339 km
2. 27
3. 68 cm
4. 47
5. C
6. H
7. B

**Reading Strategies**

1. interior
2. secant
3. because it intersects the circle at only one point
4. line $z$
5. point $B$
6. line $k$
7. $\overline{XY}$
8. point $R$
9. point $P$

## 12-2 ARCS AND CHORDS

**Practice A**

1. 90°
2. 144°
3. 108°
4. 234°
5. chords
6. chords
7. adjacent
8. perpendicular bisector
9. central angles
10. radius or diameter
11. 150°
12. 240°
13. 125°
14. 11 cm
15. 30 in.
16. 96 m

**Practice B**

1. 115.2°
2. 93.6°
3. 126°
4. 90°
5. 3.6°
6. 241.2°
7. 125°; 227°
8. 67°; 203°
9. 102°
10. 49 cm
11. 76.3 mi
12. 4.9 km

**Practice C**

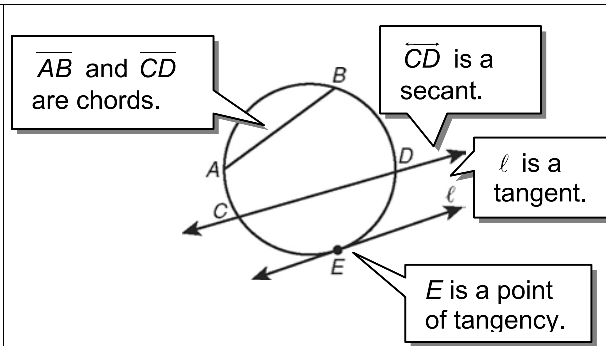
1. Possible answer: Draw $\overline{AF}$ and $\overline{EF}$. It is given that $\overline{AE}$ is perpendicular to $\overline{FG}$. Therefore $\angle ACF$ and $\angle ECF$ are right angles and are congruent. It is also given that $\overline{AC} \cong \overline{EC}$. $\overline{FC} \cong \overline{FC}$ by the Reflexive Property of Congruence. So $\triangle AFC \cong \triangle EFC$ by SAS. By CPCTC, $\overline{AF} \cong \overline{EF}$. $\overline{AF}$ and $\overline{EF}$ are radii of $\odot A$ and $\odot E$, and circles with congruent radii are congruent circles, so $\odot A \cong \odot E$.
2. Possible answer: Draw $\overline{RU}, \overline{PR}, \overline{PU}, \overline{QR}$, and $\overline{QU}$. Because $\overline{PR}$ and $\overline{PU}$ are radii of $\odot P$, they are congruent and $\triangle PRU$ is isosceles. Similar reasoning shows that $\triangle QRU$ is also isosceles. By the Base Angles Theorem, $\angle PUR \cong \angle PRU$ and $\angle QRU \cong \angle QUR$. So $m\angle PUR$ and $m\angle PRU$ are each equal to $\frac{1}{2}(180 - m\angle RPU)$. Also, $m\angle QRU$ and $m\angle QUR$ are each equal to $\frac{1}{2}(180 - m\angle RQU)$. It is given that $\overparen{RSU} \cong \overparen{RTU}$, so $m\overparen{RSU} = m\overparen{RTU}$. The measure of an arc is equal to the measure of its central angle, so $m\angle RPU = m\angle RQU$. Substitution shows that $m\angle PUR = m\angle PRU = m\angle QRU = m\angle QUR$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. So $\triangle PRU \cong \triangle QRU$ by SAS. By CPCTC, $\overline{PR} \cong \overline{QR}$ and circles with congruent radii are congruent circles, so $\odot P \cong \odot Q$.
3. 60°
4. 19.2°
5. 53.1°
6. 90°
7. 103.5°
8. 180°
9. $0.2r$
10. $0.8r$
11. $1.9r$

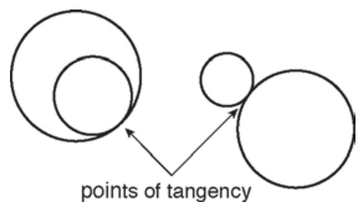
Name ______________________ Date ______________ Class ______________

LESSON 12-1

# Reteach

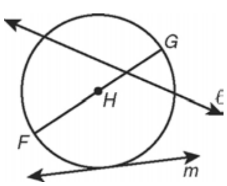
## *Lines That Intersect Circles*

| Lines and Segments That Intersect Circles | |
|---|---|
| • A **chord** is a segment whose endpoints lie on a circle.<br>• A **secant** is a line that intersects a circle at two points.<br>• A **tangent** is a line in the same plane as a circle that intersects the circle at exactly one point, called the **point of tangency.**<br>• Radii and diameters also intersect circles. |  |

| Tangent Circles | |
|---|---|
| Two coplanar circles that intersect at exactly one point are called **tangent circles**. |  |

**Identify each line or segment that intersects each circle.**

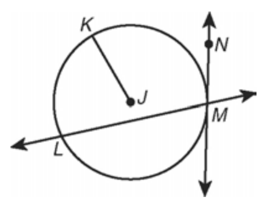
1.




________________________________

________________________________

2.

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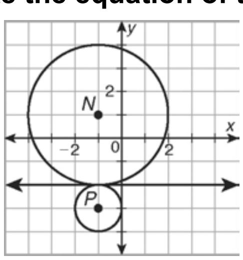
**Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at that point.**

3.

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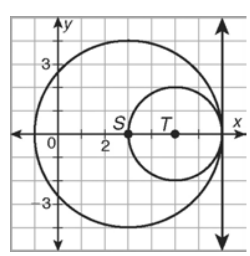
4.

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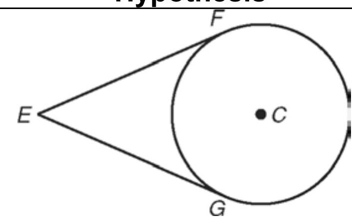
Name ______________________ Date ______________ Class______________

LESSON 12-1

# Reteach

## *Lines That Intersect Circles* continued

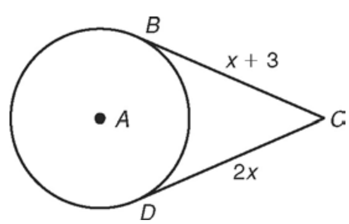
| Theorem | Hypothesis | Conclusion |
|---|---|---|
| If two segments are tangent to a circle from the same external point, then the segments are congruent. |  $\overline{EF}$ and $\overline{EG}$ are tangent to $\odot C$. | $\overline{EF} \cong \overline{EG}$ |

**In the figure above, $EF = 2y$ and $EG = y + 8$. Find $EF$.**

| | |
|---|---|
| $EF = EG$ | 2 segs. tangent to $\odot$ from same ext. pt. $\rightarrow$ segs. $\cong$. |
| $2y = y + 8$ | Substitute $2y$ for $EF$ and $y + 8$ for $EG$. |
| $y = 8$ | Subtract $y$ from each side. |
| $EF = 2(8)$ | $EF = 2y$; substitute 8 for $y$. |
| $= 16$ | Simplify. |

**The segments in each figure are tangent to the circle. Find each length.**

5. $BC$

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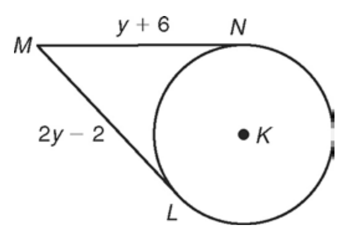
6. $LM$

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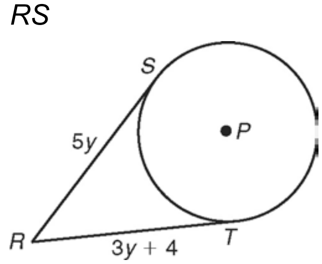
7. $RS$

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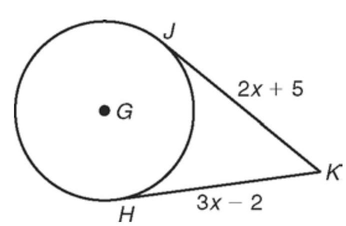
8. $JK$

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# Answers for the chapter Circles

## 12-1 LINES THAT INTERSECT CIRCLES

### Practice A

| | |
|---|---|
| 1. E | 2. B |
| 3. A | 4. C |
| 5. D | 6. 2; 1 |
| 7. 4; 2 | 8. $x = 4$ |
| 9. tangent | 10. congruent |
| 11. perpendicular | 12. 5 in. |
| 13. 24 | 14. 21 |

### Practice B

1. chords: $\overline{BC}$; secant: $\overleftrightarrow{BC}$; tangent: $\ell$; diam.: $\overline{BC}$; radii: $\overline{AB}$, $\overline{AC}$
2. chords: $\overline{RQ}$, $\overline{ST}$; secant: $\overleftrightarrow{ST}$; tangent: $\overline{UV}$; diam.: $\overline{RQ}$; radii: $\overline{PQ}$, $\overline{PR}$, $\overline{PU}$
3. radius of ⊙*D*: 4; radius of ⊙*E*: 2; pt. of tangency: (0, –4); eqn. of tangent line: $y = -4$
4. radius of ⊙*M*: 1; radius of ⊙*N*: 3; pt. of tangency: (–2, –2); eqn. of tangent line: $x = -2$
5. 385,734 km
6. 7.8 m
7. 50 ft

### Practice C

1. Possible answer: Draw $\overline{AB}$. A tangent segment is perpendicular to a radius at the point of tangency. So $\angle ACD$ and $\angle BDC$ are right angles. Two segments perpendicular to the same segment are parallel, so $\overline{AC}$ and $\overline{BD}$ are parallel. Because $\overline{AC}$ and $\overline{BD}$ are radii of ⊙*A* and ⊙*B*, they are congruent. Therefore *ABDC* is a parallelogram. Opposite sides in a parallelogram are congruent, so $\overline{CD} \cong \overline{AB}$. Similar reasoning will show that $\overline{EF} \cong \overline{AB}$. By the Transitive Property of Congruence, $\overline{CD} \cong \overline{EF}$.
2. Possible answer: It is given that $\overline{RS}$ and $\overline{TU}$ are not parallel, so they must meet at some point. Call this point *X*. $\overline{XR}$ and $\overline{XT}$ are tangent to ⊙*P* and ⊙*Q*. Because tangent segments from a common point to a circle are congruent, $XR = XT$ and $XS = XU$. The Segment Addition Postulate shows that $XR = XS + SR$ and $XT = XU + UT$. Thus, by the Transitive Property, $XS + SR = XU + UT$. By the Addition Property of Equality, $SR = UT$, and therefore $\overline{RS} \cong \overline{TU}$.
3. Possible answer: It is given that $\overline{IM}$ and $\overline{JL}$ are tangent segments. They intersect at point *K*. Because tangent segments from a common point to a circle are congruent, $KI = KL$ and $KM = KJ$. By the Addition Property of Equality, $KI + KM = KL + KJ$. The Segment Addition Postulate shows that $IM = KI + KM$ and $JL = KL + KJ$. Thus, by the Transitive Property of Equality, $IM = JL$ and therefore $\overline{IM} \cong \overline{JL}$.
4. 50 m
5. 8.5 ft or 16.5 ft

### Reteach

1. chord: $\overline{FG}$; secant: $\ell$; tangent: *m*; diam.: $\overline{FG}$; radii: $\overline{HF}$ and $\overline{HG}$
2. chord: $\overline{LM}$; secant: $\overleftrightarrow{LM}$; tangent: $\overleftrightarrow{MN}$; radius: $\overline{JK}$
3. ⊙*N*: $r = 3$; ⊙*P*: $r = 1$; pt. of tangency: (–1, –2); tangent line: $y = -2$
4. ⊙*S*: $r = 4$; ⊙*T*: $r = 2$; pt. of tangency: (7, 0); tangent line: $x = 7$
5. 6
6. 14
7. 10
8. 19

### Challenge

1. $ST = SU$
2. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency (Theorem 11-1-1).
3. If two segments are tangent to a circle from the same external point, then the

# Geometry

## Chapter 12: Circle

### Lesson 2 - Arcs and Chords

**Standards**

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

**Objectives**

Apply properties of arcs.

Apply properties of chords.

**Materials**

Protractor, compass, text book, toolbox, How To Make A Circle Graph

**Vocabulary**

Central angle – an angle whose vertex is the center of the circle

Arc – an unbroken part of a circle consisting of two endpoints and all point between.

Minor Arc – an arc whose points are on or in the interior of the central angle.

Major Arc – an arc whose points are on or in the exterior of the central angle.

Semi-Circle – an arc of a circle whose endpoints lie on the diameter

Adjacent arcs – arcs that intersect at exactly one point on the same circle.

Congruent Arcs – Two arcs that are in the same or congruent circles and have the same measure.

**Formulas**

m of arc AC = m<ABC

m of arc ADC = 360 – m<ABC

m of arc EFG = 180 when segment EG is the diameter

m of arc ABC = m of arc AB + m of arc BC

**Symbols**

None

**Postulates**

Arc Addition Postulate – the measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

**Theorems**

12-2-2 - In a circle or congruent circles:

1) congruent central angles have congruent chords

2) congruent chords have congruent arcs

3) congruent arcs have congruent central angles

12-2-3 – In a circle, if a radius or diameter is perpendicular to a chord, then it bisects the chord and its arc.

12-2-4 – In a circle, the perpendicular bisector of a chord is a radius or diameter.

**Who Uses This?**

Market analysts use circle graphs to compare sales of different products.

Comparisons of parts to the whole.

**Discussion**

Review how to create a circle graph. Have cadets write the steps in their ToolBox. Make sure to cover that 360 degrees is a circle and is used when calculating the angle measurement.

**Guided Practice**

Have cadets make a circle graph from data provided. What each cadet as they work. Make sure the percentages are correct and that they are graphing the angle and not the percentage column.

When all graphs look reasonable, look at Example 1 and discuss all of the areas. Cadets will do Check-It-Out 1.

Update the ToolBox with vocabulary for the chapter.

Review Example 2 discussing how two arcs total the entire arc and how it is like a log that has two different lengths that total the entire log.

Cadets will work on Check-it-Out 2.

Write Congruence Theorems in ToolBox and discuss how each one of them just makes sense when we read it. Look at Example 3. Discuss once again how to do the math – solving equations with variables on both sides. Have cadets do Check-It-Out 3.

Look at the relationships between radii and chords and then look at Example 4. Review again the squaring and square rooting of numbers. Cadets will do Check-it-Out 4.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 19 – 40, 45, 47 – 50

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 19 – 51

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 19 – 44, 46 – 53

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 20, 26, 30, 32, 38, 40

If this is a review and students have it, then do the following: Page 806 – 807 problems #5-35 – odd problems

**Homework**

Page 74 and 158 May be replaced with different work based on needs. Replaced by additional resources listed above.

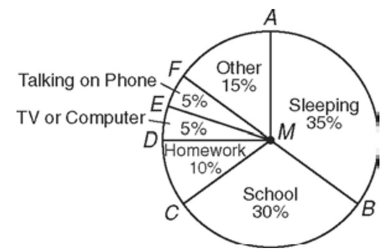
Name ______________________ Date ______________ Class______________

LESSON 12-2

# Practice A

## *Arcs and Chords*

**The circle graph shows the number of hours Rae spends on each activity in a typical weekday. Use the graph to find each of the following.**

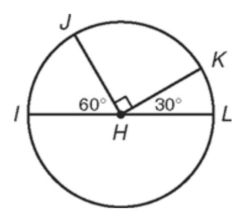



1. m$\angle AMD$ = ______________________
2. m$\angle DMB$ = ______________________
3. m$\widehat{BC}$ = ______________________
4. m$\widehat{CBA}$ = ______________________

**In Exercises 5–10, fill in the blanks to complete each postulate or theorem.**

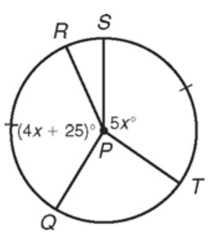
5. In a circle or congruent circles, congruent central angles have congruent ______________________.
6. In a circle or congruent circles, congruent ______________________ have congruent arcs.
7. The measure of an arc formed by two ______________________ arcs is the sum of the measures of the two arcs.
8. In a circle, the ______________________ of a chord is a radius (or diameter).
9. In a circle or congruent circles, congruent arcs have congruent ______________________.
10. In a circle, if the ______________________ is perpendicular to a chord, then it bisects the chord and its arc.

**Find each measure.**



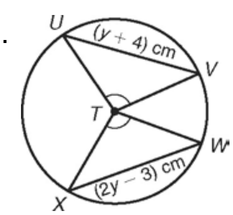

11. m$\widehat{IK}$ = ______________________
12. m$\widehat{JIL}$ = ______________________

13.




m$\widehat{QR}$ = m$\widehat{ST}$ . Find m$\angle QPR$.___________

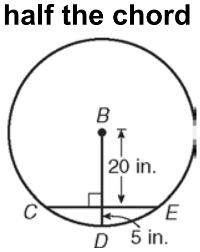
14.




$\angle UTV \cong \angle XTW$. Find $WX$.___________

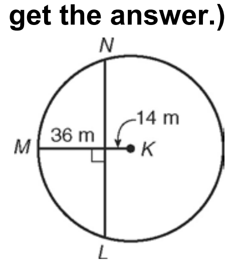
**Find the length of each chord. (*Hint:* Use the Pythagorean Theorem to find half the chord length, and then double that to get the answer.)**

15.




$CE$ = _________

16.




LN = __________

segments are congruent (Theorem 11-1-3).

4. Reflexive Property
5. Hypotenuse-Leg or SSS
6. CPCTC
7. $\frac{PQ}{PS} = \frac{QR}{RS}$
8. $\frac{15}{x} = \frac{22}{30 - x}$; $12.16 = PS$

**Problem Solving**

1. 339 km
2. 27
3. 68 cm
4. 47
5. C
6. H
7. B

**Reading Strategies**

1. interior
2. secant
3. because it intersects the circle at only one point
4. line $z$
5. point $B$
6. line $k$
7. $\overline{XY}$
8. point $R$
9. point $P$

## 12-2 ARCS AND CHORDS

**Practice A**

1. 90°
2. 144°
3. 108°
4. 234°
5. chords
6. chords
7. adjacent
8. perpendicular bisector
9. central angles
10. radius or diameter
11. 150°
12. 240°
13. 125°
14. 11 cm
15. 30 in.
16. 96 m

**Practice B**

1. 115.2°
2. 93.6°
3. 126°
4. 90°
5. 3.6°
6. 241.2°
7. 125°; 227°
8. 67°; 203°
9. 102°
10. 49 cm
11. 76.3 mi
12. 4.9 km

**Practice C**

1. Possible answer: Draw $\overline{AF}$ and $\overline{EF}$. It is given that $\overline{AE}$ is perpendicular to $\overline{FG}$. Therefore $\angle ACF$ and $\angle ECF$ are right angles and are congruent. It is also given that $\overline{AC} \cong \overline{EC}$. $\overline{FC} \cong \overline{FC}$ by the Reflexive Property of Congruence. So $\triangle AFC \cong \triangle EFC$ by SAS. By CPCTC, $\overline{AF} \cong \overline{EF}$. $\overline{AF}$ and $\overline{EF}$ are radii of $\odot A$ and $\odot E$, and circles with congruent radii are congruent circles, so $\odot A \cong \odot E$.
2. Possible answer: Draw $\overline{RU}, \overline{PR}, \overline{PU}, \overline{QR}$, and $\overline{QU}$. Because $\overline{PR}$ and $\overline{PU}$ are radii of $\odot P$, they are congruent and $\triangle PRU$ is isosceles. Similar reasoning shows that $\triangle QRU$ is also isosceles. By the Base Angles Theorem, $\angle PUR \cong \angle PRU$ and $\angle QRU \cong \angle QUR$. So $m\angle PUR$ and $m\angle PRU$ are each equal to $\frac{1}{2}(180 - m\angle RPU)$. Also, $m\angle QRU$ and $m\angle QUR$ are each equal to $\frac{1}{2}(180 - m\angle RQU)$. It is given that $\overparen{RSU} \cong \overparen{RTU}$, so $m\overparen{RSU} = m\overparen{RTU}$. The measure of an arc is equal to the measure of its central angle, so $m\angle RPU = m\angle RQU$. Substitution shows that $m\angle PUR = m\angle PRU = m\angle QRU = m\angle QUR$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. So $\triangle PRU \cong \triangle QRU$ by SAS. By CPCTC, $\overline{PR} \cong \overline{QR}$ and circles with congruent radii are congruent circles, so $\odot P \cong \odot Q$.
3. 60°
4. 19.2°
5. 53.1°
6. 90°
7. 103.5°
8. 180°
9. $0.2r$
10. $0.8r$
11. $1.9r$

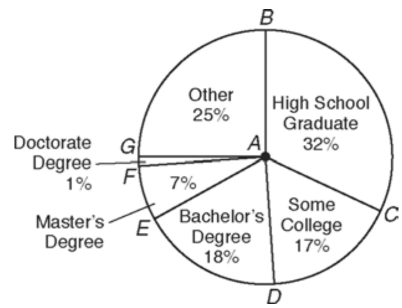
Name ______________________________ Date ______________ Class______________

LESSON 12-2

# Practice B

## *Arcs and Chords*

**The circle graph shows data collected by the U.S. Census Bureau in 2004 on the highest completed educational level for people 25 and older. Use the graph to find each of the following. Round to the nearest tenth if necessary.**

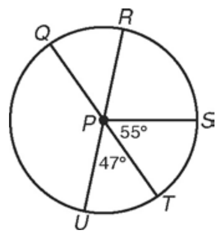



1. $m\angle CAB$ ____________ 2. $m\angle DAG$ ____________

3. $m\angle EAC$ ____________ 4. $m\overset{\frown}{BG}$ ____________

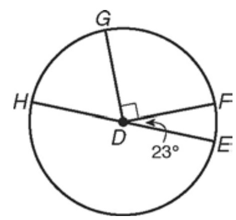
5. $m\overset{\frown}{GF}$ ____________ 6. $m\overset{\frown}{BDE}$ ____________

**Find each measure.**

7. 

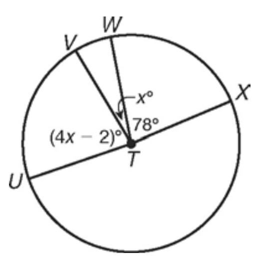

$m\overset{\frown}{QS}$ __________

$m\overset{\frown}{RQT}$ __________

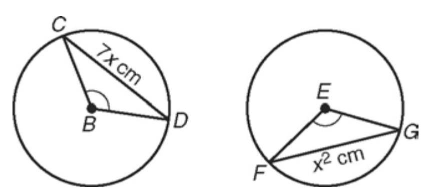
8. 


$m\overset{\frown}{HG}$ __________

$m\overset{\frown}{FEH}$ __________

9. 

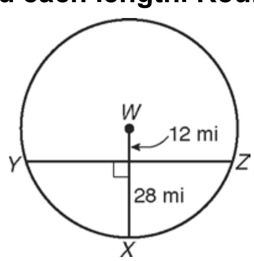

Find $m\angle UTW$. __________

10. 

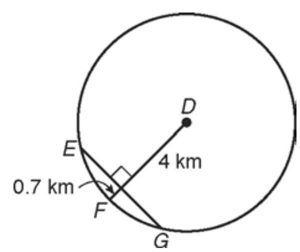

$\odot L \cong \odot E$, and $\angle CBD \cong \angle FEG$.

Find $FG$. __________

**Find each length. Round to the nearest tenth.**

11. 


$ZY$ ______________________________

12. 


$EG$ ______________________________

segments are congruent (Theorem 11-1-3).

4. Reflexive Property
5. Hypotenuse-Leg or SSS
6. CPCTC
7. $\frac{PQ}{PS} = \frac{QR}{RS}$
8. $\frac{15}{x} = \frac{22}{30 - x}$; $12.16 = PS$

**Problem Solving**

1. 339 km
2. 27
3. 68 cm
4. 47
5. C
6. H
7. B

**Reading Strategies**

1. interior
2. secant
3. because it intersects the circle at only one point
4. line *z*
5. point *B*
6. line *k*
7. $\overline{XY}$
8. point *R*
9. point *P*

## 12-2 ARCS AND CHORDS

**Practice A**

1. 90°
2. 144°
3. 108°
4. 234°
5. chords
6. chords
7. adjacent
8. perpendicular bisector
9. central angles
10. radius or diameter
11. 150°
12. 240°
13. 125°
14. 11 cm
15. 30 in.
16. 96 m

**Practice B**

1. 115.2°
2. 93.6°
3. 126°
4. 90°
5. 3.6°
6. 241.2°
7. 125°; 227°
8. 67°; 203°
9. 102°
10. 49 cm
11. 76.3 mi
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**Practice C**

1. Possible answer: Draw $\overline{AF}$ and $\overline{EF}$. It is given that $\overline{AE}$ is perpendicular to $\overline{FG}$. Therefore $\angle ACF$ and $\angle ECF$ are right angles and are congruent. It is also given that $\overline{AC} \cong \overline{EC}$. $\overline{FC} \cong \overline{FC}$ by the Reflexive Property of Congruence. So $\triangle AFC \cong \triangle EFC$ by SAS. By CPCTC, $\overline{AF} \cong \overline{EF}$. $\overline{AF}$ and $\overline{EF}$ are radii of $\odot A$ and $\odot E$, and circles with congruent radii are congruent circles, so $\odot A \cong \odot E$.
2. Possible answer: Draw $\overline{RU}$, $\overline{PR}$, $\overline{PU}$, $\overline{QR}$, and $\overline{QU}$. Because $\overline{PR}$ and $\overline{PU}$ are radii of $\odot P$, they are congruent and $\triangle PRU$ is isosceles. Similar reasoning shows that $\triangle QRU$ is also isosceles. By the Base Angles Theorem, $\angle PUR \cong \angle PRU$ and $\angle QRU \cong \angle QUR$. So $m\angle PUR$ and $m\angle PRU$ are each equal to $\frac{1}{2}(180 - m\angle RPU)$. Also, $m\angle QRU$ and $m\angle QUR$ are each equal to $\frac{1}{2}(180 - m\angle RQU)$. It is given that $\overset{\frown}{RSU} \cong \overset{\frown}{RTU}$, so $m\overset{\frown}{RSU} = m\overset{\frown}{RTU}$. The measure of an arc is equal to the measure of its central angle, so $m\angle RPU = m\angle RQU$. Substitution shows that $m\angle PUR = m\angle PRU = m\angle QRU = m\angle QUR$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. So $\triangle PRU \cong \triangle QRU$ by SAS. By CPCTC, $\overline{PR} \cong \overline{QR}$ and circles with congruent radii are congruent circles, so $\odot P \cong \odot Q$.
3. 60°
4. 19.2°
5. 53.1°
6. 90°
7. 103.5°
8. 180°
9. 0.2*r*
10. 0.8*r*
11. 1.9*r*

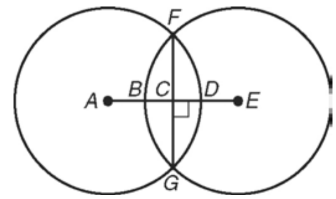
Name ________________________________ Date ______________ Class______________

LESSON 12-2

# Practice C

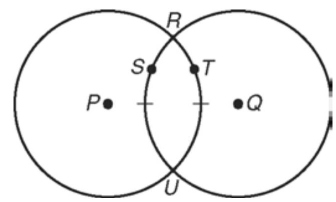
## *Arcs and Chords*

**Write proofs for Exercises 1 and 2.**



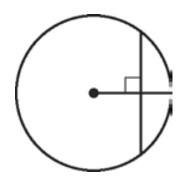

1. **Given:** $\overline{AC} \cong \overline{EC}$, $\overline{AE} \perp \overline{FG}$

   **Prove:** $\odot A \cong \odot E$




2. **Given:** $\widehat{RSU} \cong \widehat{RTU}$

   **Prove:** $\odot P \cong \odot Q$



**Give the degree measure of the arc intercepted by the chord described in Exercises 3–8. The figure is given for reference. Round to the nearest tenth if necessary.**

3. a chord congruent to the radius ____________
4. a chord one-third the length of the radius ____________
5. a chord congruent to the segment from the center to the chord ____________
6. a chord twice the length of the segment from the center to the chord ____________
7. a chord one-fourth the length of the circumference ____________
8. a chord $\frac{1}{\pi}$ multiplied by the length of the circumference ____________

**Find the length of a chord that intercepts an arc of each given measure. Give your answer in terms of the radius *r*. Round to the nearest tenth.**

9. 10° ______________ 10. 45° ______________ 11. 136° ______________

segments are congruent (Theorem 11-1-3).

4. Reflexive Property
5. Hypotenuse-Leg or SSS
6. CPCTC
7. $\frac{PQ}{PS} = \frac{QR}{RS}$
8. $\frac{15}{x} = \frac{22}{30 - x}$; $12.16 = PS$

**Problem Solving**

1. 339 km
2. 27
3. 68 cm
4. 47
5. C
6. H
7. B

**Reading Strategies**

1. interior
2. secant
3. because it intersects the circle at only one point
4. line $z$
5. point $B$
6. line $k$
7. $\overline{XY}$
8. point $R$
9. point $P$

## 12-2 ARCS AND CHORDS

**Practice A**

1. 90°
2. 144°
3. 108°
4. 234°
5. chords
6. chords
7. adjacent
8. perpendicular bisector
9. central angles
10. radius or diameter
11. 150°
12. 240°
13. 125°
14. 11 cm
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16. 96 m

**Practice B**

1. 115.2°
2. 93.6°
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**Practice C**

1. Possible answer: Draw $\overline{AF}$ and $\overline{EF}$. It is given that $\overline{AE}$ is perpendicular to $\overline{FG}$. Therefore $\angle ACF$ and $\angle ECF$ are right angles and are congruent. It is also given that $\overline{AC} \cong \overline{EC}$. $\overline{FC} \cong \overline{FC}$ by the Reflexive Property of Congruence. So $\triangle AFC \cong \triangle EFC$ by SAS. By CPCTC, $\overline{AF} \cong \overline{EF}$. $\overline{AF}$ and $\overline{EF}$ are radii of $\odot A$ and $\odot E$, and circles with congruent radii are congruent circles, so $\odot A \cong \odot E$.
2. Possible answer: Draw $\overline{RU}, \overline{PR}, \overline{PU}, \overline{QR}$, and $\overline{QU}$. Because $\overline{PR}$ and $\overline{PU}$ are radii of $\odot P$, they are congruent and $\triangle PRU$ is isosceles. Similar reasoning shows that $\triangle QRU$ is also isosceles. By the Base Angles Theorem, $\angle PUR \cong \angle PRU$ and $\angle QRU \cong \angle QUR$. So m$\angle PUR$ and m$\angle PRU$ are each equal to $\frac{1}{2}$(180 – m$\angle RPU$). Also, m$\angle QRU$ and m$\angle QUR$ are each equal to $\frac{1}{2}$(180 – m$\angle RQU$). It is given that $\overset{\frown}{RSU} \cong \overset{\frown}{RTU}$, so m$\overset{\frown}{RSU}$ = m$\overset{\frown}{RTU}$. The measure of an arc is equal to the measure of its central angle, so m$\angle RPU$ = m$\angle RQU$. Substitution shows that m$\angle PUR$ = m$\angle PRU$ = m$\angle QRU$ = m$\angle QUR$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. So $\triangle PRU \cong \triangle QRU$ by SAS. By CPCTC, $\overline{PR} \cong \overline{QR}$ and circles with congruent radii are congruent circles, so $\odot P \cong \odot Q$.
3. 60°
4. 19.2°
5. 53.1°
6. 90°
7. 103.5°
8. 180°
9. 0.2$r$
10. 0.8$r$
11. 1.9$r$

Name ______________________ Date ______________ Class______________

LESSON 12-2

# Problem Solving

## *Arcs and Chords*

1. Circle $D$ has center (–2, –7) and radius 7. What is the measure, in degrees, of the major arc that passes through points $H(-2, 0)$, $J(5, -7)$, and $K(-9, -7)$?

