Coordinate Algebra – Study Guide

Lines/Linear Functions

*Constant rate of change (slope)- adding the same amount each time

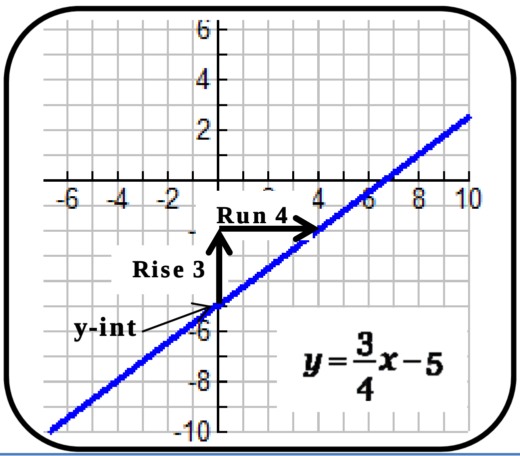
Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope Intercept Form

$$y = mx + b$$

Slope (m) — y-intercept (b)



Equation of a Line Given 2 points

-Find slope

-Substitute slope into $y = mx + b$

-Substitute the coordinates of one point

-Solve for b

-Write equation in $y = mx + b$

Example

Write the equation of the line that goes through the points (2, -3) and (4, 1).

- $m = \frac{1-(-3)}{4-2} = \frac{4}{2} = 2$
- $y = 2x + b$
- $-3 = 2(2) + b$
- $-7 = b$
- ***Answer:*** $y = 2x - 7$

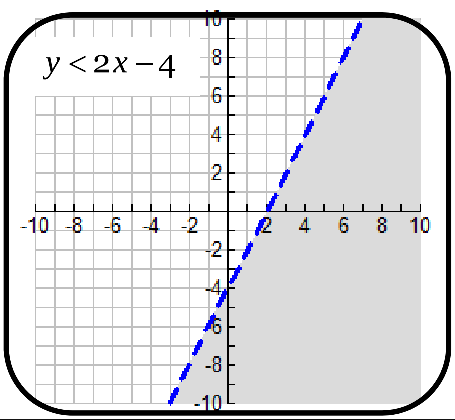
Linear Inequalities

Solid: ≤ and ≥ Dashed: < and >

When in Slope-Intercept Form:

Above: ≥ and > Below: ≤ and <

Solutions are in shaded region and on solid line



Equation of a Line Given a Table

- Slope is the change in y-values over the change in x-values
- y-intercept is the y-value when x=0.

Example

x	-1	0	1	2
y	6	8	10	12

- $m = \frac{8-6}{0-(-1)} = \frac{2}{1} = 2$
- y-int: (0,8)

Answer: $y = 2x + 8$

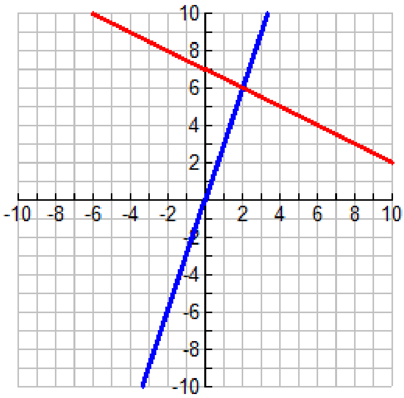
Systems of Equations

1) **Solve by graphing**

- Graph
- Solution is the intersection of the lines

Example

Solve $\begin{cases} y = 3x \\ y = -\frac{1}{2}x + 7 \end{cases}$



Answer: (2, 6)

2) **Solve by substitution**

- Solve for a variable
- Substitute into other equation
- Repeat
- Solution is ordered pair

Example

Solve $\begin{cases} 2x + y = 8 \\ 4x - 2y = 8 \end{cases}$

- $y = -2x + 8$
- $4x - 2(-2x + 8) = 8$
- $4x + 4x - 16 = 8$
- $8x - 16 = 8$
- $8x = 24$
- $x = 3$
- $y = -2(3) + 8$
- $y = -6 + 8$
- $y = 2$

Answer: (3, 2)

3) **Solve by elimination**

- Multiply equations to get one variable with opposite coefficients
- Add equations together.
- Solve/Substitute/Solve
- Solution is an ordered pair.

