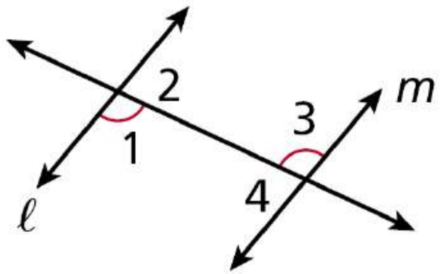
Conditions for parallelograms

Warm Up

Justify each statement.



1. $\overline{QR} \cong \overline{QR}$ Reflex Prop. of ≅

2. $\ell \parallel m$ Conv. of Alt. Int. ∠s Thm.

Evaluate each expression for *x* = 12 and *y* = 8.5.

3. $2x + 7$ 31

4. $16x - 9$ 183

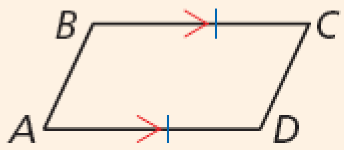
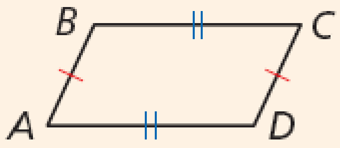
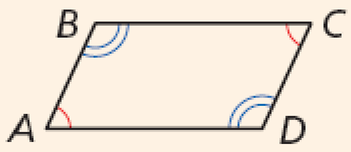
5. $(8y + 5)°$ 73°

Objective

Prove that a given quadrilateral is a parallelogram.

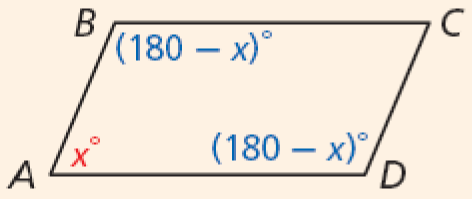
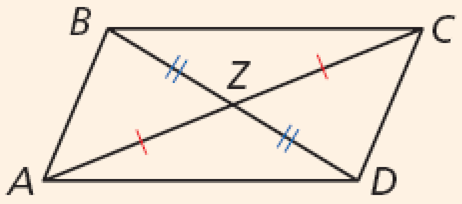
You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the conditions below.

Theorems — Conditions for Parallelograms

THEOREM	EXAMPLE
6-3-1 If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides $\parallel$ and $\cong$ $\rightarrow$ ▱)	
6-3-2 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides $\cong$ $\rightarrow$ ▱)	
6-3-3 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. $\angle$s $\cong$ $\rightarrow$ ▱)	

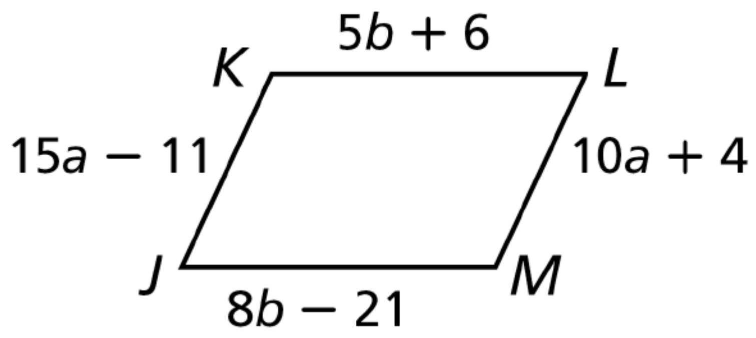
The two theorems below can also be used to show that a given quadrilateral is a parallelogram.

Theorems **Conditions for Parallelograms**

	THEOREM	EXAMPLE
6-3-4	If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with ∠ supp. to cons. ∠s → ▱)	
6-3-5	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other → ▱)	

Example 1A: Verifying Figures are Parallelograms

Show that JKLM is a parallelogram for *a* = 3 and *b* = 9.



Step 1 Find *JK* and *LM*.

$JK = 15a - 11$	*Given*	$LM = 10a + 4$
$JK = 15(3) - 11 = 34$	*Substitute and simplify.*	$LM = 10(3) + 4 = 34$

Example 1A Continued

Step 2 Find *KL* and *JM*.

