Pre-Calculus
Conics Cheat Sheet

General Form of a Conic Section: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

** You only have a B term if your conic is obliquely oriented (tilted off of a vertical or horizontal orientation)

CIRCLE:

General form: $A = C$, and both A and C are greater than 0 (positive)

Standard form for an equation of a circle with center (h, k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

PARABOLA:

General form: Either $A = 0$ or $C = 0$, but not both.

Standard form for an equation of a parabola with vertex (h, k):

Horizontal orientation

$(y-k)^2 = 4p(x-h)$ may also be presented as $x = \frac{1}{4p}(y-k)^2 + h$

The axis of symmetry has the equation $y = k$.

p is the distance between the vertex and the focus. It is also the distance between the vertex and the point of intersection of the axis of symmetry and directrix. It may be found by setting $4p$ equal to the value in front of the parenthesis of the "non-squared" side of the equation.

If p is positive:
The focus is located at $(h + p, k)$.
The directrix has the equation $x = (h - p)$
The parabola is said to be concave right

If p is negative:
The focus is located at $(h - p, k)$
The directrix has the equation $x = (h + p)$
The parabola is said to be concave left

Vertical orientation

$(x-h)^2 = 4p(y-k)$ may also be presented as $y = \frac{1}{4p}(x-h)^2 + k$

The axis of symmetry has the equation $x = h$.

p is the distance between the vertex and the focus. It is also the distance between the vertex and the point of intersection of the axis of symmetry and directrix. It may be found by setting $4p$ equal to the value in front of the parenthesis of the "non-squared" side of the equation.

If p is positive:
The focus is located at $(h, k + p)$.
The directrix has the equation $y = (k - p)$
The parabola is said to be concave up

If p is negative:
The focus is located at $(h, k - p)$
The directrix has the equation $y = (k + p)$
The parabola is said to be concave down

ELLIPSE:

General form: $A \neq C$, and both A and C are greater than 0 (positive)

Standard form for an equation of an ellipse with center (h, k):

Horizontal orientation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{where } a^2 > b^2 \text{ also may appear as } \left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$

The major axis (longer axis) is horizontally oriented and has length of $2a$.
The minor axis (shorter axis) is vertically oriented and has length of $2b$.

Vertices are located at $(h \pm a, k)$
Co-vertices are located at $(h, k \pm b)$
Foci are located at $(h \pm c, k)$ where $c = \sqrt{a^2 - b^2}$

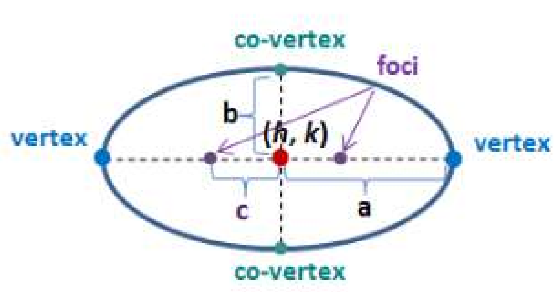
Vertical orientation

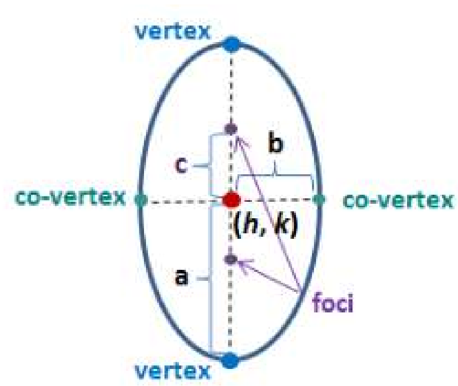
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \text{where } a^2 > b^2 \text{ also may appear as } \left(\frac{x-h}{b}\right)^2 + \left(\frac{y-k}{a}\right)^2 = 1$$

The major axis (longer axis) is vertically oriented and has length of $2a$.
The minor axis (shorter axis) is horizontally oriented and has length of $2b$.

Vertices are located at $(h, k \pm a)$
Co-vertices are located at $(h \pm b, k)$
Foci are located at $(h, k \pm c)$ where $c = \sqrt{a^2 - b^2}$

Examples, showing various parts of the ellipses…





HYPERBOLA:

General form: $A \neq C$, and either A or C is less than 0 (negative), but not both

Standard form for an equation of a hyperbola with center at (h, k):

Horizontal orientation

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{also may appear as} \quad \left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=1$$

Foci are located at $(h \pm c, k)$ where $c=\sqrt{a^2+b^2}$

Vertices are located at $(h \pm a, k)$

Fundamental rectangle vertices located at

$(h + a, k + b)$, $(h - a, k + b)$, $(h - a, k - b)$, $(h + a, k - b)$

Asymptotes are the lines with equations $y=\pm\frac{b}{a}(x-h)+k$ (this is slope-intercept form)

or $(y-k)=\pm\frac{b}{a}(x-h)$ (this is the point-slope form)

Vertical orientation

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1 \quad \text{also may appear as} \quad \left(\frac{y-k}{a}\right)^2-\left(\frac{x-h}{b}\right)^2=1$$

Foci are located at $(h, k \pm c)$ where $c=\sqrt{a^2+b^2}$

Vertices are located at $(h, k \pm a)$

Fundamental rectangle vertices located at

$(h + b, k + a)$, $(h - b, k + a)$, $(h - b, k - a)$, $(h + b, k - a)$

Asymptotes are the lines with equations $y=\pm\frac{a}{b}(x-h)+k$ (this is the slope-intercept form)

or $(y-k)=\pm\frac{a}{b}(x-h)$ (this is the point-slope form)

The transverse axis joins the vertices, and has length $2a$.

The conjugate axis is the perpendicular bisector of the transverse axis, joins the co-vertices, and has length $2b$.

An example, showing various parts of a hyperbola…