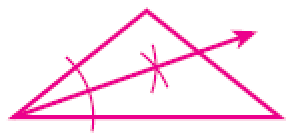
Warm Up

1. Draw a triangle and construct the bisector of one angle.

2. *JK* is perpendicular to *M*



. List the congruent segments.

$$\overline{JM} \cong \overline{JL}, \overline{MK} \cong \overline{KL}$$

Objectives

Prove and apply properties of perpendicular bisectors of a triangle.

Prove and apply properties of angle bisectors of a triangle.

Vocabulary

concurrent

point of concurrency

circumcenter of a triangle

circumscribed

incenter of a triangle

inscribed

Since a triangle has three sides, it has three perpendicular bisectors. When you construct the perpendicular bisectors, you find that they have an interesting property.

Helpful Hint

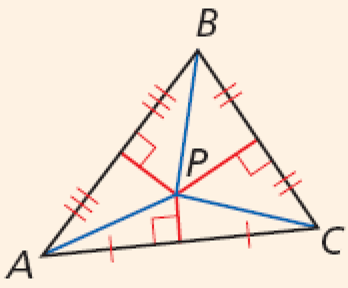
The perpendicular bisector of a side of a triangle does not always pass through the opposite vertex.

When three or more lines intersect at one point, the lines are said to be **concurrent**. The **point of concurrency** is the point where they intersect. In the construction, you saw that the three perpendicular bisectors of a triangle are concurrent. This point of concurrency is the **circumcenter of the triangle**.

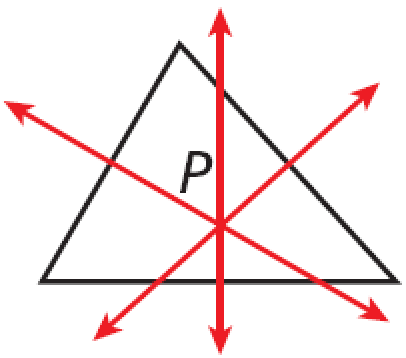
Theorem 5-2-1 **Circumcenter Theorem**

The circumcenter of a triangle is equidistant from the vertices of the triangle.

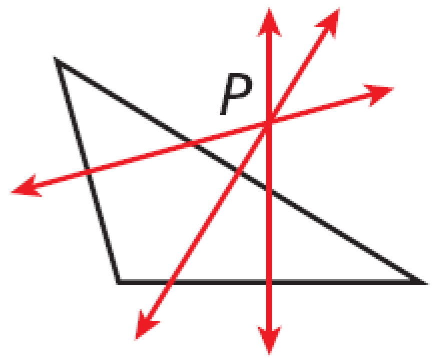
$$PA = PB = PC$$



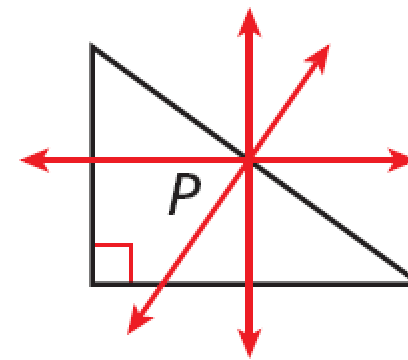
The circumcenter can be inside the triangle, outside the triangle, or on the triangle.