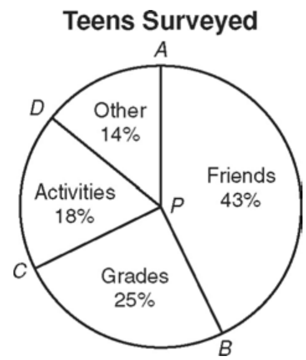
________________________

2. A circle graph is composed of sectors with central angles that measure $3x°$, $3x°$, $4x°$, and $5x°$. What is the measure, in degrees, of the smallest minor arcs?

________________________

**Use the following information for Exercises 3 and 4.**

The circle graph shows the results of a survey in which teens were asked what says the most about them at school. Find each of the following.




3. $m\widehat{AB}$

________________________

4. $m\angle APC$

________________________

**Choose the best answer.**

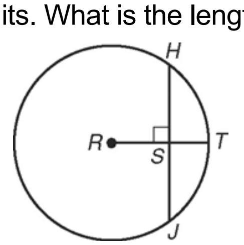
5. Students were asked to name their favorite cafeteria food. The results of the survey are shown in the table. In a circle graph showing these results, which is closest to the measure of the central angle for the section representing chicken tenders?

| Favorite Lunch | Number of Students |
|---|---|
| Pizza | 108 |
| Chicken tenders | 75 |
| Taco salad | 90 |
| Other | 54 |

A 21° C 83°

B 75° D 270°

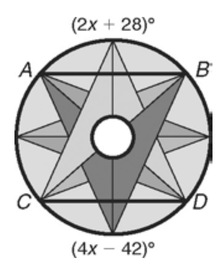
6. The diameter of $\odot R$ is 15 units, and $HJ = 12$ units. What is the length of $\overline{ST}$?




F 2.1 units H 4.5 units

G 3 units J 9.6 units

7. In the stained glass window, $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. What is $m\widehat{CBD}$?




A 35° C 98°

B 70° D 262°

### Reteach

1. 63° 2. 117°
3. 130° 4. 140°
5. 75° 6. 225°
7. 88° 8. 21
9. 16.0 10. 30.0

### Challenge

1. 86° 2. 47°
3. 43° 4. 14 cm
5. a. $\sin 43° = \frac{AD}{14}$
   b. AD ≈ 9.5 cm
   c. AB ≈ 19.1 cm
6. 1.9 in. 7. 3.0 m
8. 1.3 ft 9. $\ell = d\left(\sin\left[\frac{n}{2}\right]^{\circ}\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in. 11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in. 13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.
    $A = \frac{1}{4}nd^2\left(\cos\left[\frac{180}{n}\right]^{\circ}\right)\left(\sin\left[\frac{180}{n}\right]^{\circ}\right)$

### Problem Solving

1. 270° 2. 72°
3. 154.8° 4. 115.2°
5. C 6. G
7. D

### Reading Strategies

1. 60° 2. 360°
3. central angles 4. 32°
5. 263° 6. 328°
7. 295° 8. 32°
9. 65°

## 12-3 SECTOR AREA AND ARC LENGTH

### Practice A

1. $\pi r^3\left(\frac{m°}{360°}\right)$ 2. $2\pi r\left(\frac{m°}{360°}\right)$
3. $9\pi$ mm$^2$; 28.27 mm$^2$
4. $27\pi$ mi$^2$; 84.82 mi$^2$ 5. 982 yd$^2$
6. 1173 yd$^2$ 7. $25\pi$ in$^2$
8. 50 in$^2$ 9. 28.54 in$^2$
10. $4\pi$ cm; 12.57 cm 11. $3\pi$ km; 9.42 km

### Practice B

1. sector *BAC* 126 $\pi$ mm$^2$; 395.84 mm$^2$
2. sector *UTV* 30 $\pi$ in$^2$; 94.25 in$^2$
3. sector *KJL* $\pi$ ft$^2$; 3.14 ft$^2$
4. sector *FEG* $100\pi$ m$^2$; 314.16 m$^2$
5. 4.54 in$^2$ 6. 10.96 km$^2$
7. 24.47 yd$^2$ 8. 0.29 cm$^2$
9. 9.83 mi$^2$ 10. $\pi$ ft; 3.14 ft
11. $14\pi$ m; 43.98 m 12. $\frac{\pi}{2}$ mi; 1.57 mi
13. $10\pi$ mm; 31.42 mm

### Practice C

1. Possible answer: The area of a sector of a circle with radius *r* and central angle *m* is $A = \pi r^2\left(\frac{m}{360}\right)$. Half this area is $\pi r^2\left(\frac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\frac{m}{2}\right)$, and the length of the base is $2r\sin\left(\frac{m}{2}\right)$. The area of the triangle is $\frac{bh}{2}$ or $r^2\sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Set this equal to the area of half the sector:

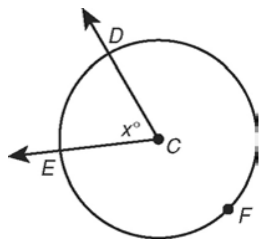
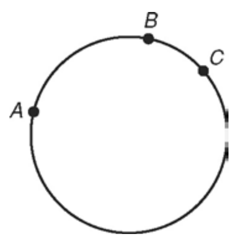
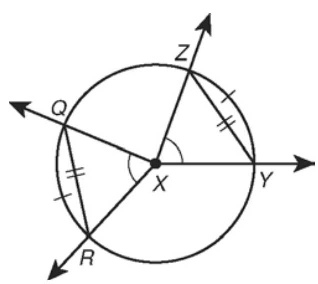
Name ______________________________ Date ______________ Class______________

LESSON 12-2

# Reading Strategies

## *Use a Table*

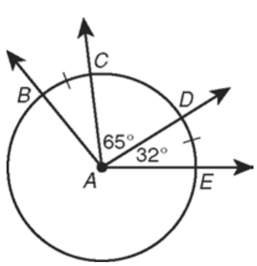
The table below shows some of the relationships among arcs, chords, and central angles.

| Words | Diagram | Mathematical Symbols |
|---|---|---|
| A minor arc is equal to the measure of its central angle. |  | $m\overparen{DE} = m\angle DCE = x°$ |
| A major arc is equal to 360° minus the measure of its central angle. | | $m\overparen{DFE} = 360° - m\angle DCE$ <br> $= 360° - x°$ |
| The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. |  | $m\overparen{ABC} = m\overparen{AB} + m\overparen{BC}$ |
| Congruent central angles have congruent chords. |  | $\overline{RQ} \cong \overline{YZ}$ |
| Congruent chords have congruent arcs. | | $\overparen{RQ} \cong \overparen{YZ}$ |
| Congruent arcs have congruent central angles. | | $\angle QXR \cong \angle ZXY$ |

**Answer the following.**

1. The measure of a central angle is 60°. What is the measure of its minor arc? ________________
2. What will be the sum of a central angle's minor arc and major arc? ________________
3. Congruent ________________ have congruent chords.

**Use circle *A* to find each measure.**




4. $m\overparen{DE}$ ____________
5. $m\overparen{CBE}$ ____________
6. $m\overparen{EBD}$ ____________
7. $m\overparen{CBD}$ ____________
8. $m\angle CAB$ ____________
9. $m\overparen{CD}$ ____________

**Reteach**

1. 63°
2. 117°
3. 130°
4. 140°
5. 75°
6. 225°
7. 88°
8. 21
9. 16.0
10. 30.0

**Challenge**

1. 86°
2. 47°
3. 43°
4. 14 cm
5. a. $\sin 43° = \dfrac{AD}{14}$

   b. AD ≈ 9.5 cm

   c. AB ≈ 19.1 cm
6. 1.9 in.
7. 3.0 m
8. 1.3 ft
9. $\ell = d\left(\sin\left[\dfrac{n}{2}\right]^\circ\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in.
11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in.
13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.

    $A = \dfrac{1}{4}nd^2\left(\cos\left[\dfrac{180}{n}\right]^\circ\right)\left(\sin\left[\dfrac{180}{n}\right]^\circ\right)$

**Problem Solving**

1. 270°
2. 72°
3. 154.8°
4. 115.2°
5. C
6. G
7. D

**Reading Strategies**

1. 60°
2. 360°
3. central angles
4. 32°
5. 263°
6. 328°
7. 295°
8. 32°
9. 65°

## 12-3 SECTOR AREA AND ARC LENGTH

**Practice A**

1. $\pi r^3\left(\dfrac{m°}{360°}\right)$
2. $2\pi r\left(\dfrac{m°}{360°}\right)$
3. 9π mm$^2$; 28.27 mm$^2$
4. 27π mi$^2$; 84.82 mi$^2$
5. 982 yd$^2$
6. 1173 yd$^2$
7. 25π in$^2$
8. 50 in$^2$
9. 28.54 in$^2$
10. 4π cm; 12.57 cm
11. 3π km; 9.42 km

**Practice B**

1. sector *BAC* 126 π mm$^2$; 395.84 mm$^2$
2. sector *UTV* 30 π in$^2$; 94.25 in$^2$
3. sector *KJL* π ft$^2$; 3.14 ft$^2$
4. sector *FEG* 100π m$^2$; 314.16 m$^2$
5. 4.54 in$^2$
6. 10.96 km$^2$
7. 24.47 yd$^2$
8. 0.29 cm$^2$
9. 9.83 mi$^2$
10. π ft; 3.14 ft
11. 14π m; 43.98 m
12. $\dfrac{\pi}{2}$ mi; 1.57 mi
13. 10π mm; 31.42 mm

**Practice C**

1. Possible answer: The area of a sector of a circle with radius *r* and central angle *m* is $A = \pi r^2\left(\dfrac{m}{360}\right)$. Half this area is $\pi r^2\left(\dfrac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\dfrac{m}{2}\right)$, and the length of the base is $2r\sin\left(\dfrac{m}{2}\right)$. The area of the triangle is $\dfrac{bh}{2}$ or $r^2\sin\left(\dfrac{m}{2}\right)\cos\left(\dfrac{m}{2}\right)$. Set this equal to the area of half the sector:

Name ______________________________ Date ______________ Class______________

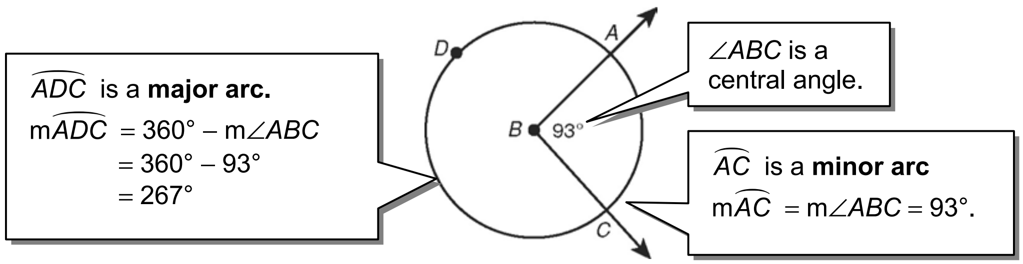
**LESSON 12-2**

# Reteach

## *Arcs and Chords*

### Arcs and Their Measure

- A **central angle** is an angle whose vertex is the center of a circle.
- An **arc** is an unbroken part of a circle consisting of two points on a circle and all the points on the circle between them.

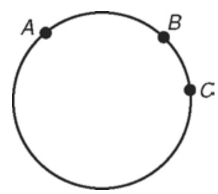



- If the endpoints of an arc lie on a diameter, the arc is a semicircle and its measure is 180°.

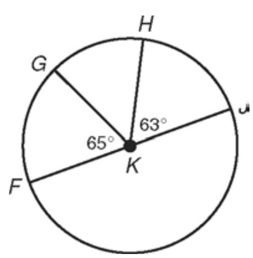
### Arc Addition Postulate

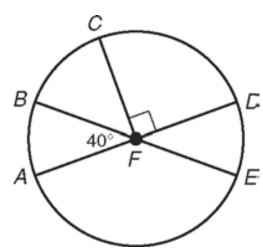
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

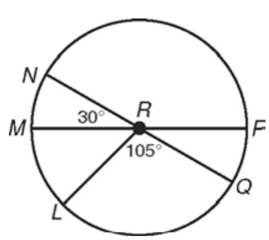



**Find each measure.**




1. $m\overset{\frown}{HJ}$ ______________________
2. $m\overset{\frown}{FGH}$ ______________________
3. $m\overset{\frown}{CDE}$ ______________________
4. $m\overset{\frown}{BCD}$ ______________________
5. $m\overset{\frown}{LMN}$ ______________________
6. $m\overset{\frown}{LNP}$ ______________________

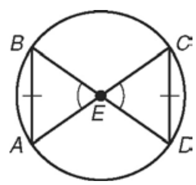
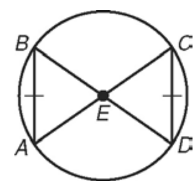
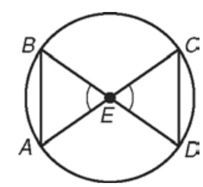


LESSON 12-2

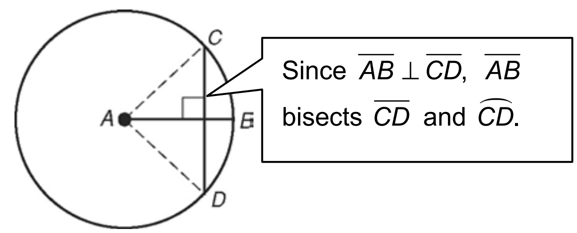
# Reteach

## *Arcs and Chords* continued

**Congruent arcs** are arcs that have the same measure.

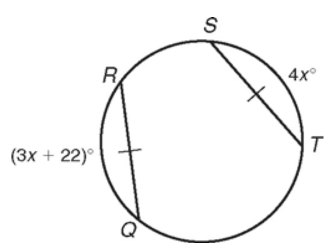
| Congruent Arcs, Chords, and Central Angles | | |
|---|---|---|
|   If $m\angle BEA \cong m\angle CED$, then $\overline{BA} \cong \overline{CD}$. |   If $\overline{BA} \cong \overline{CD}$, then $\overset{\frown}{BA} \cong \overset{\frown}{CD}$. |   If $\overset{\frown}{BA} \cong \overset{\frown}{CD}$, then $m\angle BEA \cong m\angle CED$. |
| Congruent central angles have congruent chords. | Congruent chords have congruent arcs. | Congruent arcs have congruent central angles. |

In a circle, if a radius or diameter is perpendicular to a chord, then it bisects the chord and its arc.




**Find each measure.**

7. $\overline{QR} \cong \overline{ST}$. Find $m\overset{\frown}{QR}$.

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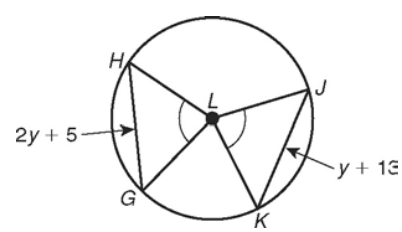
8. $\angle HLG \cong \angle KLJ$. Find $GH$.

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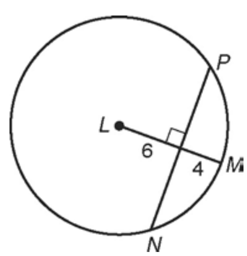
**Find each length to the nearest tenth.**

9. $NP$

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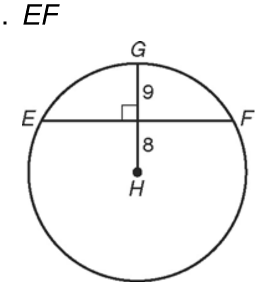
10. $EF$




_______________________

## Reteach

1. 63°
2. 117°
3. 130°
4. 140°
5. 75°
6. 225°
7. 88°
8. 21
9. 16.0
10. 30.0

## Challenge

1. 86°
2. 47°
3. 43°
4. 14 cm
5. a. $\sin 43° = \frac{AD}{14}$

   b. AD ≈ 9.5 cm

   c. AB ≈ 19.1 cm
6. 1.9 in.
7. 3.0 m
8. 1.3 ft
9. $\ell = d\left(\sin\left[\frac{n}{2}\right]^{\circ}\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in.
11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in.
13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.

$$A = \frac{1}{4}nd^2\left(\cos\left[\frac{180}{n}\right]^{\circ}\right)\left(\sin\left[\frac{180}{n}\right]^{\circ}\right)$$

## Problem Solving

1. 270°
2. 72°
3. 154.8°
4. 115.2°
5. C
6. G
7. D

## Reading Strategies

1. 60°
2. 360°
3. central angles
4. 32°
5. 263°
6. 328°
7. 295°
8. 32°
9. 65°

# 12-3 SECTOR AREA AND ARC LENGTH

## Practice A

1. $\pi r^3\left(\frac{m°}{360°}\right)$
2. $2\pi r\left(\frac{m°}{360°}\right)$
3. $9\pi$ mm$^2$; 28.27 mm$^2$
4. $27\pi$ mi$^2$; 84.82 mi$^2$
5. 982 yd$^2$
6. 1173 yd$^2$
7. $25\pi$ in$^2$
8. 50 in$^2$
9. 28.54 in$^2$
10. $4\pi$ cm; 12.57 cm
11. $3\pi$ km; 9.42 km

## Practice B

1. sector *BAC* 126 $\pi$ mm$^2$; 395.84 mm$^2$
2. sector *UTV* 30 $\pi$ in$^2$; 94.25 in$^2$
3. sector *KJL* $\pi$ ft$^2$; 3.14 ft$^2$
4. sector *FEG* $100\pi$ m$^2$; 314.16 m$^2$
5. 4.54 in$^2$
6. 10.96 km$^2$
7. 24.47 yd$^2$
8. 0.29 cm$^2$
9. 9.83 mi$^2$
10. $\pi$ ft; 3.14 ft
11. $14\pi$ m; 43.98 m
12. $\frac{\pi}{2}$ mi; 1.57 mi
13. $10\pi$ mm; 31.42 mm

## Practice C

1. Possible answer: The area of a sector of a circle with radius $r$ and central angle $m$ is $A = \pi r^2\left(\frac{m}{360}\right)$. Half this area is $\pi r^2\left(\frac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\frac{m}{2}\right)$, and the length of the base is $2r\sin\left(\frac{m}{2}\right)$. The area of the triangle is $\frac{bh}{2}$ or $r^2\sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Set this equal to the area of half the sector:

# Geometry

## Chapter 12: Circle

### Lesson 3 - Sector Area and Arc Length

**Standards**

G.C.5 - Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

**Objectives**

Find the area of sectors.

Find arc lengths.

**Materials**

Calculator, ToolBox, Text Book, paper, pencil

**Vocabulary**

Sector of a circle – a region bounded by two radii of the circle and their intercepted arc.

Segment of a circle – region bounded by an arc and its chord.

Arc length – distance along an arc measured in linear units.

**Formulas**

Area of a Sector – $A = \pi r^2(m/360)$

Area of a Segment - $A = \pi r^2(m/360) – 1/2bh$

Arc length – $L = 2\pi r(m/360)$

**Symbols**

None

**Postulates**

None

**Theorems**

None

**Who Uses This?**

Farmers use irrigation radii to calculate areas of sectors.

**Discussion**

Update ToolBox with vocabulary. Go over formulas carefully and explain why the 360 degrees. Most cadets seem to be confused about "the fraction" so be sure to go over that it is the ratio or the portion of the complete circle that the arc takes up. Refer it a slice of pizza – cadets get that analogy.

Review example 1 with cadets. May need to keep reviewing the fraction idea. Have cadets work the Check It Out #1.

Review Example 2. This is another example of why we would need to know the amount of arc for a circle – farming and irrigation. Have cadets work the Check-it-Out #2.

Review Example 4. Be sure to show that the line drawn is the radius but so are the other segments from the center to the points on the circle. Notice that the radius is not squared – Why is this? Have cadets work the Check-It-Out #4.

**Guided Practice**

Check work from the Check-It-Out problems. If still need some help, go over problems 2, 3, 4, 5, 9, 10 as needed.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 12 – 34

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 35

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 37

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 15, 18, 20, 21, 24

Problems 12,13,14,16, 19, 20 Extra credit #22

**Homework**

Page 75 and 159 May be replaced with different work based on needs. Replaced by additional resources listed above.

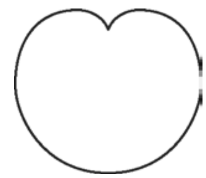
Name ______________________________________ Date ________________ Class________________

LESSON 12-3

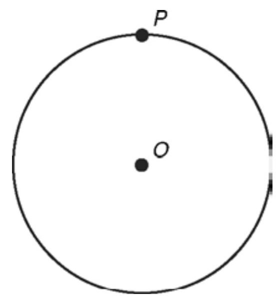
# Challenge

## *Investigating Cardioids*

The unusual curve at right is called a **cardioid.** The cardioid derives its name from its resemblance to the classic shape of a heart. A surprising fact about the cardioid is that it can be generated by constructing a set of circles that satisfies a certain set of conditions.



1. In the figure at right, $P$ is a point on circle $O$. Use a compass and straightedge to perform the following construction:

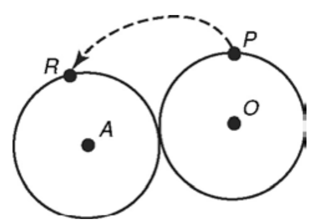



   a. Divide circle $O$ into 24 congruent arcs, with point $P$ being the common endpoint of two of the arcs. (*Hint:* Begin by constructing six congruent arcs.)

   b. Place the metal tip of the compass at any endpoint of an arc *except point P*. Open the compass so that the pencil tip is at point $P$ and draw a circle.

   c. Repeat part **b** for the other 22 points from part **a.** The outline of the figure that results will approximate a cardioid.

2. In the figure at right, circle $A$ is congruent to circle $O$ and is being rolled counterclockwise around it. Point $R$ is on circle $A$; its path is shown by the dashed arrow. After one complete revolution of circle $A$, the path of point $R$ will trace a cardioid. The length of this cardioid is a whole-number multiple of the diameter of circle $O$. Make a conjecture about the value of that whole number. (*Hint:* Use your drawing from Exercise 1, or act out the roll, using two coins of the same type.)




________________________________________________________________________

________________________________________________________________________

3. Shade your cardioid construction from Exercise 1 to create the figure shown at right.

4. Research the meaning of the term *nephroid*. Find out how to construct a nephroid by using circles that satisfy a given set of conditions. Perform the construction on a separate sheet of paper. Also, find how the nephroid compares to the cardioid in terms of rolling one circle around another, and make a report of your findings on a separate sheet of paper.

$m\ \frac{\pi}{720} = \sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

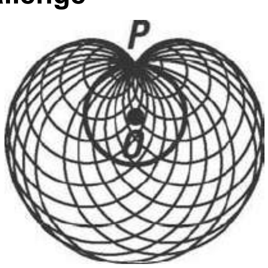
2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\frac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\frac{m}{360}\right) = r$. This simplifies to $m = \frac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

**Reteach**

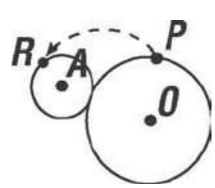
1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
5. $11\pi$ in.; 34.56 in.
6. $7\pi$ cm; 21.99 cm
7. $4\pi$ in.; 12.57 in.
8. $\frac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

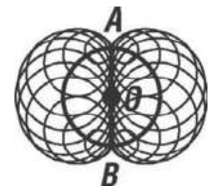
**Challenge**

1.




2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle $O$.
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, $O$, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle $O$ and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)




**Problem Solving**

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

**Reading Strategies**

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\frac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\frac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

**Practice A**

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

**Practice B**

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

Name ______________________ Date ______________ Class______________

LESSON 12-3

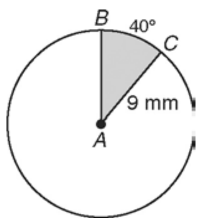
# Practice A

## *Sector Area and Arc Length*

**In Exercises 1 and 2, fill in the blanks to complete each formula.**

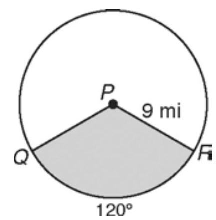
1. The area of a sector of a circle with radius $r$ and central angle $m°$ is $A =$ ____________________.
2. The length of an arc with central angle $m°$ on a circle with radius $r$ is $L =$ ____________________.

**Find the area of each sector. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**

3. 


sector $BAC$

________________________________________

4. 

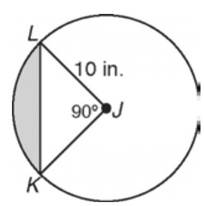

sector $QPR$

________________________________________

**Different animals have different fields of view. Humans can generally see a 180° arc in front of them. Horses can see a 215° arc. A horse and rider are in heavy fog, so they can see for only 25 yards in any direction. Round your answers to Exercises 5 and 6 to the nearest square yard.**

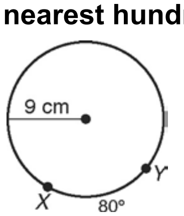
5. Find the area of the rider's field of view. ____________________
6. Find the area of the horse's field of view. ____________________

**Complete Exercises 7–9 to find the area of segment *KJL*.**

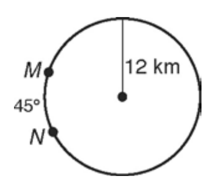



7. Find the area of sector $KJL$. Give your answer in terms of $\pi$. ____________________
8. Find the area of $\triangle KJL$. ____________________
9. Subtract the area of $\triangle KJL$ from the area of sector $KJL$ to find the area of segment $KJL$. Round to the nearest hundredth. ____________________

**Find each arc length. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**

10. 


$\overset{\frown}{XY}$ ________________________________

11. 


$\overset{\frown}{MN}$ ________________________________

**Reteach**

1. 63°
2. 117°
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8. 21
9. 16.0
10. 30.0

**Challenge**

1. 86°
2. 47°
3. 43°
4. 14 cm
5. a. $\sin 43° = \dfrac{AD}{14}$

   b. AD ≈ 9.5 cm

   c. AB ≈ 19.1 cm
6. 1.9 in.
7. 3.0 m
8. 1.3 ft
9. $\ell = d\left(\sin\left[\dfrac{n}{2}\right]^{\circ}\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in.
11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in.
13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.

   $A = \dfrac{1}{4}nd^2\left(\cos\left[\dfrac{180}{n}\right]^{\circ}\right)\left(\sin\left[\dfrac{180}{n}\right]^{\circ}\right)$

**Problem Solving**

1. 270°
2. 72°
3. 154.8°
4. 115.2°
5. C
6. G
7. D

**Reading Strategies**

1. 60°
2. 360°
3. central angles
4. 32°
5. 263°
6. 328°
7. 295°
8. 32°
9. 65°

## 12-3 SECTOR AREA AND ARC LENGTH

**Practice A**

1. $\pi r^3\left(\dfrac{m°}{360°}\right)$
2. $2\pi r\left(\dfrac{m°}{360°}\right)$
3. $9\pi$ mm$^2$; 28.27 mm$^2$
4. $27\pi$ mi$^2$; 84.82 mi$^2$
5. 982 yd$^2$
6. 1173 yd$^2$
7. $25\pi$ in$^2$
8. 50 in$^2$
9. 28.54 in$^2$
10. $4\pi$ cm; 12.57 cm
11. $3\pi$ km; 9.42 km

**Practice B**

1. sector *BAC* $126\pi$ mm$^2$; 395.84 mm$^2$
2. sector *UTV* $30\pi$ in$^2$; 94.25 in$^2$
3. sector *KJL* $\pi$ ft$^2$; 3.14 ft$^2$
4. sector *FEG* $100\pi$ m$^2$; 314.16 m$^2$
5. 4.54 in$^2$
6. 10.96 km$^2$
7. 24.47 yd$^2$
8. 0.29 cm$^2$
9. 9.83 mi$^2$
10. $\pi$ ft; 3.14 ft
11. $14\pi$ m; 43.98 m
12. $\dfrac{\pi}{2}$ mi; 1.57 mi
13. $10\pi$ mm; 31.42 mm

**Practice C**

1. Possible answer: The area of a sector of a circle with radius $r$ and central angle $m$ is $A = \pi r^2\left(\dfrac{m}{360}\right)$. Half this area is $\pi r^2\left(\dfrac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\dfrac{m}{2}\right)$, and the length of the base is $2r\sin\left(\dfrac{m}{2}\right)$. The area of the triangle is $\dfrac{bh}{2}$ or $r^2\sin\left(\dfrac{m}{2}\right)\cos\left(\dfrac{m}{2}\right)$. Set this equal to the area of half the sector:

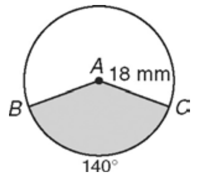
Name ______________________ Date ______________ Class______________

**LESSON 12-3**

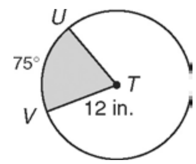
# Practice B

## *Sector Area and Arc Length*

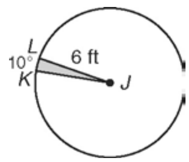
**Find the area of each sector. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**

1. 

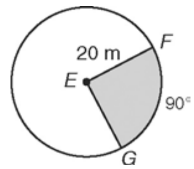

sector *BAC* ______________________

2. 


sector *UTV* ______________________

3. 

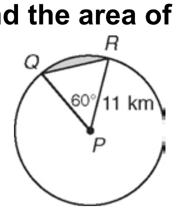

sector *KJL* ______________________

4. 


sector *FEG* ______________________

5. The speedometer needle in Ignacio's car is 2 inches long. The needle sweeps out a 130° sector during acceleration from 0 to 60 mi/h. Find the area of this sector. Round to the nearest hundredth. ____________

**Find the area of each segment to the nearest hundredth.**

6. 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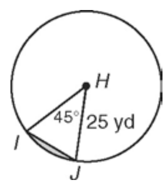
7. 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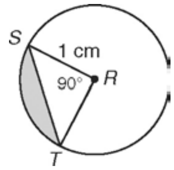
8. 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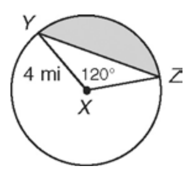
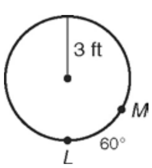
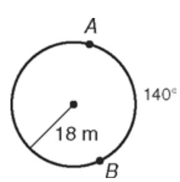
9. 


