Warm Up

Simplify each expression.

1. $\sqrt{108}$ $6\sqrt{3}$

2. $\sqrt{6} \bullet \sqrt{24}$ 12

3. $\dfrac{\sqrt{42}}{-\sqrt{3}}$ $-\sqrt{14}$

Find the zeros of each function.

4. $f(x) = x^2 - 18x + 16$ $9 \pm \sqrt{65}$

5. $f(x) = x^2 + 8x - 24$ $-4 \pm 2\sqrt{10}$

Objectives

Define and use imaginary and complex numbers.

Solve quadratic equations with complex roots.

Vocabulary

imaginary unit

imaginary number

complex number

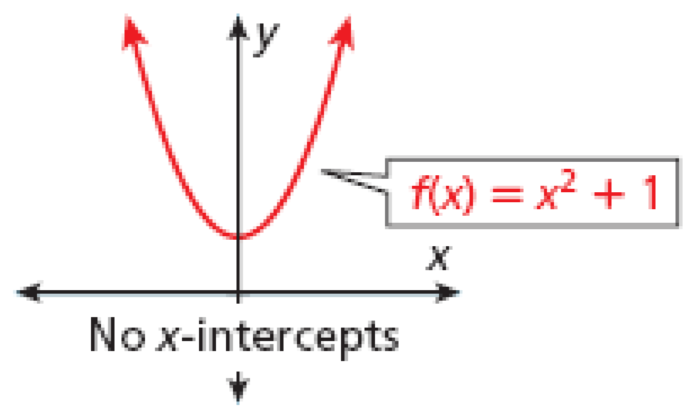
real part

imaginary part

complex conjugate

You can see in the graph of $f(x) = x^2 + 1$ below that f has no real zeros. If you solve the corresponding equation $0 = x^2 + 1$, you find that $x = \pm\sqrt{-1}$,which has no *real* solutions.

However, you can find solutions if you define the square root of negative numbers, which is why *imaginary numbers* were invented. The **imaginary unit** i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.



Imaginary Numbers

WORDS	NUMBERS	ALGEBRA
An **imaginary number** is the square root of a negative number. Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit. The square of an imaginary number is the original negative number.	$\sqrt{-1} = i$ $\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$ $\left(\sqrt{-1}\right)^2 = i^2 = -1$	If b is a positive real number, then $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$. $\left(\sqrt{-b}\right)^2 = -b$

Example 1A: Simplifying Square Roots of Negative Numbers

Express the number in terms of *i*.

$5\sqrt{(121)(-1)}$ *Factor out –1.*

Product Property.

$5 \cdot 11\sqrt{-1}$ *Simplify.*

$55\sqrt{-1}$ *Multiply.*

$55i$ *Express in terms of i.*

Example 1B: Simplifying Square Roots of Negative Numbers

Express the number in terms of *i*.

$-\sqrt{-96}$

$-\sqrt{(96)(-1)}$ *Factor out –1.*

$-\sqrt{16}\sqrt{6}\sqrt{-1}$ *Product Property.*

$-4\sqrt{6}\sqrt{-1}$ *Simplify.*

Express in terms of i.

Check It Out! Example 1a

Express the number in terms of *i*.

$$\sqrt{-12}$$

$$\sqrt{(12)(-1)}$$ *Factor out –1.*

$$\sqrt{12}\,\sqrt{-1}$$ *Product Property.*

$$\sqrt{4}\,\sqrt{3}\,\sqrt{-1}$$ *Product Property.*

$$2\sqrt{3}\,\sqrt{-1}$$ *Simplify.*

$$2\sqrt{3}i = 2i\sqrt{3}$$ *Express in terms of i.*

Check It Out! Example 1b

Express the number in terms of *i*.

Factor out –1.

Product Property.

Simplify.

$$12\sqrt{-1}$$

Multiply.

Express in terms of i.

Check It Out! Example 1c

Express the number in terms of *i*.

$$-\frac{1}{3}\sqrt{-63}$$

$$-\frac{1}{3}\sqrt{(63)(-1)}$$ *Factor out –1.*

$$-\frac{1}{3}\sqrt{63}\sqrt{-1}$$ *Product Property.*

$$-\frac{1}{3}\sqrt{9}\sqrt{7}\sqrt{-1}$$ *Simplify.*

$$-\frac{1}{3}\cdot 3\sqrt{7}\sqrt{-1}$$ *Multiply.*

$$-i\sqrt{7}$$ *Express in terms of i.*

Example 2A: Solving a Quadratic Equation with Imaginary Solutions

Solve the equation.

$$x^2 = -144$$

$$x = \pm\sqrt{-144}$$ *Take square roots.*

$$x = \pm 12i$$ *Express in terms of i.*

Check

x^2	$= -144$
$(12i)^2$	-144
$144i^2$	-144
$144(-1)$	-144 ✓

x^2	$= -144$
$(-12i)^2$	-144
$144i^2$	-144
$144(-1)$	-144 ✓

Example 2B: Solving a Quadratic Equation with Imaginary Solutions

Solve the equation.

$$5x^2 + 90 = 0$$

$5x^2 = -90$ *Add –90 to both sides.*

$x^2 = -18$ *Divide both sides by 5.*

$x = \pm\sqrt{-18}$ *Take square roots.*

$x = \pm 3i\sqrt{2}$ *Express in terms of i.*

Check

$5x^2 + 90 = 0$	
$5\left(\pm 3i\sqrt{2}\right)^2 + 90$	0
$5(18)i^2 + 90$	0
$90(-1) + 90$	0 ✓

Check It Out! Example 2a

Solve the equation.

$$\boldsymbol{x^2 = -36}$$

$x = \pm\sqrt{-36}$ *Take square roots.*

$x = \pm 6i$ *Express in terms of i.*

Check

x^2	$= -36$
$(6i)^2$	-36
$36i^2$	-36
$36(-1)$	-36 ✓

x^2	$= -36$
$(-6i)^2$	-36
$36i^2$	-36
$36(-1)$	-36 ✓

Check It Out! Example 2b

Solve the equation.

$$\boldsymbol{x^2 + 48 = 0}$$

$x^2 = -48$ — *Add –48 to both sides.*

$x = \pm\sqrt{-48}$ — *Take square roots.*

$x = \pm 4i\sqrt{3}$ — *Express in terms of i.*

Check

$x^2 + 48$	$= 0$
$(\pm 4i\sqrt{3})^2 + 48$	0
$(48)i^2 + 48$	0
$48(-1) + 48$	0 ✓

Check It Out! Example 2c

Solve the equation.