$KL = 5b + 6$

$KL = 5(9) + 6 = 51$

Given

Substitute and simplify.

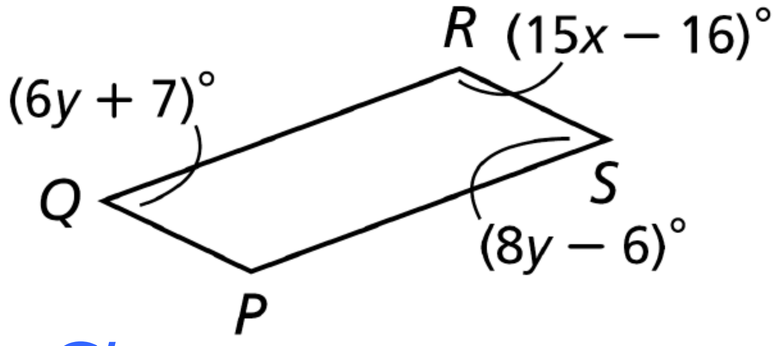
$JM = 8b - 21$

$JM = 8(9) - 21 = 51$

Since *JK* = *LM* and *KL* = *JM*, *JKLM* is a parallelogram by Theorem 6-3-2.

Example 1B: Verifying Figures are Parallelograms

Show that *PQRS* is a parallelogram for *x* = 10 and *y* = 6.5.



$m\angle Q = (6y + 7)°$ *Given*

$m\angle Q = [(6(6.5) + 7)]° = 46°$ *Substitute 6.5 for y and simplify.*

$m\angle S = (8y - 6)°$ *Given*

$m\angle S = [(8(6.5) - 6)]° = 46°$ *Substitute 6.5 for y and simplify.*

$m\angle R = (15x - 16)°$ *Given*

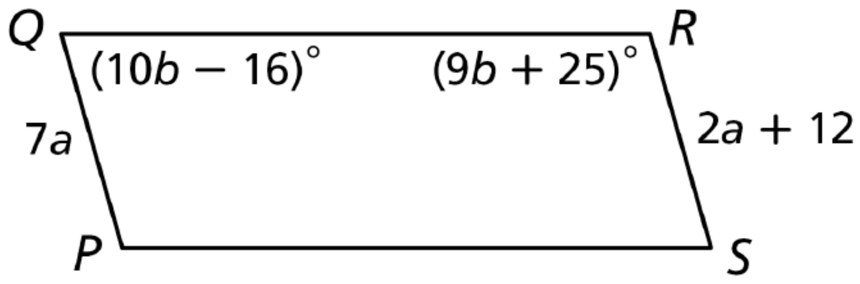
$m\angle R = [(15(10) - 16)]° = 134°$ *Substitute 10 for x and simplify.*

Example 1B Continued

Since $46° + 134° = 180°$, $\angle R$ is supplementary to both $\angle Q$ and $\angle S$. *PQRS* is a parallelogram by Theorem 6-3-4.

Check It Out! Example 1

Show that *PQRS* is a parallelogram for *a* = 2.4 and *b* = 9.



$PQ = RS = 16.8$, so $\overline{PQ} \cong \overline{RS}$.

$m\angle Q = 74°$, and $m\angle R = 106°$, so $\angle Q$ and $\angle R$ are supplementary.

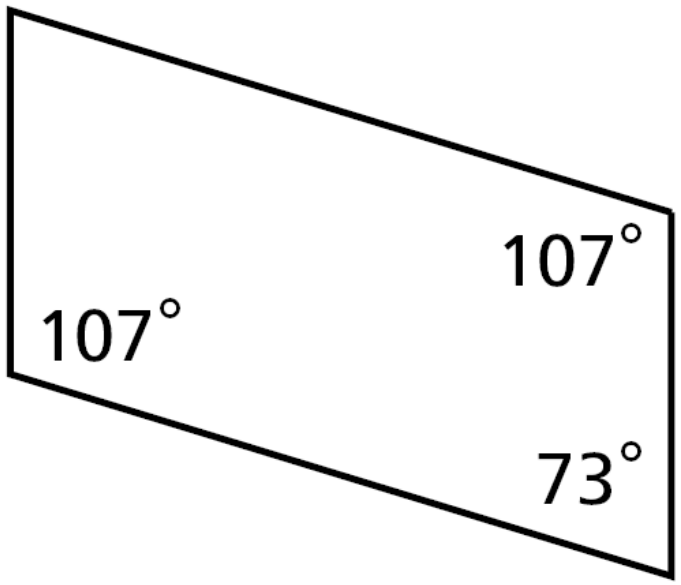
Therefore, $\overline{PQ} \parallel \overline{RS}$.