______________________

**Find each arc length. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**

10. 


______________________

11. 


______________________

12. an arc with measure 45° in a circle with radius 2 mi ______________

13. an arc with measure 120° in a circle with radius 15 mm ______________

## Reteach

1. 63°
2. 117°
3. 130°
4. 140°
5. 75°
6. 225°
7. 88°
8. 21
9. 16.0
10. 30.0

## Challenge

1. 86°
2. 47°
3. 43°
4. 14 cm
5. a. $\sin 43° = \dfrac{AD}{14}$

   b. AD ≈ 9.5 cm

   c. AB ≈ 19.1 cm
6. 1.9 in.
7. 3.0 m
8. 1.3 ft
9. $\ell = d\left(\sin\left[\dfrac{n}{2}\right]^{\circ}\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in.
11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in.
13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.

$$A = \frac{1}{4}nd^2\left(\cos\left[\frac{180}{n}\right]^{\circ}\right)\left(\sin\left[\frac{180}{n}\right]^{\circ}\right)$$

## Problem Solving

1. 270°
2. 72°
3. 154.8°
4. 115.2°
5. C
6. G
7. D

## Reading Strategies

1. 60°
2. 360°
3. central angles
4. 32°
5. 263°
6. 328°
7. 295°
8. 32°
9. 65°

# 12-3 SECTOR AREA AND ARC LENGTH

## Practice A

1. $\pi r^3\left(\dfrac{m°}{360°}\right)$
2. $2\pi r\left(\dfrac{m°}{360°}\right)$
3. $9\pi$ mm$^2$; 28.27 mm$^2$
4. $27\pi$ mi$^2$; 84.82 mi$^2$
5. 982 yd$^2$
6. 1173 yd$^2$
7. $25\pi$ in$^2$
8. 50 in$^2$
9. 28.54 in$^2$
10. $4\pi$ cm; 12.57 cm
11. $3\pi$ km; 9.42 km

## Practice B

1. sector *BAC* 126 $\pi$ mm$^2$; 395.84 mm$^2$
2. sector *UTV* 30 $\pi$ in$^2$; 94.25 in$^2$
3. sector *KJL* $\pi$ ft$^2$; 3.14 ft$^2$
4. sector *FEG* $100\pi$ m$^2$; 314.16 m$^2$
5. 4.54 in$^2$
6. 10.96 km$^2$
7. 24.47 yd$^2$
8. 0.29 cm$^2$
9. 9.83 mi$^2$
10. $\pi$ ft; 3.14 ft
11. $14\pi$ m; 43.98 m
12. $\dfrac{\pi}{2}$ mi; 1.57 mi
13. $10\pi$ mm; 31.42 mm

## Practice C

1. Possible answer: The area of a sector of a circle with radius $r$ and central angle $m$ is $A = \pi r^2\left(\dfrac{m}{360}\right)$. Half this area is $\pi r^2\left(\dfrac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\dfrac{m}{2}\right)$, and the length of the base is $2r\sin\left(\dfrac{m}{2}\right)$. The area of the triangle is $\dfrac{bh}{2}$ or $r^2\sin\left(\dfrac{m}{2}\right)\cos\left(\dfrac{m}{2}\right)$. Set this equal to the area of half the sector:

Name ______________________ Date ______________ Class______________

LESSON 12-3

# Practice C

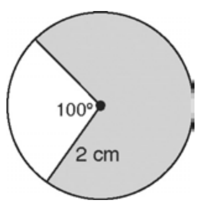
## *Sector Area and Arc Length*

1. Find the measure of a central angle in a circle so that the segment has half the area of the sector. First derive an equation, and then use trial and error to estimate the measure of the central angle to within 1 degree. Explain your answer.

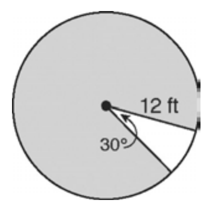
2. The circumference of a circle is $18\pi$ m. Find the central angle of a sector of the circle whose area is $40.5\pi$ $m^2$. ________________

**Find the shaded area of each figure. Round to the nearest hundredth.**

3. ________________




4. ________________




5. Find the measure of the central angle of an arc so that the length of the arc is equal to the radius of the circle. Round to the nearest tenth. Explain your answer.

**Angela is wrapping 1 meter of twine around a spool with a 2-centimeter diameter. The spool is thin and accommodates only one wrap of twine before the twine stacks on top of itself. The twine has a diameter of $\frac{1}{2}$ cm.**

6. Find how many complete times Angela will wrap the twine around the spool. ________________

7. Find the percentage of a complete circle that the last wrapping of the twine will make. Round to the nearest tenth. ________________

## Reteach

1. 63°
2. 117°
3. 130°
4. 140°
5. 75°
6. 225°
7. 88°
8. 21
9. 16.0
10. 30.0

## Challenge

1. 86°
2. 47°
3. 43°
4. 14 cm
5. a. $\sin 43° = \frac{AD}{14}$
   b. AD ≈ 9.5 cm
   c. AB ≈ 19.1 cm
6. 1.9 in.
7. 3.0 m
8. 1.3 ft
9. $\ell = d\left(\sin\left[\frac{n}{2}\right]^{\circ}\right)$

Students' answers may vary slightly.

10. $S \approx 5.9$ in.
11. $P \approx 29.4$ in.
12. $a \approx 4.1$ in.
13. $A \approx 59.4$ in$^2$
14. Formulas may vary in form.
   $A = \frac{1}{4}nd^2\left(\cos\left[\frac{180}{n}\right]^{\circ}\right)\left(\sin\left[\frac{180}{n}\right]^{\circ}\right)$

## Problem Solving

1. 270°
2. 72°
3. 154.8°
4. 115.2°
5. C
6. G
7. D

## Reading Strategies

1. 60°
2. 360°
3. central angles
4. 32°
5. 263°
6. 328°
7. 295°
8. 32°
9. 65°

# 12-3 SECTOR AREA AND ARC LENGTH

## Practice A

1. $\pi r^3\left(\frac{m°}{360°}\right)$
2. $2\pi r\left(\frac{m°}{360°}\right)$
3. $9\pi$ mm$^2$; 28.27 mm$^2$
4. $27\pi$ mi$^2$; 84.82 mi$^2$
5. 982 yd$^2$
6. 1173 yd$^2$
7. $25\pi$ in$^2$
8. 50 in$^2$
9. 28.54 in$^2$
10. $4\pi$ cm; 12.57 cm
11. $3\pi$ km; 9.42 km

## Practice B

1. sector *BAC* 126 $\pi$ mm$^2$; 395.84 mm$^2$
2. sector *UTV* 30 $\pi$ in$^2$; 94.25 in$^2$
3. sector *KJL* $\pi$ ft$^2$; 3.14 ft$^2$
4. sector *FEG* $100\pi$ m$^2$; 314.16 m$^2$
5. 4.54 in$^2$
6. 10.96 km$^2$
7. 24.47 yd$^2$
8. 0.29 cm$^2$
9. 9.83 mi$^2$
10. $\pi$ ft; 3.14 ft
11. $14\pi$ m; 43.98 m
12. $\frac{\pi}{2}$ mi; 1.57 mi
13. $10\pi$ mm; 31.42 mm

## Practice C

1. Possible answer: The area of a sector of a circle with radius *r* and central angle *m* is $A = \pi r^2\left(\frac{m}{360}\right)$. Half this area is $\pi r^2\left(\frac{m}{720}\right)$. The measure of the segment cannot be calculated directly. But if the segment has half the area of the sector, then the triangle must have the other half of the area, and the area of the triangle can be calculated. The height of the triangle is $r\cos\left(\frac{m}{2}\right)$, and the length of the base is $2r\sin\left(\frac{m}{2}\right)$. The area of the triangle is $\frac{bh}{2}$ or $r^2\sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Set this equal to the area of half the sector:

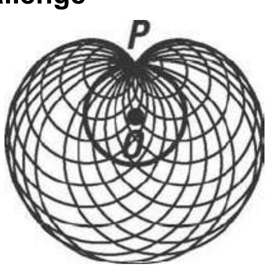
$m\frac{\pi}{720} = \sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\frac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\frac{m}{360}\right) = r$. This simplifies to $m = \frac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

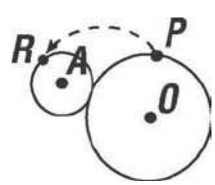
### Reteach

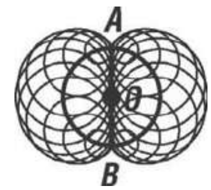
1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
5. $11\pi$ in.; 34.56 in.
6. $7\pi$ cm; 21.99 cm
7. $4\pi$ in.; 12.57 in.
8. $\frac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

### Challenge

1. 

2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle $O$.
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, $O$, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle $O$ and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)




### Problem Solving

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

### Reading Strategies

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\frac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\frac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

### Practice A

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

### Practice B

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

Name ______________________________ Date ______________ Class______________

LESSON 12-3

# Problem Solving

## *Sector Area and Arc Length*

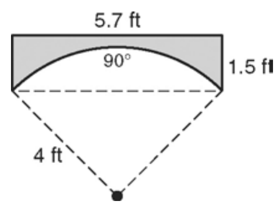
1. A circle with a radius of 20 centimeters has a sector that has an arc measure of 105°. What is the area of the sector? Round to the nearest tenth.

_________________________

2. A sector whose central angle measures 72° has an area of $16.2\pi$ square feet. What is the radius of the circle?

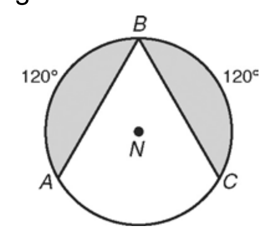
_________________________

3. The archway below is to be painted. What is the area of the archway to the nearest tenth?




_________________________

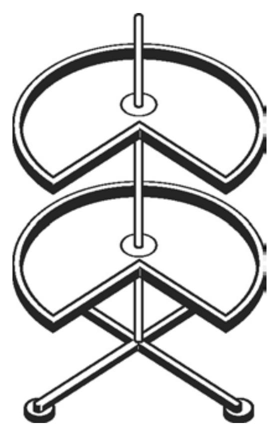
4. Circle $N$ has a circumference of $16\pi$ millimeters. What is the area of the shaded region to the nearest tenth?




_________________________

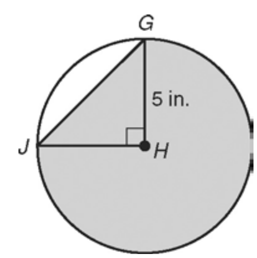
**Choose the best answer.**

5. The circular shelves in diagram are each 28 inches in diameter. The "cut-out" portion of each shelf is 90°. Approximately how much shelf paper is needed to cover both shelves?



   A 154 $in^2$
   B 308 $in^2$
   C 462 $in^2$
   D 924 $in^2$

6. Find the area of the shaded region. Round to the nearest tenth.




   F 8.2 $in^2$ H 71.4 $in^2$
   G 19.6 $in^2$ J 78.5 $in^2$

7. A semicircular garden with a diameter of 6 feet is to have 2 inches of mulch spread over it. To the nearest tenth, what is the volume of mulch that is needed?

   A 2.4 $ft^3$ C 14.1 $ft^3$
   B 4.8 $ft^3$ D 28.3 $ft^3$

8. A round cheesecake 12 inches in diameter and 3 inches high is cut into 8 equal-sized pieces. If five pieces have been taken, what is the approximate volume of the cheesecake that remains?

   F 42.4 $in^3$ H 127.2 $in^3$
   G 70.7 $in^3$ J 212.1 $in^3$

$m\,\dfrac{\pi}{720} = \sin\left(\dfrac{m}{2}\right)\cos\left(\dfrac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

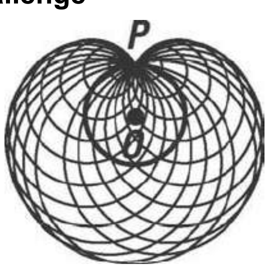
2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\dfrac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\dfrac{m}{360}\right) = r$. This simplifies to $m = \dfrac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

**Reteach**

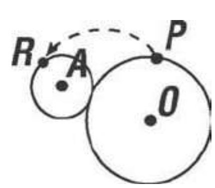
1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
5. $11\pi$ in.; 34.56 in.
6. $7\pi$ cm; 21.99 cm
7. $4\pi$ in.; 12.57 in.
8. $\dfrac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

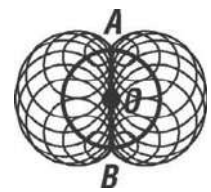
**Challenge**

1.




2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle *O*.
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, *O*, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle *O* and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)




**Problem Solving**

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

**Reading Strategies**

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\dfrac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\dfrac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

**Practice A**

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

**Practice B**

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

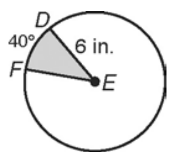
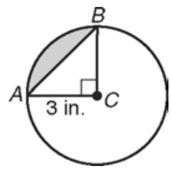
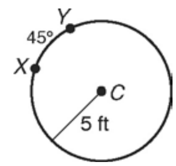
Name ________________________ Date ______________ Class______________

LESSON 12-3

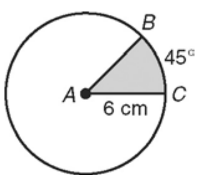
# Reading Strategies

## *Use a Formula*

The table below shows you how to use formulas for sector area and arc length.

| Area of a Sector | Area of a Segment | Arc Length |
|---|---|---|
| $A = \pi r^2 \left(\dfrac{m^\circ}{360^\circ}\right)$ | $A = \pi r^2 \left(\dfrac{m^\circ}{360^\circ}\right) - \dfrac{1}{2}bh$ | $L = 2\pi r \left(\dfrac{m^\circ}{360^\circ}\right)$ |
|  |  |  |
| Find the area of sector *DEF*.<br>$A = \pi(6)^2\left(\dfrac{40^\circ}{360^\circ}\right)$<br>$= 36\pi\left(\dfrac{1}{9}\right)$<br>$= 4\pi \text{ cm}^2$<br>$\approx 12.57 \text{ cm}^2$ | Find the area of segment *ACB*.<br>$A = \pi(3)^2\left(\dfrac{90^\circ}{360^\circ}\right) - \dfrac{1}{2}(3)(3)$<br>$= 9\pi\left(\dfrac{1}{4}\right) - 4.5$<br>$= 2.25\pi - 4.5$<br>$\approx 2.57 \text{ in}^2$ | Find the length of $\overset{\frown}{XY}$.<br>$L = 2\pi(5)\left(\dfrac{45^\circ}{360^\circ}\right)$<br>$= 10\pi\left(\dfrac{1}{8}\right)$<br>$= \dfrac{5\pi}{4}$ ft<br>$\approx 3.93$ ft |

**Find the area of each sector. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**




1. sector *BAC*

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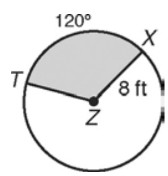


2. sector *TZX*

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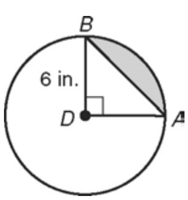
**Find the area of each segment. Round your answer to the nearest hundredth.**




3. segment *BDA*

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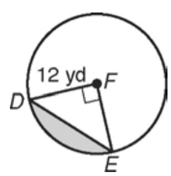


4. segment *DFE*

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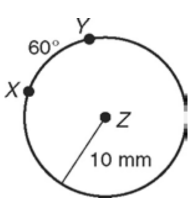
**Find each arc length. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**




5. $\overset{\frown}{XY}$

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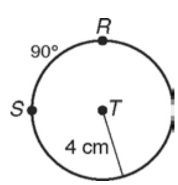


6. $\overset{\frown}{RS}$

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________________________

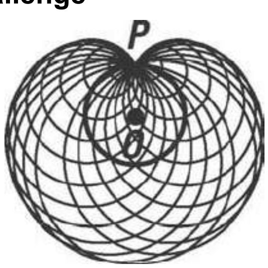
$m\left(\frac{\pi}{720}\right) = \sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\frac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\frac{m}{360}\right) = r$. This simplifies to $m = \frac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

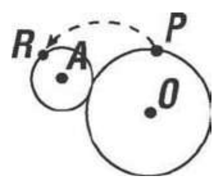
**Reteach**

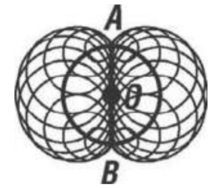
1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
5. $11\pi$ in.; 34.56 in.
6. $7\pi$ cm; 21.99 cm
7. $4\pi$ in.; 12.57 in.
8. $\frac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

**Challenge**

1. 


2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle $O$.
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, $O$, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle $O$ and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)




**Problem Solving**

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

**Reading Strategies**

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\frac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\frac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

**Practice A**

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

**Practice B**

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

Name ________________________________________ Date __________________ Class__________________

LESSON 12-3

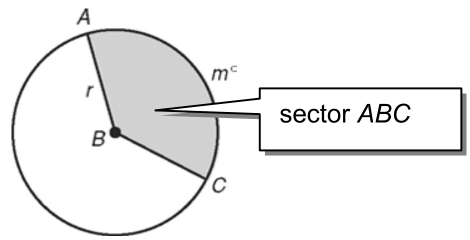
# Reteach

## *Sector Area and Arc Length*

### Sector of a Circle

A **sector of a circle** is a region bounded by two radii of the circle and their intercepted arc.

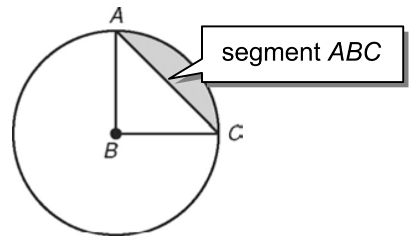
The area of a sector of a circle is given by the formula $A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$.




### Segment of a Circle

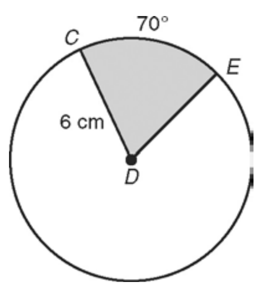
A **segment of a circle** is a region bounded by an arc and its chord.

area of segment *ABC* = area of sector *ABC* − area of $\Delta ABC$




**Find the area of each sector. Give your answer in terms of $\pi$ and rounded to the nearest hundredth.**

1. sector *CDE*

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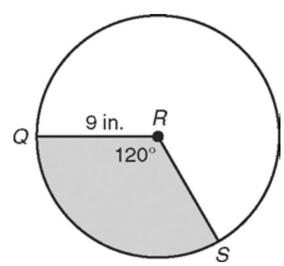
2. sector *QRS*

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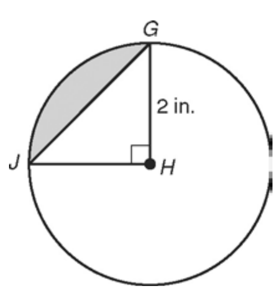
**Find the area of each segment to the nearest hundredth.**

3.

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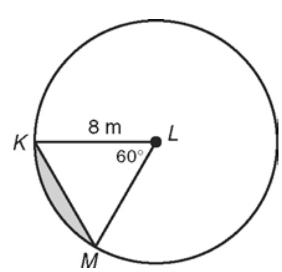
4.

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Name ______________________ Date ______________ Class ______________

LESSON 12-3

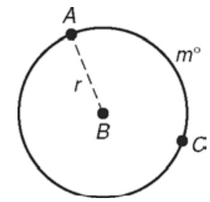
# Reteach

## *Sector Area and Arc Length* *continued*

### Arc Length

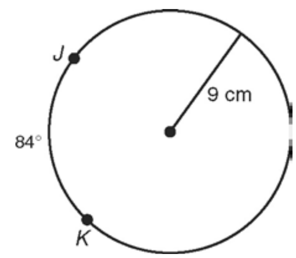
**Arc length** is the distance along an arc measured in linear units.

The arc length of a circle is given by the formula $L = 2\pi r\left(\frac{m°}{360°}\right)$.



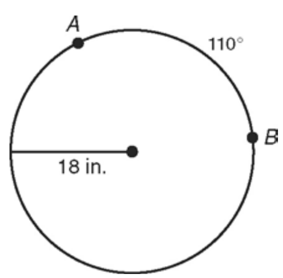

**Find the arc length of $\overset{\frown}{JK}$.**

| | |
|---|---|
| $L = 2\pi r\left(\frac{m°}{360°}\right)$ | Formula for arc length |
| $= 2\pi(9\text{ cm})\left(\frac{84°}{360°}\right)$ | Substitute 9 cm for *r* and 84° for *m*°. |
| $= \frac{21}{5}\pi\text{ cm}$ | Simplify. |
| $\approx 13.19\text{ cm}$ | Round to the nearest hundredth. |




**Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.**

5. $\overset{\frown}{AB}$

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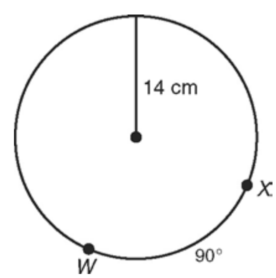
6. $\overset{\frown}{WX}$

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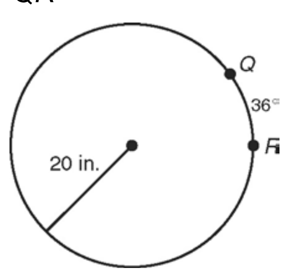
7. $\overset{\frown}{QR}$

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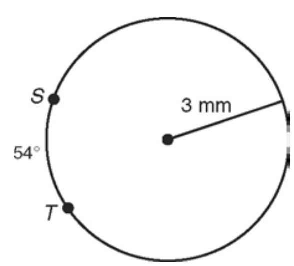
8. $\overset{\frown}{ST}$

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$m\frac{\pi}{720} = \sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

2. 180°
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4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\frac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\frac{m}{360}\right) = r$. This simplifies to $m = \frac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

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1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
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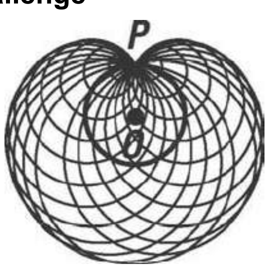
8. $\frac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

### Challenge

1. 


2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle $O$.
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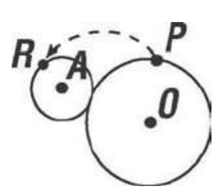


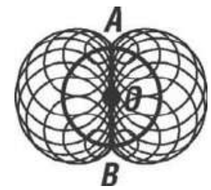


### Problem Solving

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

### Reading Strategies

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## 12-4 INSCRIBED ANGLES

### Practice A

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2. congruent
3. half
4. right angle
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6. 45°; 40°
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9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

### Practice B

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

# Geometry

## Chapter 12: Circle

### Lesson 4 - Inscribed Angles

**Standards**

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.C.3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.CO.13 – Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

**Objectives**

Find the measure of an inscribed angle.

Use inscribed angles and their properties to solve problems.

**Materials**

ToolBox, Pencil, Paper, Text Book

**Vocabulary**

Inscribed angle – an angle whose vertex is on a circle and whose sides contain chords of the circle.  The measure of the angle is half the measure of the arc.

Intercepted arc – consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them.

Subtends – a chord or arc will subtend an angle if its endpoints lie on the sides of the angle.

**Formulas**

Inscribed angle – m<ABC = ½ m of arc AC

**Symbols**

None

**Postulates**

None

**Theorems**

12-4-1 - Inscribed angle theory - The measure of an inscribed angle is half the measure of its intercepted arc.

12-4-2 – If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent.

12-4-3 - An inscribed angle subtends a semicircle if and only if the angle is a right angle.

12-4-4 - If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

**Who Uses This?**

You can use inscribed angles to find measures of angles in string art.

**Discussion**

Update ToolBox with vocabulary, theorems and formulas.

Work through example 1. Be real careful in helping cadets find the arc that is listed. Remind them that multiplying by ½ is the same as dividing by 2. Have cadets work Check-It-Out 1 and monitor cadets carefully. Review the answers.

Work through example 2. Before working through it, discuss the picture and the way the circle is represented. Have cadets work the Check-It-Out 2 and monitor carefully. This picture confuses many cadets so make sure each has the proper perspective on the picture.

Example 3 involves variables so make sure to review how to solve equations. Work through the example making sure cadets follow which angle is being used. Make clear the difference between finding the value of the variable and finding the measure of the angle. The values of the variable will have no units on it. The measure of the angle will have the units of degrees. Have cadets work on the Check-It-Out #3.

Example 4 is good for working more with solving equations. Stress once again that when working the problem, they are a where of which angle they are using and which they are solving for. Review Check-It-Out 4.

**Guided Practice**

Have cadets select from problems 2-11 that they would like to see worked through. Pick one from each example section and work through them. Cadets that have it may start on the classwork.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 12 – 30, 33, 35, 36, 39 – 42

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 44

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 47

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 13, 16, 18, 22, 26

**Homework**

Page 76 and 160 May be replaced with different work based on needs. Replaced by additional resources listed above.

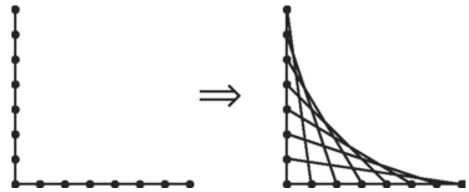
Name ______________________________________ Date ________________ Class________________

LESSON 12-4

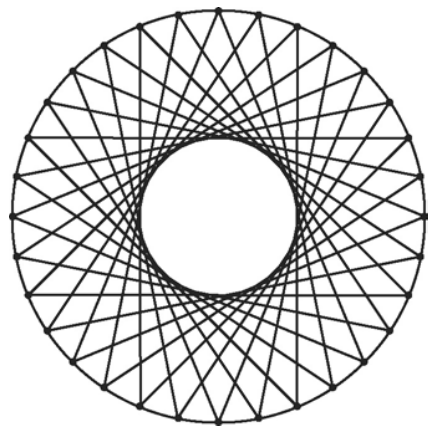
# Challenge

## *Line Designs*

Line designs are sets of segments arranged in a pattern that creates the illusion of a curve. Different figures can be used as the basis of a line design. For instance, the simple line design at right is formed by connecting equally spaced *anchor points* on the sides of a right angle. On this page, you will investigate line designs that are based on circles.



**In the design at right, 32 anchor points are equally spaced on a circle. Segments connect each anchor point to the two anchor points that are exactly 12 spaces away from it. There is an illusion of a circle within the circle.**



1. Fill in the blanks to make a true statement:

   Each segment that joins two anchor points is a(n) ____________________of the circle. Two segments that share a common anchor point form a(n) ____________________ angle of the circle.

2. Let $x$ represent the degree measure of each angle formed by two segments with a common endpoint.

   a. What is the value of $x$ in the figure above? ______________

   b. Suppose that the segments connected each anchor point to the two anchor points exactly 10 spaces away from it. What is value of $x$? ______________

   c. Let $n$ represent the number of spaces between two anchor points that are connected by a segment. Write an expression for $x$. (*Hint:* Be sure to account for values of $n$ that are greater than 16.) ______________

   d. What restrictions must be placed on the value of $n$ in part **c**? ________________

   e. Let $p$ represent the number of equally spaced anchor points, and let $n$ represent the number of spaces between two anchor points connected by a segment. Write an expression for $x$. State any restrictions on $n$.

   ________________________________________________________________________

   f. Find values of $n$ and $p$, $p \neq 32$, for which $x = 30$.

   ________________________________________________________________________

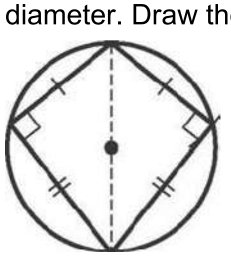
   ________________________________________________________________________

3. The type of line design that you investigated on this page is just one of countless types of line designs based on circles. Using the library or the Internet as a resource, find a different type of circle line design and re-create it on a separate paper, using only a compass and straightedge.

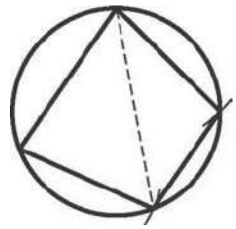
11. 90°; 90°; 90°; 90°
12. 68°; 95°; 112°; 85°
13. 59°; 73°; 121°; 107°

**Practice C**

1. Possible answer: It is given that $\overline{AC} \cong \overline{AD}$. In a circle, congruent chords intercept congruent arcs, so $\overset{\frown}{ABC} \cong \overset{\frown}{AED}$. $\overset{\frown}{DC}$ is congruent to itself by the Reflexive Property of Congruence. By the Arc Addition Postulate and the Addition Property of Congruence, $\overset{\frown}{ACD} \cong \overset{\frown}{ADC}$. $\angle ABC$ intercepts $\overset{\frown}{ADC}$, so $m\angle ABC = \frac{1}{2} m\overset{\frown}{ADC}$. $\angle AED$ intercepts $\overset{\frown}{ACD}$, so $m\angle AED = \frac{1}{2} m\overset{\frown}{ACD}$. By substitution, $m\angle ABC = m\angle AED$. Therefore $\angle ABC \cong \angle AED$.
2. Possible answer: It is given that $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. By the definition of congruent arcs, $m\overset{\frown}{PQ} = m\overset{\frown}{RS}$. $\angle PSQ$ intercepts $\overset{\frown}{PQ}$, and $\angle RQS$ intercepts $\overset{\frown}{RS}$. So $m\angle PSQ$ must equal $m\angle RQS$. Therefore $\angle PSQ \cong \angle RQS$. $\angle PSQ$ and $\angle RQS$ are congruent alternate interior angles of $\overline{QR}$ and $\overline{PS}$. So $\overline{QR} \parallel \overline{PS}$.
3. cannot be inscribed in a circle
4. Can be inscribed in a circle; possible answer: The two congruent angles of the kite are opposite, so they must be right angles. Draw a diameter. Draw segments from opposite ends of the diameter to any point on the circle. Use the compass to copy one of the segments across the diameter. Draw the fourth side.



5. Can be inscribed in a circle; possible answer: The pairs of base angles of a trapezoid inscribed in a circle must be congruent. Draw any inscribed angle. Use the compass to copy the arc that this angle intercepts. Mark off the same arc from the vertex of the inscribed angle. Connect the points.