$$9x^2 + 25 = 0$$

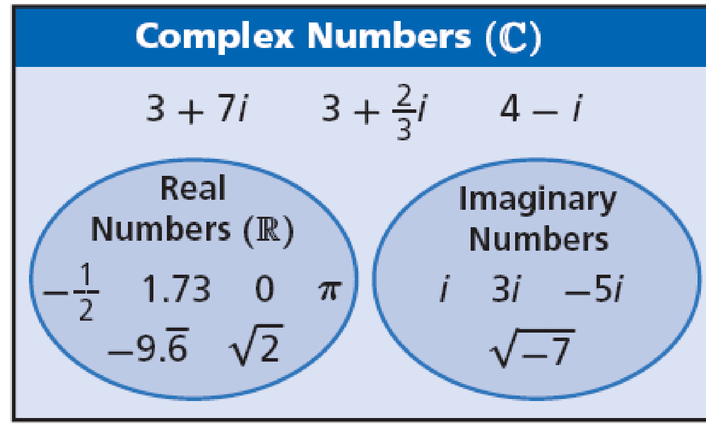
$$9x^2 = -25$$ *Add –25 to both sides.*

$$x^2 = -\frac{25}{9}$$ *Divide both sides by 9.*

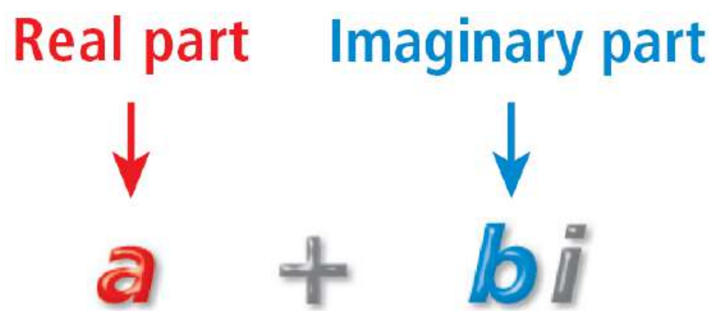
$$x = \pm\sqrt{-\frac{25}{9}}$$ *Take square roots.*

$$x = \pm\frac{5}{3}i$$ *Express in terms of i.*

A **complex number** is a number that can be written in the form *a* + *bi*, where *a* and *b* are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers C.



Every complex number has a **real part** *a* and an **imaginary part** *b*.



Real numbers are complex numbers where $b = 0$. Imaginary numbers are complex numbers where $a = 0$ and $b \neq 0$. These are sometimes called *pure imaginary numbers*.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

Example 3: Equating Two Complex Numbers

Find the values of *x* and *y* that make the equation $4x + 10i = 2 - (4y)i$ true .

Real parts

$$4x + 10i = 2 - (4y)i$$

Imaginary parts

$4x = 2$ *Equate the real parts.*

$x = \frac{1}{2}$ *Solve for x.*

$10 = -4y$ *Equate the imaginary parts.*

$-\frac{5}{2} = y$ *Solve for y.*

Check It Out! Example 3a

Find the values of x and y that make each equation true.

$$\mathbf{2x - 6i = -8 + (20y)i}$$

Real parts

$$2x - 6i = -8 + (20y)i$$

Imaginary parts

$2x = -8$ *Equate the real parts.*

$x = -4$ *Solve for x.*

$-6 = 20y$ *Equate the imaginary parts.*

$-\frac{3}{10} = y$ *Solve for y.*

Check It Out! Example 3b

Find the values of x and y that make each equation true.

$$-8 + (6y)i = 5x - i$$

Real parts

$$-8 + (6y)i = 5x - i$$

Imaginary parts

$-8 = 5x$ *Equate the real parts.*

$-\frac{8}{5} = x$ *Solve for x.*

$6y = -\sqrt{6}$ *Equate the imaginary parts.*

$y = -\frac{\sqrt{6}}{6}$ *Solve for y.*

Example 4A: Finding Complex Zeros of Quadratic Functions

Find the zeros of the function.

$$\boldsymbol{f(x) = x^2 + 10x + 26}$$

$x^2 + 10x + 26 = 0$ *Set equal to 0.*

$x^2 + 10x + \square = -26 + \square$ *Rewrite.*

$x^2 + 10x + 25 = -26 + 25$ *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$(x + 5)^2 = -1$ *Factor.*

$x + 5 = \pm\sqrt{-1}$ *Take square roots.*

$x = -5 \pm i$ *Simplify.*

Example 4B: Finding Complex Zeros of Quadratic Functions

Find the zeros of the function.

$\boldsymbol{g(x) = x^2 + 4x + 12}$

$x^2 + 4x + 12 = 0$ *Set equal to 0.*

$x^2 + 4x + \square = -12 + \square$ *Rewrite.*

$x^2 + 4x + 4 = -12 + 4$ *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$(x + 2)^2 = -8$ *Factor.*

$x + 2 = \pm\sqrt{-8}$ *Take square roots.*

$x = -2 \pm 2i\sqrt{2}$ *Simplify.*

Check It Out! Example 4a

Find the zeros of the function.

$$f(x) = x^2 + 4x + 13$$

$x^2 + 4x + 13 = 0$	*Set equal to 0.*
$x^2 + 4x + \square = -13 + \square$	*Rewrite.*
$x^2 + 4x + 4 = -13 + 4$	*Add $\left(\frac{b}{2}\right)^2$ to both sides.*
$(x + 2)^2 = -9$	*Factor.*
$x + 2 = \pm\sqrt{-9}$	*Take square roots.*
$x = -2 \pm 3i$	*Simplify.*

Check It Out! Example 4b

Find the zeros of the function.

$$g(x) = x^2 - 8x + 18$$

$x^2 - 8x + 18 = 0$ — *Set equal to 0.*

$x^2 - 8x + \square = -18 + \square$ — *Rewrite.*

$x^2 - 8x + 16 = -18 + 16$ — *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$(x - 4)^2 = -2$ — *Factor.*

$x - 4 = \pm\sqrt{-2}$ — *Take square roots.*

$x = 4 \pm i\sqrt{2}$ — *Simplify.*

The solutions $-5 + i\sqrt{10}$ and $-5 - i\sqrt{10}$ are related. These solutions are a *complex conjugate* pair. Their real parts are equal and their imaginary parts are opposites. The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

If a quadratic equation with real coefficients has nonreal roots, those roots are complex conjugates.

Helpful Hint

When given one complex root, you can always find the other by finding its conjugate.

Example 5: Finding Complex Zeros of Quadratic Functions

Find each complex conjugate.

A. $8 + 5i$

$8 + 5i$ *Write as a + bi.*

$8 - 5i$ *Find a – bi.*

B. $6i$

$0 + 6i$ *Write as a + bi.*

$0 - 6i$ *Find a – bi.*

$-6i$ *Simplify.*

Check It Out! Example 5

Find each complex conjugate.

A. $9 - i$

$9 + (-i)$ *Write as a + bi.*

$9 - (-i)$ *Find a – bi.*

$9 + i$ *Simplify.*

B. $i + \sqrt{3}$

$\sqrt{3} + i$ *Write as a + bi.*

$\sqrt{3} - i$ *Find a – bi.*

C. $-8i$

$0 + (-8)i$ *Write as a + bi.*

$0 - (-8)i$ *Find a – bi.*

$8i$ *Simplify.*

Warm Up

Express each number in terms of *i*.