Example

Solve $\begin{cases} 3x + 2y = 5 \\ 4x + 3y = 6 \end{cases}$

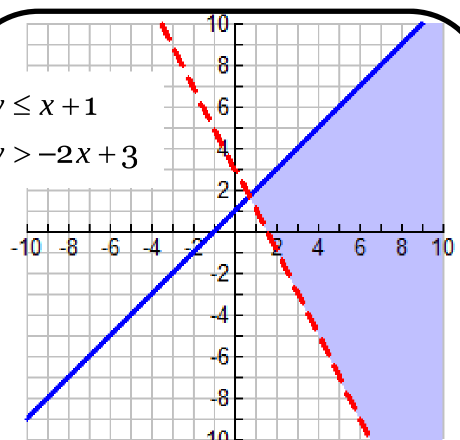
- $3 \cdot (3x + 2y) = (5) \cdot 3$
$-2 \cdot (4x + 3y) = (6) \cdot -2$
- $9x + 6y = 15$
$-8x - 6y = -12$
- $x = 3$
- $3(3) + 2y = 5$
- $9 + 2y = 5$
- $2y = -4$
- $y = -2$

Answer: (3, - 2)

Systems of Inequalities

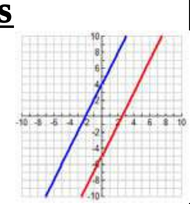
- Graph both inequalities
- Solutions lie in the overlapping region

$\begin{cases} y \le x + 1 \\ y > -2x + 3 \end{cases}$

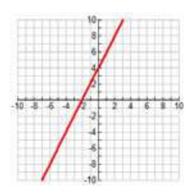


Types of Systems of Equations

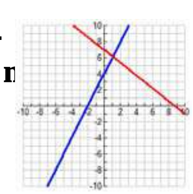
1) **Inconsistent- No Solution**
Parallel lines (same slope, different *y*-int)



2) **Consistent Dep-endent- Infinitely Many Solutions**
Same line (same slope, same *y*-int)



3) **Consistent Indep-endent- One Solution**
Intersecting lines (different slopes)



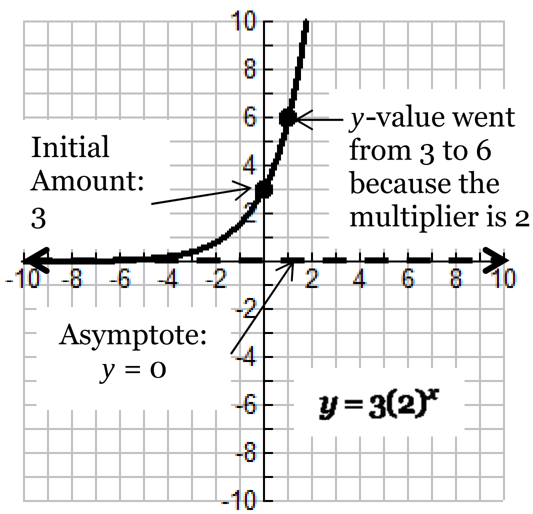
Exponential Functions

*Rate of change varies- multiplying the same number each time

Exponential Function

$y = ab^x$

Initial amount (a) — Multiplier (b)



Exponential Growth- when $b > 1$
Exponential Decay- when $0 < b < 1$

The average 2nd grade student starts off with 100 cooties. If the number of cooties a 2nd grade student has doubles every hour, how many cooties will they have in 7 hours?

- $y = 100(2)^x$
- $y = 100(2)^7$

Answer: 128,000 cooties

At his 3 year doctor's appointment, Graham weighed 29 pounds. If he is expected to increase his weight 10% per year, how much will he weigh 10 years later?

- $y = 29(1 + .1)^x$
- $y = 29(1.1)^x$
- $y = 29(1.1)^{10}$

Answer: 75.22 pounds

Equation of an Exponential Given 2 points

-Substitute both points into $y = ab^x$
-Divide the equations (larger exp. over smaller)
-Solve for b
-Substitute b into either equation.
-Solve for a
-Write equation in $y = ab^x$

Example

Write the equation of the exponential that goes through the points (3, 28) and (4, 56).