6. cannot be inscribed in a circle

**Reteach**

1. $m\angle LMP = 18°$; $m\overset{\frown}{MN} = 96°$
2. $m\angle GFJ = 55°$; $m\overset{\frown}{FH} = 72°$
3. 16.4
4. 45°
5. 70°; 88°; 110°; 92°
6. 120°; 75°; 60°; 105°
7. 132°; 90°; 48°; 90°
8. 101°; 86°; 79°; 94°

**Challenge**

1. chord; inscribed
2. a. 45°
   b. 67.5°
   c. $|180 - 11.25n|$
   d. $0 < |n - 16| < 16$
   e. $\left|180 - \frac{360n}{p}\right|$, or $\left[180\left(1 - \frac{2n}{p}\right)\right]$, where $0 < n - \left|n - \frac{p}{2}\right| < \frac{p}{2}$
   f. Answers will vary. Students may choose any values of $n$ and $p$ for which $\frac{n}{p} = \frac{5}{12}$. Sample answer: $n = 15$, $p = 36$

**Problem Solving**

1. 160°
2. 112°; 52°; 68°; 128°
3. C
4. G
5. B
6. G

Name ______________________________________ Date _________________ Class_________________

LESSON **12-4**

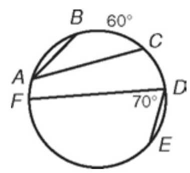
# Practice A

## *Inscribed Angles*

**In Exercises 1–4, fill in the blanks to complete each theorem.**

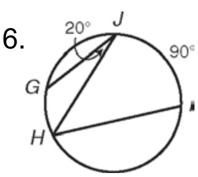
1. If a quadrilateral is inscribed in a circle, then its opposite angles are ______________________.
2. If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are ______________________.
3. The measure of an inscribed angle is ______________________ the measure of its intercepted arc.
4. An inscribed angle subtends a semicircle if and only if the angle is a ______________________.

**Find each measure.**

5. 


$m\angle BAC =$ ___________

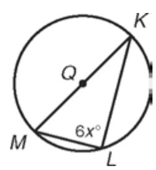
$m\overset{\frown}{FE} =$ ___________

6. 

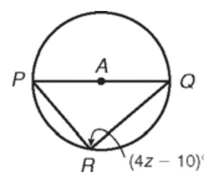

$m\angle IHJ =$ ___________

$m\overset{\frown}{GH} =$ ___________

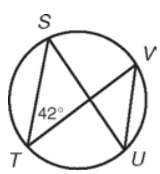
**Find each value.**

7. 

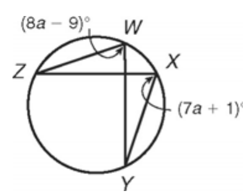

$x =$ ___________

8. 


$z =$ ___________

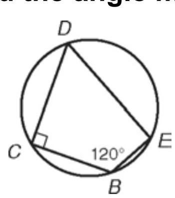
9. 


$m\angle VUS =$ ___________

10. 


$m\angle ZWY =$ ___________

**Find the angle measures of each inscribed quadrilateral.**

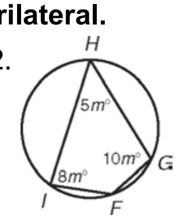
11. 


$m\angle B =$ ___________

$m\angle C =$ ___________

$m\angle D =$ ___________

$m\angle E =$ ___________

12. 

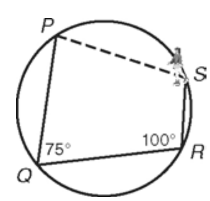

$m\angle F =$ ___________

$m\angle G =$ ___________

$m\angle H =$ ___________

$m\angle I =$ ___________

13. Iyla has not learned how to stop on ice skates yet, so she just skates straight across the circular rink until she hits a wall. She starts at $P$, turns 75° at $Q$, and turns 100° at $R$. Find how many degrees Iyla will turn at $S$ to get back to her starting point.




______________________

$m\dfrac{\pi}{720} = \sin\left(\dfrac{m}{2}\right)\cos\left(\dfrac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

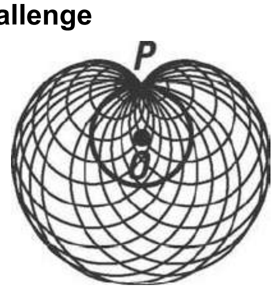
2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\dfrac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\dfrac{m}{360}\right) = r$. This simplifies to $m = \dfrac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

**Reteach**

1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
5. $11\pi$ in.; 34.56 in.
6. $7\pi$ cm; 21.99 cm
7. $4\pi$ in.; 12.57 in.
8. $\dfrac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

**Challenge**

1.




2. Conjectures may vary. The length of the cardioid is in fact eight times the diameter of circle $O$.
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, $O$, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle $O$ and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)


**Problem Solving**

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

**Reading Strategies**

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\dfrac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\dfrac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

**Practice A**

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

**Practice B**

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

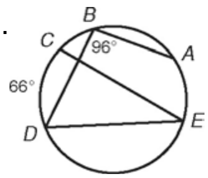
Name ________________________ Date ______________ Class______________

LESSON 12-4

# Practice B

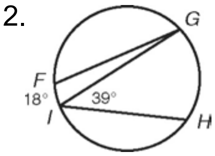
## *Inscribed Angles*

**Find each measure.**

1. 

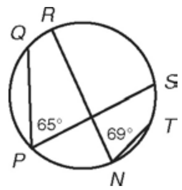

m∠*CED* = ________

m$\widehat{DEA}$ = ________

2. 

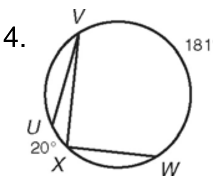

m∠*FGI* = ________

m$\widehat{GH}$ = ________

3. 


m$\widehat{QRS}$ = ________

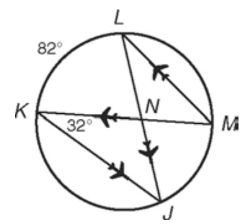
m$\widehat{TSR}$ = ________

4. 

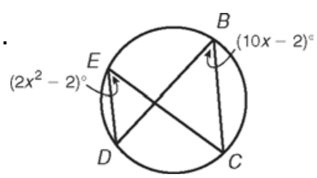

m∠*XVU* = ________

m∠*VXW* = ________

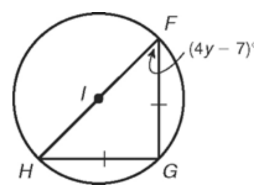
5. A circular radar screen in an air traffic control tower shows these flight paths. Find m∠*LNK*. ________



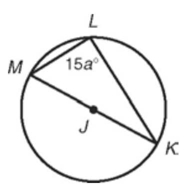

**Find each value.**

6. 

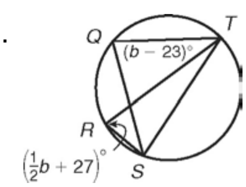

m∠*CED* = ________

7. 


*y* = ________

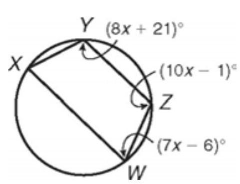
8. 


*a* = ________

9. 


m∠*SRT* = ________

**Find the angle measures of each inscribed quadrilateral.**

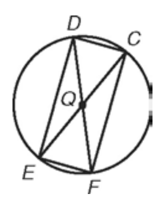
10. 


m∠*X* = ________

m∠*Y* = ________

m∠*Z* = ________

m∠*W* = ________

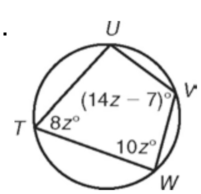
11. 


m∠*C* = ________

m∠*D* = ________

m∠*E* = ________

m∠*F* = ________

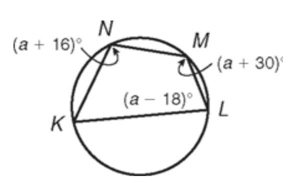
12. 


m∠*T* = ________

m∠*U* = ________

m∠*V* = ________

m∠*W* = ________

13. 


m∠*K* = ________

m∠*L* = ________

m∠*M* = ________

m∠*N* = ________

$m\frac{\pi}{720} = \sin\left(\frac{m}{2}\right)\cos\left(\frac{m}{2}\right)$. Notice that this equation is independent of the radius $r$. Using trial and error with this equation shows that the measure of a central angle whose segment has half the area of the sector is between 108° and 109°.

2. 180°
3. 9.08 $cm^2$
4. 414.69 $ft^2$
5. Possible answer: The length of an arc with central angle measure $m$ in a circle with radius $r$ is equal to $2\pi r\left(\frac{m}{360}\right)$. Set this equal to the radius: $2\pi r\left(\frac{m}{360}\right) = r$. This simplifies to $m = \frac{180}{\pi}$. The measure of the central angle is 57.3°.
6. 6 times
7. 60.4%

**Reteach**

1. $7\pi$ $cm^2$; 21.99 $cm^2$
2. $27\pi$ $in^2$; 84.82 $in^2$
3. 1.14 $in^2$
4. 5.80 $m^2$
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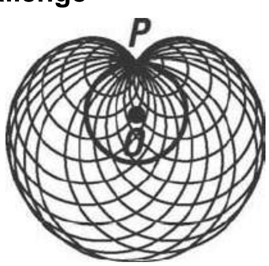
8. $\frac{9}{10}\pi$ or $0.9\pi$ mm; 2.83 mm

**Challenge**

1. 

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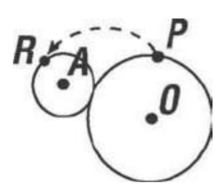
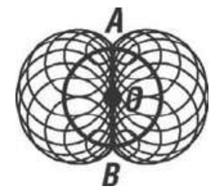
3. Check students' work.
4. The term nephroid comes from the Greek word nephros, which means "kidney." The nephroid is a kidney-shaped curve. Start with a circle, $O$, and draw one diameter. The nephroid is then generated by the set of circles whose centers are on circle $O$ and that intersect the chosen diameter in exactly one point. (Circles are tangent to the chosen diameter.)




**Problem Solving**

1. 366.5 $cm^2$
2. 9 ft
3. 4.0 $ft^2$
4. 78.6 $mm^2$
5. D
6. H
7. A
8. H

**Reading Strategies**

1. $4.5\pi$ $cm^2 \approx 14.14$ $cm^2$
2. $\frac{64\pi}{3}$ $ft^2 \approx 67.02$ $ft^2$
3. 10.27 $in^2$
4. 41.10 $yd^2$
5. $\frac{10}{3}\pi$ mm $\approx 10.47$ mm
6. $2\pi$ cm $\approx 6.28$ cm

## 12-4 INSCRIBED ANGLES

**Practice A**

1. supplementary
2. congruent
3. half
4. right angle
5. 30°; 140°
6. 45°; 40°
7. 15
8. 15
9. 42°
10. 71°
11. 120°; 90°; 60°; 90°
12. 130°; 100°; 50°; 80°
13. 105°

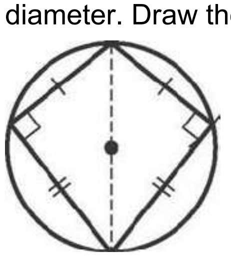
**Practice B**

1. 33°; 192°
2. 9°; 78°
3. 130°; 138°
4. 10°; 90.5°
5. 73°
6. 48°
7. 13
8. 6
9. 77°
10. 71°; 109°; 109°; 71°

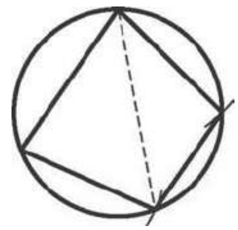
11. 90°; 90°; 90°; 90°
12. 68°; 95°; 112°; 85°
13. 59°; 73°; 121°; 107°

**Practice C**

1. Possible answer: It is given that $\overline{AC} \cong \overline{AD}$. In a circle, congruent chords intercept congruent arcs, so $\overparen{ABC} \cong \overparen{AED}$. $\overparen{DC}$ is congruent to itself by the Reflexive Property of Congruence. By the Arc Addition Postulate and the Addition Property of Congruence, $\overparen{ACD} \cong \overparen{ADC}$. $\angle ABC$ intercepts $\overparen{ADC}$, so $m\angle ABC = \frac{1}{2} m\overparen{ADC}$. $\angle AED$ intercepts $\overparen{ACD}$, so $m\angle AED = \frac{1}{2} m\overparen{ACD}$. By substitution, $m\angle ABC = m\angle AED$. Therefore $\angle ABC \cong \angle AED$.
2. Possible answer: It is given that $\overparen{PQ} \cong \overparen{RS}$. By the definition of congruent arcs, $m\overparen{PQ} = m\overparen{RS}$. $\angle PSQ$ intercepts $\overparen{PQ}$, and $\angle RQS$ intercepts $\overparen{RS}$. So $m\angle PSQ$ must equal $m\angle RQS$. Therefore $\angle PSQ \cong \angle RQS$. $\angle PSQ$ and $\angle RQS$ are congruent alternate interior angles of $\overline{QR}$ and $\overline{PS}$. So $\overline{QR} \parallel \overline{PS}$.
3. cannot be inscribed in a circle
4. Can be inscribed in a circle; possible answer: The two congruent angles of the kite are opposite, so they must be right angles. Draw a diameter. Draw segments from opposite ends of the diameter to any point on the circle. Use the compass to copy one of the segments across the diameter. Draw the fourth side.



5. Can be inscribed in a circle; possible answer: The pairs of base angles of a trapezoid inscribed in a circle must be congruent. Draw any inscribed angle. Use the compass to copy the arc that this angle intercepts. Mark off the same arc from the vertex of the inscribed angle. Connect the points.



6. cannot be inscribed in a circle

**Reteach**

1. $m\angle LMP = 18°$; $m\overparen{MN} = 96°$
2. $m\angle GFJ = 55°$; $m\overparen{FH} = 72°$
3. 16.4
4. 45°
5. 70°; 88°; 110°; 92°
6. 120°; 75°; 60°; 105°
7. 132°; 90°; 48°; 90°
8. 101°; 86°; 79°; 94°

**Challenge**

1. chord; inscribed
2. a. 45°
   b. 67.5°
   c. $|180 - 11.25n|$
   d. $0 < |n - 16| < 16$
   e. $\left|180 - \frac{360n}{p}\right|$, or $\left[180\left(1 - \frac{2n}{p}\right)\right]$,
   where $0 < n - \left|n - \frac{p}{2}\right| < \frac{p}{2}$
   f. Answers will vary. Students may choose any values of $n$ and $p$ for which $\frac{n}{p} = \frac{5}{12}$. Sample answer: $n = 15$, $p = 36$

**Problem Solving**

1. 160°
2. 112°; 52°; 68°; 128°
3. C
4. G
5. B
6. G

LESSON 12-4

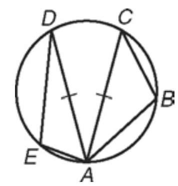
# Practice C

## *Inscribed Angles*

**Write paragraph proofs for Exercises 1 and 2.**

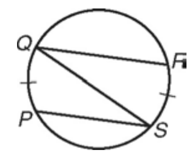
1. **Given:** $\overline{AC} \cong \overline{AD}$

   **Prove:** $\angle ABC \cong \angle AED$




2. **Given:** $\overparen{PQ} \cong \overparen{RS}$

   **Prove:** $\overline{QR} \parallel \overline{PS}$




**For each quadrilateral described, tell whether it can be inscribed in a circle. If so, describe a method for doing so using a compass and straightedge, and draw an example.**

3. a parallelogram that is not a rectangle or a square

4. a kite

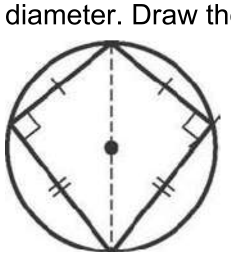
5. a trapezoid

6. a rhombus that is not a square

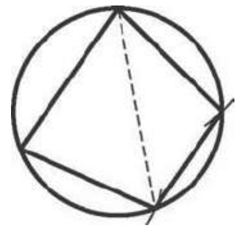
11. 90°; 90°; 90°; 90°
12. 68°; 95°; 112°; 85°
13. 59°; 73°; 121°; 107°

**Practice C**

1. Possible answer: It is given that $\overline{AC} \cong \overline{AD}$. In a circle, congruent chords intercept congruent arcs, so $\overset{\frown}{ABC} \cong \overset{\frown}{AED}$. $\overset{\frown}{DC}$ is congruent to itself by the Reflexive Property of Congruence. By the Arc Addition Postulate and the Addition Property of Congruence, $\overset{\frown}{ACD} \cong \overset{\frown}{ADC}$. $\angle ABC$ intercepts $\overset{\frown}{ADC}$, so $m\angle ABC = \frac{1}{2} m\overset{\frown}{ADC}$. $\angle AED$ intercepts $\overset{\frown}{ACD}$, so $m\angle AED = \frac{1}{2} m\overset{\frown}{ACD}$. By substitution, $m\angle ABC = m\angle AED$. Therefore $\angle ABC \cong \angle AED$.
2. Possible answer: It is given that $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. By the definition of congruent arcs, $m\overset{\frown}{PQ} = m\overset{\frown}{RS}$. $\angle PSQ$ intercepts $\overset{\frown}{PQ}$, and $\angle RQS$ intercepts $\overset{\frown}{RS}$. So $m\angle PSQ$ must equal $m\angle RQS$. Therefore $\angle PSQ \cong \angle RQS$. $\angle PSQ$ and $\angle RQS$ are congruent alternate interior angles of $\overline{QR}$ and $\overline{PS}$. So $\overline{QR} \parallel \overline{PS}$.
3. cannot be inscribed in a circle
4. Can be inscribed in a circle; possible answer: The two congruent angles of the kite are opposite, so they must be right angles. Draw a diameter. Draw segments from opposite ends of the diameter to any point on the circle. Use the compass to copy one of the segments across the diameter. Draw the fourth side.



5. Can be inscribed in a circle; possible answer: The pairs of base angles of a trapezoid inscribed in a circle must be congruent. Draw any inscribed angle. Use the compass to copy the arc that this angle intercepts. Mark off the same arc from the vertex of the inscribed angle. Connect the points.



6. cannot be inscribed in a circle

**Reteach**

1. $m\angle LMP = 18°$; $m\overset{\frown}{MN} = 96°$
2. $m\angle GFJ = 55°$; $m\overset{\frown}{FH} = 72°$
3. 16.4
4. 45°
5. 70°; 88°; 110°; 92°
6. 120°; 75°; 60°; 105°
7. 132°; 90°; 48°; 90°
8. 101°; 86°; 79°; 94°

**Challenge**

1. chord; inscribed
2. a. 45°

   b. 67.5°

   c. $|180 - 11.25n|$

   d. $0 < |n - 16| < 16$

   e. $\left|180 - \frac{360n}{p}\right|$, or $\left[180\left(1 - \frac{2n}{p}\right)\right]$,

   where $0 < n - \left|n - \frac{p}{2}\right| < \frac{p}{2}$

   f. Answers will vary. Students may choose any values of $n$ and $p$ for which $\frac{n}{p} = \frac{5}{12}$. Sample answer: $n = 15$, $p = 36$

**Problem Solving**

1. 160°
2. 112°; 52°; 68°; 128°
3. C
4. G
5. B
6. G

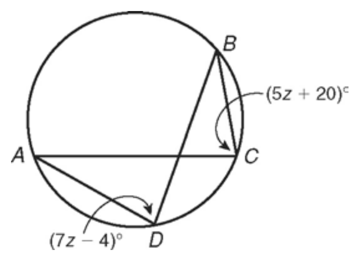
Name ______________________________ Date ______________ Class______________

**LESSON 12-4**

# Problem Solving

## *Inscribed Angles*

1. Find $m\overset{\frown}{AB}$.

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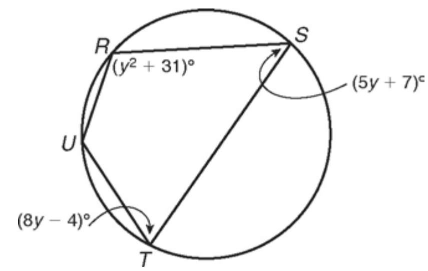
2. Find the angle measures of *RSTU*.

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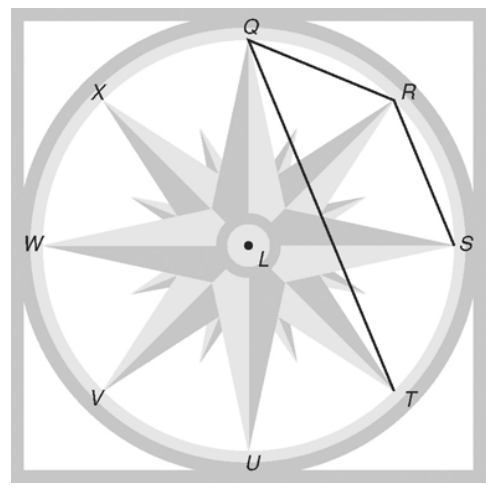
**Choose the best answer.**

Use the diagram of a floor tile for Exercises 3 and 4. Points *Q*, *R*, *S*, *T*, *U*, *V*, *W*, and *X* are equally spaced around ⊙*L*.




3. Find $m\angle RQT$.

   A 15° C 45°

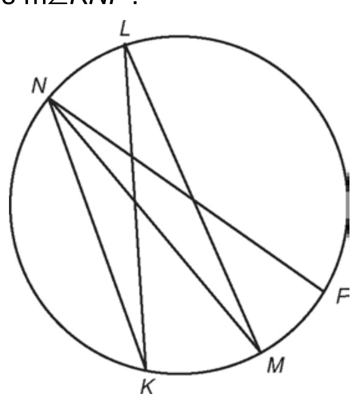
   B 30° D 60°

4. Find $m\angle QRS$.

   F 67.5° H 180°

   G 135° J 270°

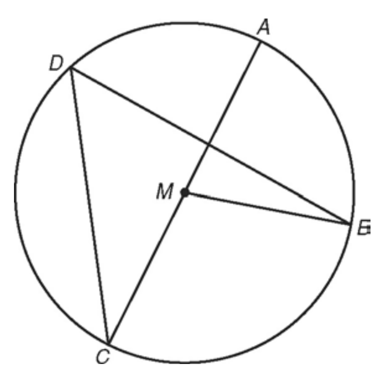
5. If $m\angle KLM = 20°$ and $m\overset{\frown}{MP} = 30°$, what is $m\angle KNP$?




   A 25° C 50°

   B 35° D 70°

6. In ⊙*M*, $m\angle AMB = 74°$. What is $m\angle CDB$?



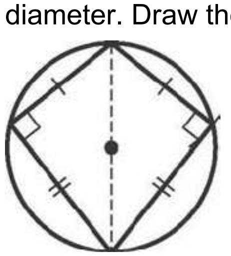

   F 37° H 74°

   G 53° J 106°

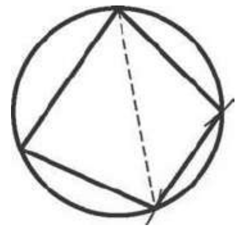
11. 90°; 90°; 90°; 90°
12. 68°; 95°; 112°; 85°
13. 59°; 73°; 121°; 107°

**Practice C**

1. Possible answer: It is given that $\overline{AC} \cong \overline{AD}$. In a circle, congruent chords intercept congruent arcs, so $\overset{\frown}{ABC} \cong \overset{\frown}{AED}$. $\overset{\frown}{DC}$ is congruent to itself by the Reflexive Property of Congruence. By the Arc Addition Postulate and the Addition Property of Congruence, $\overset{\frown}{ACD} \cong \overset{\frown}{ADC}$. $\angle ABC$ intercepts $\overset{\frown}{ADC}$, so $m\angle ABC = \frac{1}{2} m\overset{\frown}{ADC}$. $\angle AED$ intercepts $\overset{\frown}{ACD}$, so $m\angle AED = \frac{1}{2} m\overset{\frown}{ACD}$. By substitution, $m\angle ABC = m\angle AED$. Therefore $\angle ABC \cong \angle AED$.
2. Possible answer: It is given that $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. By the definition of congruent arcs, $m\overset{\frown}{PQ} = m\overset{\frown}{RS}$. $\angle PSQ$ intercepts $\overset{\frown}{PQ}$, and $\angle RQS$ intercepts $\overset{\frown}{RS}$. So $m\angle PSQ$ must equal $m\angle RQS$. Therefore $\angle PSQ \cong \angle RQS$. $\angle PSQ$ and $\angle RQS$ are congruent alternate interior angles of $\overline{QR}$ and $\overline{PS}$. So $\overline{QR} \parallel \overline{PS}$.
3. cannot be inscribed in a circle
4. Can be inscribed in a circle; possible answer: The two congruent angles of the kite are opposite, so they must be right angles. Draw a diameter. Draw segments from opposite ends of the diameter to any point on the circle. Use the compass to copy one of the segments across the diameter. Draw the fourth side.



5. Can be inscribed in a circle; possible answer: The pairs of base angles of a trapezoid inscribed in a circle must be congruent. Draw any inscribed angle. Use the compass to copy the arc that this angle intercepts. Mark off the same arc from the vertex of the inscribed angle. Connect the points.



6. cannot be inscribed in a circle

**Reteach**

1. $m\angle LMP = 18°$; $m\overset{\frown}{MN} = 96°$
2. $m\angle GFJ = 55°$; $m\overset{\frown}{FH} = 72°$
3. 16.4
4. 45°
5. 70°; 88°; 110°; 92°
6. 120°; 75°; 60°; 105°
7. 132°; 90°; 48°; 90°
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**Challenge**

1. chord; inscribed
2. a. 45°
   b. 67.5°
   c. $|180 - 11.25n|$
   d. $0 < |n - 16| < 16$
   e. $\left|180 - \frac{360n}{p}\right|$, or $\left[180\left(1 - \frac{2n}{p}\right)\right]$,
   where $0 < n - \left|n - \frac{p}{2}\right| < \frac{p}{2}$
   f. Answers will vary. Students may choose any values of $n$ and $p$ for which $\frac{n}{p} = \frac{5}{12}$. Sample answer: $n = 15$, $p = 36$

**Problem Solving**

1. 160°
2. 112°; 52°; 68°; 128°
3. C
4. G
5. B
6. G

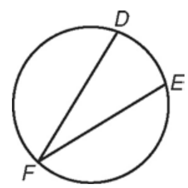
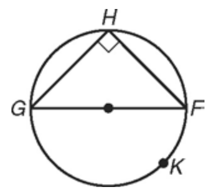
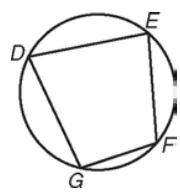
Name ______________________________ Date ______________ Class______________

**LESSON 12-4**

# Reading Strategies

## *Use a Table*

The table below shows properties of inscribed angles.

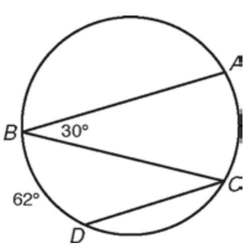
| Words | Diagram | Mathematical Symbols |
|---|---|---|
| The measure of an angle inscribed in a circle is half the measure of the intercepted arc. |   | $m\angle DFE = \frac{1}{2}m\widehat{DE}$ |
| An inscribed angle intercepts a semicircle if and only if the angle is a right angle. |   | $m\angle GHF = 90°$ <br> $\widehat{GKF}$ is a semicircle (180°) |
| If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. |   | $m\angle D + m\angle F = 180°$ <br> $m\angle E + m\angle G = 180°$ |

**Answer the following.**

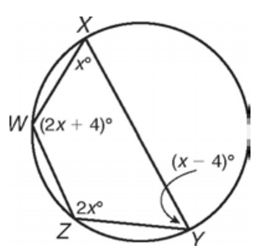
1. The measure of an angle inscribed in a circle is ______________________ the measure of the intercepted arc.
2. Quadrilateral *ABCD* is inscribed in a circle. Write two equations that show the relationships of the angles of the quadrilateral.

________________________________________________________________________

**Find each measure.**




3. $m\angle BCD$ _________
4. $m\widehat{AC}$ _________




5. $m\angle W$ _________
6. $m\angle X$ _________
7. $m\angle Y$ _________
8. $m\angle Z$ _________

**Reading Strategies**

1. half
2. $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$
3. 31°
4. 60°
5. 124°
6. 60°
7. 56°
8. 120°

## 12-5 ANGLE RELATIONSHIPS IN CIRCLES

**Practice A**

1. B
2. C
3. A
4. 45°
5. 150°
6. 55°
7. 116°
8. 82°
9. 40
10. 67
11. 96°
12. 134°
13. 38°

**Practice B**

1. 64°; 96°
2. 119°; 42°
3. 130°
4. 99°
5. 64
6. 47
7. 8
8. 45
9. 60
10. 66.5°; 115°
11. 84°; 192°

**Practice C**

1. Possible answer: It is given that $\overline{AB} \cong \overline{EB}$. So $\triangle ABE$ is an isosceles triangle, and $\angle BAC \cong \angle BEA$. $\angle BEA$ is an inscribed angle, so $m\angle BEA = \frac{1}{2} m\widehat{BC}$. By substitution, $m\angle BAC = \frac{1}{2} m\widehat{BC}$. $\overline{AD}$ and $\overline{AE}$ are secants that intersect in the exterior of the circle. So $m\angle BAC = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. Substitution leads to $\frac{1}{2} m\widehat{BC} = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. This simplifies to $m\widehat{DE} = 2m\widehat{BC}$.
2. Possible answer: Draw chords $\overline{RU}$ and $\overline{ST}$. It is given that $\widehat{RU} \cong \widehat{ST}$. Because congruent arcs have congruent chords, $\overline{RU} \cong \overline{ST}$. It is given that $\widehat{RS} \cong \widehat{TU}$. $\angle RUS$, $\angle URT$, $\angle TSU$, and $\angle STR$ are all inscribed angles that intercept either $\widehat{RS}$ or $\widehat{TU}$. Therefore all four angles have the same measure and are congruent. By SAS, $\triangle QRU$ and $\triangle QST$ are congruent triangles. Furthermore, the base angles are all the same, so they are isosceles triangles. So $\overline{RQ}, \overline{UQ}$, and $\overline{SQ}$ (and $\overline{TQ}$) are congruent by CPCTC and the Isosceles Triangle Theorem. Congruent segments have equal lengths, so $Q$ is equidistant from points $R$, $U$, and $S$ (and $T$) that lie on the circle. Therefore $Q$ is the center of the circle.
3. Possible answer: Draw chord $\overline{KM}$. Assume that $m\widehat{KM} = 180°$. Because a circle contains 360°, $m\widehat{KLM}$ would also equal 180°. $\overrightarrow{JK}$ and $\overrightarrow{JM}$ intersect outside the circle, thus $m\angle KJM = \frac{1}{2}(m\widehat{KLM} - m\widehat{KM}) = 0°$. A triangle cannot contain a 0° angle, so $\triangle JKM$ does not exist, and $m\widehat{KM} = 180°$ cannot be true. Assume that $m\widehat{KM} > 180°$. Tangent $\overrightarrow{JM}$ and chord $\overrightarrow{KM}$ intersect at the point of tangency, $M$. So $m\angle JMK = \frac{1}{2} m\widehat{KM}$, which means $m\angle JMK > 90°$. Similar reasoning shows that $m\angle JKM$ is also greater than 90°. A triangle cannot contain two obtuse angles, so $\triangle JKM$ does not exist, and $m\widehat{KM} > 180°$ cannot be true. Therefore $m\widehat{KM} < 180°$.
4. 120°
5. 80°

**Reteach**

1. 108°
2. 128°
3. 61°
4. 103°
5. 33
6. 52
7. 23
8. 38

Name ______________________ Date ______________ Class______________

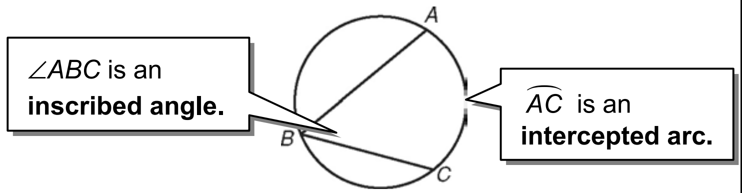
LESSON 12-4

# Reteach

## *Inscribed Angles*

### Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

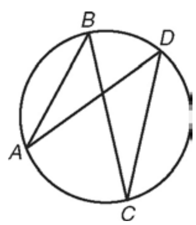



$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$$

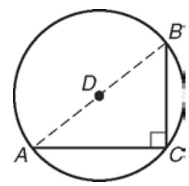
### Inscribed Angles

If inscribed angles of a circle intercept the same arc, then the angles are congruent.