1. $\sqrt{-81}$ $9i$

2. $-\sqrt{-18}$ $-3i\sqrt{2}$

Find each complex conjugate.

3. $6 - i\sqrt{3}$ $6 + i\sqrt{3}$

4. $4i + \sqrt{2}$ $\sqrt{2} - 4i$

Find each product.

5. $\left(2 + \sqrt{2}\right)\left(3 - \sqrt{2}\right)$

$4 + \sqrt{2}$

6. $\left(6 + 3\sqrt{3}\right)\left(1 + \sqrt{3}\right)$

$15 + 9\sqrt{3}$

Objective

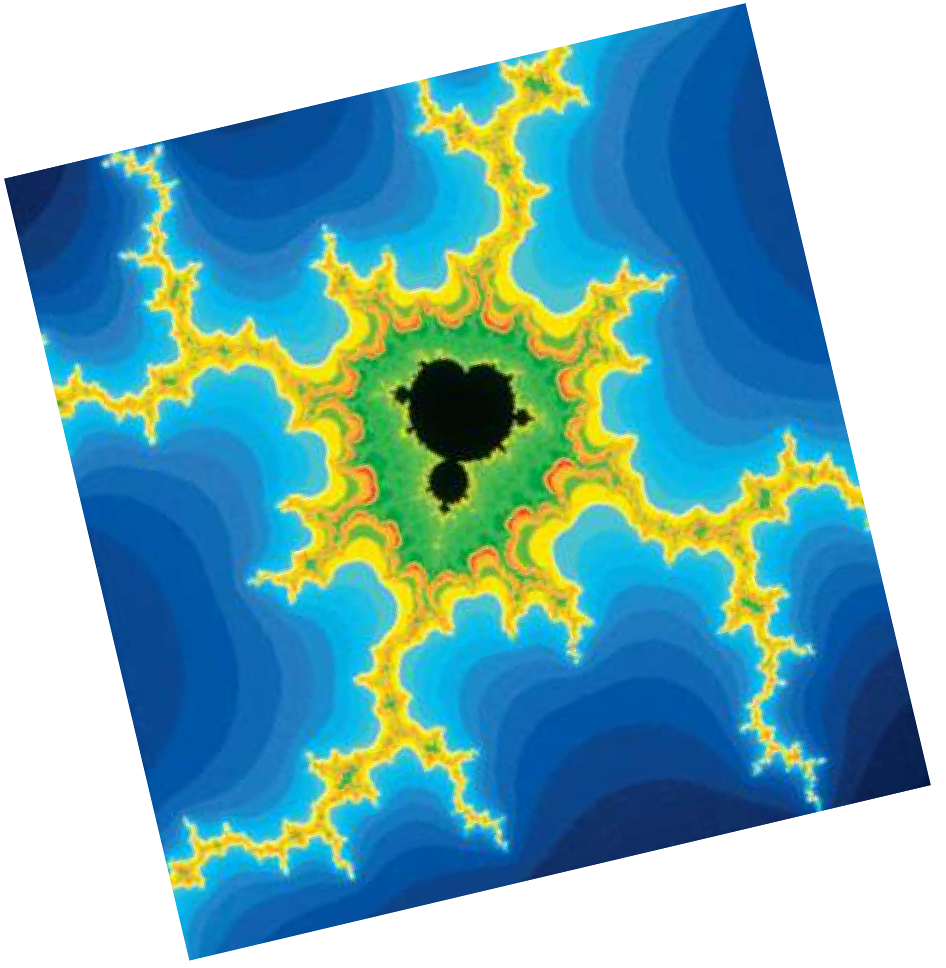
Perform operations with complex numbers.

Vocabulary

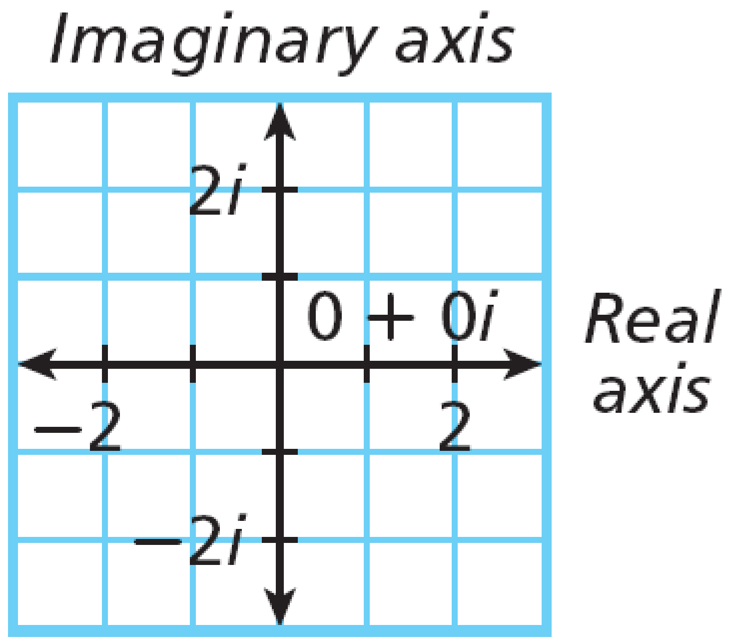
complex plane

absolute value of a complex number

Just as you can represent real numbers graphically as points on a number line, you can represent complex numbers in a special coordinate plane.



The **complex plane** is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.



Helpful Hint

The real axis corresponds to the *x*-axis, and the imaginary axis corresponds to the *y*-axis. Think of $a + bi$ as $x + yi$.

Example 1: Graphing Complex Numbers

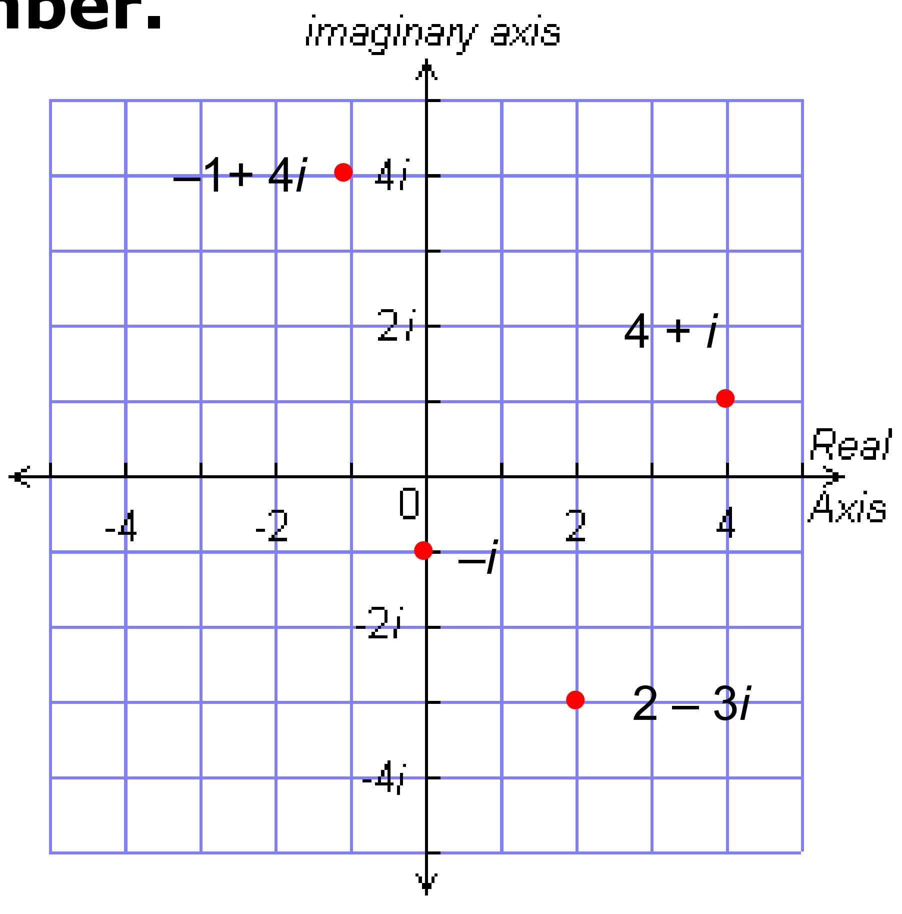
Graph each complex number.

