Congruent triangles

Warm Up

Lesson Presentation

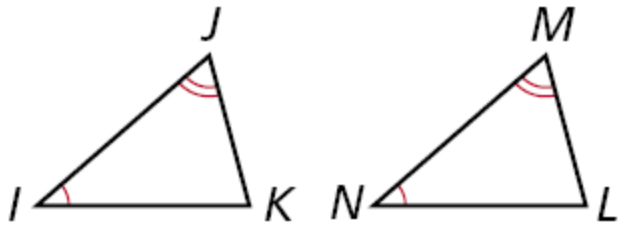
Lesson Quiz

Warm Up

1. Name all sides and angles of ΔFGH.

$\overline{FG}, \overline{GH}, \overline{FH}, \angle F, \angle G, \angle H$

2. What is true about $\angle K$ and $\angle L$? Why?



$\cong$;Third $\angle$s Thm.

3. What does it mean for two segments to be congruent?

They have the same length.

Objectives

Use properties of congruent triangles.

Prove triangles congruent by using the definition of congruence.

Vocabulary

corresponding angles

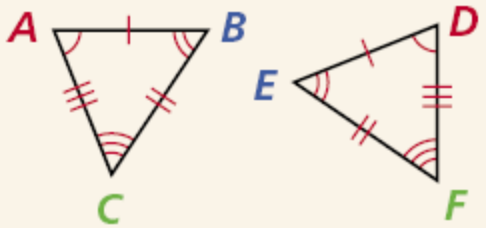
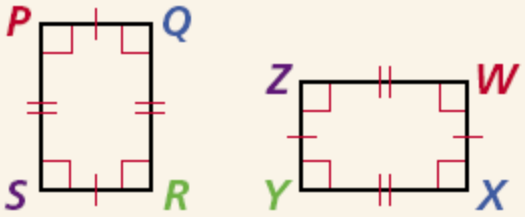
corresponding sides

congruent polygons

Geometric figures are congruent if they are the same size and shape. **Corresponding angles** and **corresponding sides** are in the same position in polygons with an equal number of sides.

Two polygons are **congruent polygons** if and only if their corresponding sides are congruent. Thus triangles that are the same size and shape are congruent.

Properties of Congruent Polygons

DIAGRAM	CORRESPONDING ANGLES	CORRESPONDING SIDES
 $\triangle ABC \cong \triangle DEF$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$
 polygon $PQRS \cong$ polygon $WXYZ$	$\angle P \cong \angle W$ $\angle Q \cong \angle X$ $\angle R \cong \angle Y$ $\angle S \cong \angle Z$	$\overline{PQ} \cong \overline{WX}$ $\overline{QR} \cong \overline{XY}$ $\overline{RS} \cong \overline{YZ}$ $\overline{PS} \cong \overline{WZ}$

Helpful Hint

Two vertices that are the endpoints of a side are called consecutive vertices. For example, P and Q are consecutive vertices.

To name a polygon, write the vertices in consecutive order. For example, you can name polygon *PQRS* as *QRSP* or *SRQP*, but **not** as *PRQS*.
In a congruence statement, the order of the vertices indicates the corresponding parts.

Helpful Hint

When you write a statement such as $\Delta ABC \cong \Delta DEF$, you are also stating which parts are congruent.

Example 1: Naming Congruent Corresponding Parts

Given: $\Delta PQR \cong \Delta STW$

Identify all pairs of corresponding congruent parts.

Angles: $\angle P \cong \angle S,\ \angle Q \cong \angle T,\ \angle R \cong \angle W$

Sides: $\overline{PQ} \cong \overline{ST},\ \overline{QR} \cong \overline{TW},\ \overline{PR} \cong \overline{SW}$

Check It Out! Example 1

If polygon *LMNP* ≅ polygon *EFGH*, identify all pairs of corresponding congruent parts.