$\angle ABC$ and $\angle ADC$ intercept $\overset{\frown}{AC}$, so $\angle ABC \cong \angle ADC$.

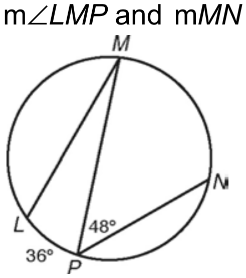



An inscribed angle subtends a semicircle if and only if the angle is a right angle.




**Find each measure.**

1. $m\angle LMP$ and $m\overset{\frown}{MN}$

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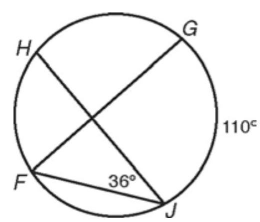
2. $m\angle GFJ$ and $m\overset{\frown}{FH}$

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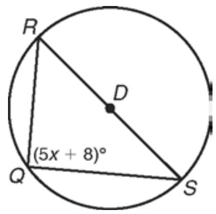
**Find each value.**

3. $x$

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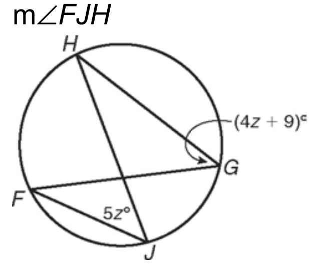
4. $m\angle FJH$

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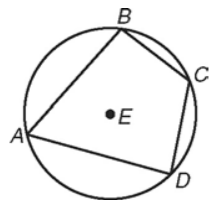
Name ______________________________ Date ______________ Class______________

LESSON 12-4

# Reteach

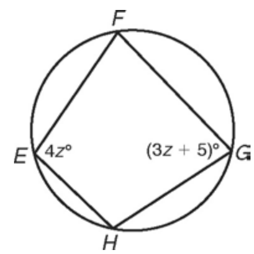
## *Inscribed Angles* continued

| Inscribed Angle Theorem | |
|---|---|
| If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. |   $\angle A$ and $\angle C$ are supplementary.<br>$\angle B$ and $\angle D$ are supplementary.<br>*ABCD* is inscribed in $\odot E$. |

**Find m$\angle G$.**

**Step 1** Find the value of $z$.

| | |
|---|---|
| $m\angle E + m\angle G = 180°$ | *EFGH* is inscribed in a circle. |
| $4z + 3z + 5 = 180$ | Substitute the given values. |
| $7z = 175$ | Simplify. |
| $z = 25$ | Divide both sides by 7. |



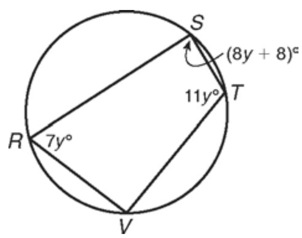

**Step 2** Find the measure of $\angle G$.

$m\angle G = 3z + 5$

$= 3(25) + 5 = 80°$ Substitute 25 for $z$.

**Find the angle measures of each quadrilateral.**

5. *RSTV*

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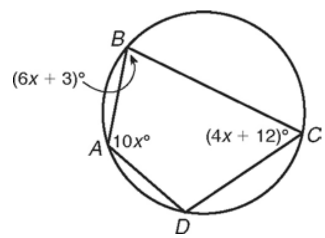
6. *ABCD*

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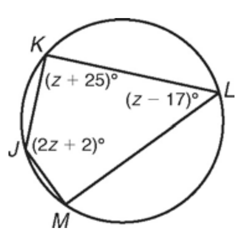
7. *JKLM*

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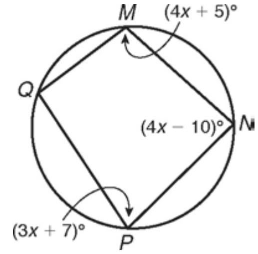
8. *MNPQ*

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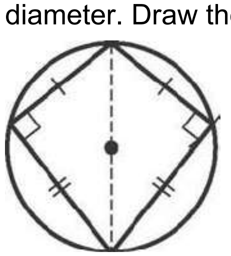
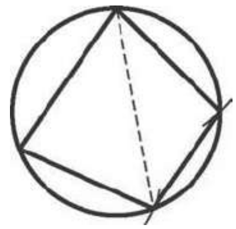
11. 90°; 90°; 90°; 90°
12. 68°; 95°; 112°; 85°
13. 59°; 73°; 121°; 107°

**Practice C**

1. Possible answer: It is given that $\overline{AC} \cong \overline{AD}$. In a circle, congruent chords intercept congruent arcs, so $\overset{\frown}{ABC} \cong \overset{\frown}{AED}$. $\overset{\frown}{DC}$ is congruent to itself by the Reflexive Property of Congruence. By the Arc Addition Postulate and the Addition Property of Congruence, $\overset{\frown}{ACD} \cong \overset{\frown}{ADC}$. $\angle ABC$ intercepts $\overset{\frown}{ADC}$, so $m\angle ABC = \frac{1}{2} m\overset{\frown}{ADC}$. $\angle AED$ intercepts $\overset{\frown}{ACD}$, so $m\angle AED = \frac{1}{2} m\overset{\frown}{ACD}$. By substitution, $m\angle ABC = m\angle AED$. Therefore $\angle ABC \cong \angle AED$.
2. Possible answer: It is given that $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. By the definition of congruent arcs, $m\overset{\frown}{PQ} = m\overset{\frown}{RS}$. $\angle PSQ$ intercepts $\overset{\frown}{PQ}$, and $\angle RQS$ intercepts $\overset{\frown}{RS}$. So $m\angle PSQ$ must equal $m\angle RQS$. Therefore $\angle PSQ \cong \angle RQS$. $\angle PSQ$ and $\angle RQS$ are congruent alternate interior angles of $\overline{QR}$ and $\overline{PS}$. So $\overline{QR} \parallel \overline{PS}$.
3. cannot be inscribed in a circle
4. Can be inscribed in a circle; possible answer: The two congruent angles of the kite are opposite, so they must be right angles. Draw a diameter. Draw segments from opposite ends of the diameter to any point on the circle. Use the compass to copy one of the segments across the diameter. Draw the fourth side.



5. Can be inscribed in a circle; possible answer: The pairs of base angles of a trapezoid inscribed in a circle must be congruent. Draw any inscribed angle. Use the compass to copy the arc that this angle intercepts. Mark off the same arc from the vertex of the inscribed angle. Connect the points.



6. cannot be inscribed in a circle

**Reteach**

1. $m\angle LMP = 18°$; $m\overset{\frown}{MN} = 96°$
2. $m\angle GFJ = 55°$; $m\overset{\frown}{FH} = 72°$
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**Challenge**

1. chord; inscribed
2. a. 45°
   b. 67.5°
   c. $|180 - 11.25n|$
   d. $0 < |n - 16| < 16$
   e. $\left|180 - \frac{360n}{p}\right|$, or $\left[180\left(1 - \frac{2n}{p}\right)\right]$,
   where $0 < n - \left|n - \frac{p}{2}\right| < \frac{p}{2}$
   f. Answers will vary. Students may choose any values of $n$ and $p$ for which $\frac{n}{p} = \frac{5}{12}$. Sample answer: $n = 15$, $p = 36$

**Problem Solving**

1. 160°
2. 112°; 52°; 68°; 128°
3. C
4. G
5. B
6. G

# Geometry

## Chapter 12: Circle

### Lesson 5 - Angle Relationships in Circles

**Standards**

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

**Objectives**

Find the measures of angles formed by lines that intersect circles.

Use angle measures to solve problems.

**Materials**

ToolBox, Pencil, Papers, Text Book

**Vocabulary**

None

**Formulas**

m<AC = 1/2m of arc AB (B is tangent point) (on a circle)

m<1 = ½(m of arc AB + m of arc CD) (AD and BC are secants) (inside a circle)

m<1 = ½(m of arc AD – m of arc BD) (tangent and secant) (outside a circle)

**Symbols**

None

**Postulates**

None

**Theorems**

12-5-1 – If a tangent and a secant or chord intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.

12-5-2 – If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.

12-5-3 – If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.

**Who Uses This?**

Circles and angles help optometrists correct vision problems.

**Discussion**

ToolBox Update: Make a new page and label it 'Angle Relationships in Circles'. Have cadets write the chart in their ToolBox – this is the best example of the three angles that we will be looking at.

Review the first theorem and work through Example 1. This should be review so cadets should be able to work through Check-It-Out 1. The second theorem is also a good review as Example 2 is gone over. Cadets should be able to do the Check-It-Out #2 with any problem. Review quickly with them. The third theorem is confusing as presented in the book because it looks like there are three formulas to use. There is only one so help cadets see that there is in fact only one formula. Work through the Example 3. Cadets should have no problem since there is only one formula. Have them work through Check-It-Out #3.

Example 5 is a good example for having cadets pick out different arcs and different angles. Have cadets pick out the arcs and angles but do not necessarily work through the problem.

**Guided Practice**

Work problems 1 – 15 with cadets. Have them pick one from each example that they would like to see worked through.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 16 – 34, 39 – 44

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 16 – 45

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 16 – 48

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 16, 20, 24, 26, 28, 30

**Homework**

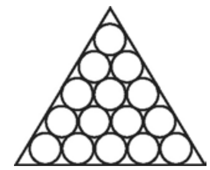
Page 77 and 161 May be replaced with different work based on needs. Replaced by additional resources listed above.

Name ______________________ Date ______________ Class______________

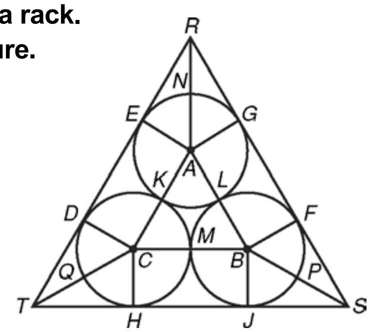
LESSON 12-5

# Challenge

## *Racking Billiard Balls*



A regulation pocket billiard ball is a perfect sphere with a diameter of 2.25 inches, and a tolerance of 0.005 inch. At the start of a game of pocket billiards, the 15 balls must be arranged in five rows in a triangular rack as shown at right. On this page, you will see how the properties of circles determine the shape and size of the rack.




**Below at right is a figure depicting just two rows of billiard balls in a rack. On a separate sheet of paper, justify each statement about this figure.**

1. $\triangle ABC$ is an equilateral triangle.
2. $ACDE$ is a rectangle.
3. $m\widehat{KL} = 60°$
4. $m\widehat{EK} = m\widehat{GL} = 90°$
5. $m\widehat{ENG} = 120°$
6. $m\angle ERG = 60°$
7. $\triangle AER \cong \triangle AGR$
8. $m\angle ERA = 30°$
9. $AC = 2.25$ inches
10. $ED = 2.25$ inches
11. $AE = 1.125$ inches
12. $ER = (1.125)\sqrt{3}$ inches
13. $TD = (1.125)\sqrt{3}$ inches
14. $TR = \left(2.25 + 2.25\sqrt{3}\right)$ inches $\approx 6.1$ inches

Following similar reasoning, $m\angle HTD = m\angle FSJ = 60°$ and $RS = ST \approx 6.1$ inches. So a triangular rack for two rows of pocket billiard balls would be an equilateral triangle with sides that are each slightly longer than 6.1 inches in length.

**Suppose that a rack shaped like an equilateral triangle encloses the given number of pocket billiard balls. Find the length of each side of the rack. (*Hint:* How many rows of balls will there be?)**

15. 6 balls ________________________________________

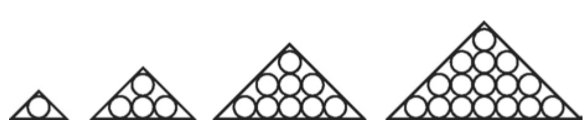
16. 10 balls ________________________________________

17. 15 balls ________________________________________

18. Write an expression for the length, in inches, of each side of the equilateral triangular rack that would enclose *n* rows of pocket billiard balls in the manner shown above.

________________________________________

19. Suppose that billiard balls of diameter *d* inches were racked in the pattern shown at right. Describe the rack that would enclose *n* rows of billiard balls in this way.



________________________________________

________________________________________

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

Name ______________________________ Date _______________ Class_______________

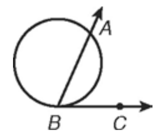
**LESSON 12-5**

# Practice A

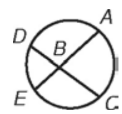
## *Angle Relationships in Circles*

**In Exercises 1–3, match the letter of the drawing to the formula for finding the measure of the angle.**

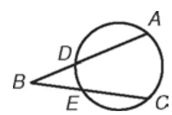
1. $m\angle ABC = \frac{1}{2}\left(m\overset{\frown}{AC} + m\overset{\frown}{DE}\right)$ __________

A. 


2. $m\angle ABC = \frac{1}{2}\left(m\overset{\frown}{AC} - m\overset{\frown}{DE}\right)$ __________

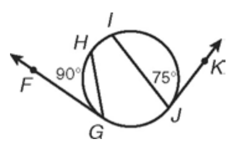
B. 

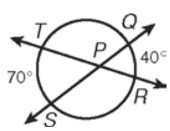

3. $m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$ __________

C. 

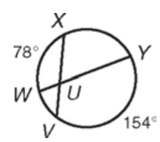

**Find each measure.**

4. $m\angle FGH =$ ____________
5. $m\overset{\frown}{IJ} =$ ____________



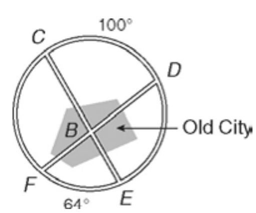

6. 


$m\angle QPR =$ _________

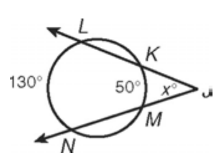
7. 


$m\angle YUV =$ _________

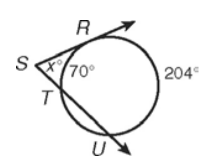
8. Some cities in Europe are thousands of years old. Often the small center of the old city is surrounded by a newer "ring road" that allows traffic to bypass the old streets. The figure shows a circular ring road and two roads that provide access to the old city. Find $m\angle CBD$. ____________




**Find the value of *x*.**

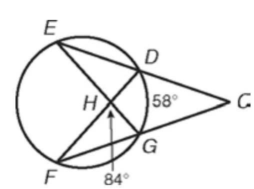
9. 


_____________

10. 


_____________

**Complete Exercises 11–13 in order to find $m\angle ECF$.**




11. Find $m\angle DHG$. (*Hint:* $\overline{DF}$ is a straight segment.) _____________
12. Find $m\overset{\frown}{EF}$. _____________
13. Find $m\angle ECF$. _____________

**Reading Strategies**

1. half
2. $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$
3. 31°
4. 60°
5. 124°
6. 60°
7. 56°
8. 120°

## 12-5 ANGLE RELATIONSHIPS IN CIRCLES

**Practice A**

1. B
2. C
3. A
4. 45°
5. 150°
6. 55°
7. 116°
8. 82°
9. 40
10. 67
11. 96°
12. 134°
13. 38°

**Practice B**

1. 64°; 96°
2. 119°; 42°
3. 130°
4. 99°
5. 64
6. 47
7. 8
8. 45
9. 60
10. 66.5°; 115°
11. 84°; 192°

**Practice C**

1. Possible answer: It is given that $\overline{AB} \cong \overline{EB}$. So $\triangle ABE$ is an isosceles triangle, and $\angle BAC \cong \angle BEA$. $\angle BEA$ is an inscribed angle, so $m\angle BEA = \frac{1}{2} m\widehat{BC}$. By substitution, $m\angle BAC = \frac{1}{2} m\widehat{BC}$. $\overline{AD}$ and $\overline{AE}$ are secants that intersect in the exterior of the circle. So $m\angle BAC = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. Substitution leads to $\frac{1}{2} m\widehat{BC} = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. This simplifies to $m\widehat{DE} = 2m\widehat{BC}$.
2. Possible answer: Draw chords $\overline{RU}$ and $\overline{ST}$. It is given that $\widehat{RU} \cong \widehat{ST}$. Because congruent arcs have congruent chords, $\overline{RU} \cong \overline{ST}$. It is given that $\widehat{RS} \cong \widehat{TU}$. $\angle RUS$, $\angle URT$, $\angle TSU$, and $\angle STR$ are all inscribed angles that intercept either $\widehat{RS}$ or $\widehat{TU}$. Therefore all four angles have the same measure and are congruent. By SAS, $\triangle QRU$ and $\triangle QST$ are congruent triangles. Furthermore, the base angles are all the same, so they are isosceles triangles. So $\overline{RQ}, \overline{UQ}$, and $\overline{SQ}$ (and $\overline{TQ}$) are congruent by CPCTC and the Isosceles Triangle Theorem. Congruent segments have equal lengths, so $Q$ is equidistant from points $R$, $U$, and $S$ (and $T$) that lie on the circle. Therefore $Q$ is the center of the circle.
3. Possible answer: Draw chord $\overline{KM}$. Assume that $m\widehat{KM} = 180°$. Because a circle contains 360°, $m\widehat{KLM}$ would also equal 180°. $\overrightarrow{JK}$ and $\overrightarrow{JM}$ intersect outside the circle, thus $m\angle KJM = \frac{1}{2}(m\widehat{KLM} - m\widehat{KM}) = 0°$. A triangle cannot contain a 0° angle, so $\triangle JKM$ does not exist, and $m\widehat{KM} = 180°$ cannot be true. Assume that $m\widehat{KM} > 180°$. Tangent $\overrightarrow{JM}$ and chord $\overrightarrow{KM}$ intersect at the point of tangency, $M$. So $m\angle JMK = \frac{1}{2} m\widehat{KM}$, which means $m\angle JMK > 90°$. Similar reasoning shows that $m\angle JKM$ is also greater than 90°. A triangle cannot contain two obtuse angles, so $\triangle JKM$ does not exist, and $m\widehat{KM} > 180°$ cannot be true. Therefore $m\widehat{KM} < 180°$.
4. 120°
5. 80°

**Reteach**

1. 108°
2. 128°
3. 61°
4. 103°
5. 33
6. 52
7. 23
8. 38

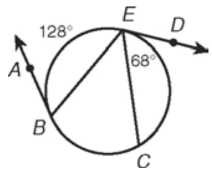
Name ________________________ Date ______________ Class______________

LESSON 12-5

# Practice B

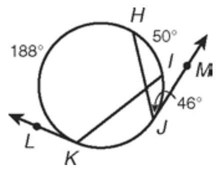
## *Angle Relationships in Circles*

**Find each measure.**

1. 

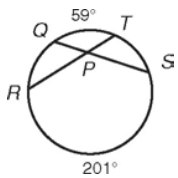

$m\angle ABE =$ ________

$m\widehat{BC} =$ ________

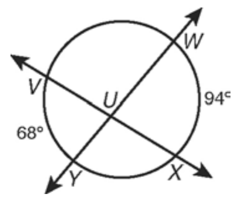
2. 


$m\angle LKI =$ ________

$m\widehat{IJ} =$ ________

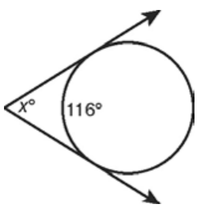
3. 


$m\angle RPS =$ ________

4. 


$m\angle YUX =$ ________

**Find the value of *x*.**

5. 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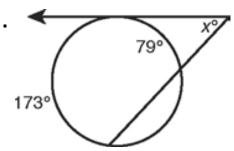
6. 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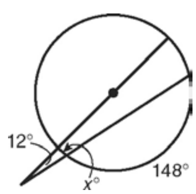
7. 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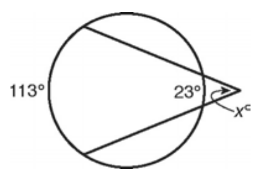
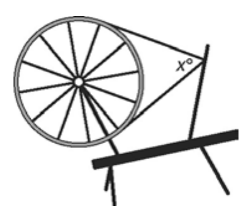
8. 


__________

9. The figure shows a spinning wheel. The large wheel is turned by hand or with a foot trundle. A belt attaches to a small bobbin that turns very quickly. The bobbin twists raw materials into thread, twine, or yarn. Each pair of spokes intercepts a 30° arc. Find the value of *x*.

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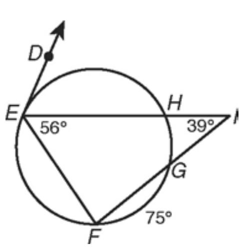
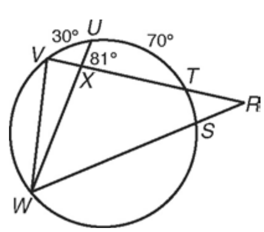
**Find each measure.**

10. 


$m\angle DEI =$ __________

$m\widehat{EF} =$ __________

11. 


$m\angle WVR =$ __________

$m\widehat{TUW} =$ __________

**Reading Strategies**

1. half
2. $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$
3. 31°
4. 60°
5. 124°
6. 60°
7. 56°
8. 120°

## 12-5 ANGLE RELATIONSHIPS IN CIRCLES

**Practice A**

1. B
2. C
3. A
4. 45°
5. 150°
6. 55°
7. 116°
8. 82°
9. 40
10. 67
11. 96°
12. 134°
13. 38°

**Practice B**

1. 64°; 96°
2. 119°; 42°
3. 130°
4. 99°
5. 64
6. 47
7. 8
8. 45
9. 60
10. 66.5°; 115°
11. 84°; 192°

**Practice C**

1. Possible answer: It is given that $\overline{AB} \cong \overline{EB}$. So $\triangle ABE$ is an isosceles triangle, and $\angle BAC \cong \angle BEA$. $\angle BEA$ is an inscribed angle, so $m\angle BEA = \frac{1}{2}m\overset{\frown}{BC}$. By substitution, $m\angle BAC = \frac{1}{2}m\overset{\frown}{BC}$. $\overline{AD}$ and $\overline{AE}$ are secants that intersect in the exterior of the circle. So $m\angle BAC = \frac{1}{2}(m\overset{\frown}{DE} - m\overset{\frown}{BC})$. Substitution leads to $\frac{1}{2}m\overset{\frown}{BC} = \frac{1}{2}(m\overset{\frown}{DE} - m\overset{\frown}{BC})$. This simplifies to $m\overset{\frown}{DE} = 2m\overset{\frown}{BC}$.
2. Possible answer: Draw chords $\overline{RU}$ and $\overline{ST}$. It is given that $\overset{\frown}{RU} \cong \overset{\frown}{ST}$. Because congruent arcs have congruent chords, $\overline{RU} \cong \overline{ST}$. It is given that $\overset{\frown}{RS} \cong \overset{\frown}{TU}$. $\angle RUS$, $\angle URT$, $\angle TSU$, and $\angle STR$ are all inscribed angles that intercept either $\overset{\frown}{RS}$ or $\overset{\frown}{TU}$. Therefore all four angles have the same measure and are congruent. By SAS, $\triangle QRU$ and $\triangle QST$ are congruent triangles. Furthermore, the base angles are all the same, so they are isosceles triangles. So $\overline{RQ}, \overline{UQ}$, and $\overline{SQ}$ (and $\overline{TQ}$) are congruent by CPCTC and the Isosceles Triangle Theorem. Congruent segments have equal lengths, so $Q$ is equidistant from points $R$, $U$, and $S$ (and $T$) that lie on the circle. Therefore $Q$ is the center of the circle.
3. Possible answer: Draw chord $\overline{KM}$. Assume that $m\overset{\frown}{KM} = 180°$. Because a circle contains 360°, $m\overset{\frown}{KLM}$ would also equal 180°. $\overrightarrow{JK}$ and $\overrightarrow{JM}$ intersect outside the circle, thus $m\angle KJM = \frac{1}{2}(m\overset{\frown}{KLM} - m\overset{\frown}{KM}) = 0°$. A triangle cannot contain a 0° angle, so $\triangle JKM$ does not exist, and $m\overset{\frown}{KM} = 180°$ cannot be true. Assume that $m\overset{\frown}{KM} > 180°$. Tangent $\overrightarrow{JM}$ and chord $\overrightarrow{KM}$ intersect at the point of tangency, $M$. So $m\angle JMK = \frac{1}{2}m\overset{\frown}{KM}$, which means $m\angle JMK > 90°$. Similar reasoning shows that $m\angle JKM$ is also greater than 90°. A triangle cannot contain two obtuse angles, so $\triangle JKM$ does not exist, and $m\overset{\frown}{KM} > 180°$ cannot be true. Therefore $m\overset{\frown}{KM} < 180°$.
4. 120°
5. 80°

**Reteach**

1. 108°
2. 128°
3. 61°
4. 103°
5. 33
6. 52
7. 23
8. 38

Name ________________________________ Date ______________ Class______________

LESSON 12-5

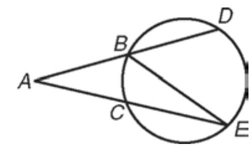
# Practice C

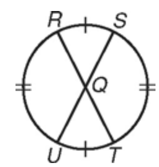
## *Angle Relationships in Circles*

**Write paragraph proofs for Exercises 1–3.**

1. **Given:** $\overline{AB} \cong \overline{EB}$

   **Prove:** $m\overparen{DE} = 2m\overparen{BC}$



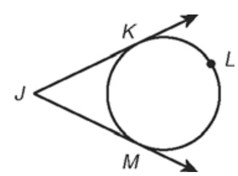




2. **Given:** $\overparen{RS} \cong \overparen{TU}, \overparen{RU} \cong \overparen{ST}$

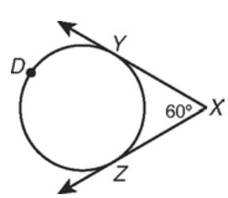
   **Prove:** $Q$ is the center of the circle.
   (*Hint:* Show that $Q$ is equidistant from three points on the circle.)

3. **Given:** $\overrightarrow{JK}$ and $\overrightarrow{JM}$ are tangent to the circle.

   **Prove:** $m\overparen{KM} < 180°$ (*Hint:* Use an indirect proof and consider two cases.)




**Find the measure of the minor arc intercepted by the two tangents.**

4. 

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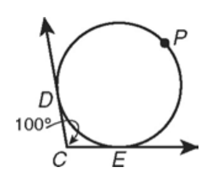
5. 

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**Reading Strategies**

1. half
2. $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$
3. 31°
4. 60°
5. 124°
6. 60°
7. 56°
8. 120°

## 12-5 ANGLE RELATIONSHIPS IN CIRCLES

**Practice A**

1. B
2. C
3. A
4. 45°
5. 150°
6. 55°
7. 116°
8. 82°
9. 40
10. 67
11. 96°
12. 134°
13. 38°

**Practice B**

1. 64°; 96°
2. 119°; 42°
3. 130°
4. 99°
5. 64
6. 47
7. 8
8. 45
9. 60
10. 66.5°; 115°
11. 84°; 192°

**Practice C**

1. Possible answer: It is given that $\overline{AB} \cong \overline{EB}$. So $\triangle ABE$ is an isosceles triangle, and $\angle BAC \cong \angle BEA$. $\angle BEA$ is an inscribed angle, so $m\angle BEA = \frac{1}{2}m\widehat{BC}$. By substitution, $m\angle BAC = \frac{1}{2}m\widehat{BC}$. $\overline{AD}$ and $\overline{AE}$ are secants that intersect in the exterior of the circle. So $m\angle BAC = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. Substitution leads to $\frac{1}{2}m\widehat{BC} = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$. This simplifies to $m\widehat{DE} = 2m\widehat{BC}$.
2. Possible answer: Draw chords $\overline{RU}$ and $\overline{ST}$. It is given that $\widehat{RU} \cong \widehat{ST}$. Because congruent arcs have congruent chords, $\overline{RU} \cong \overline{ST}$. It is given that $\widehat{RS} \cong \widehat{TU}$. $\angle RUS$, $\angle URT$, $\angle TSU$, and $\angle STR$ are all inscribed angles that intercept either $\widehat{RS}$ or $\widehat{TU}$. Therefore all four angles have the same measure and are congruent. By SAS, $\triangle QRU$ and $\triangle QST$ are congruent triangles. Furthermore, the base angles are all the same, so they are isosceles triangles. So $\overline{RQ}, \overline{UQ}$, and $\overline{SQ}$ (and $\overline{TQ}$) are congruent by CPCTC and the Isosceles Triangle Theorem. Congruent segments have equal lengths, so $Q$ is equidistant from points $R$, $U$, and $S$ (and $T$) that lie on the circle. Therefore $Q$ is the center of the circle.
3. Possible answer: Draw chord $\overline{KM}$. Assume that $m\widehat{KM} = 180°$. Because a circle contains 360°, $m\widehat{KLM}$ would also equal 180°. $\overrightarrow{JK}$ and $\overrightarrow{JM}$ intersect outside the circle, thus $m\angle KJM = \frac{1}{2}(m\widehat{KLM} - m\widehat{KM}) = 0°$. A triangle cannot contain a 0° angle, so $\triangle JKM$ does not exist, and $m\widehat{KM} = 180°$ cannot be true. Assume that $m\widehat{KM} > 180°$. Tangent $\overrightarrow{JM}$ and chord $\overrightarrow{KM}$ intersect at the point of tangency, $M$. So $m\angle JMK = \frac{1}{2}m\widehat{KM}$, which means $m\angle JMK > 90°$. Similar reasoning shows that $m\angle JKM$ is also greater than 90°. A triangle cannot contain two obtuse angles, so $\triangle JKM$ does not exist, and $m\widehat{KM} > 180°$ cannot be true. Therefore $m\widehat{KM} < 180°$.
4. 120°
5. 80°

**Reteach**

1. 108°
2. 128°
3. 61°
4. 103°
5. 33
6. 52
7. 23
8. 38

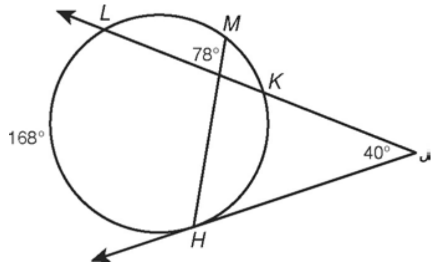
Name ________________________________ Date ______________ Class______________

LESSON 12-5

# Problem Solving

## *Angle Relationships in Circles*

1. What is $m\overset{\frown}{LM}$ ?

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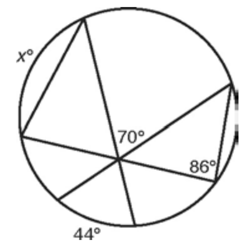
2. An artist painted the design shown below. What is the value of *x*?