A. $2 - 3i$

B. $-1 + 4i$

C. $4 + i$

D. $-i$



Check It Out! Example 1

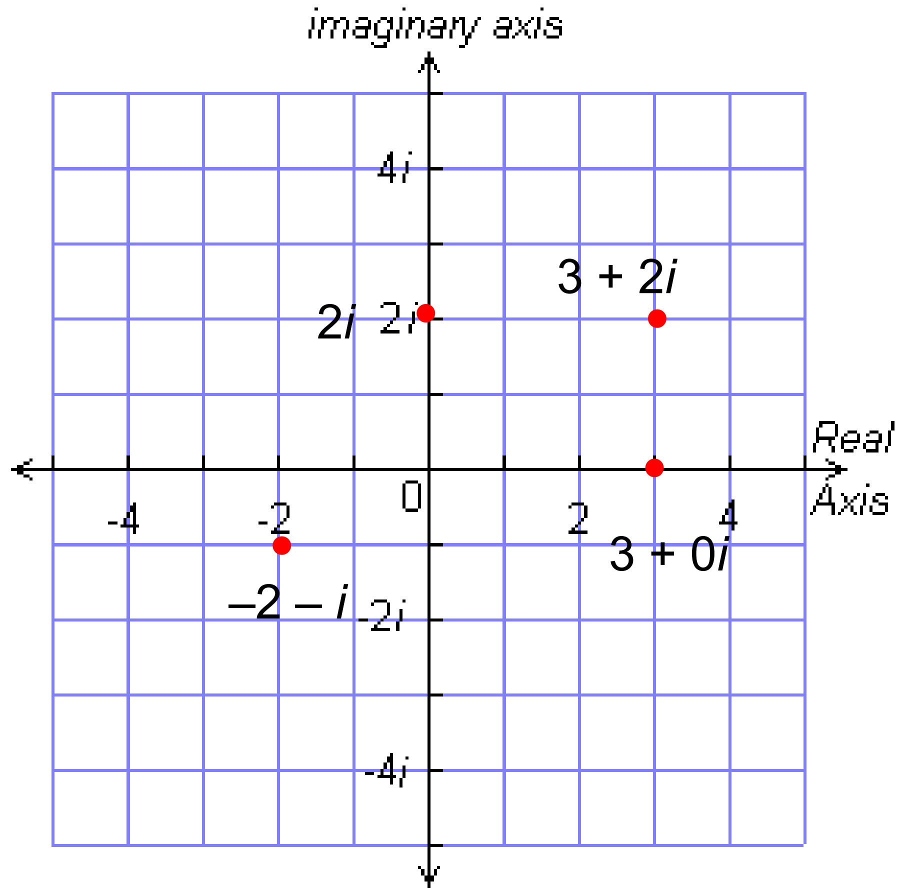
Graph each complex number.

a. 3 + 0*i*

b. 2*i*

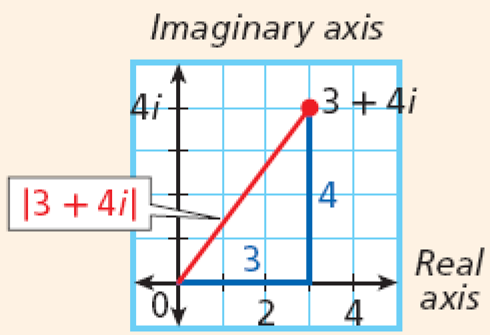
c. −2 − *i*

d. 3 + 2*i*



Recall that absolute value of a real number is its distance from 0 on the real axis, which is also a number line. Similarly, the absolute value of an imaginary number is its distance from 0 along the imaginary axis.

Absolute Value of a Complex Number

WORDS	ALGEBRA	EXAMPLE
The **absolute value of a complex number** $a + bi$ is the distance from the origin to the point (a, b) in the complex plane, and is denoted $\lvert a + bi \rvert$.	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$	 $\lvert 3 + 4i \rvert = \sqrt{3^2 + 4^2}$ $= \sqrt{9 + 16}$ $= 5$

Example 2: Determining the Absolute Value of Complex Numbers

Find each absolute value.

A. $|3 + 5i|$

$\sqrt{3^2 + 5^2}$

$\sqrt{9 + 25}$

$\sqrt{34}$

B. $|-13|$

$|-13 + 0i|$

$\sqrt{(-13)^2 + 0^2}$

$\sqrt{169}$

13

C. $|-7i|$

$|0 + (-7)i|$

$\sqrt{0^2 + (-7)^2}$

$\sqrt{49}$

7

Check It Out! Example 2

Find each absolute value.

a. $|1 - 2i|$

$$\sqrt{1^2 + (-2)^2}$$

$$\sqrt{1 + 4}$$

$$\sqrt{5}$$

b. $\left|-\frac{1}{2}\right|$

$$\left|-\frac{1}{2} + 0i\right|$$

$$\sqrt{\left(-\frac{1}{2}\right)^2 + 0^2}$$

$$\sqrt{\frac{1}{4}}$$

$$\frac{1}{2}$$

c. $|23i|$

$$|0 + 23i|$$

$$\sqrt{0^2 + 23^2}$$

$$\sqrt{529}$$

$$23$$

Adding and subtracting complex numbers is similar to adding and subtracting variable expressions with like terms. Simply combine the real parts, and combine the imaginary parts.

The set of complex numbers has all the properties of the set of real numbers. So you can use the Commutative, Associative, and Distributive Properties to simplify complex number expressions.

Helpful Hint

Complex numbers also have additive inverses. The additive inverse of $a + bi$ is $-(a + bi)$, or $-a - bi$.