Acute triangle

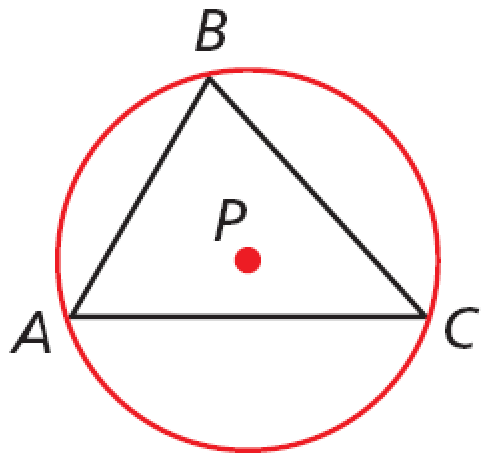


Obtuse triangle



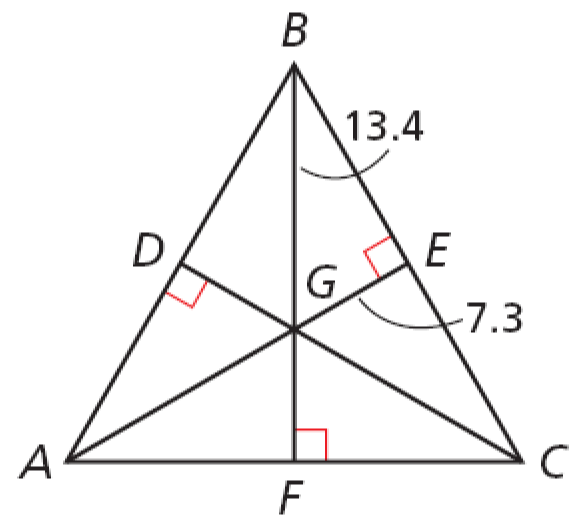
Right triangle

The circumcenter of ΔABC is the center of its circumscribed circle. A circle that contains all the vertices of a polygon is **circumscribed** about the polygon.



Example 1: Using Properties of Perpendicular Bisectors

$\overline{DG}$, $\overline{EG}$, and $\overline{FG}$ are the perpendicular bisectors of Δ*ABC*. Find *GC*.



G is the circumcenter of Δ*ABC*. By the Circumcenter Theorem, *G* is equidistant from the vertices of Δ*ABC*.

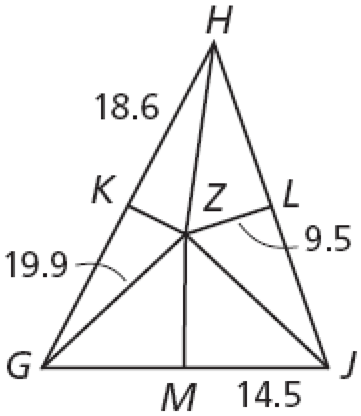
GC = *CB* *Circumcenter Thm.*

GC = 13.4 *Substitute 13.4 for GB.*

Check It Out! Example 1a

Use the diagram. Find *GM*.

$\overline{MZ}$ is a perpendicular bisector of ΔGHJ.



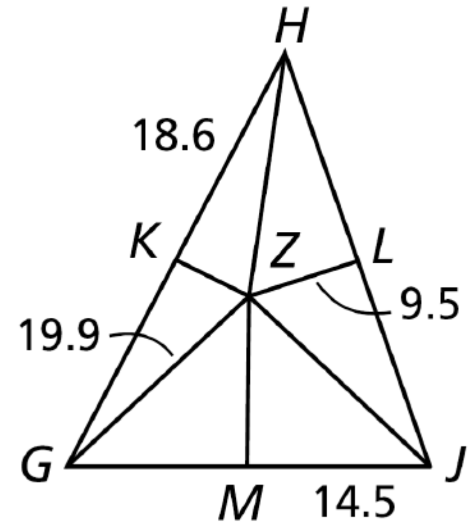
$GM = MJ$ *Circumcenter Thm.*

$GM = 14.5$ *Substitute 14.5 for MJ.*

Check It Out! Example 1b

Use the diagram. Find *GK*.

$\overline{KZ}$ is a perpendicular bisector of ΔGHJ.



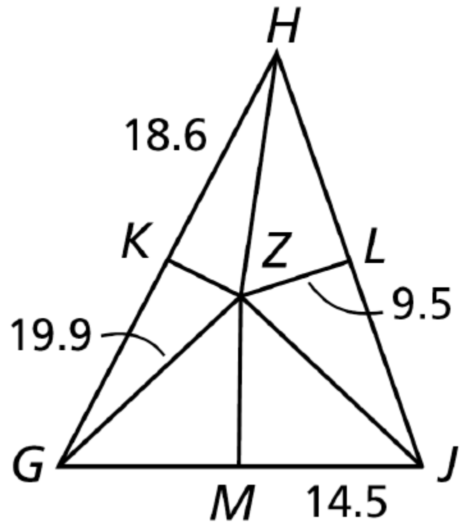
$GK = KH$ *Circumcenter Thm.*

$GK = 18.6$ *Substitute 18.6 for KH.*

Check It Out! Example 1c

Use the diagram. Find *JZ*.