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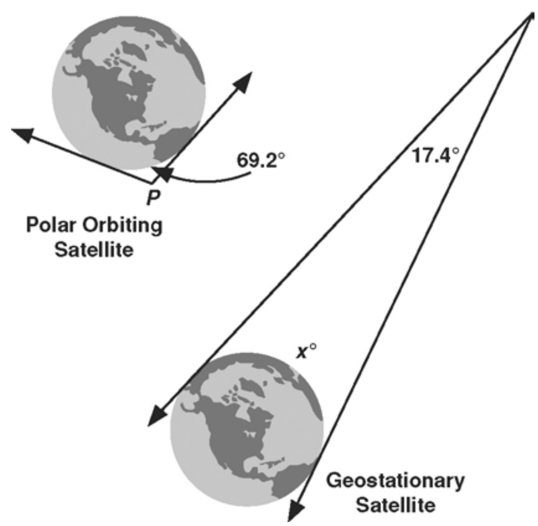
**For Exercises 3 and 4, use the diagrams.**

3. A polar orbiting satellite is about 850 kilometers above Earth. About 69.2 arc degrees of the planet are visible to a camera in the satellite. What is $m\angle P$?

______________________________

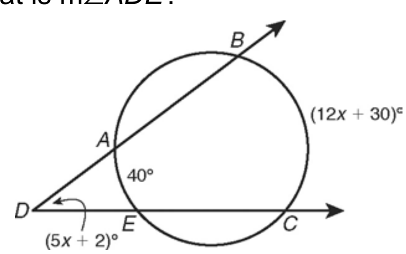
4. A geostationary satellite is about 35,800 kilometers above Earth. How many arc degrees of the planet are visible to a camera in the satellite?

______________________________




**Choose the best answer.**

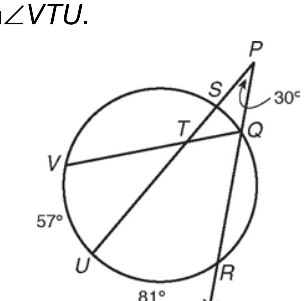
5. What is $m\angle ADE$?




A 7° C 37°

B 33° D 114°

6. Find $m\angle VTU$.




F 21° H 36°

G 29° J 39°

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

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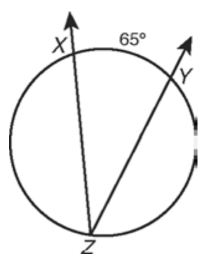
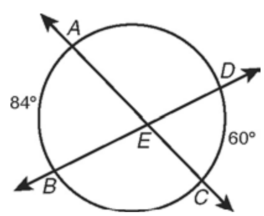
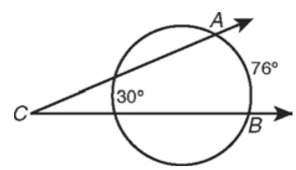
LESSON 12-5

# Reading Strategies

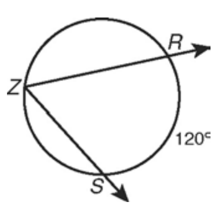
## *Use a Graphic Aid*

The graphic aid below summarizes angle relationships in circles.

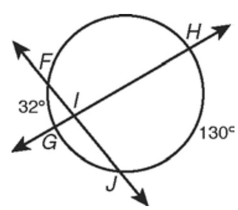
**Angle Relationships in Circles**

| Vertex lies **on a circle.** | Vertex lies **inside a circle.** | Vertex lies **outside a circle.** |
|---|---|---|
| Angle measure is **half** the measure of the intercepted arc. | Angle measure is **half the sum** of the measures of the intercepted arcs. | Angle measure is **half the difference** of the measures of the intercepted arcs. |
|  |  |  |
| $m\angle XYZ = \frac{1}{2}(65°) = 32.5°$ | $m\angle AEB = m\angle DEC = \frac{1}{2}(84° + 60°) = 72°$ | $m\angle ACB = \frac{1}{2}(76° - 30°) = 23°$ |

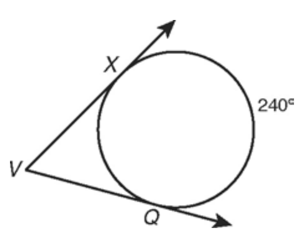
**Find each measure.**



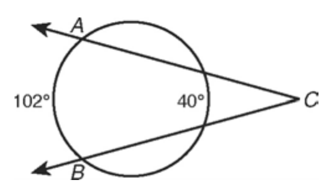

1. $m\angle RZS$ ________________




2. $m\angle HIJ$ ________________




3. $m\angle XVQ$ ________________




4. $m\angle ACB$ ________________

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

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2. B
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6. 9.75 m
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8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

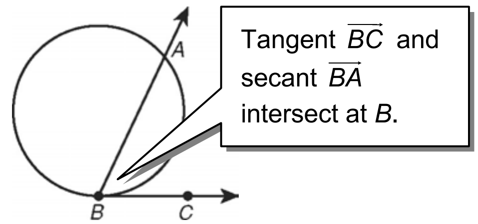
Name ________________________________________ Date ________________ Class________________

LESSON 12-5

# Reteach

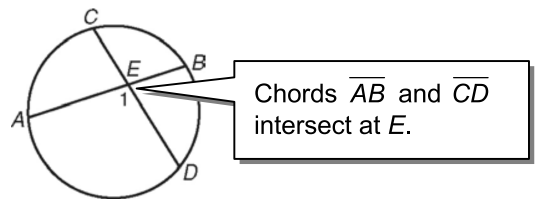
## *Angle Relationships in Circles*

If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.




$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AB}$$

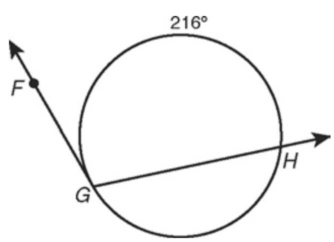
If two secants or chords intersect in the interior of a circle, then the measure of the angle formed is half the sum of the measures of its intercepted arcs.




$$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AD} + m\overset{\frown}{BC}\right)$$

**Find each measure.**

1. $m\angle FGH$

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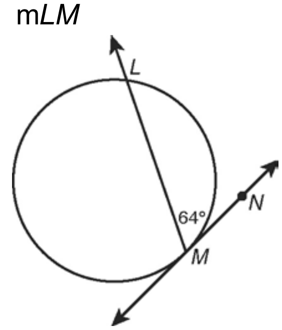
2. $m\overset{\frown}{LM}$

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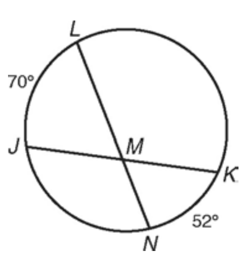
3. $m\angle JML$

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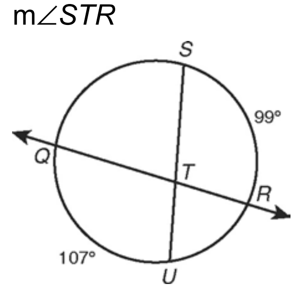
4. $m\angle STR$

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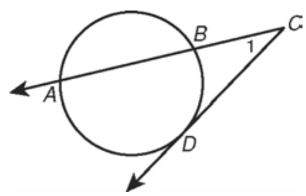
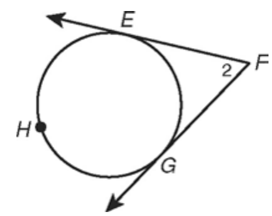
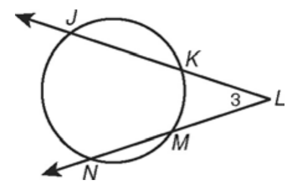
Name ______________________ Date ______________ Class______________

LESSON 12-5

# Reteach

## *Angle Relationships in Circles* *continued*

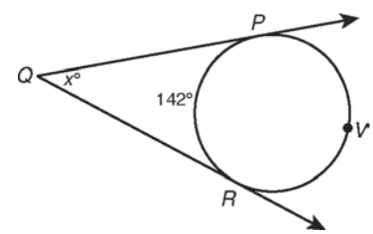
If two segments intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.

| A Tangent and a Secant | Two Tangents | Two Secants |
|---|---|---|
|   |   |   |
| $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BD}\right)$ | $m\angle 2 = \frac{1}{2}\left(m\overset{\frown}{EHG} - m\overset{\frown}{EG}\right)$ | $m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{JN} - m\overset{\frown}{KM}\right)$ |

**Find the value of *x*.**

Since $m\overset{\frown}{PVR} + m\overset{\frown}{PR} = 360°$, $m\overset{\frown}{PVR} + 142° = 360°$,

and $m\overset{\frown}{PVR} = 218°$.




$$x° = \frac{1}{2}\left(m\overset{\frown}{PVR} - m\overset{\frown}{PR}\right)$$

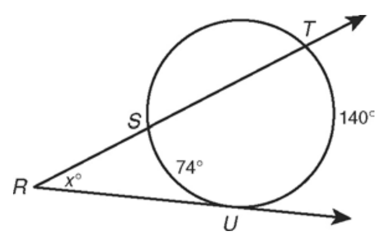
$$= \frac{1}{2}(218° - 142°)$$

$$x° = 38°$$

$$x = 38$$

**Find the value of *x*.**

5.

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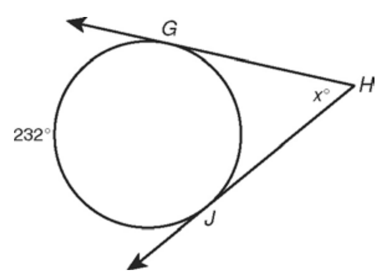
6.

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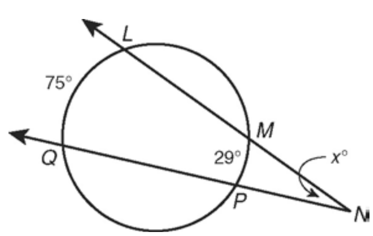
7.

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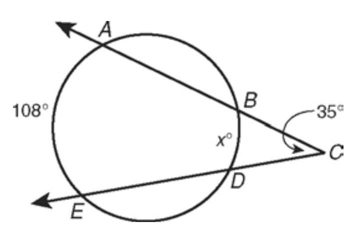
8.

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### Reading Strategies

1. half
2. $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$
3. 31°
4. 60°
5. 124°
6. 60°
7. 56°
8. 120°

## 12-5 ANGLE RELATIONSHIPS IN CIRCLES

### Practice A

1. B
2. C
3. A
4. 45°
5. 150°
6. 55°
7. 116°
8. 82°
9. 40
10. 67
11. 96°
12. 134°
13. 38°

### Practice B

1. 64°; 96°
2. 119°; 42°
3. 130°
4. 99°
5. 64
6. 47
7. 8
8. 45
9. 60
10. 66.5°; 115°
11. 84°; 192°

### Practice C

1. Possible answer: It is given that $\overline{AB} \cong \overline{EB}$. So $\triangle ABE$ is an isosceles triangle, and $\angle BAC \cong \angle BEA$. $\angle BEA$ is an inscribed angle, so $m\angle BEA = \frac{1}{2} m\overset{\frown}{BC}$. By substitution, $m\angle BAC = \frac{1}{2} m\overset{\frown}{BC}$. $\overline{AD}$ and $\overline{AE}$ are secants that intersect in the exterior of the circle. So $m\angle BAC = \frac{1}{2}(m\overset{\frown}{DE} - m\overset{\frown}{BC})$. Substitution leads to $\frac{1}{2} m\overset{\frown}{BC} = \frac{1}{2}(m\overset{\frown}{DE} - m\overset{\frown}{BC})$. This simplifies to $m\overset{\frown}{DE} = 2m\overset{\frown}{BC}$.
2. Possible answer: Draw chords $\overline{RU}$ and $\overline{ST}$. It is given that $\overset{\frown}{RU} \cong \overset{\frown}{ST}$. Because congruent arcs have congruent chords, $\overline{RU} \cong \overline{ST}$. It is given that $\overset{\frown}{RS} \cong \overset{\frown}{TU}$. $\angle RUS$, $\angle URT$, $\angle TSU$, and $\angle STR$ are all inscribed angles that intercept either $\overset{\frown}{RS}$ or $\overset{\frown}{TU}$. Therefore all four angles have the same measure and are congruent. By SAS, $\triangle QRU$ and $\triangle QST$ are congruent triangles. Furthermore, the base angles are all the same, so they are isosceles triangles. So $\overline{RQ}, \overline{UQ}$, and $\overline{SQ}$ (and $\overline{TQ}$) are congruent by CPCTC and the Isosceles Triangle Theorem. Congruent segments have equal lengths, so $Q$ is equidistant from points $R$, $U$, and $S$ (and $T$) that lie on the circle. Therefore $Q$ is the center of the circle.
3. Possible answer: Draw chord $\overline{KM}$. Assume that $m\overset{\frown}{KM} = 180°$. Because a circle contains 360°, $m\overset{\frown}{KLM}$ would also equal 180°. $\overleftrightarrow{JK}$ and $\overrightarrow{JM}$ intersect outside the circle, thus $m\angle KJM = \frac{1}{2}(m\overset{\frown}{KLM} - m\overset{\frown}{KM}) = 0°$. A triangle cannot contain a 0° angle, so $\triangle JKM$ does not exist, and $m\overset{\frown}{KM} = 180°$ cannot be true. Assume that $m\overset{\frown}{KM} > 180°$. Tangent $\overrightarrow{JM}$ and chord $\overleftrightarrow{KM}$ intersect at the point of tangency, $M$. So $m\angle JMK = \frac{1}{2} m\overset{\frown}{KM}$, which means $m\angle JMK > 90°$. Similar reasoning shows that $m\angle JKM$ is also greater than 90°. A triangle cannot contain two obtuse angles, so $\triangle JKM$ does not exist, and $m\overset{\frown}{KM} > 180°$ cannot be true. Therefore $m\overset{\frown}{KM} < 180°$.
4. 120°
5. 80°

### Reteach

1. 108°
2. 128°
3. 61°
4. 103°
5. 33
6. 52
7. 23
8. 38

# Geometry

## Chapter 12: Circle

### Lesson 6 - Segment Relationships in Circles

**Standards**

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

**Objectives**

Find the lengths of segments formed by lines that intersect circles.

Use the lengths of segments in circles to solve problems.

**Materials**

ToolBox, pencil, paper, Text Book

**Vocabulary**

Secant segment – A segment of a secant with at least one endpoint on the circle.

External secant segment - A segment of a secant that lies in the exterior of the circle with one endpoint on the circle.

Tangent Segment – A segment of a tangent with one endpoint on the circle.

**Formulas**

Two chords – AE X EB + CE X ED Chords AB and CD intersect at E

Two secants - AE X BE = CE X DE Secants AE and CE intersect at E

Secant and tangent – AC X BC = $DC^2$ – Secant AC and tangent DC

**Symbols**

None

**Postulates**

None

**Theorems**

12-6-1 – Chord-Chord Product Theorem – If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.

12-6-2 – Secant-Secant product Theorem – If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

12-6-3 - Secant-Tangent Product Theorem – If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.

**Who Uses This?**

Archaeologists use facts about segments in circles to help them understand ancient objects.

**Discussion**

Cadets have a difficult time with this lesson because there is a switch from angles to segments. Take time to be sure cadets have made that transition.

Present the first theorem and have cadets write it in their ToolBox. Work through the example 1 problem with students making sure that they are looking at the lengths of the segments and not the angles. Have cadets work through Check-It-Out #1.

Example #2 is great for giving cadets an example of how to use this theorem in the real world.  Work through the example and then have them try the Check-It-Out #2.

Look at the next theorem and be sure that cadets understand that secants can extend outside the circle.  Some have the impression that secants cannot intersect, so make sure everyone understands the model with the secants.   Work through Example #3.  Change the numbers in the problem and work another example of this theorem before having cadets work Check-It-Out #3.

The next theorem is the secant and a tangent.  Write the theorem in the ToolBox and then discuss the example.  Make sure to help cadets see the difference between each example in the way the secant, chord and tangent look when involved with the circle.  Work example #4.  May need to review once more about finding the square root to get the final answer.  Have cadets work Check-it-Out #4.

**Guided Practice**

Pick some problems from 2-11 and work them until cadets feel they have the process and then they can work on the classwork.  Number 5 is a great problem to work through.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 12 – 28, 32 – 35

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 36

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 36, 28 – 39

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 14, 15, 16, 20, 22

**Homework**

Page 78 and 162 May be replaced with different work based on needs. Replaced by additional resources listed above.

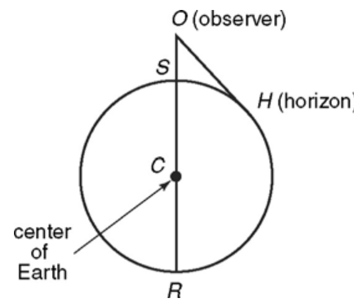
Name ______________________ Date ______________ Class______________

LESSON 12-6

# Challenge

## *Finding the Distance to the Horizon*

For an observer at a point *O* above Earth, the horizon is the place where Earth appears to “meet the sky.” The higher above Earth’s surface the observer is, the farther away the horizon appears to be. It may surprise you to learn that you can calculate this distance to the horizon by applying your knowledge of tangents and secants.




**Refer to the diagram of Earth at right.**

1. Name the segment that represents each measure.

   a. the diameter of Earth ______________

   b. the observer’s altitude above Earth’s surface ______________

   c. the distance the observer can see to the horizon ______________

2. Justify the following equation: $(OH)^2 = OR \cdot OS$

   ________________________________________________

   ________________________________________________

When the observer’s altitude above Earth’s surface is small relative to the diameter of Earth, you can replace *OR* with *RS* in the equation from Exercise 2. Then, since the diameter of Earth is approximately 7920 miles, you obtain the formula for *OH* shown at right. In this formula, the unit for both *OH* and *OS* is miles.

$$(OH)^2 = OR \cdot OS$$
$$(OH)^2 \approx RS \cdot OS$$
$$(OH)^2 \approx 7920 \cdot OS$$
$$OH \approx \sqrt{7920 \cdot OS}$$

**Use the formula above to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.**

3. 2.5 miles ______________

4. 30,000 feet ______________

5. Rewrite the formula above so that you can input *OS* as a number of feet and find the distance to the horizon in *miles.*

   $$\left(Hint: OH \text{ miles} = \sqrt{7920 \text{ miles} \cdot \frac{1 \text{ mile}}{? \text{ feet}} \cdot OS \text{ feet}}\right)$$ ______________

**Use your formula from Exercise 5 to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.**

6. 10 feet ______________

7. 200 feet ______________

**Find the altitude above Earth’s surface that an observer must attain in order to see the given distance to the horizon. Round answers to the nearest tenth.**

8. 1 mile ______________

9. 300 miles ______________

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

Name ______________________ Date ______________ Class______________

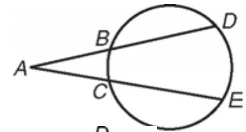
LESSON 12-6

# Practice A

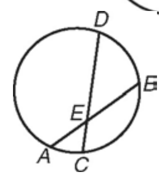
## *Segment Relationships in Circles*

**In Exercises 1–3, match the letter of the drawing to the formula that relates the lengths of the segments in the drawing.**

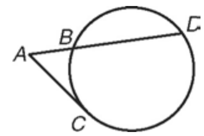
1. $AC^2 = AB(AD)$ __________

A. 

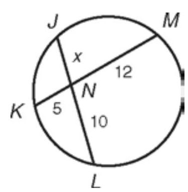

2. $AE(BE) = CE(DE)$ __________

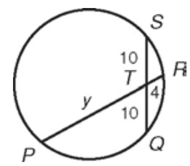
B. 


3. $AB(AD) = AC(AE)$ __________

C. 


**Find the value of the variable and the length of each chord.**

4. 


5. 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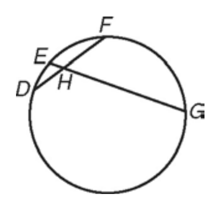
6. Henri is riding a carousel at an amusement park. Devon, Emile, Francis, and George are looking on from around the edge of the carousel. At the moment shown in the figure, Devon is 2.5 meters from Henri, Emile is 1 meter from Henri, and Francis is 3.5 meters from Henri. Find the distance from Emile to George.

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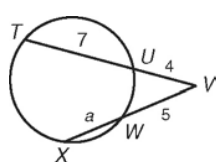
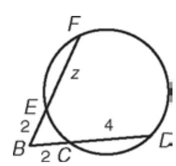
**Find the value of the variable and the length of each secant segment.**

7. 


8. 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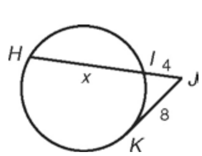
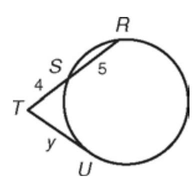
**Find the value of the variable.**

9. 


10. 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### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

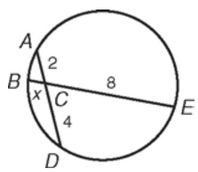
Name ______________________ Date ______________ Class ______________

LESSON 12-6

# Practice B

## *Segment Relationships in Circles*

**Find the value of the variable and the length of each chord.**

1. 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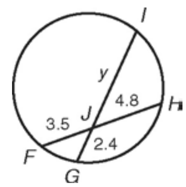
2. 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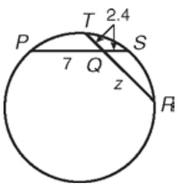
3. 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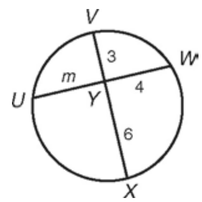
4. 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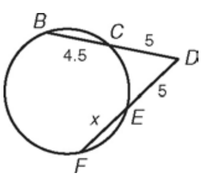
**Find the value of the variable and the length of each secant segment.**

5. 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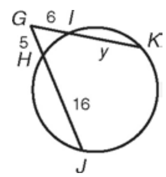
6. 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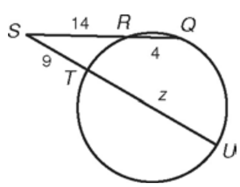
7. 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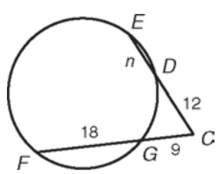
8. 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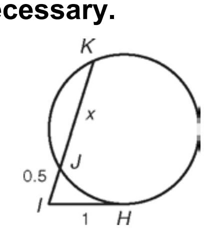
**Find the value of the variable. Give answers in simplest radical form if necessary.**

9. 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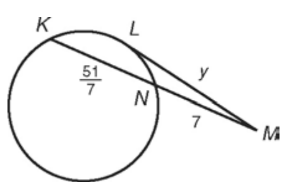
10. 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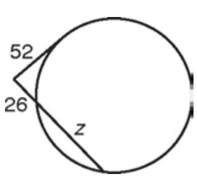
11. 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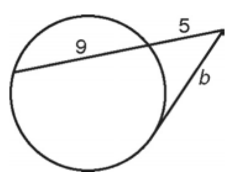
12. 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## Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

## Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

## Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

# 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

## Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

## Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

## Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

## Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

## Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

## Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

## Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

Name ______________________ Date ______________ Class______________

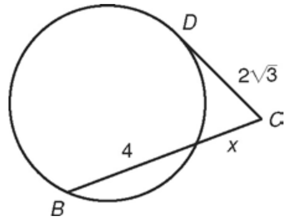
**LESSON 12-6**

# Practice C

## *Segment Relationships in Circles*

**Find the value of *x*. Round to the nearest tenth if necessary.**

1.

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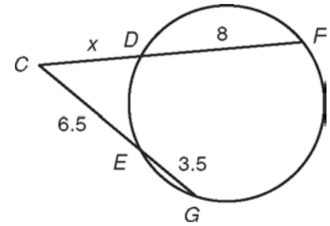
2.

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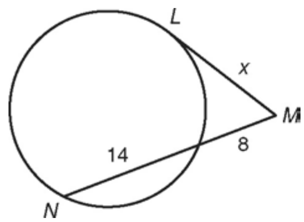
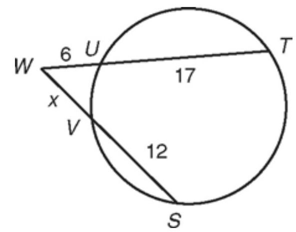
3.



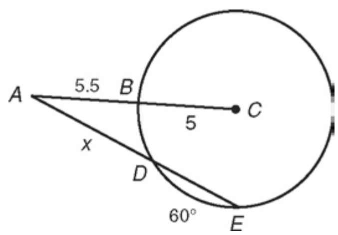

______________________

4.




______________________

5.

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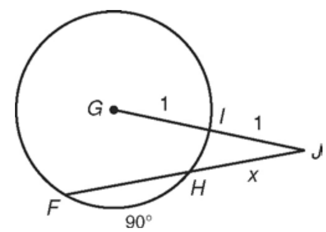
6.

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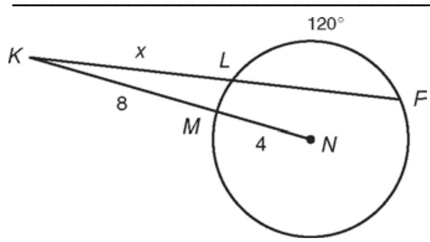
7.

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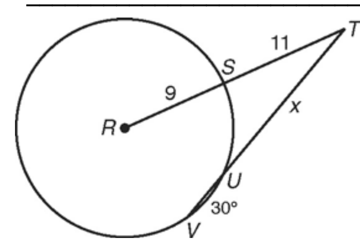
8.

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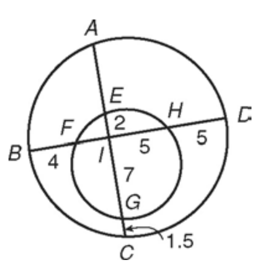
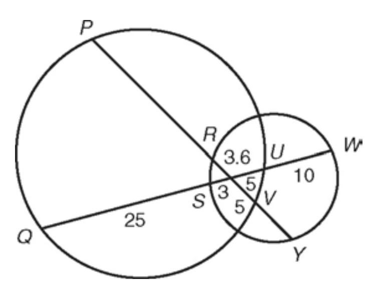
**Find each length.**

9.




$AC =$ __________ $BD =$ __________

10.




$PY =$ ______________________

**Challenge**

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

**Problem Solving**

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

**Reading Strategies**

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

**Practice A**

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

**Practice B**

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

**Practice C**

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

**Reteach**

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

**Challenge**

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

**Problem Solving**

1. 24.5
2. 20
3. A
4. H
5. D
6. F

**Reading Strategies**

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

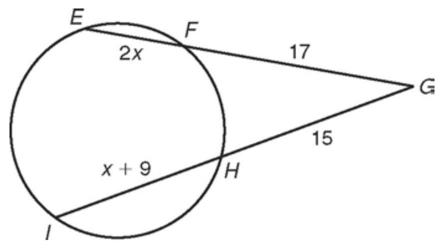
Name ______________________ Date ______________ Class______________

LESSON 12-6

# Problem Solving

## *Segment Relationships in Circles*

1. Find *EG* to the nearest tenth.

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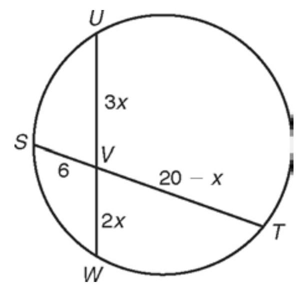
2. What is the length of $\overline{UW}$?

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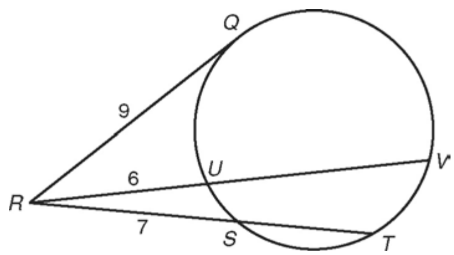
**Choose the best answer.**

3. Which of these is closest to the length of $\overline{ST}$?




A 4.6 C 7.5

B 5.4 D 11.6

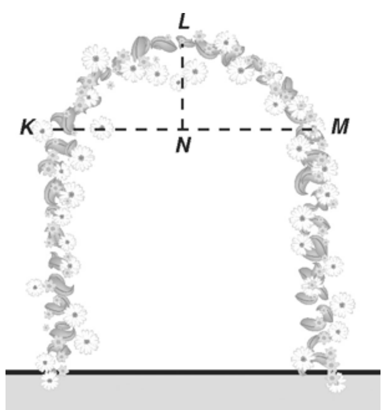
4. Floral archways like the one shown below are going to be used for the prom. $\overline{LN}$ is the perpendicular bisector of $\overline{KM}$. $KM = 6$ feet and $LN = 2$ feet. What is the diameter of the circle that contains $\overparen{KM}$?

F 4.5 ft

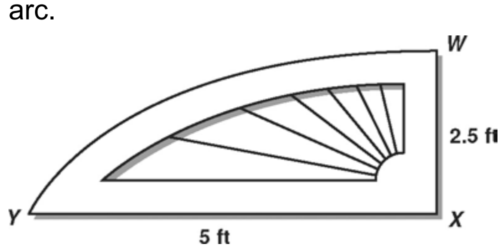
G 5.5 ft

H 6.5 ft

J 8 ft



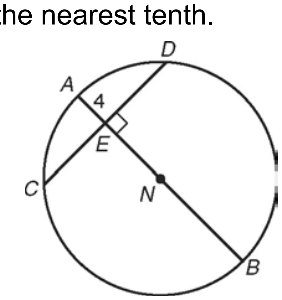

5. The figure is a "quarter" wood arch used in architecture. $\overline{WX}$ is the perpendicular bisector of the chord containing $\overline{YX}$. Find the diameter of the circle containing the arc.




A 5 ft C 10 ft

B 8.5 ft D 12.5 ft

6. In $\odot N$, $CD = 18$. Find the radius of the circle to the nearest tenth.




F 12.1 H 20.3

G 16.3 J 24.3

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

Name ______________________ Date ____________ Class____________

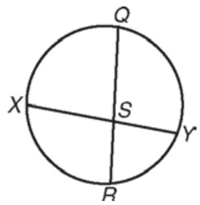
LESSON **12-6**

# Reading Strategies

## *Use a Model*

The models below show segment relationships in circles.