Example 3A: Adding and Subtracting Complex Numbers

Add or subtract. Write the result in the form $a + bi$**.**

$$(4 + 2i) + (-6 - 7i)$$

$$(4 - 6) + (2i - 7i)$$

Add real parts and imaginary parts.

$$-2 - 5i$$

Example 3B: Adding and Subtracting Complex Numbers

Add or subtract. Write the result in the form *a* + *bi*.

$$\mathbf{(5 - 2\boldsymbol{i}) - (-2 - 3\boldsymbol{i})}$$

$(5 - 2i) + 2 + 3i$ *Distribute.*

$(5 + 2) + (-2i + 3i)$ *Add real parts and imaginary parts.*

$7 + i$

Example 3C: Adding and Subtracting Complex Numbers

Add or subtract. Write the result in the form *a* + *bi*.

$\mathbf{(1 - 3}\boldsymbol{i}\mathbf{) + (-1 + 3}\boldsymbol{i}\mathbf{)}$

$(1 - 1) + (-3i + 3i)$

Add real parts and imaginary parts.

0

Check It Out! Example 3a

Add or subtract. Write the result in the form *a* + *bi*.

$$(-3 + 5i) + (-6i)$$

$$(-3) + (5i - 6i)$$ *Add real parts and imaginary parts.*

$$-3 - i$$

Check It Out! Example 3b

Add or subtract. Write the result in the form $a + bi$.

$2i - (3 + 5i)$

$(2i) - 3 - 5i$ *Distribute.*

$(-3) + (2i - 5i)$ *Add real parts and imaginary parts.*

$-3 - 3i$

Check It Out! Example 3c

Add or subtract. Write the result in the form *a* + *bi*.

$$(4 + 3i) + (4 - 3i)$$

$$(4 + 4) + (3i - 3i)$$ *Add real parts and imaginary parts.*

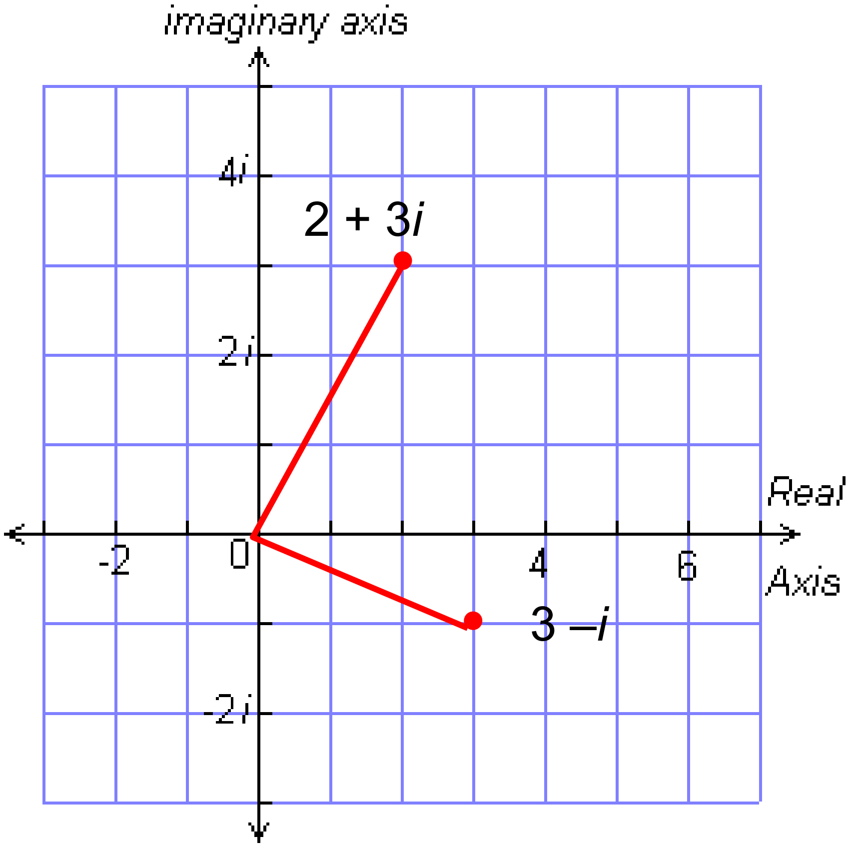
$$8$$

You can also add complex numbers by using coordinate geometry.

Example 4: Adding Complex Numbers on the Complex Plane

Find (3 – *i*) + (2 + 3*i*) by graphing.

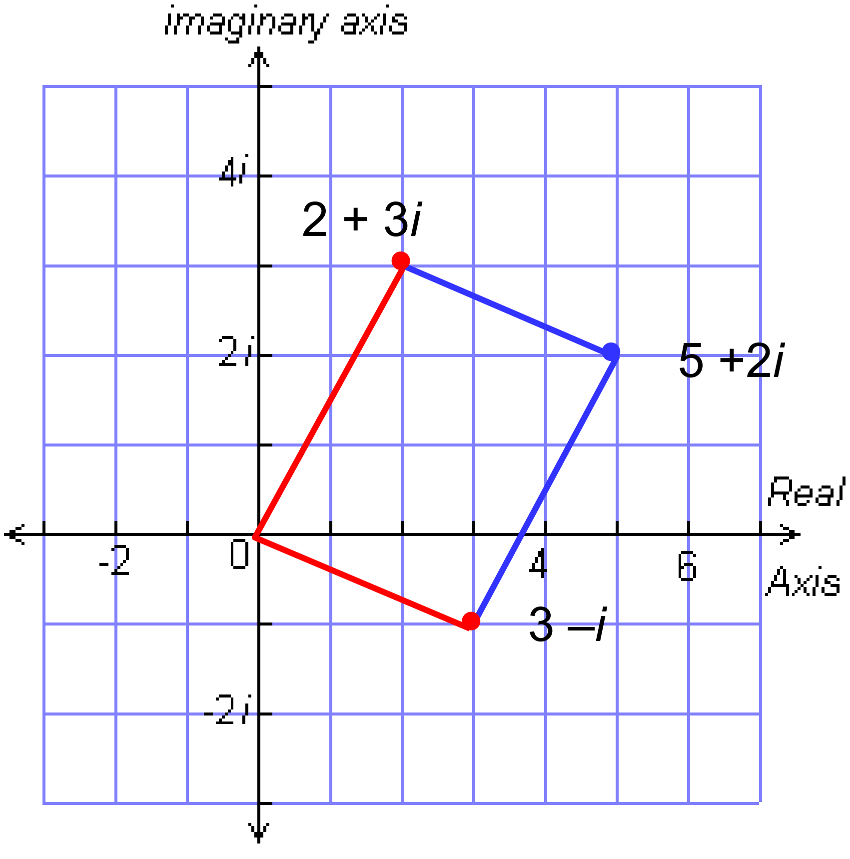
Step 1 Graph 3 – *i* and 2 + 3*i* on the complex plane. Connect each of these numbers to the origin with a line segment.



Example 4 Continued

Find (3 – *i*) + (2 + 3*i*) by graphing.