Z is the circumcenter of Δ*GHJ*. By the Circumcenter Theorem, *Z* is equidistant from the vertices of Δ*GHJ*.



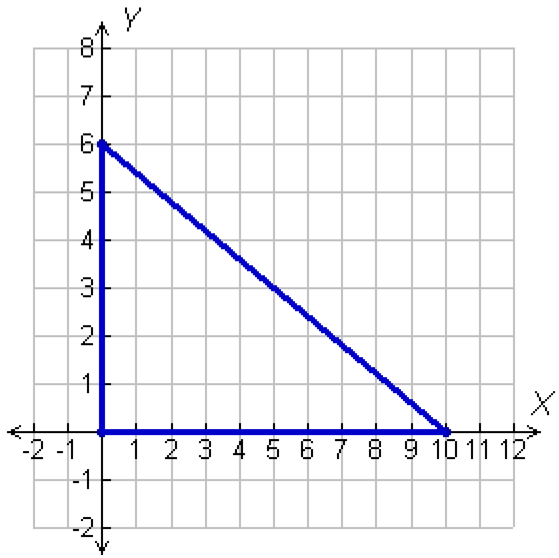
$JZ = GZ$ *Circumcenter Thm.*

$JZ = 19.9$ *Substitute 19.9 for GZ.*

Example 2: Finding the Circumcenter of a Triangle

Find the circumcenter of ∆*HJK* with vertices *H*(0, 0), *J*(10, 0), and *K*(0, 6).

Step 1 Graph the triangle.



Example 2 Continued

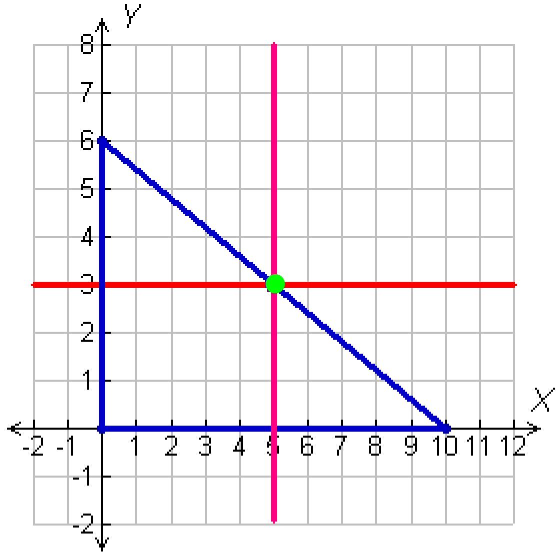
Step 2 Find equations for two perpendicular bisectors.

Since two sides of the triangle lie along the axes, use the graph to find the perpendicular bisectors of these two sides. The perpendicular bisector of $\overline{HJ}$ is $x = 5$, and the perpendicular bisector of $\overline{HK}$ is $y = 3$.

Example 2 Continued

Step 3 Find the intersection of the two equations.

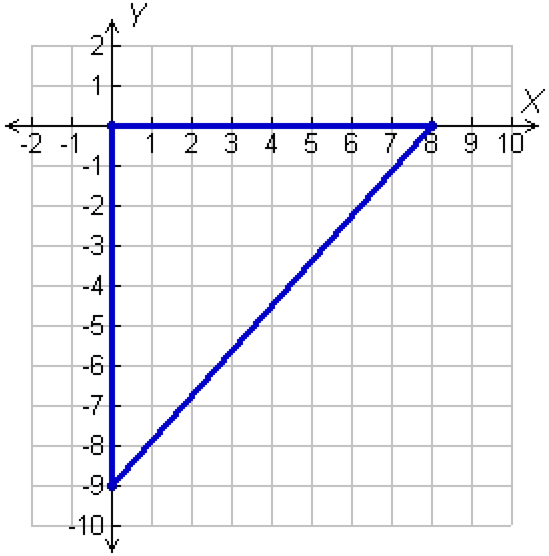
The lines $x = 5$ and $y = 3$ intersect at (5, 3), the circumcenter of Δ*HJK*.