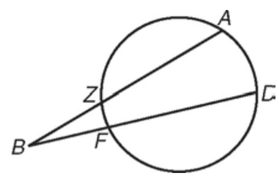
**Chord-Chord**




Chords $\overline{XY}$ and $\overline{QR}$ intersect at $S$.

$RS \cdot SQ = XS \cdot SY$

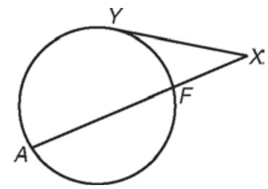
**Secant-Secant**




Secants $\overline{AB}$ and $\overline{DB}$ intersect at $B$.

$AB \cdot ZB = DB \cdot FB$

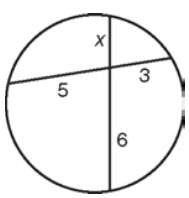
**Secant-Tangent**



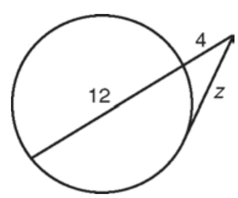

Secant $\overline{AX}$ and tangent $\overline{YX}$ intersect at $X$.

$AX \cdot FX = YX^2$

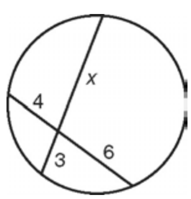
**Find the value of each variable.**



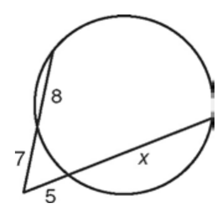

1. __________



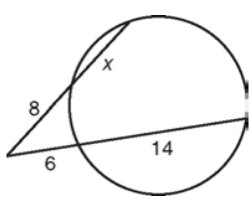

2. __________



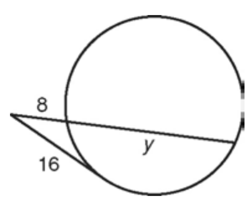

3. __________




4. __________




5. __________




6. __________

### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
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7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
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### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

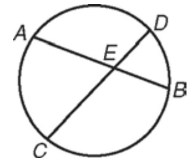
1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

Name ________________________________________ Date ________________ Class________________

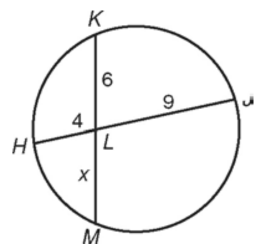
LESSON 12-6

# Reteach

## *Segment Relationships in Circles*

| Chord-Chord Product Theorem | |
|---|---|
| If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal. |   $AE \cdot EB = CE \cdot ED$ |

**Find the value of *x* and the length of each chord.**




$HL \cdot LJ = KL \cdot LM$ Chord-Chord Product Thm.

$4 \cdot 9 = 6 \cdot x$ $HL = 4, LJ = 9, KL = 6, LM = x$

$36 = 6x$ Simplify.

$6 = x$ Divide each side by 6.

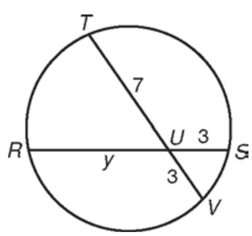
$HJ = 4 + 9 = 13$

$KM = 6 + x$

$= 6 + 6 = 12$

**Find the value of the variable and the length of each chord.**

1.

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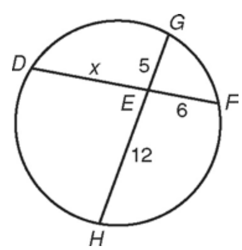
2.

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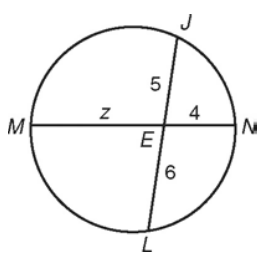
3.

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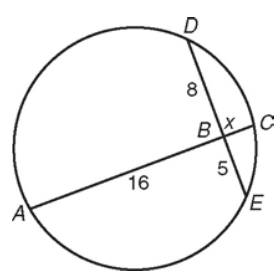
4.

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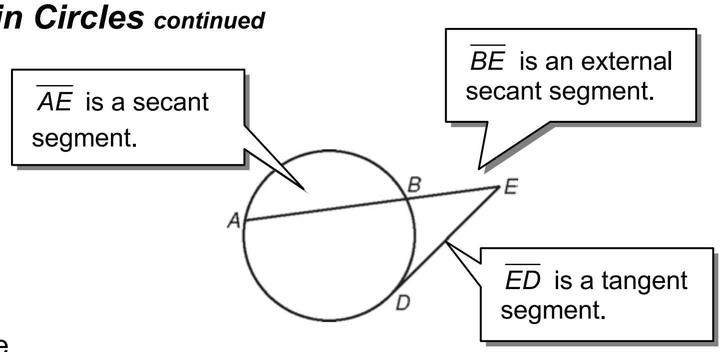
Name ________________________ Date ______________ Class______________

**LESSON 12-6**

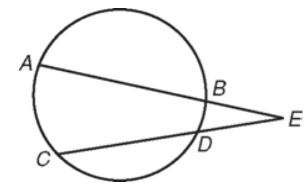
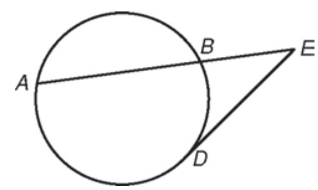
# Reteach

## *Segment Relationships in Circles* continued

- A **secant segment** is a segment of a secant with at least one endpoint on the circle.
- An **external secant segment** is the part of the secant segment that lies in the exterior of the circle.
- A **tangent segment** is a segment of a tangent with one endpoint on the circle.

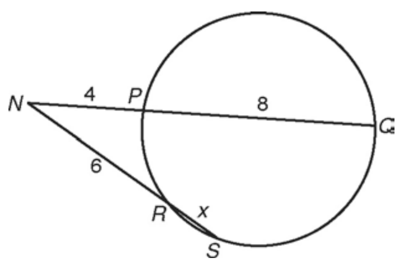



If two segments intersect outside a circle, the following theorems are true.

| | |
|---|---|
| **Secant-Secant Product Theorem**<br>The product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.<br>whole · outside = whole · outside<br>$AE \cdot BE = CE \cdot DE$ |  |
| **Secant-Tangent Product Theorem**<br>The product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.<br>whole · outside = tangent$^2$<br>$AE \cdot BE = DE^2$ |  |

**Find the value of the variable and the length of each secant segment.**

5.

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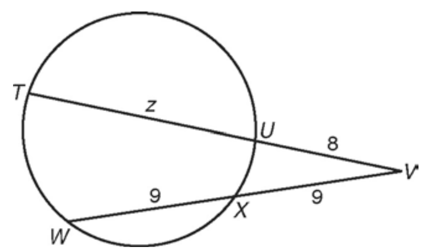
6.

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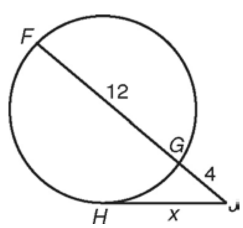
**Find the value of the variable.**

7.

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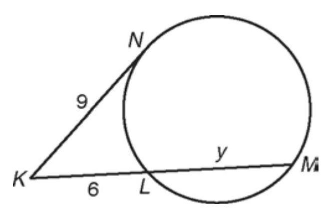


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### Challenge

Student justifications will vary.

15. $(4.5 + 2.25\sqrt{3})$ inches ≈ 8.4 in.
16. $(6.75 + 2.25\sqrt{3})$ inches ≈ 10.6 in.
17. $(9 + 2.25\sqrt{3})$ inches ≈ 12.9 in.
18. $2.25(n - 1) + 2.25\sqrt{3}$, or $2.25(n - 1 + \sqrt{3})$
19. Isosceles right triangle with legs of $\left[d\left(1+\frac{\sqrt{2}}{2}+(n-1)\sqrt{2}\right)\right]$ in.; hypotenuse of $[d(2n - 1 + \sqrt{2})]$ in.

### Problem Solving

1. 68°
2. 76
3. 110.8°
4. 162.6°
5. C
6. J

### Reading Strategies

1. 60°
2. 81°
3. 60°
4. 31°

## 12-6 SEGMENT RELATIONSHIPS IN CIRCLES

### Practice A

1. C
2. B
3. A
4. $x = 6$; $JL = 16$; $KM = 17$
5. $y = 25$; $PR = 29$; $QS = 20$
6. 9.75 m
7. $a = 3.8$; $VT = 11$; $VX = 8.8$
8. $z = 4$; $BD = 6$; $BF = 6$
9. 12
10. 6

### Practice B

1. $x = 1$; $AD = 6$; $BE = 9$
2. $y = 7$; $FH = 8.3$; $GI = 9.4$
3. $z = 7$; $PS = 9.4$; $TR = 9.4$
4. $m = 4.5$; $UW = 8.5$; $VX = 9$
5. $x = 4.5$; $BD = 9.5$; $FD = 9.5$
6. $y = 11.5$; $GJ = 21$; $GK = 17.5$
7. $z = 19$; $SQ = 18$; $SU = 28$
8. $n = 8.25$; $CE = 20.25$; $CF = 27$
9. 1.5
10. 10
11. 78
12. $\sqrt{70}$

### Practice C

1. 2
2. 5
3. 13.3
4. 7.2
5. 7.1
6. 1.2
7. 8.4
8. 15.7
9. 16.5; 16.8
10. 40.5

### Reteach

1. $y = 7$; $RS = 10$; $TV = 10$
2. $x = 10$; $DF = 16$; $GH = 17$
3. $z = 7.5$; $JL = 11$; $MN = 11.5$
4. $x = 2.5$; $AC = 18.5$; $DE = 13$
5. $x = 2$; $NQ = 12$; $NS = 8$
6. $z = 12.25$; $TV = 20.25$; $WV = 18$
7. 8
8. 7.5

### Challenge

1. $\overline{RS}$; $\overline{OS}$; $\overline{OH}$
2. $\overline{OH}$ is a tangent segment of circle $C$, $\overline{OR}$ is a secant segment, and $\overline{OS}$ is its external secant segment. So $(OH)^2 = OR \cdot OS$.
3. 140.7 mi
4. 212.1 mi
5. $OH \approx \sqrt{1.5 \cdot OS}$
6. 3.9 mi
7. 17.3 mi
8. 0.7 ft, or about 8 in.
9. 11.4 mi

### Problem Solving

1. 24.5
2. 20
3. A
4. H
5. D
6. F

### Reading Strategies

1. 2.5
2. 8
3. 8
4. 16
5. 7
6. 24

# Geometry

## Chapter 12: Circle

### Lesson 7 - Circles in the Coordinate Plane

**Standards**

G.GPE.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

**Objectives**

Write equations and graph circles in the coordinate plane.

Use the equation and graph of a circle to solve problems

**Materials**

Graph paper, pencil, Text Book, ToolBox, Compass, straight edge,

**Vocabulary**

None

**Formulas**

Equation of a circle – $(x - h)^2 + (y - k)^2 = r^2$

(h, k) is center of circle and r is the radius.

**Symbols**

None

**Postulates**

None

**Theorems**

12-7-1 – Equation of a Circle – The equation of a circle with center (h, k) and radius r is $(x – h)^2 + (y – k)^2 = r^2$.

**Who Uses This?**

Meteorologists use circles and coordinates to plan the location of weather stations.

**Discussion**

Write the theorem for the equation of a circle. Discuss how the formula is derived from the distance formula that has been already studied. May need to review the fact that the opposite of square rooting is to square. This is a concept that cadets often forget.

Review Example #1. May have to spend some time discussing the variables and what they stand for in the circle. Work through carefully step by step. Work the Check-It-Out #1 and monitor cadets carefully. Make adjustments and reteach where appropriate.

Example #2 is graphing a circle on the coordinate graph. Review with the cadets how to create a table of values that they can then use as points to graph the circle. Make sure to review which is the x and which is the y axis. Work the two Check-It-Out #2 problems.

The graphing calculator (TI83 and higher) will not graph a circle. However, the cadet can solve the equation for y and then enter the equation in the calculator with a positive value and a second equation with a negative value and the result will be a circle.

Found many cadets find example #3 very confusing and then have problems working on other problems. You can present triangulation with cadets that have a good handle on the circle. There are many uses for this skill in today's world. Cell phone towers are just one. Tread here carefully so cadets are not over whelmed.

**Guided Practice**

Look at problems 1-9 and work those that cadets select. Once cadets think they can work on their own, they can start on the classwork problems.

**Practice**

**Classwork** (additional resources used for differentiation)**:**

Non-proficient Cadets – Problems 10 – 35, 37 – 40, 42 – 44

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 10 – 45

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 10 – 47

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 11, 15, 18, 20, 24

**Homework**

Page 79 and 163 May be replaced with different work based on needs. Replaced by additional resources listed above.

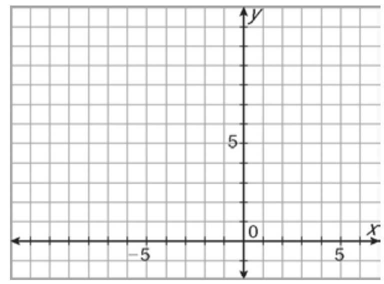
Name ______________________ Date ______________ Class______________

LESSON 12-7

# Challenge

## *Circles in the Coordinate Plane*

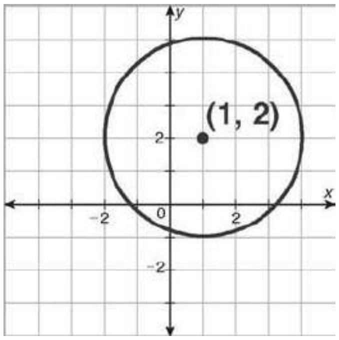
**Find the area of the intersection of two circles. The equation for circle $A$ is $(x - 2)^2 + (y - 3)^2 = 16$. The equation for circle $B$ is $(x + 5)^2 + (y - 5)^2 = 36$.**




1. Graph each circle in the coordinate grid. Shade the area of intersection of the two circles.
2. Draw $\overline{AB}$.
3. Find $AB$ to the nearest whole number. ________________
4. Label the points of intersection of the two circles $C$ and $D$.
5. a. Solve each equation for $y$.
   ______________________________________
   b. Graph the positive square root equation for each curve on a graphing calculator and find the point of intersection, $C$. Round coordinates to the nearest hundredth. ________________
   c. Graph the negative square root equation for each curve on a graphing calculator and find the point of intersection, $D$. Round coordinates to the nearest hundredth. ________________
6. Draw radii $\overline{BC}$, $\overline{AC}$, $\overline{BD}$, and $\overline{AD}$.
7. Find $AC$ and $BC$. ________________
8. Find m$\angle CBA$. Round to the nearest whole degree. (*Hint:* Use the Law of Cosines.) ________________
9. Find m$\angle CBD$. Round to the nearest whole degree. ________________
10. Find m$\angle CAB$. Round to the nearest whole degree. ________________
11. Find m$\angle CAD$. Round to the nearest whole degree. ________________
12. Draw chord $\overline{CD}$.

**For Exercises 13–19, round to the nearest hundredth.**

13. Find the area of sector $BCD$. ________________
14. Find the area of $\triangle BCD$. (*Hint:* Assume $\overline{BD}$ is the base of $\triangle BCD$. Find the height of the triangle first. You know that $\overline{BC}$ and $\overline{BD}$ are both radii of circle $B$.) ________________
15. Subtract the area of $\triangle BCD$ from the area of sector $BCD$. ________________
16. Find the area of sector $ACD$. ________________
17. Find the area of $\triangle ACD$. (*Hint:* Assume $\overline{AD}$ is the base of $\triangle ACD$. $\overline{AD}$ and $\overline{AC}$ are both radii of circle $A$.) ________________
18. Subtract the area of $\triangle ACD$ from the area of sector $ACD$. ________________
19. Find the sum of your answers to Exercises 15 and 18. This is the area of the intersection of the two circles. ________________

5.
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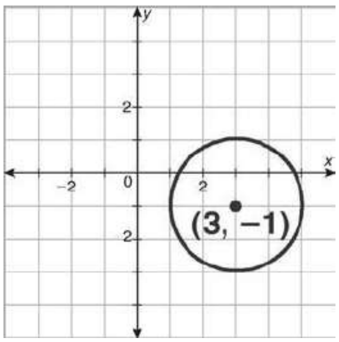


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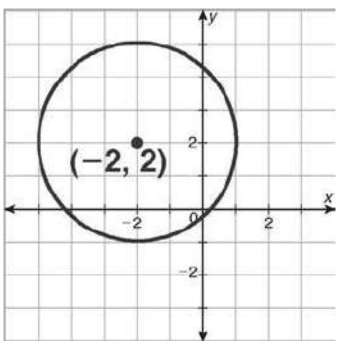


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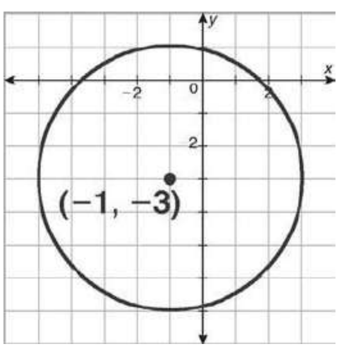
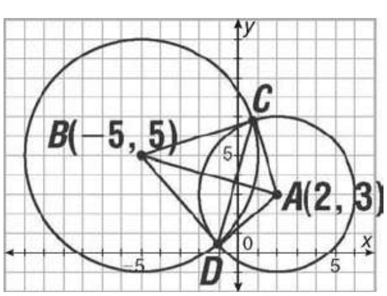



**Challenge**

1, 2, 4, 6, 12.



3. $AB \approx 7.28 \approx 7$

5. a. $y = 3 \pm \sqrt{16} - (x - 2)^2$; $y = 5 \pm \sqrt{36} - (x + 5)^2$

   b. (0.73, 6.79)

   c. (–1.08, 0.45)

7. $AC = 4$, $BC = 6$

8. 33°

9. 66°

10. 55°

11. 110°

13. 20.73 square units

14. 16.44 square units

15. 4.29 square units

16. 15.36 square units

17. 7.52 square units

18. 7.84 square units

19. 12.13 square units

**Problem Solving**

1. $(x + 1)^2 + (y - 4)^2 = 16$

2. $A = 74\pi$

3. B

4. J

5. A

6. G

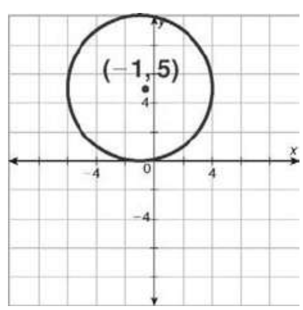
7. C

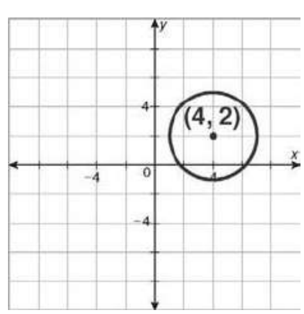
**Reading Strategies**

1. $(x + 2)^2 + (y - 10)^2 = 36$

2. $x^2 + y^2 = 9$

3. $(x - 8)^2 + (y - 2)^2 = 7$

4.



5.


Name ______________________________ Date ______________ Class______________

LESSON **12-7**

# Practice A

## *Circles in the Coordinate Plane*

1. Write the equation of a circle with center $(h, k)$ and radius $r$. ________________________

**Write the equation of each circle.**

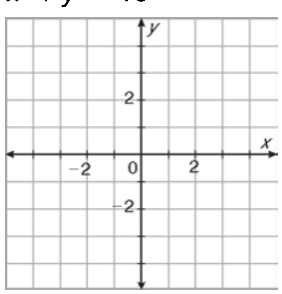
2. $\odot A$ centered at the origin with radius 6 ________________________
3. $\odot D$ with center $D(3, 3)$ and radius 2 ________________________
4. $\odot L$ with center $L(–3, –3)$ and radius 1 ________________________
5. $\odot M$ with center $M(0, –2)$ and radius 9 ________________________
6. $\odot Q$ with center $Q(7, 0)$ and radius 3 ________________________

**Complete Exercises 7 and 8 to write the equation of $\odot F$ with center $F(2, –1)$ that passes through (10, 5).**

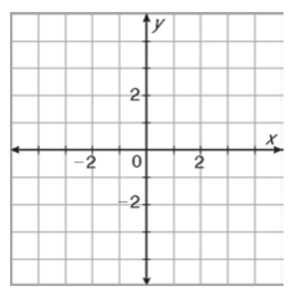
7. Use the distance formula with the two given points to find the radius of $\odot F$. ____________
8. Write the equation of $\odot F$. ________________________

**Graph each equation. First locate the center point, and use the radius to plot four points around the center that lie on the circle. Then draw a circle through the four points.**

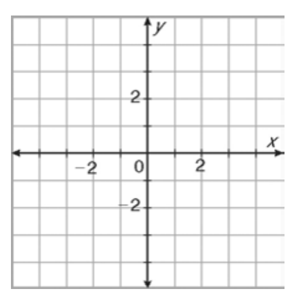
9. $x^2 + y^2 = 16$




10. $x^2 + y^2 = 4$



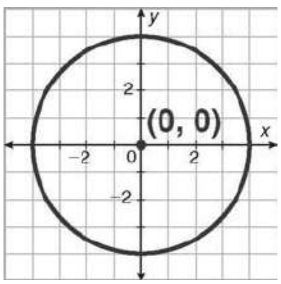

**A county planning department is meeting to choose the location of a rural fire station. The fire station needs to be the same distance from each of the three towns it will serve. The towns are located at $A(–3, 2)$, $B(–3, –4)$, and $C(1, –4)$. Complete Exercises 11–13 in order to find the best location for the fire station.**

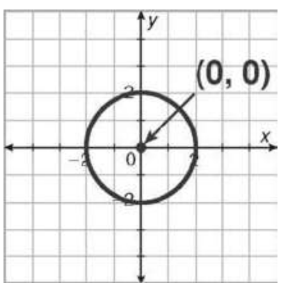


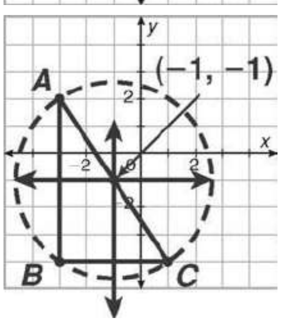

11. Plot $A$, $B$, and $C$. Draw $\triangle ABC$.
12. Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$.
13. The intersection point of the perpendicular bisectors is the same distance from the three points. So it is the center of a circle that intersects $A$, $B$, and $C$. Find the coordinates where the fire station should be built. ________________________

## 12-7 CIRCLES IN THE COORDINATE PLANE

### Practice A

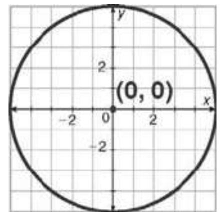
1. $(x - h)^2 + (y - k)^2 = r^2$
2. $x^2 + y^2 = 36$
3. $(x - 3)^2 + (y - 3)^2 = 4$
4. $(x + 3)^2 + (y + 3)^2 = 1$
5. $x^2 + (y + 2)^2 = 81$ 6. $(x - 7)^2 + y^2 = 9$
7. 10
8. $(x - 2)^2 + (y + 1)^2 = 100$
9. 

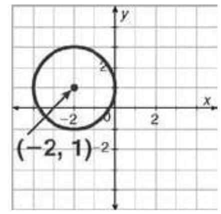
10. 

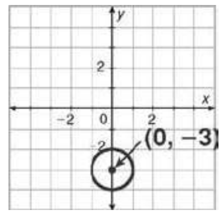
11. 

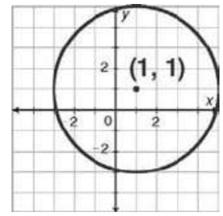
13. (–1, –1)

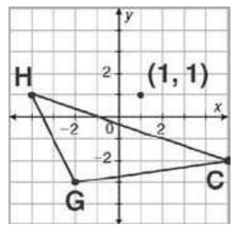
### Practice B

1. $x^2 + y^2 = 100$
2. $(x + 1)^2 + (y - 8)^2 = 25$
3. $(x + 5)^2 + (y + 5)^2 = 20$
4. $x^2 + y^2 = 85$
5. $x^2 + (y + 2)^2 = 40$
6. $(x - 11)^2 + (y - 4)^2 = 170$
7. 

8. 

9. 

10. 

11. (1, 1)


12. 6 miles

### Practice C

1. 90°
2. obtuse
3. $3\sqrt{5}$
4. (3.4, –2.7); (8.6, 7.8)
5. $\left(\dfrac{4+2\sqrt{19}}{5}, \dfrac{7-4\sqrt{19}}{5}\right), \left(\dfrac{4-2\sqrt{19}}{5}, \dfrac{7+4\sqrt{19}}{5}\right)$
6. (2, –3)
7. The line is tangent to the circle at the point (2, –3).
8. (3, 0)
9. The circles are internally tangent at the point (3, 0).

### Reteach

1. $(x + 1)^2 + (y + 1)^2 = 9$
2. $(x - 3)^2 + (y + 1)^2 = 16$
3. $(x - 4)^2 + (y - 5)^2 = 64$
4. $(x + 2)^2 + (y - 6)^2 = 25$

Name ________________________________ Date ______________ Class______________

LESSON 12-7

# Practice B

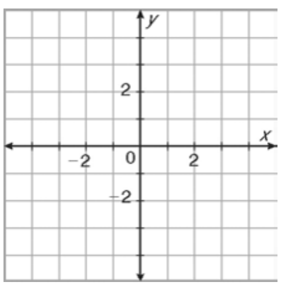
## *Circles in the Coordinate Plane*

**Write the equation of each circle.**

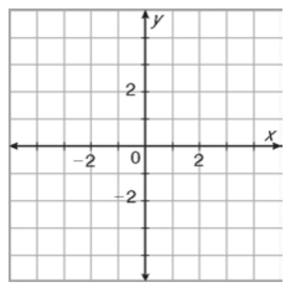
1. $\odot X$ centered at the origin with radius 10 ________________________
2. $\odot R$ with center $R(-1, 8)$ and radius 5 ________________________
3. $\odot P$ with center $P(-5, -5)$ and radius $2\sqrt{5}$ ________________________
4. $\odot O$ centered at the origin that passes through $(9, -2)$ ________________________
5. $\odot B$ with center $B(0, -2)$ that passes through $(-6, 0)$ ________________________
6. $\odot F$ with center $F(11, 4)$ that passes through $(-2, 5)$. ________________________

**Graph each equation.**

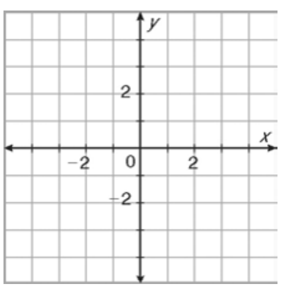
7. $x^2 + y^2 = 25$



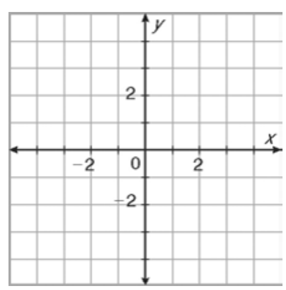

8. $(x + 2)^2 + (y - 1)^2 = 4$



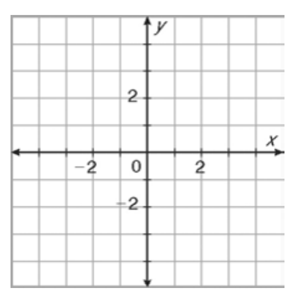

9. $x^2 + (y + 3)^2 = 1$




10. $(x - 1)^2 + (y - 1)^2 = 16$




**Crater Lake in Oregon is a roughly circular lake. The lake basin formed about 7000 years ago when the top of a volcano exploded in an immense explosion. Hillman Peak, Garfield Peak, and Cloudcap are three mountain peaks on the rim of the lake. The peaks are located in a coordinate plane at *H*(–4, 1), *G*(–2, –3), and *C*(5, –2).**




11. Find the coordinates of the center of the lake.

________________________

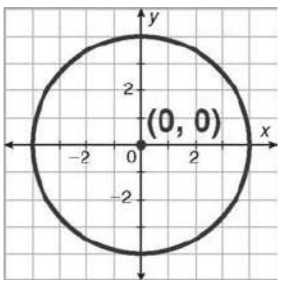
12. Each unit of the coordinate plane represents $\frac{3}{5}$ mile.

Find the diameter of the lake. ________________________

## 12-7 CIRCLES IN THE COORDINATE PLANE

### Practice A

1. $(x - h)^2 + (y - k)^2 = r^2$
2. $x^2 + y^2 = 36$
3. $(x - 3)^2 + (y - 3)^2 = 4$
4. $(x + 3)^2 + (y + 3)^2 = 1$
5. $x^2 + (y + 2)^2 = 81$ 6. $(x - 7)^2 + y^2 = 9$
7. 10
8. $(x - 2)^2 + (y + 1)^2 = 100$
9.

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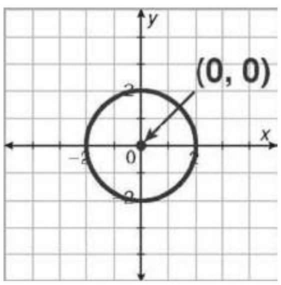


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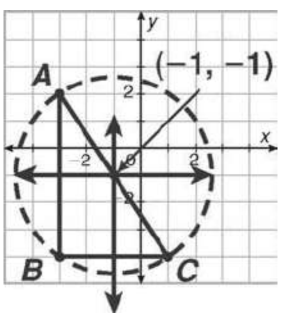
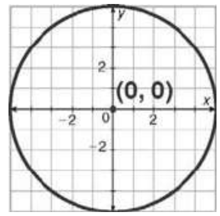



13. (–1, –1)

### Practice B

1. $x^2 + y^2 = 100$
2. $(x + 1)^2 + (y - 8)^2 = 25$
3. $(x + 5)^2 + (y + 5)^2 = 20$
4. $x^2 + y^2 = 85$
5. $x^2 + (y + 2)^2 = 40$
6. $(x - 11)^2 + (y - 4)^2 = 170$
7.

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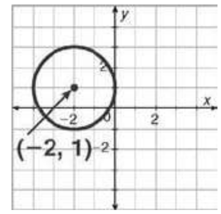


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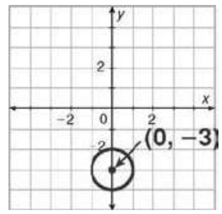


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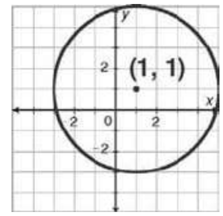
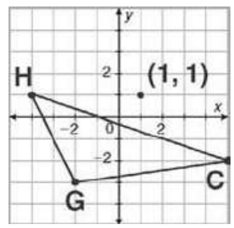



11. (1, 1)




12. 6 miles

### Practice C

1. 90°
2. obtuse
3. $3\sqrt{5}$
4. (3.4, –2.7); (8.6, 7.8)
5. $\left(\frac{4+2\sqrt{19}}{5}, \frac{7-4\sqrt{19}}{5}\right), \left(\frac{4-2\sqrt{19}}{5}, \frac{7+4\sqrt{19}}{5}\right)$
6. (2, –3)
7. The line is tangent to the circle at the point (2, –3).
8. (3, 0)
9. The circles are internally tangent at the point (3, 0).