Step 2 Draw a parallelogram that has these two line segments as sides. The vertex that is opposite the origin represents the sum of the two complex numbers, 5 + 2*i*. Therefore, (3 – *i*) + (2 + 3*i*) = 5 + 2*i*.



Example 4 Continued

Find $(3 - i) + (2 + 3i)$ by graphing.

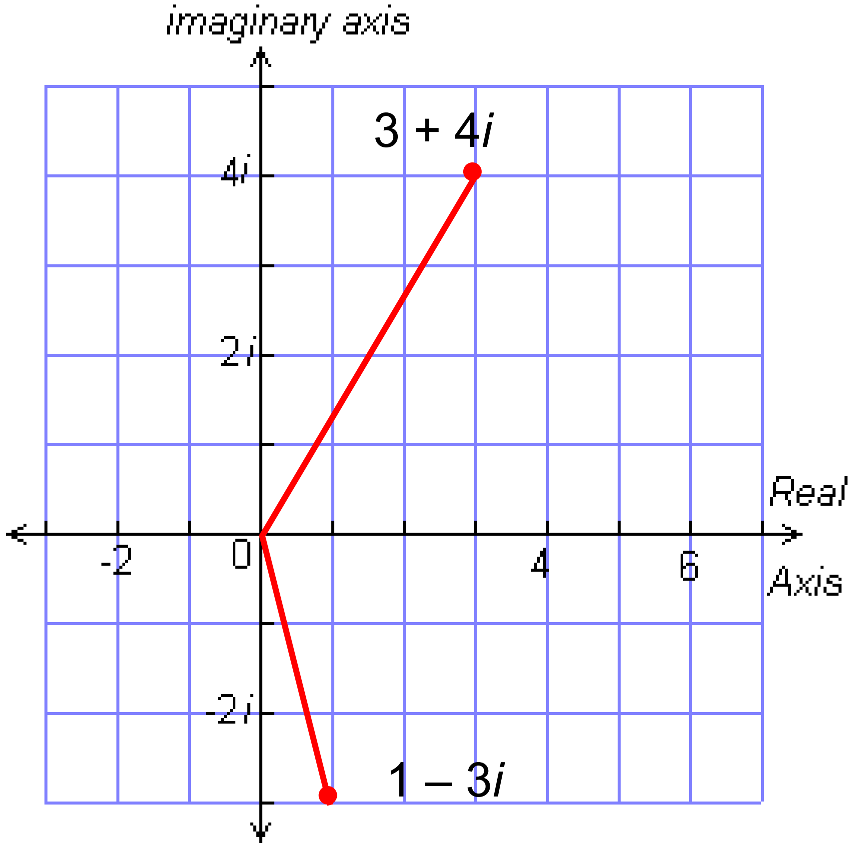
Check Add by combining the real parts and combining the imaginary parts.

$$(3 - i) + (2 + 3i) = (3 + 2) + (-i + 3i) = 5 + 2i$$

Check It Out! Example 4a

Find (3 + 4*i*) + (1 – 3*i*) by graphing.

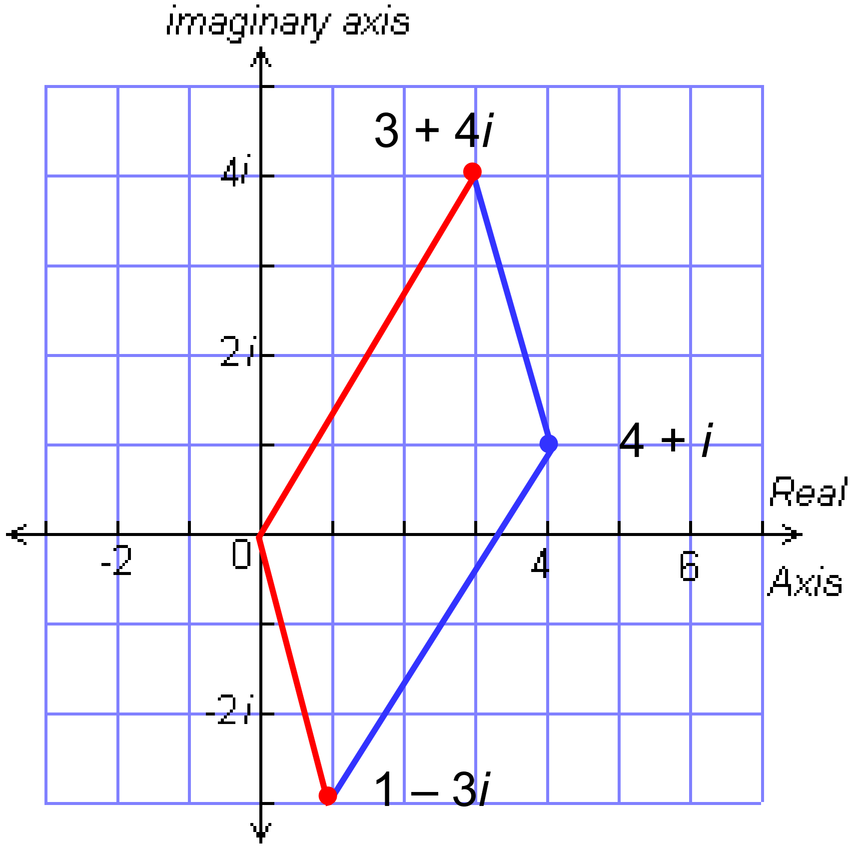
Step 1 Graph 3 + 4*i* and 1 – 3*i* on the complex plane. Connect each of these numbers to the origin with a line segment.



Check It Out! Example 4a Continued

Find (3 + 4*i*) + (1 – 3*i*) by graphing.

Step 2 Draw a parallelogram that has these two line segments as sides. The vertex that is opposite the origin represents the sum of the two complex numbers, 4 + *i*. Therefore, $(3 + 4i) + (1 - 3i) = 4 + i$.



Check It Out! Example 4a Continued

Find (3 + 4*i*) + (1 – 3*i*) by graphing.