Check It Out! Example 2

Find the circumcenter of ∆*GOH* with vertices *G*(0, –9), *O*(0, 0), and *H*(8, 0) .

Step 1 Graph the triangle.



Check It Out! Example 2 Continued

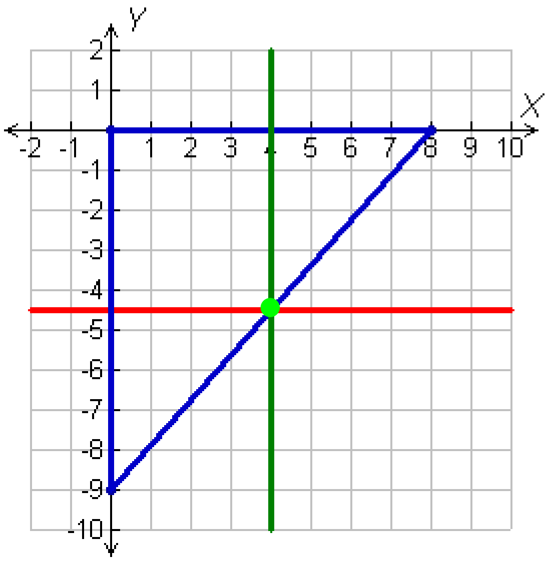
Step 2 Find equations for two perpendicular bisectors.

Since two sides of the triangle lie along the axes, use the graph to find the perpendicular bisectors of these two sides. The perpendicular bisector of *GO* is $y = -4.5$, and the perpendicular bisector of *OH* is
$x = 4$.

Check It Out! Example 2 Continued

Step 3 Find the intersection of the two equations.

The lines $x = 4$ and $y = -4.5$ intersect at $(4, -4.5)$, the circumcenter of ΔGOH.

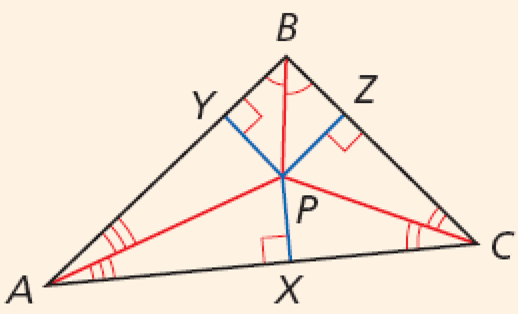


A triangle has three angles, so it has three angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the **incenter of the triangle** .

Theorem 5-2-2 **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

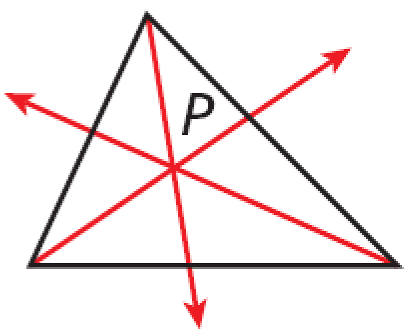
$$PX = PY = PZ$$



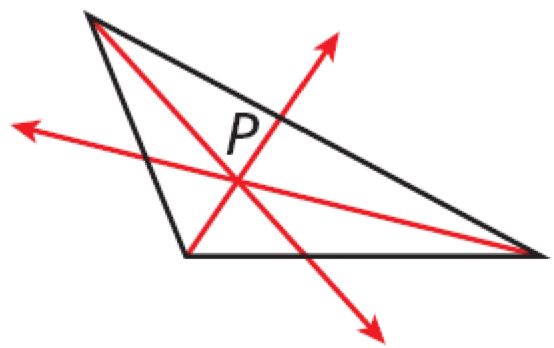
Remember!