- $28 = ab^3$ and $56 = ab^4$
- $\frac{56}{28} = \frac{ab^4}{ab^3}$
- $2 = b$
- $28 = a(2)^3$
- $28 = a(8)$ $\quad \frac{28}{8} = a$ $\quad \frac{7}{2} = a$

Answer: $y = \frac{7}{2}(2)^x$

Equation of an Exponential Given a Table

- b is the multiplier
- a is the initial amount. If you use the y-intercept, the exponent is x. If you use the y-value when x is 1, then the exponent is $x - 1$.

Example

x	0	1	2	3
y	5	10	20	40

- $b = 2$
- $a = 5$ or $a = 10$

Answer: $y = 5(2)^x$ or $y = 10(2)^{x-1}$

Functions and Transformations

Function- Each x-value is assigned one y-value
Domain- The set of all x-values
Range- The set of all y-values

Example

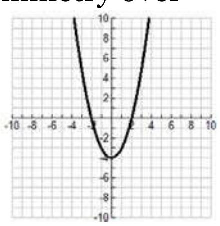
Determine if the following relation is a function. Determine the domain and range.

x	2	4	2	6
y	3	1	2	5

Answer: Not a function, because the x-value of 2 is assigned two different y-values.
Domain: {2, 4, 6}
Range {1, 2, 3, 5}

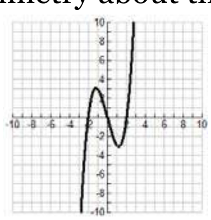
Even Functions- Symmetry over y-axis

$$f(-x) = f(x)$$



Odd Functions- Symmetry about the origin

$$f(-x) = -f(x)$$



Increase- when (at which x-values) is the graph going uphill from left to right
Decrease- when (at which x-values) is the graph going downhill from left to right
Positive- when (at which x-values) is the graph above the x-axis
Negative- when (at which x-values) is the graph below the x-axis

<u>**Transformations**</u>

Outside- Vertical (Same)
Inside- Horizontal (Opposite)

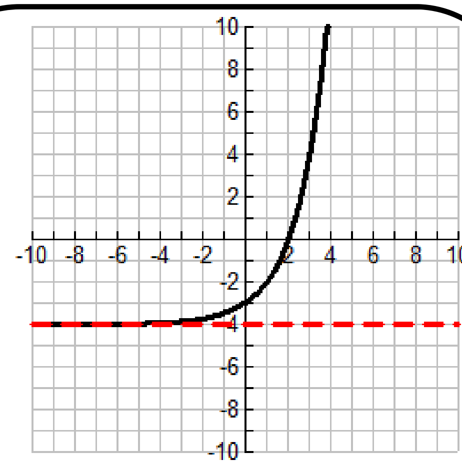
Add/Subtract- Shift
Multiply/Divide- Stretch (>1)/Shrink(<1)
Negative- Reflection

Example

Describe the transformations of the function from the parent graph $y = x^2$.

a) $y = -x^2 + 4$
b) $y = 2x^2 - 3$
c) $y = \frac{1}{2}(x+8)^2$

Answers: a) reflection over x-axis, vertical shift up 4 units
b) vertical stretch by a factor of 2, vertical shift down 3 units
c) vertical shrink by a factor of ½, horizontal shift left 8 units



Domain: All real numbers or $(-\infty, \infty)$
Range: $y > -4$ or $(-4, \infty)$
Increase: Always increasing
Decrease: Never decreasing
Positive: $x > 2$ or $(2, \infty)$
Negative: $x < 2$ or $(-\infty, 2)$

<u>Sequences</u>

Example

Find the first five terms of the sequence $a_n = 3n$.

Answer: 3, 6, 9, 12, 15

a_n (Range) → 3, 6, 9, 12, 15, ...

1 2 3 4 5 ← n (Domain)

Range is the terms of the sequence

Domain is the position (1st, 2nd, 3rd, etc), so whole numbers only

Arithmetic- a sequence with the difference between two consecutive terms constant. The difference is called the **common difference, *d*.** (Linear Sequence)

Explicit Formula: $a_n = a_1 + (n-1) \bullet d$ or $y = mx + b$

Recursive Formula: $a_n = a_{n-1} + d$

a_{n-1} means previous term

Example

Write a rule for the *n*th term of the arithmetic sequence.