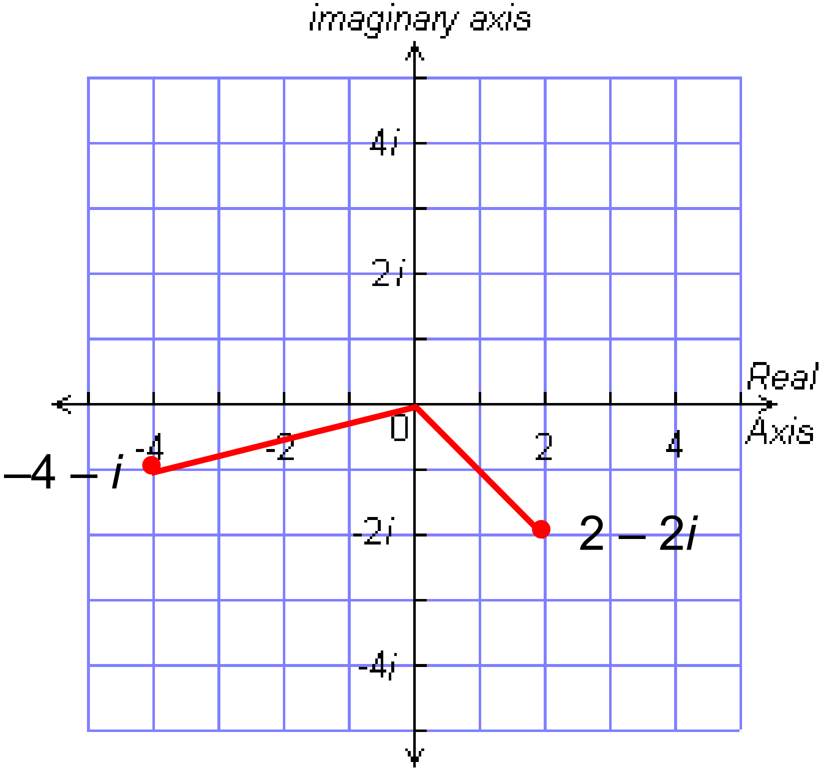
Check Add by combining the real parts and combining the imaginary parts.

$(3 + 4i) + (1 - 3i) = (3 + 1) + (4i - 3i) = 4 + i$

Check It Out! Example 4b

Find (–4 – *i*) + (2 – 2*i*) by graphing.

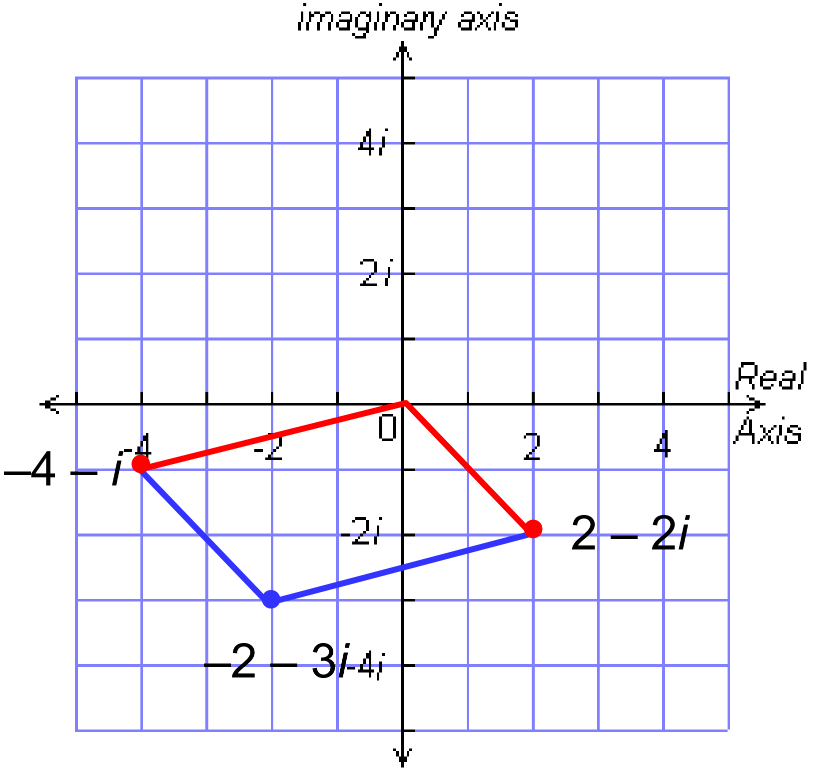
Step 1 Graph –4 – *i* and 2 – 2*i* on the complex plane. Connect each of these numbers to the origin with a line segment.



Check It Out! Example 4b

Find (–4 – *i*) + (2 – 2*i*) by graphing.

Step 2 Draw a parallelogram that has these two line segments as sides. The vertex that is opposite represents the sum of the two complex numbers, –2 –3*i*. Therefore,(–4 – *i*) + (2 – 2*i*) = –2 – 3*i*.



Check It Out! Example 4b

Find $(-4 - i) + (2 - 2i)$ by graphing.

Check Add by combining the real parts and combining the imaginary parts.

$$(-4 - i) + (2 - 2i) = (-4 + 2) + (-i - 2i) = -2 - 3i$$

You can multiply complex numbers by using the Distributive Property and treating the imaginary parts as like terms. Simplify by using the fact $i^2 = -1$.

Example 5A: Multiplying Complex Numbers

Multiply. Write the result in the form *a* + *bi*.

$$-2i(2 - 4i)$$

$-4i + 8i^2$ *Distribute.*

$-4i + 8(-1)$ *Use $i^2 = -1$.*

$-8 - 4i$ *Write in a + bi form.*

Example 5B: Multiplying Complex Numbers

Multiply. Write the result in the form $a + bi$.

$$(3 + 6i)(4 - i)$$

$12 + 24i - 3i - 6i^2$ *Multiply.*

$12 + 21i - 6(-1)$ *Use $i^2 = -1$.*

$18 + 21i$ *Write in $a + bi$ form.*

Example 5C: Multiplying Complex Numbers

Multiply. Write the result in the form $a + bi$.

$$(2 + 9i)(2 - 9i)$$

$4 - 18i + 18i - 81i^2$ *Multiply.*

$4 - 81(-1)$ *Use $i^2 = -1$.*

85 *Write in $a + bi$ form.*

Example 5D: Multiplying Complex Numbers

Multiply. Write the result in the form *a* + *bi*.

$$(-5i)(6i)$$

$-30i^2$ *Multiply.*

$-30(-1)$ *Use $i^2 = -1$*

30 *Write in a + bi form.*

Check It Out! Example 5a

Multiply. Write the result in the form *a* + *bi*.

$\mathbf{2i(3 - 5i)}$

$6i - 10i^2$ *Distribute.*

$6i - 10(-1)$ *Use $i^2 = -1$.*

$10 + 6i$ *Write in a + bi form.*

Check It Out! Example 5b

Multiply. Write the result in the form $a + bi$.

$$(4 - 4i)(6 - i)$$

$24 - 4i - 24i + 4i^2$ *Distribute.*

$24 - 28i + 4(-1)$ *Use $i^2 = -1$.*

$20 - 28i$ *Write in $a + bi$ form.*

Check It Out! Example 5c

Multiply. Write the result in the form *a* + *bi*.

$(3 + 2i)(3 - 2i)$

$9 + 6i - 6i - 4i^2$ *Distribute.*

$9 - 4(-1)$ *Use $i^2 = -1$.*

13 *Write in a + bi form.*

The imaginary unit i can be raised to higher powers as shown below.

Powers of i		
$i^1 = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

Helpful Hint

Notice the repeating pattern in each row of the table. The pattern allows you to express any power of i as one of four possible values: i, -1, $-i$, or 1.

Example 6A: Evaluating Powers of *i*

Simplify $-6i^{14}$.