So one pair of opposite sides of *PQRS* are || and ≅.

By Theorem 6-3-1, *PQRS* is a parallelogram.

Example 2A: Applying Conditions for Parallelograms

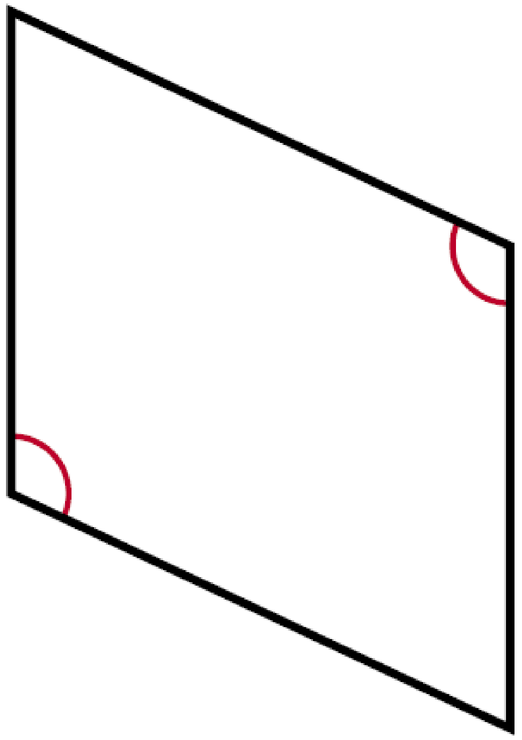
Determine if the quadrilateral must be a parallelogram. Justify your answer.



Yes. The 73° angle is supplementary to both its corresponding angles. By Theorem 6-3-4, the quadrilateral is a parallelogram.

Example 2B: Applying Conditions for Parallelograms

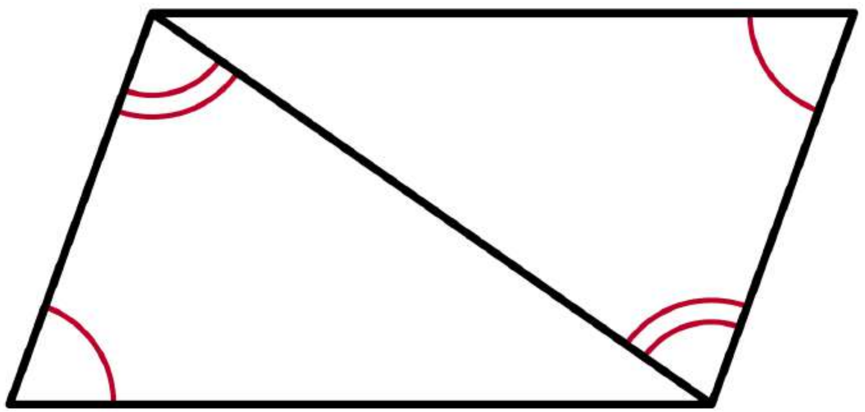
Determine if the quadrilateral must be a parallelogram. Justify your answer.



No. One pair of opposite angles are congruent. The other pair is not. The conditions for a parallelogram are not met.

Check It Out! Example 2a

Determine if the quadrilateral must be a parallelogram. Justify your answer.



Yes

The diagonal of the quadrilateral forms 2 triangles.