2, 5, 8, 11, 14, . . .

- Common difference (d or m): 3
- The "y-int" would be the term before the first term: -1
- $y = 3x - 1$

Answer: Explicit: $a_n = 3n - 1$

Recursive: $a_n = a_{n-1} + 3, a_1 = 2$

Example

Write a rule for arithmetic sequence.

a_5=20 , a_{13}= 36

- (5, 20) and (13, 36)
- $m = \dfrac{36-20}{13-5} = \dfrac{16}{8} = 2$
- $y = 2x + b$
- $20 = 2(5) + b$
- $20 = 10 + b$
- $10 = b$
- $y = 2x + 10$

Answer: $a_n = 2n + 10$

Geometric- a sequence with the ratio between two consecutive terms constant. This ratio is called the **common ratio, *r*.** (Exponential Sequence)

Explicit Formula: $a_n = a_1 \bullet r^{n-1}$

Recursive Formula: $a_n = r \bullet a_{n-1}$

Example

Write a rule for the *n*th term of the geometric sequence.

2, 6, 18, 54, 162, . . .

- Common ratio (r): 3
- First term (a_1): 2

Answer: Explicit: $a_n = 2(3)^{n-1}$

Recursive: $a_n = a_{n-1} \bullet 3, a_1 = 2$

Example

Write a rule for geometric sequence.

a_2=20 , a_3= 40

- (2, 20) and (3, 40)
- $20 = a_1 r^{2-1}$ and $40 = a_1 r^{3-1}$
- $20 = a_1 r^1$ and $40 = a_1 r^2$
- $\dfrac{40}{20} = \dfrac{a_1 r^2}{a_1 r^1}$
- $2 = r$
- $20 = a_1 (2)^1$
- $10 = a_1$

Answer: $a_n = 10(2)^{n-1}$

Data Analysis- One Variable

Measures of Central Tendency: Mean ($\overline{x}$), Median, Mode

Measures of Dispersion: Range, Interquartile Range, Mean Absolute Deviation

Example

Find the mean, median, mode, range, interquartile range, and mean absolute deviation.

5, 6, 8, 10, 12, 14, 15

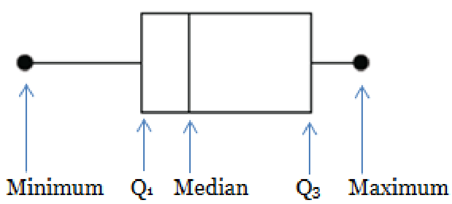
Mean (Average) $\overline{x} = \dfrac{5+6+8+10+12+14+15}{7} = 10$

5, 6, 8, 10, 12, 14, 15
↑ Q_1 ↑ Median ↑ Q_3

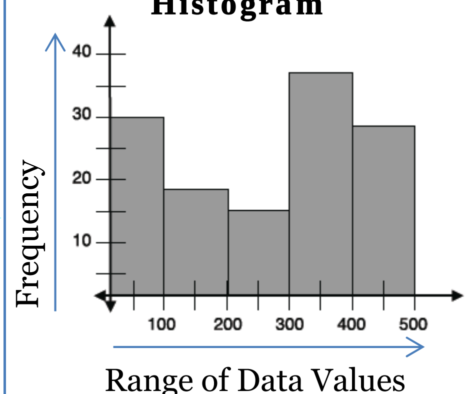
Median (Middle number): 10
Mode (Occurs most often): None
Range (Largest – smallest): 15 – 5 = 10
Interquartile Range ($Q_3 - Q_1$): 14 – 6 = 8

Mean Absolute Deviation (Average of the deviations from $\overline{x}$): $\dfrac{5+4+2+0+2+4+5}{7} = \dfrac{22}{7} \approx 3.14$