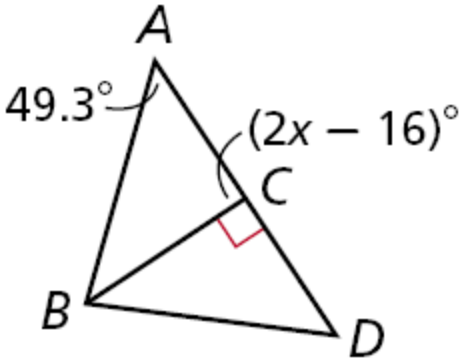
Angles: $\angle L \cong \angle E,\ \angle M \cong \angle F,\ \angle N \cong \angle G,\ \angle P \cong \angle H$

Sides: $\overline{LM} \cong \overline{EF},\ \overline{MN} \cong \overline{FG},\ \overline{NP} \cong \overline{GH},\ \overline{LP} \cong \overline{EH}$

Example 2A: Using Corresponding Parts of Congruent Triangles

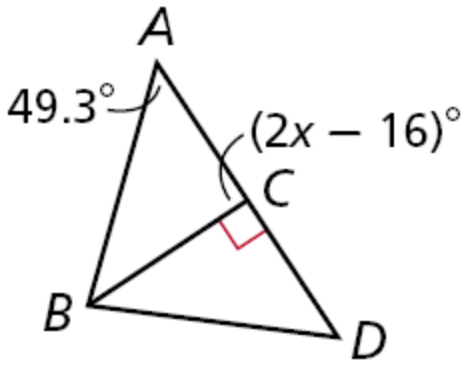
Given: $\Delta ABC \cong \Delta DBC$.

Find the value of x.



$\angle BCA$ and $\angle BCD$ are rt. $\angle$s.	*Def. of $\perp$ lines.*
$\angle BCA \cong \angle BCD$	*Rt. $\angle \cong$ Thm.*
$m\angle BCA = m\angle BCD$	*Def. of $\cong \angle$s*
$(2x - 16)° = 90°$	*Substitute values for $m\angle BCA$ and $m\angle BCD$.*
$2x = 106$	*Add 16 to both sides.*
$x = 53$	*Divide both sides by 2.*

Example 2B: Using Corresponding Parts of Congruent Triangles



Given: $\Delta ABC \cong \Delta DBC$.

Find $m\angle DBC$.

$m\angle ABC + m\angle BCA + m\angle A = 180°$ *Δ Sum Thm.*

$m\angle ABC + 90 + 49.3 = 180$ *Substitute values for $m\angle BCA$ and $m\angle A$.*

$m\angle ABC + 139.3 = 180$ *Simplify.*

$m\angle ABC = 40.7$ *Subtract 139.3 from both sides.*

$\angle DBC \cong \angle ABC$ *Corr. ∠s of ≅ Δs are ≅ .*

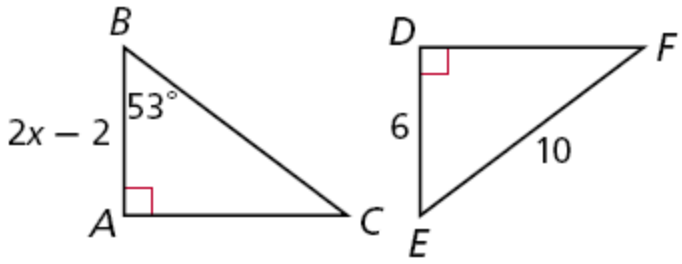
$m\angle DBC = m\angle ABC$ *Def. of ≅ ∠s.*

$m\angle DBC = 40.7°$ *Trans. Prop. of =*

Check It Out! Example 2a

Given: $\Delta ABC \cong \Delta DEF$

Find the value of x.

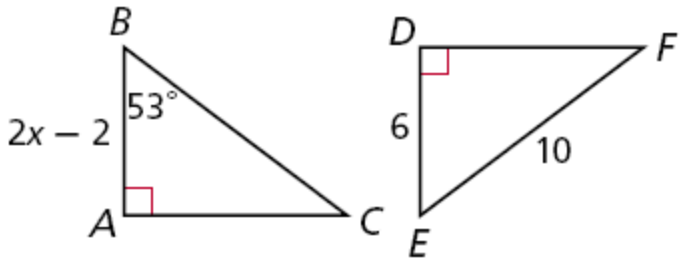


$\overline{AB} \cong \overline{DE}$	*Corr. sides of ≅ Δs are ≅.*
$AB = DE$	*Def. of ≅ parts.*
$2x - 2 = 6$	*Substitute values for AB and DE.*
$2x = 8$	*Add 2 to both sides.*
$x = 4$	*Divide both sides by 2.*

Check It Out! Example 2b

Given: $\Delta ABC \cong \Delta DEF$

Find $m\angle F$.