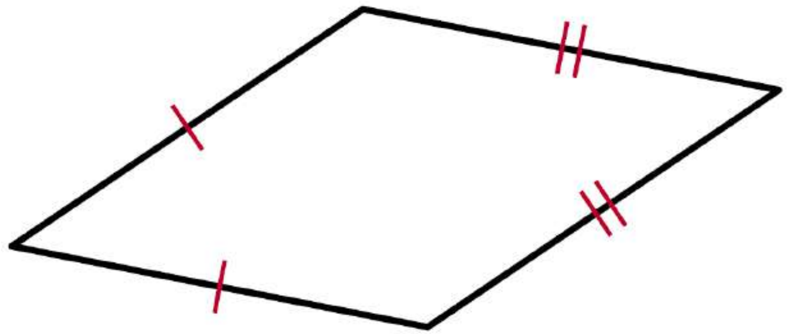
Two angles of one triangle are congruent to two angles of the other triangle, so the third pair of angles are congruent by the Third Angles Theorem.

So both pairs of opposite angles of the quadrilateral are congruent .

By Theorem 6-3-3, the quadrilateral is a parallelogram.

Check It Out! Example 2b

Determine if each quadrilateral must be a parallelogram. Justify your answer.



No. Two pairs of consective sides are congruent.

None of the sets of conditions for a parallelogram are met.

Helpful Hint

To say that a quadrilateral is a parallelogram *by definition*, you must show that both pairs of opposite sides are parallel.

Example 3A: Proving Parallelograms in the Coordinate Plane

Show that quadrilateral *JKLM* is a parallelogram by using the definition of parallelogram. *J*(–1, –6), *K*(–4, –1), *L*(4, 5), *M*(7, 0).

Find the slopes of both pairs of opposite sides.

$$\text{slope of } \overline{JK} = \frac{-1-(-6)}{-4-(-1)} = \frac{5}{-3} = -\frac{5}{3} \quad \text{slope of } \overline{LM} = \frac{0-5}{7-4} = \frac{-5}{3} = -\frac{5}{3}$$

$$\text{slope of } \overline{JM} = \frac{0-(-6)}{7-(-1)} = \frac{6}{8} = \frac{3}{4} \quad \text{slope of } \overline{KL} = \frac{5-(-1)}{4-(-4)} = \frac{6}{8} = \frac{3}{4}$$

Since both pairs of opposite sides are parallel, *JKLM* is a parallelogram by definition.

Example 3B: Proving Parallelograms in the Coordinate Plane

Show that quadrilateral *ABCD* is a parallelogram by using Theorem 6-3-1. *A*(2, 3), *B*(6, 2), *C*(5, 0), *D*(1, 1).

Find the slopes and lengths of one pair of opposite sides.

$$\text{slope of } \overline{AB} = \frac{2-3}{6-2} = \frac{-1}{4} = -\frac{1}{4} \qquad \text{slope of } \overline{CD} = \frac{1-0}{1-5} = \frac{1}{-4} = -\frac{1}{4}$$

$$AB = \sqrt{(6-2)^2 + (2-3)^2} = \sqrt{17} \qquad CD = \sqrt{(1-5)^2 + (1-0)^2} = \sqrt{17}$$

$\overline{AB}$ and $\overline{CD}$ have the same slope, so $\overline{AB} \parallel \overline{CD}$. Since $AB = CD$, $\overline{AB} \cong \overline{CD}$. So by Theorem 6-3-1, *ABCD* is a parallelogram.

Check It Out! Example 3

Use the definition of a parallelogram to show that the quadrilateral with vertices *K*(–3, 0), *L*(–5, 7), *M*(3, 5), and *N*(5, –2) is a parallelogram.

$$\text{slope of } \overline{KL} = \frac{7-0}{-5-(-3)} = \frac{7}{-2} = -\frac{7}{2} \qquad \text{slope of } \overline{MN} = \frac{-2-5}{5-3} = \frac{-7}{2} = -\frac{7}{2}$$

$$\text{slope of } \overline{LM} = \frac{5-7}{3-(-5)} = \frac{-2}{8} = -\frac{1}{4} \qquad \text{slope of } \overline{NK} = \frac{-2-0}{5-(-3)} = \frac{-2}{8} = -\frac{1}{4}$$

Both pairs of opposite sides have the same slope so $\overline{KL} \parallel \overline{MN}$ and $\overline{LM} \parallel \overline{NK}$ by definition, *KLMN* is a parallelogram.