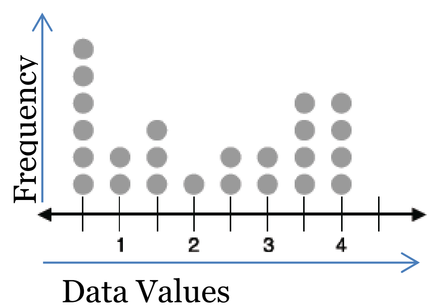
Box- And-Whisker Plots

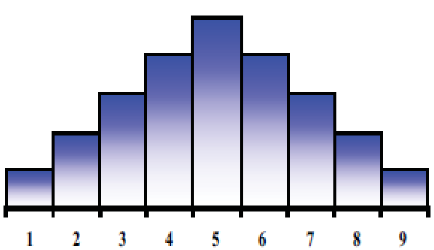


Histogram

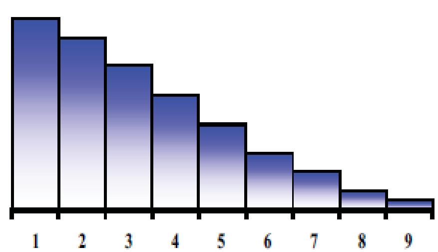


Dot Plot

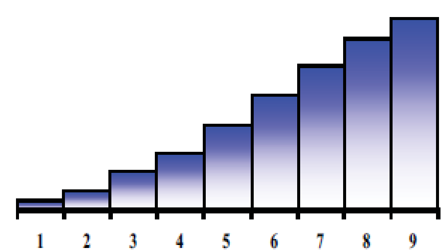




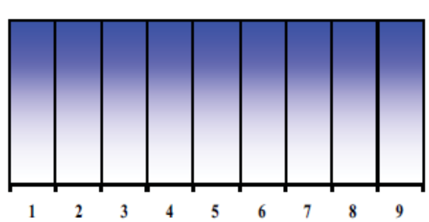
symmetric, unimodal, bell-shaped



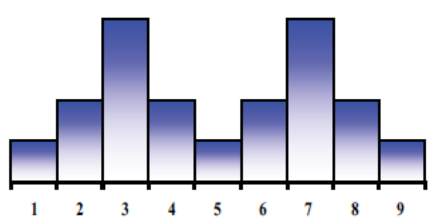
skewed right



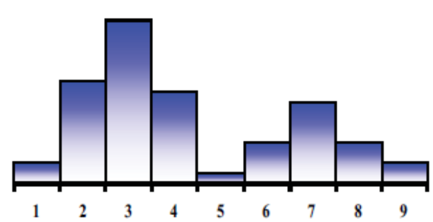
skewed left



uniform



symmetric, bimodal



non-symmetric, bimodal

Data Analysis- Two Variables

Marginal Frequency- the **total** number of responses with a specific characteristic (a total for a row or column)

Joint Frequency- the number of responses for a given characteristic in a row AND a given characteristic in a column (a cell in the table)

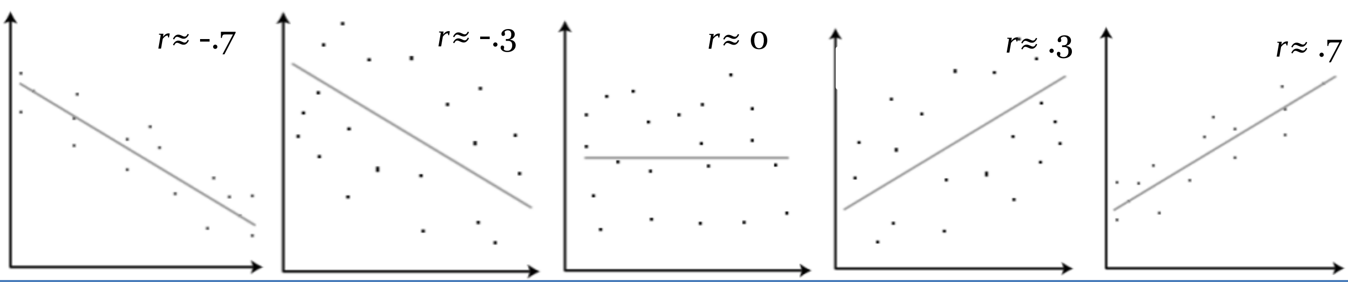
	Studied for EOCT	Didn't Study	Total
Passed EOCT	30	2	32
Didn't Pass EOCT	1	7	8
Total	31	9	40