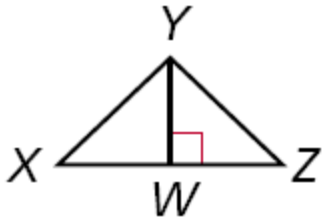
$m\angle EFD + m\angle DEF + m\angle FDE = 180°$	*Δ Sum Thm.*
$\angle ABC \cong \angle DEF$	*Corr. ∠s of ≅ Δ are ≅.*
$m\angle ABC = m\angle DEF$	*Def. of ≅ ∠s.*
$m\angle DEF = 53°$	*Transitive Prop. of =.*
$m\angle EFD + 53 + 90 = 180$	*Substitute values for m∠DEF and m∠FDE.*
$m\angle F + 143 = 180$	*Simplify.*
$m\angle F = 37°$	*Subtract 143 from both sides.*

Example 3: Proving Triangles Congruent

Given: $\angle YWX$ and $\angle YWZ$ are right angles.

$\overline{YW}$ bisects $\angle XYZ$. W is the midpoint of $\overline{XZ}$. $\overline{XY} \cong \overline{YZ}$.

Prove: $\Delta XYW \cong \Delta ZYW$



Statements	Reasons
1. $\angle YWX$ and $\angle YWZ$ are rt. $\angle$s.	**1.** Given
2. $\angle YWX \cong \angle YWZ$	**2.** Rt. $\angle \cong$ Thm.
3. YW bisects $\angle XYZ$	**3.** Given
4. $\angle XYW \cong \angle ZYW$	**4.** Def. of bisector
5. W is mdpt. of $\overline{XZ}$	**5.** Given
6. $\overline{XW} \cong \overline{ZW}$	**6.** Def. of mdpt.
7. $\overline{YW} \cong \overline{YW}$	**7.** Reflex. Prop. of $\cong$
8. $\angle X \cong \angle Z$	**8.** Third $\angle$s Thm.
9. $\overline{XY} \cong \overline{YZ}$	**9.** Given
10. $\Delta XYW \cong \Delta ZYW$	**10.** Def. of $\cong$ Δ

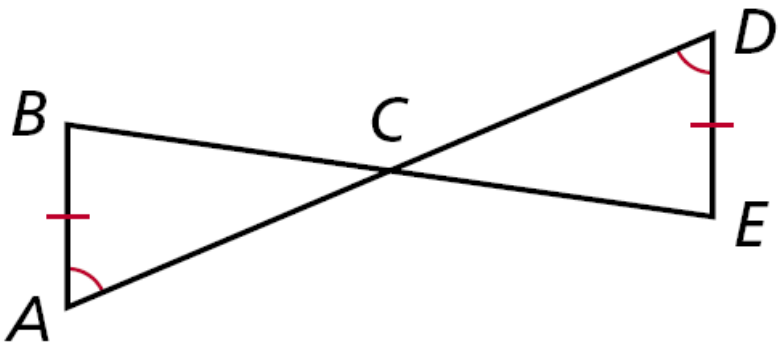
Check It Out! Example 3

Given: $\overline{AD}$ bisects $\overline{BE}$.

$\overline{BE}$ bisects $\overline{AD}$.

$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$

Prove: $\Delta ABC \cong \Delta DEC$



Statements	Reasons
1. $\angle A \cong \angle D$	**1.** Given
2. $\angle BCA \cong \angle DCE$	**2.** Vertical $\angle$s are $\cong$.
3. $\angle ABC \cong \angle DEC$	**3.** Third $\angle$s Thm.
4. $\overline{AB} \cong \overline{DE}$	**4.** Given
5. $\overline{AD}$ bisects $\overline{BE}$, $\overline{BE}$ bisects $\overline{AD}$	**5.** Given
6. $\overline{BC} \cong \overline{EC}$, $\overline{AC} \cong \overline{DC}$	**6.** Def. of bisector
7. $\Delta ABC \cong \Delta DEC$	**7.** Def. of $\cong$ Δs

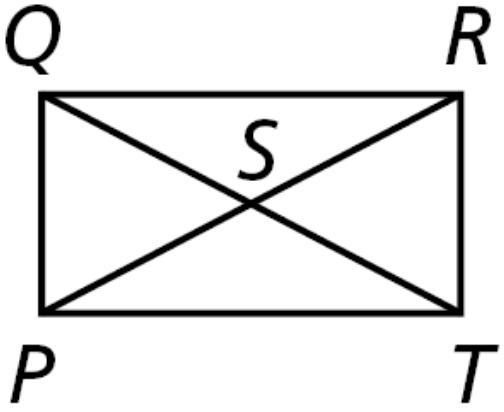
Example 4: Engineering Application

The diagonal bars across a gate give it support. Since the angle measures and the lengths of the corresponding sides are the same, the triangles are congruent.