You have learned several ways to determine whether a quadrilateral is a parallelogram. You can use the given information about a figure to decide which condition is best to apply.

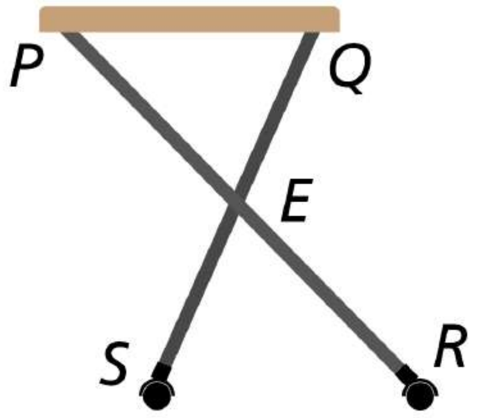
Conditions for Parallelograms
Both pairs of opposite sides are parallel. (definition)
One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)
Both pairs of opposite sides are congruent. (Theorem 6-3-2)
Both pairs of opposite angles are congruent. (Theorem 6-3-3)
One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)
The diagonals bisect each other. (Theorem 6-3-5)

Helpful Hint

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

Example 4: Application

The legs of a keyboard tray are connected by a bolt at their midpoints, which allows the tray to be raised or lowered. Why is *PQRS* always a parallelogram?



Since the bolt is at the midpoint of both legs, *PE* = *ER* and *SE* = *EQ*. So the diagonals of *PQRS* bisect each other, and by Theorem 6-3-5, *PQRS* is always a parallelogram.

Check It Out! Example 4

The frame is attached to the tripod at points *A* and *B* such that *AB* = *RS* and *BR* = *SA*. So *ABRS* is also a parallelogram. How does this ensure that the angle of the binoculars stays the same?

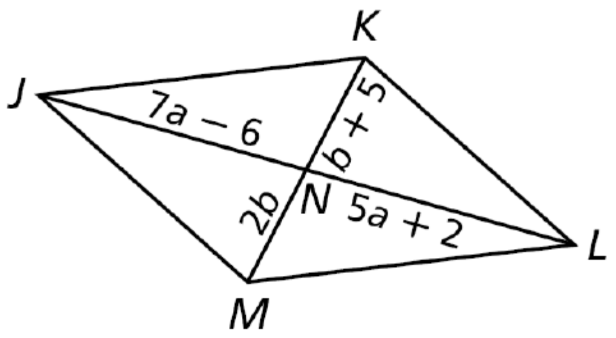
Since *ABRS* is a parallelogram, it is always true that $\overline{AB} \parallel \overline{RS}$.

Since $\overline{AB}$ stays vertical, $\overline{RS}$ also remains vertical no matter how the frame is adjusted.

Therefore the viewing $\angle$ never changes.

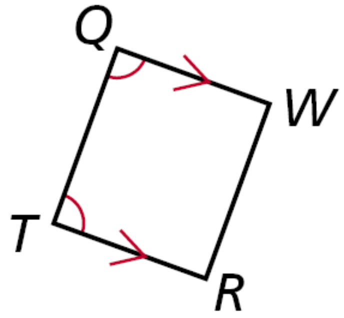
Lesson Quiz: Part I

1. Show that *JKLM* is a parallelogram for $a = 4$ and $b = 5$.



$JN = LN = 22$; $KN = MN = 10$; so *JKLM* is a parallelogram by Theorem 6-3-5.

2. Determine if *QWRT* must be a parallelogram. Justify your answer.



No; One pair of consecutive ∠s are ≅, and one pair of opposite sides are ||. The conditions for a parallelogram are not met.

Lesson Quiz: Part II

3. Show that the quadrilateral with vertices *E*(–1, 5), *F*(2, 4), *G*(0, –3), and *H*(–3, –2) is a parallelogram.

$$\text{slope of } \overline{EF} = \text{slope of } \overline{GH} = -\frac{1}{3}$$

$$EF = GH = \sqrt{10}$$

Since one pair of opposite sides are || and ≅, *EFGH* is a parallelogram by Theorem 6-3-1.