Joint Frequencies　　　　Marginal Frequencies

Example
What fraction of the people who studied for the EOCT passed?
Answer: $\frac{30}{31}$

Example
Given that a person didn't study, what is the probability they passed?
Answer: $\frac{2}{9} = .\overline{2} \approx 22\%$

Correlation Coefficient, *r*- is a number between -1 and 1 that measures how well a line fits a set of data

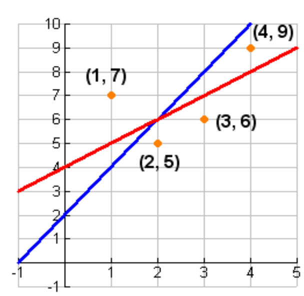


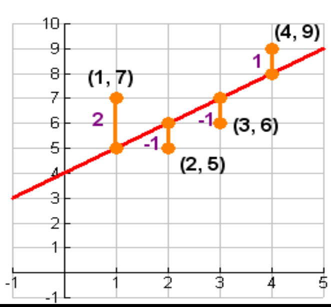
Line of Best Fit (on calculator)
Data
L_1: *x*- values
L_2: *y*- values
2nd Data (Stat)
2-Vars
Calculate
a= slope
b= *y*-int

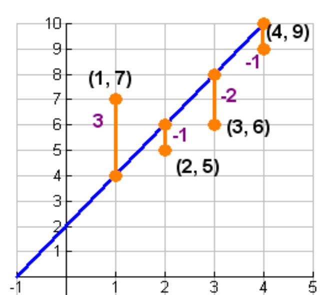
Residual- signed vertical distance between a data point and a line of fit.

Example
Two lines of fit for this data are $y = 2x + 2$ and $y = x + 4$. For each line, find the sum of the squares of the residuals. Which line is a better fit?

x	1	2	3	4
y	7	5	6	9







For $y = x + 4$:
Sum of squared residuals:
$(2)^2 + (-1)^2 + (-1)^2 + (1)^2$
$4 + 1 + 1 + 1 = 7$

For $y = 2x + 2$:
Sum of squared residuals:
$(3)^2 + (-1)^2 + (-2)^2 + (-1)^2$
$9 + 1 + 4 + 1 = 15$

Answer: *The line $y = x + 4$ is a better fit.*

Transformations in Geometry

Translations

$(x, y) \rightarrow (x + a, y + b)$
*Right a, Up b

$(x, y) \rightarrow (x - a, y - b)$
*Left a, Down b

Vector Notation: $<a, b>$

T-Notation: $T_{a,b}$

Reflection

$(x, y) \rightarrow (x, -y)$
*Across the x –axis
- Same order, opposite y.

$(x, y) \rightarrow (-x, y)$
*Across the y –axis
- Same order, opposite x.

$(x, y) \rightarrow (y, x)$
*Across the line $y = x$
-Switch x and y.

$(x, y) \rightarrow (-y, -x)$
*Across the line $y = -x$
-Switch x and y, opposite signs for both

Rotation about the Origin

$(x, y) \rightarrow (-y, x)$
*90° counterclockwise or 270° clockwise
- Switch x & y, opposite first #.

$(x, y) \rightarrow (-x, -y)$
*180° counterclockwise or clockwise
- Same order, opposite x & y.

$(x, y) \rightarrow (y, -x)$
*270° counterclockwise or 90° counterclockwise
- Switch x & y, opposite second #.

Coordinate Geometry

Parallel Lines- Same Slope
Perpendicular Lines- Opposite Reciprocal Slopes

Area

Rectangle and Parallelogram $A = bh$

Triangle $A = \frac{1}{2}bh$

Circle $A = \pi r^2$

Trapezoid $A = \frac{1}{2}(h)(b_1 + b_2)$

Circumference

$C = \pi d \quad \pi \approx 3.14$

Pythagorean Theorem

$a^2 + b^2 = c^2$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

*Distance Formula and Pythagorean Theorem can both be used to find length or distance between two points.
*Perimeter- Distance around a figure

Example

Find a point that partitions a directed line segment from C(4,3) to D(10,15) to a given ratio of 5:1.

C(4,3)
D(10,15)

$+6 \times \frac{5}{6}$
+5

$+12 \times \frac{5}{6}$
+10

Answer: (9, 13)