Given: $\overline{PR}$ and $\overline{QT}$ bisect each other.

$\angle PQS \cong \angle RTS$, $\overline{QP} \cong \overline{RT}$

Prove: $\Delta QPS \cong \Delta TRS$



Example 4 Continued

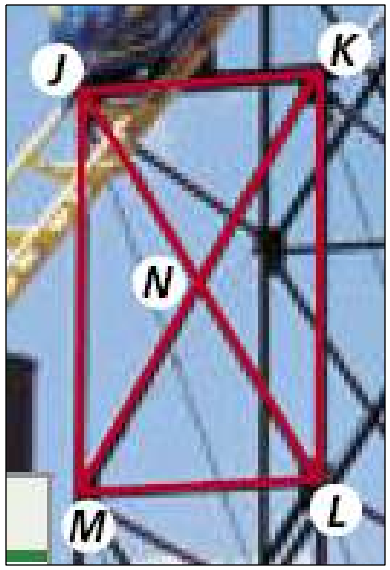
Statements	Reasons
1. $\overline{QP} \cong \overline{RT}$	**1.** Given
2. $\angle PQS \cong \angle RTS$	**2.** Given
3. $\overline{PR}$ and $\overline{QT}$ bisect each other.	**3.** Given
4. $\overline{QS} \cong \overline{TS}$, $\overline{PS} \cong \overline{RS}$	**4.** Def. of bisector
5. $\angle QSP \cong \angle TSR$	**5.** Vert. $\angle$s Thm.
6. $\angle QSP \cong \angle TRS$	**6.** Third $\angle$s Thm.
7. $\Delta QPS \cong \Delta TRS$	**7.** Def. of $\cong$ Δs

Check It Out! Example 4

Use the diagram to prove the following.

Given: $\overline{MK}$ bisects $\overline{JL}$. $\overline{JL}$ bisects $\overline{MK}$. $\overline{JK} \cong \overline{ML}$. $\overline{JK} \parallel \overline{ML}$.

Prove: $\Delta JKN \cong \Delta LMN$



Check It Out! Example 4 Continued

Statements	Reasons
1. $\overline{JK} \cong \overline{ML}$	**1.** Given
2. $\overline{JK} \parallel \overline{ML}$	**2.** Given
3. $\angle JKN \cong \angle NML$	**3.** Alt int. $\angle$s are $\cong$.
4. $\overline{JL}$ and $\overline{MK}$ bisect each other.	**4.** Given
5. $\overline{JN} \cong \overline{LN}$, $\overline{MN} \cong \overline{KN}$	**5.** Def. of bisector
6. $\angle KNJ \cong \angle MNL$	**6.** Vert. $\angle$s Thm.
7. $\angle KJN \cong \angle MLN$	**7.** Third $\angle$s Thm.
8. $\Delta JKN \cong \Delta LMN$	**8.** Def. of $\cong$ Δs

Lesson Quiz

1. $\Delta ABC \cong \Delta JKL$ and $AB = 2x + 12$. $JK = 4x - 50$. Find x and AB. 31, 74

Given that polygon *MNOP* ≅ polygon *QRST*, identify the congruent corresponding part.

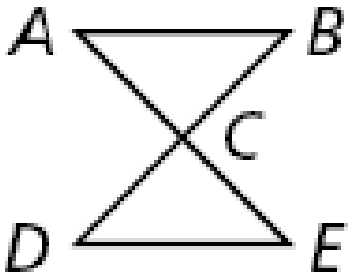
2. $\overline{NO} \cong$ ____ **RS**

3. $\angle T \cong$ _____ $\angle P$

4. Given: C is the midpoint of $\overline{BD}$ and $\overline{AE}$.

$\angle A \cong \angle E$, $\overline{AB} \cong \overline{ED}$

Prove: $\Delta ABC \cong \Delta EDC$



Lesson Quiz

4.

Statements	Reasons
1. $\angle A \cong \angle E$	1. Given
2. C is mdpt. of BD and AE	2. Given
3. $\overline{AC} \cong \overline{EC}$; $\overline{BC} \cong \overline{DC}$	3. Def. of mdpt.
4. $\overline{AB} \cong \overline{ED}$	4. Given
5. $\angle ACB \cong \angle ECD$	5. Vert. $\angle$s Thm.
6. $\angle B \cong \angle D$	6. Third $\angle$s Thm.
7. $\Delta ABC \cong \Delta EDC$	7. Def. of $\cong$ Δs