### Reteach

1. $(x + 1)^2 + (y + 1)^2 = 9$
2. $(x - 3)^2 + (y + 1)^2 = 16$
3. $(x - 4)^2 + (y - 5)^2 = 64$
4. $(x + 2)^2 + (y - 6)^2 = 25$

Name ______________________________________ Date _________________ Class_________________

LESSON 12-7

# Practice C

## *Circles in the Coordinate Plane*

1. Points $A$, $B$, and $C$ lie on the circumference of a circle. $AB$ is twice the radius of the circle. Find $m\angle ACB$.

_______________________________________________________________________

2. Points $A$, $B$, and $C$ lie on the circumference of a circle. The center of the circle lies in the exterior of $\triangle ABC$. Classify $\triangle ABC$ by its angles.

_______________________________________________________________________

**Give answers in simplest radical form if necessary.**

3. The points $X(3, 4)$ and $Y(9, 1)$ lie on the circumference of a circle. There is exactly 60° of arc between $X$ and $Y$. Find the radius of the circle.

_______________________________________________________________________

4. Find the coordinates of all possible centers of the circle in Exercise 3.

_______________________________________________________________________

5. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the line $2x + y = 3$.

_______________________________________________________________________

6. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the line $y = \frac{4}{3}x - \frac{17}{3}$.

_______________________________________________________________________

7. Describe the relationship between the circle and the line in Exercise 6.

_______________________________________________________________________

_______________________________________________________________________

8. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the circle $x^2 + y^2 = 9$.

_______________________________________________________________________

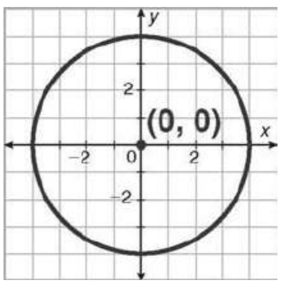
9. Describe the relationship between the two circles in Exercise 8.

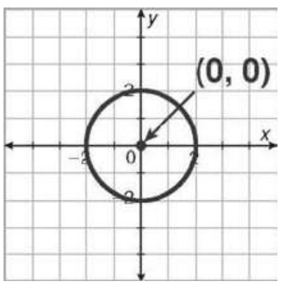
_______________________________________________________________________

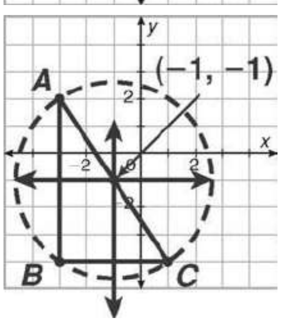
_______________________________________________________________________

## 12-7 CIRCLES IN THE COORDINATE PLANE

### Practice A

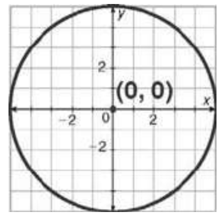
1. $(x - h)^2 + (y - k)^2 = r^2$
2. $x^2 + y^2 = 36$
3. $(x - 3)^2 + (y - 3)^2 = 4$
4. $(x + 3)^2 + (y + 3)^2 = 1$
5. $x^2 + (y + 2)^2 = 81$ 6. $(x - 7)^2 + y^2 = 9$
7. 10
8. $(x - 2)^2 + (y + 1)^2 = 100$
9. 

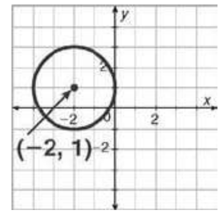
10. 

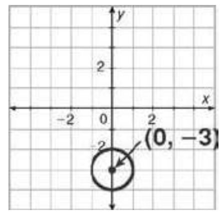
11. 

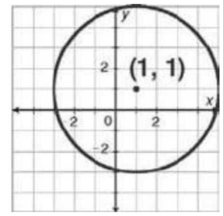
13. (–1, –1)

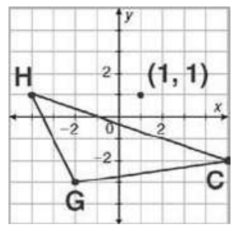
### Practice B

1. $x^2 + y^2 = 100$
2. $(x + 1)^2 + (y - 8)^2 = 25$
3. $(x + 5)^2 + (y + 5)^2 = 20$
4. $x^2 + y^2 = 85$
5. $x^2 + (y + 2)^2 = 40$
6. $(x - 11)^2 + (y - 4)^2 = 170$
7. 

8. 

9. 

10. 

11. (1, 1)


12. 6 miles

### Practice C

1. 90° 2. obtuse
3. $3\sqrt{5}$
4. (3.4, –2.7); (8.6, 7.8)
5. $\left(\dfrac{4+2\sqrt{19}}{5}, \dfrac{7-4\sqrt{19}}{5}\right), \left(\dfrac{4-2\sqrt{19}}{5}, \dfrac{7+4\sqrt{19}}{5}\right)$
6. (2, –3)
7. The line is tangent to the circle at the point (2, –3).
8. (3, 0)
9. The circles are internally tangent at the point (3, 0).

### Reteach

1. $(x + 1)^2 + (y + 1)^2 = 9$
2. $(x - 3)^2 + (y + 1)^2 = 16$
3. $(x - 4)^2 + (y - 5)^2 = 64$
4. $(x + 2)^2 + (y - 6)^2 = 25$

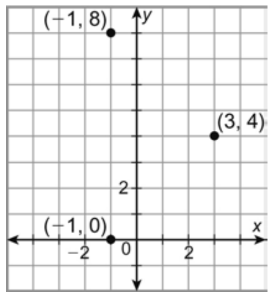
Name ________________________________________ Date __________________ Class__________________

**LESSON 12-7**

# Problem Solving

## *Circles in the Coordinate Plane*

1. Write the equation of the circle that contains the points graphed below.

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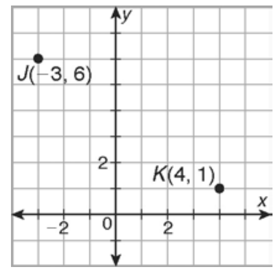
2. Find the area of a circle that has center $J$ and passes through $K$. Express your answer in terms of $\pi$.




_________________________________________

**Choose the best answer.**

3. An English knot garden has hedges planted to form geometric shapes. A blueprint of a knot garden contains three circular hedges as described in the table. Flowers are to be planted in the space that is within all three circles. Which is a point that could be planted with flowers?

   A (7, 1) C (0, 5)
   B (5, 1) D (0, 0)

| Circular Hedge | Center | Radius |
|---|---|---|
| A | (3, 2) | 3 ft |
| B | (7, 2) | 4 ft |
| C | (5, –1) | 3 ft |

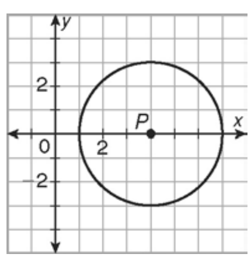
4. Which of these circles intersects the circle that has center (0, 6) and radius 1?

   F $(x - 5)^2 + (y + 3)^2 = 4$
   G $(x - 4)^2 + (y - 3)^2 = 9$
   H $(x + 5)^2 + (y + 1)^2 = 16$
   J $(x + 1)^2 + (y - 4)^2 = 4$

5. The center of $\odot B$ is (9, 2), and the radius of the circle is 5 units. Which is a point on the circle?

   A (4, 2) C (9, 4)
   B (14, 0) D (9, –5)

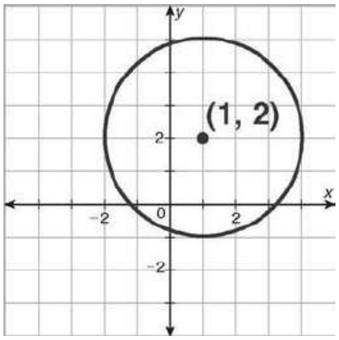
6. Which is an equation for a circle that has the same center as $\odot P$ but has a circumference that is four times as great?

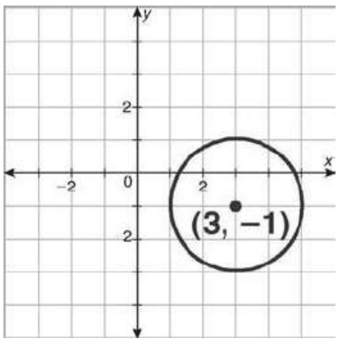


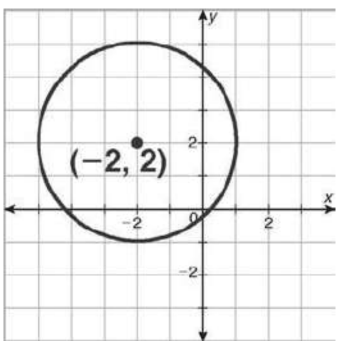

   F $(x - 4)^2 + y^2 = 36$
   G $(x - 4)^2 + y^2 = 144$
   H $x^2 + (y - 4)^2 = 36$
   J $x^2 + (y - 4)^2 = 144$

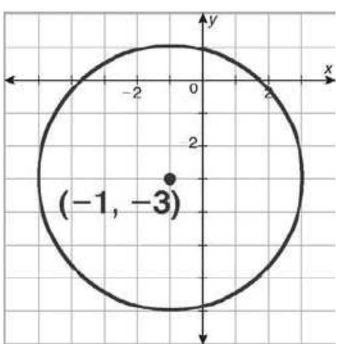
7. The Maxair amusement park ride consists of a circular ring that holds 50 riders. Suppose that the center of the ride is at the origin and that one of the riders on the circular ring is at (16, 15.1). If one unit on the coordinate plane equals 1 foot, which is a close approximation of the circumference of the ride?

   A 22 ft C 138 ft
   B 44 ft D 1521 ft

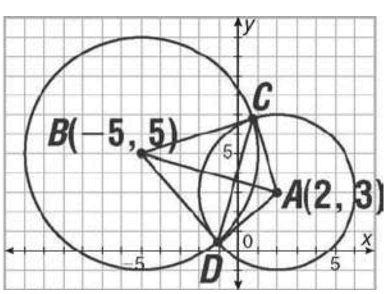
5. 


6. 


7. 


8. 


**Challenge**

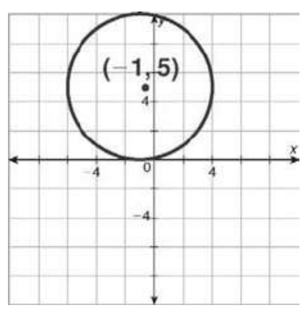
1, 2, 4, 6, 12. 

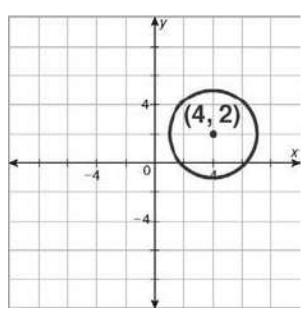

3. $AB \approx 7.28 \approx 7$
5. a. $y = 3 \pm \sqrt{16} - (x - 2)^2$; $y = 5 \pm \sqrt{36} - (x + 5)^2$
   b. (0.73, 6.79)
   c. (–1.08, 0.45)
7. $AC = 4$, $BC = 6$
8. 33°
9. 66°
10. 55°
11. 110°
13. 20.73 square units
14. 16.44 square units
15. 4.29 square units
16. 15.36 square units
17. 7.52 square units
18. 7.84 square units
19. 12.13 square units

**Problem Solving**

1. $(x + 1)^2 + (y - 4)^2 = 16$
2. $A = 74\pi$
3. B
4. J
5. A
6. G
7. C

**Reading Strategies**

1. $(x + 2)^2 + (y - 10)^2 = 36$
2. $x^2 + y^2 = 9$
3. $(x - 8)^2 + (y - 2)^2 = 7$
4. 


5. 

Name ________________________ Date ______________ Class______________

LESSON 12-7

# Reading Strategies

## *Use a Concept Map*

Use the concept map below to help you understand circles in the coordinate plane.

**Circles in the Coordinate Plane**

**Equation**

The equation of a circle with center ($h$, $k$) and radius $r$ is

$(x - h)^2 + (y - k)^2 = r^2$.

**Example**

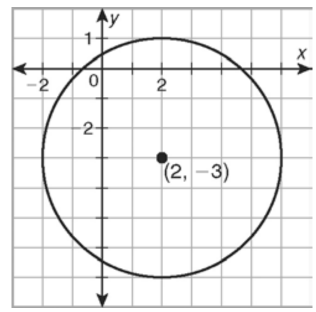
The equation of a circle with center (2, –3) and radius 4:

$(x - 2)^2 + (y - (-3))^2 = 4^2$

or

$(x - 2)^2 + (y + 3)^2 = 16$

**Graph**




**Write the equation of each circle.**

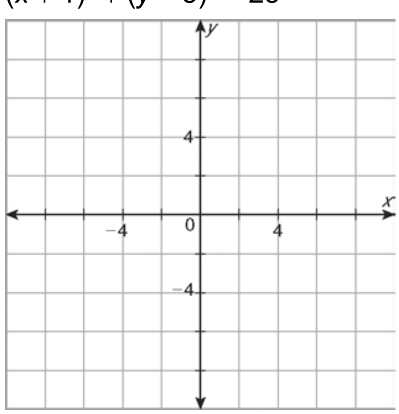
1. a circle with center (–2, 10) and radius 6 ____________________

2. a circle with center (0, 0) and radius 3 ____________________

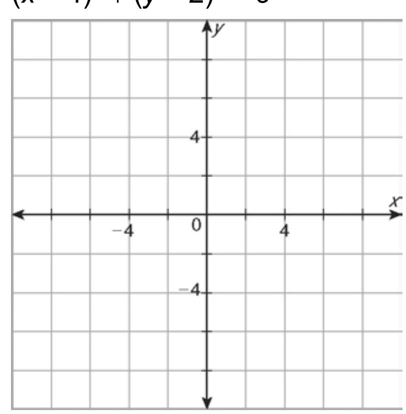
3. a circle with center (8, 2) and radius $\sqrt{7}$ ____________________

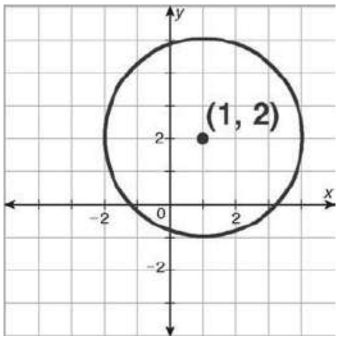
**Graph each circle.**

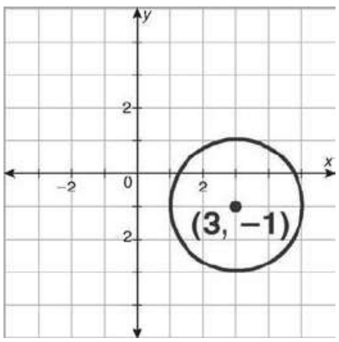
4. $(x + 1)^2 + (y - 5)^2 = 25$

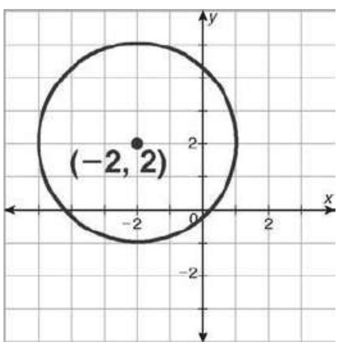


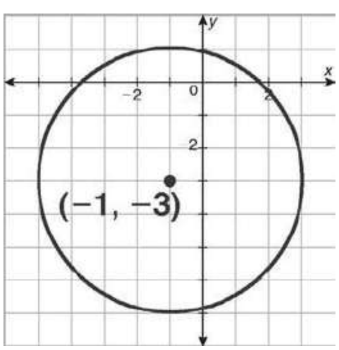

5. $(x - 4)^2 + (y - 2)^2 = 9$



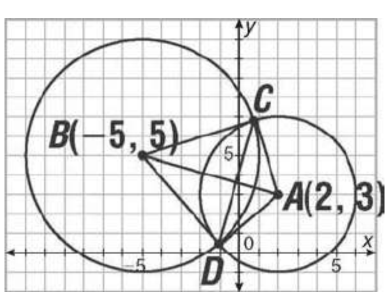
5. 


6. 


7. 


8. 


**Challenge**

1, 2, 4, 6, 12. 


3. $AB \approx 7.28 \approx 7$

5. a. $y = 3 \pm \sqrt{16} - (x - 2)^2$; $y = 5 \pm \sqrt{36} - (x + 5)^2$

   b. (0.73, 6.79)

   c. (–1.08, 0.45)

7. $AC = 4$, $BC = 6$

8. 33° 9. 66°

10. 55° 11. 110°

13. 20.73 square units

14. 16.44 square units

15. 4.29 square units

16. 15.36 square units

17. 7.52 square units

18. 7.84 square units

19. 12.13 square units

**Problem Solving**

1. $(x + 1)^2 + (y - 4)^2 = 16$

2. $A = 74\pi$ 3. B

4. J 5. A

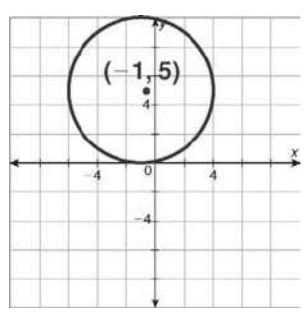
6. G 7. C

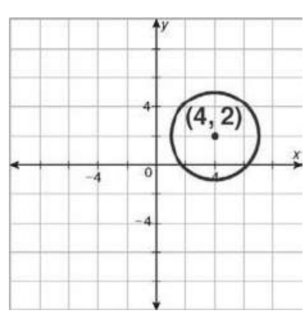
**Reading Strategies**

1. $(x + 2)^2 + (y - 10)^2 = 36$

2. $x^2 + y^2 = 9$

3. $(x - 8)^2 + (y - 2)^2 = 7$

4. 


5. 

Name ______________________________ Date ______________ Class______________

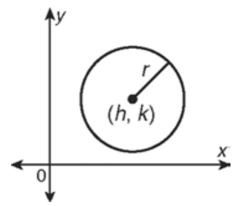
LESSON 12-7

# Reteach

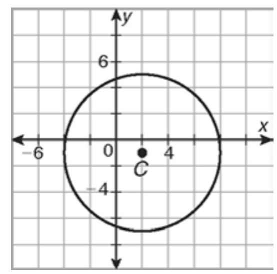
## *Circles in the Coordinate Plane*

### Equation of a Circle

The equation of a circle with center ($h$, $k$) and radius $r$ is $(x - h)^2 + (y - k)^2 = r^2$.




**Write the equation of ⊙*C* with center *C*(2, –1) and radius 6.**




| | |
|---|---|
| $(x - h)^2 + (y - k)^2 = r^2$ | Equation of a circle |
| $(x - 2)^2 + (y - (-1))^2 = 6^2$ | Substitute 2 for $h$, –1 for $k$, and 6 for $r$. |
| $(x - 2)^2 + (y + 1)^2 = 36$ | Simplify. |

You can also write the equation of a circle if you know the center and one point on the circle.

**Write the equation of ⊙*L* that has center *L*(3, 7) and passes through (1, 7).**

**Step 1** Find the radius.

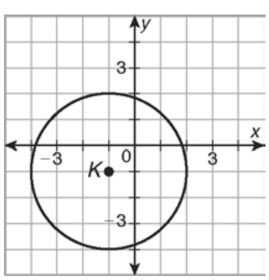
| | |
|---|---|
| $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | Distance Formula |
| $r = \sqrt{(1 - 3)^2 + (7 - 7)^2}$ | Substitution |
| $r = \sqrt{4} = 2$ | Simplify. |

**Step 2** Use the equation of a circle.

| | |
|---|---|
| $(x - h)^2 + (y - k)^2 = r^2$ | Equation of a circle |
| $(x - 3)^2 + (y - 7)^2 = 2^2$ | $(h, k) = (3, 7)$ |
| $(x - 3)^2 + (y - 7)^2 = 4$ | Simplify. |

**Write the equation of each circle.**

1.

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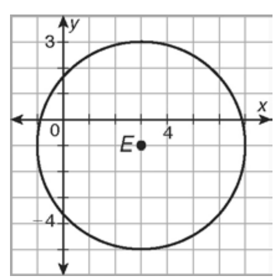
2.

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3. ⊙*T* with center *T*(4, 5) and radius 8

________________________________________

4. ⊙*B* that passes through (3, 6) and has center *B*(–2, 6)

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Name ________________________ Date ______________ Class______________

LESSON 12-7

# Reteach

## *Circles in the Coordinate Plane* *continued*

You can use an equation to graph a circle by making a table or by identifying its center and radius.

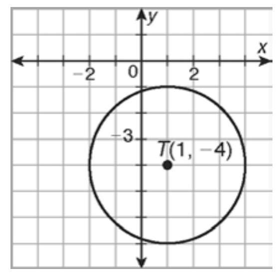
**Graph $(x - 1)^2 + (y + 4)^2 = 9$.**

The equation of the given circle can be rewritten.

$$\begin{array}{c} (x - h)^2 + (y - k)^2 = r^2 \\ \downarrow \qquad\quad \downarrow \qquad \downarrow \\ (x - 1)^2 + (y - (-4))^2 = 3^2 \end{array}$$

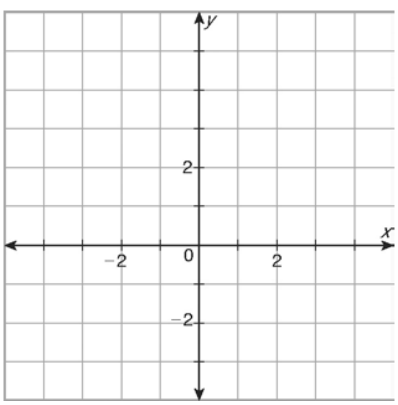
$h = 1$, $k = -4$, and $r = 3$

The center is at $(h, k)$ or $(1, -4)$, and the radius is 3. Plot the point $(1, -4)$. Then graph a circle having this center and radius 3.

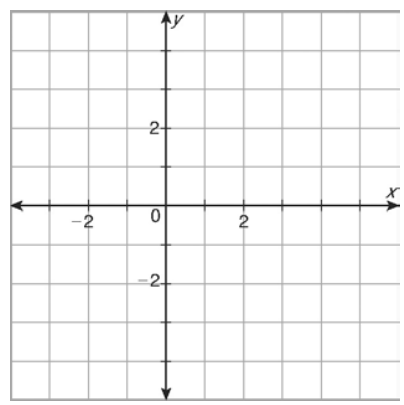



**Graph each equation.**

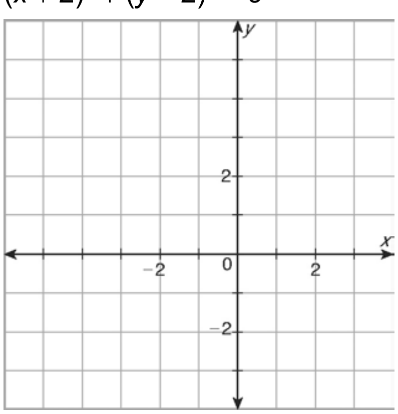
5. $(x - 1)^2 + (y - 2)^2 = 9$



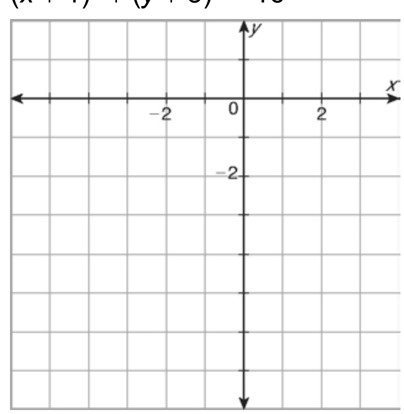

6. $(x - 3)^2 + (y + 1)^2 = 4$




7. $(x + 2)^2 + (y - 2)^2 = 9$

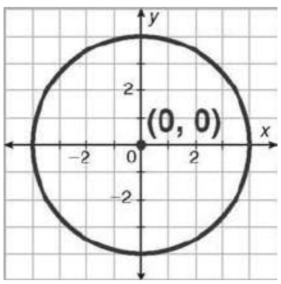


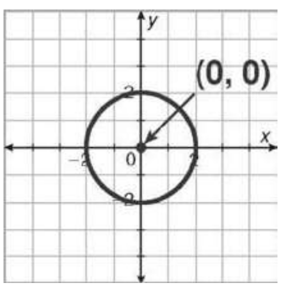

8. $(x + 1)^2 + (y + 3)^2 = 16$

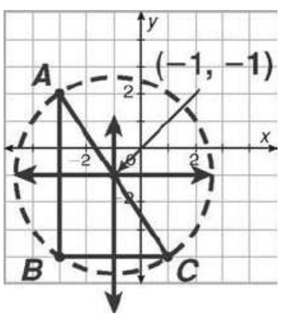


## 12-7 CIRCLES IN THE COORDINATE PLANE

### Practice A

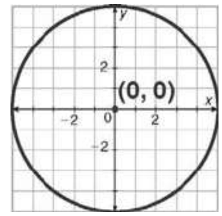
1. $(x - h)^2 + (y - k)^2 = r^2$
2. $x^2 + y^2 = 36$
3. $(x - 3)^2 + (y - 3)^2 = 4$
4. $(x + 3)^2 + (y + 3)^2 = 1$
5. $x^2 + (y + 2)^2 = 81$ 6. $(x - 7)^2 + y^2 = 9$
7. 10
8. $(x - 2)^2 + (y + 1)^2 = 100$
9. 

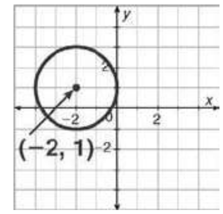
10. 

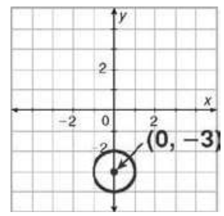
11. 

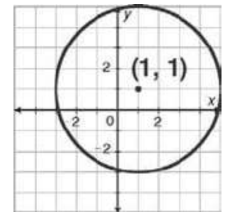

13. (–1, –1)

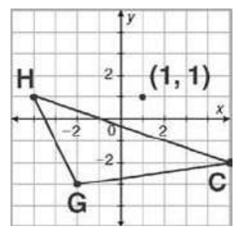
### Practice B

1. $x^2 + y^2 = 100$
2. $(x + 1)^2 + (y - 8)^2 = 25$
3. $(x + 5)^2 + (y + 5)^2 = 20$
4. $x^2 + y^2 = 85$
5. $x^2 + (y + 2)^2 = 40$
6. $(x - 11)^2 + (y - 4)^2 = 170$
7. 

8. 

9. 

10. 

11. (1, 1)


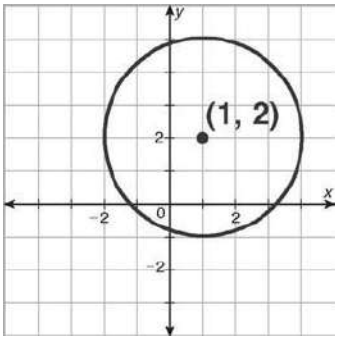
12. 6 miles

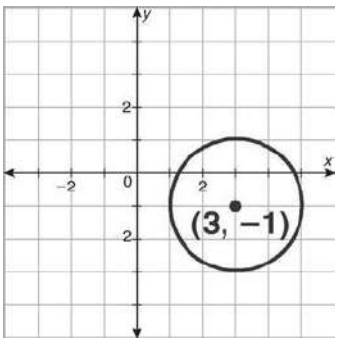
### Practice C

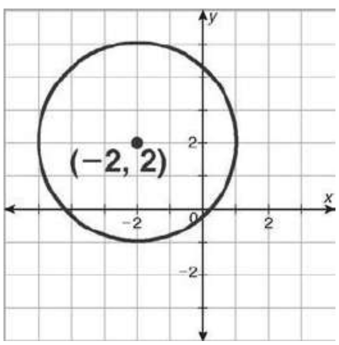
1. 90°
2. obtuse
3. $3\sqrt{5}$
4. (3.4, –2.7); (8.6, 7.8)
5. $\left(\frac{4+2\sqrt{19}}{5}, \frac{7-4\sqrt{19}}{5}\right), \left(\frac{4-2\sqrt{19}}{5}, \frac{7+4\sqrt{19}}{5}\right)$
6. (2, –3)
7. The line is tangent to the circle at the point (2, –3).
8. (3, 0)
9. The circles are internally tangent at the point (3, 0).

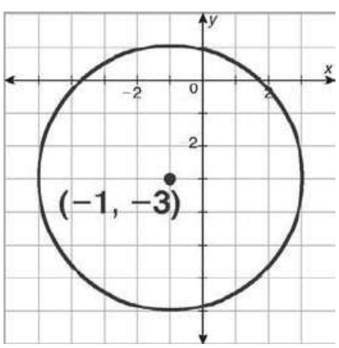
### Reteach

1. $(x + 1)^2 + (y + 1)^2 = 9$
2. $(x - 3)^2 + (y + 1)^2 = 16$
3. $(x - 4)^2 + (y - 5)^2 = 64$
4. $(x + 2)^2 + (y - 6)^2 = 25$

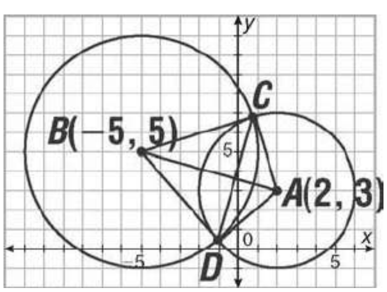
5. 


6. 


7. 


8. 


**Challenge**

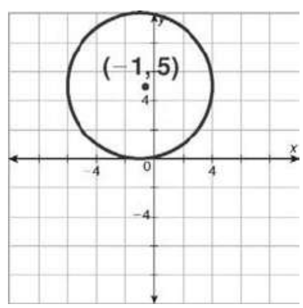
1, 2, 4, 6, 12. 


3. $AB \approx 7.28 \approx 7$
5. a. $y = 3 \pm \sqrt{16} - (x - 2)^2$; $y = 5 \pm \sqrt{36} - (x + 5)^2$
   b. (0.73, 6.79)
   c. (–1.08, 0.45)
7. $AC = 4$, $BC = 6$
8. 33°
9. 66°
10. 55°
11. 110°
13. 20.73 square units
14. 16.44 square units
15. 4.29 square units
16. 15.36 square units
17. 7.52 square units
18. 7.84 square units
19. 12.13 square units

**Problem Solving**

1. $(x + 1)^2 + (y - 4)^2 = 16$
2. $A = 74\pi$
3. B
4. J
5. A
6. G
7. C

**Reading Strategies**

1. $(x + 2)^2 + (y - 10)^2 = 36$
2. $x^2 + y^2 = 9$
3. $(x - 8)^2 + (y - 2)^2 = 7$
4. 


5.