$-6i^{14} = -6(i^2)^7$ *Rewrite i^{14} as a power of i^2.*

$= -6(-1)^7$

$= -6(-1) = 6$ *Simplify.*

Example 6B: Evaluating Powers of *i*

Simplify i^{63}.

$i^{63} = i \bullet i^{62}$ *Rewrite as a product of i and an even power of i.*

$= i \bullet (i^2)^{31}$ *Rewrite i^{62} as a power of i^2.*

$= i \bullet (-1)^{31} = i \bullet -1 = -i$ *Simplify.*

Check It Out! Example 6a

Simplify $\frac{1}{2}i^7$.

$\frac{1}{2}i^7 = \frac{1}{2}i \cdot i^6$ *Rewrite as a product of i and an even power of i.*

$= \frac{1}{2}i \cdot \left(i^2\right)^3$ *Rewrite i^6 as a power of i^2.*

$= \frac{1}{2}i\left(-1\right)^3 = \frac{1}{2} \cdot -i = -\frac{1}{2}i$ *Simplify.*

Check It Out! Example 6b

Simplify i^{42}.

$i^{42} = (i^2)^{21}$ *Rewrite i^{42} as a power of i^2.*

$= (-1)^{21} = -1$ *Simplify.*

Recall that expressions in simplest form cannot have square roots in the denominator (Lesson 1-3). Because the imaginary unit represents a square root, you must rationalize any denominator that contains an imaginary unit. To do this, multiply the numerator and denominator by the complex conjugate of the denominator.

Helpful Hint

The complex conjugate of a complex number $a + bi$ is $a - bi$. (Lesson 5-5)

Example 7A: Dividing Complex Numbers

Simplify.

$$\frac{3+10i}{5i}$$

$$\frac{3+10i}{5i}\left(\frac{-5i}{-5i}\right)$$ *Multiply by the conjugate.*

$$\frac{-15i-50i^2}{-25i^2}$$ *Distribute.*

$$\frac{-15i+50}{25}$$ *Use $i^2 = -1$.*

$$\frac{-3i+10}{5}=2-\frac{3}{5}i$$ *Simplify.*

Example 7B: Dividing Complex Numbers

Simplify.

$$\frac{2+8i}{4-2i}$$

$$\frac{2+8i}{4-2i}\left(\frac{4+2i}{4+2i}\right)$$ *Multiply by the conjugate.*

$$\frac{8+4i+32i+16i^2}{16-4i^2}$$ *Distribute.*

$$\frac{-8+36i}{20}$$ *Use $i^2 = -1$.*

$$\frac{-2+9i}{5}=-\frac{2}{5}+\frac{9}{5}i$$ *Simplify.*

Check It Out! Example 7a

Simplify.

$$\frac{3+8i}{-i}$$

$$\frac{3+8i}{-i}\left(\frac{i}{i}\right)$$ *Multiply by the conjugate.*

$$\frac{3i+8i^2}{-i^2}$$ *Distribute.*

$$\frac{3i-8}{1}$$ *Use $i^2 = -1$.*

$$-8+3i$$ *Simplify.*

Check It Out! Example 7b

Simplify.

$$\frac{3-i}{2-i}$$

$$\frac{3-i}{2-i}\left(\frac{2+i}{2+i}\right)$$ *Multiply by the conjugate.*

$$\frac{6-2i+3i-i^2}{4-i^2}$$ *Distribute.*

$$\frac{7+i}{5}$$ *Use $i^2 = -1$.*

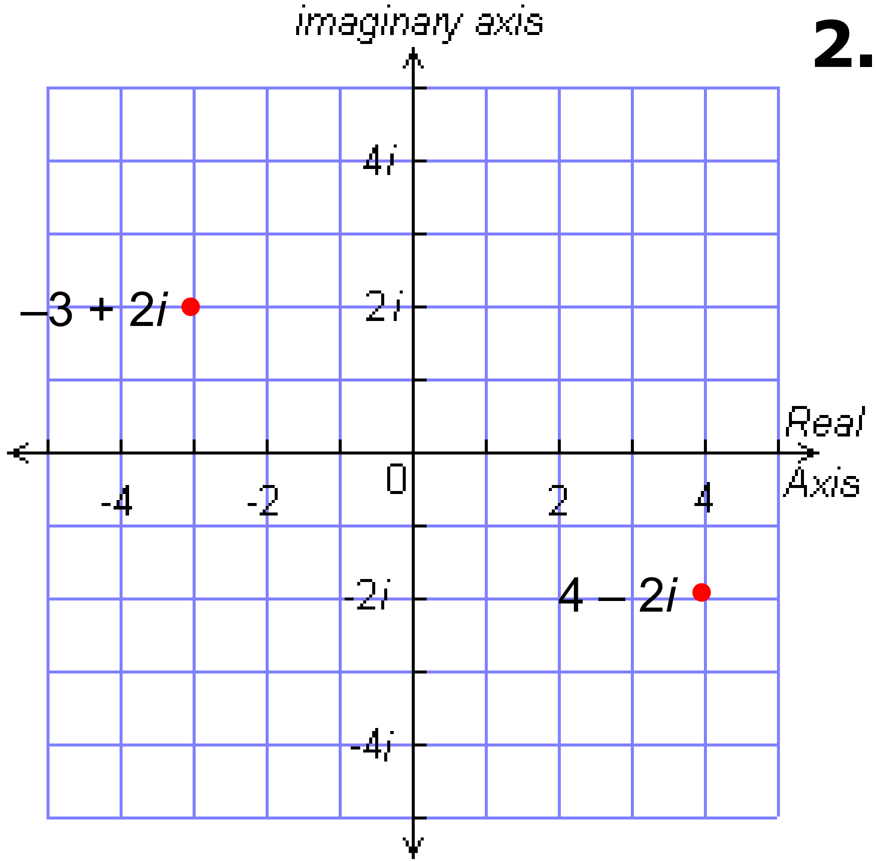
$$\frac{7}{5}+\frac{1}{5}i$$ *Simplify.*

Lesson Quiz: Part I

Graph each complex number.

1. $-3 + 2i$

2. $4 - 2i$



Lesson Quiz: Part II

3. Find $|7 + 3i|$. $\sqrt{58}$

Perform the indicated operation. Write the result in the form *a* + *bi*.

4. $(2 + 4i) + (-6 - 4i)$ -4

5. $(5 - i) - (8 - 2i)$ $-3 + i$

6. $(2 + 5i)(3 - 2i)$

$16 + 11i$

7. $\dfrac{4-2i}{1-i}$ $3 + i$

8. Simplify i^{31}. $-i$

Lesson Quiz

1. Express $\sqrt{-300}$ **in terms of *i*.** $10i\sqrt{3}$

Solve each equation.

2. $3x^2 + 96 = 0$ $\pm 4i\sqrt{2}$

3. $x^2 + 8x + 20 = 0$ $-4 \pm 2i$

4. Find the values of x and y that make the equation $3x + 8i = 12 - (12y)i$ true.

5. Find the complex conjugate of $1 - i\sqrt{2}$.