The distance between a point and a line is the length of the perpendicular segment from the point to the line.

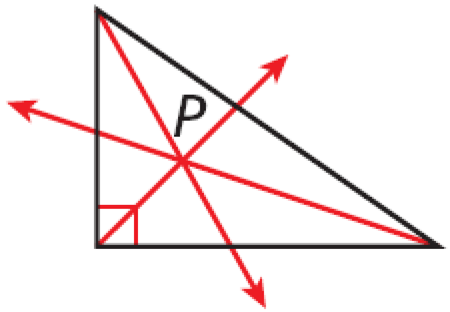
Unlike the circumcenter, the incenter is always inside the triangle.



Acute triangle

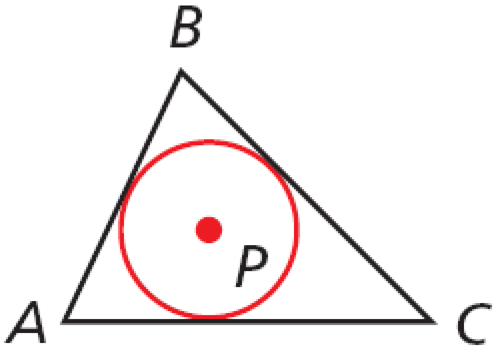


Obtuse triangle



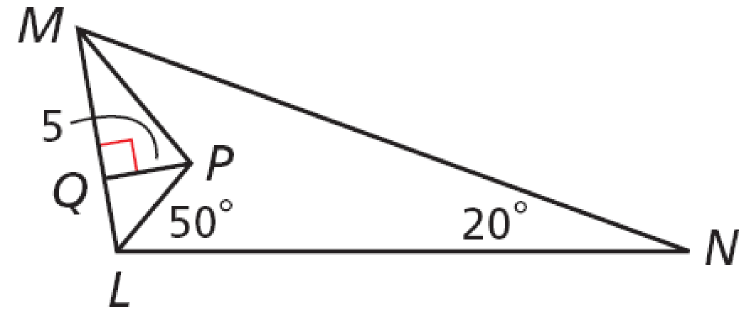
Right triangle

The incenter is the center of the triangle's *inscribed circle*. A circle **inscribed** in a polygon intersects each line that contains a side of the polygon at exactly one point.



Example 3A: Using Properties of Angle Bisectors

$\overline{MP}$ and $\overline{LP}$ are angle bisectors of ΔLMN. Find the distance from *P* to $\overline{MN}$.

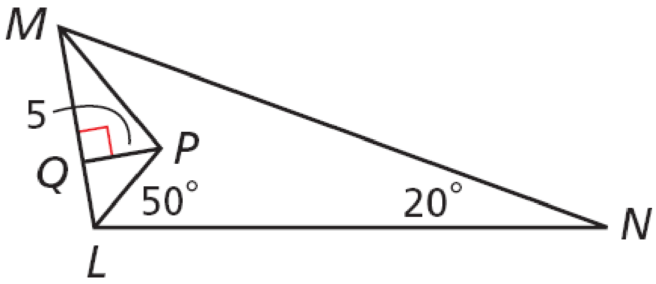


P is the incenter of Δ*LMN*. By the Incenter Theorem, *P* is equidistant from the sides of Δ*LMN*.

The distance from P to $\overline{LM}$ is 5. So the distance from P to $\overline{MN}$ is also 5.

Example 3B: Using Properties of Angle Bisectors

$\overline{MP}$ and $\overline{LP}$ are angle bisectors of ΔLMN. Find $m\angle PMN$.



$m\angle MLN = 2m\angle PLN$ — $\overline{PL}$ *is the bisector of* $\angle MLN$.

$m\angle MLN = 2(50°) = 100°$ — *Substitute 50° for* $m\angle PLN$.

$m\angle MLN + m\angle LNM + m\angle LMN = 180°$ — *Δ Sum Thm.*

$100 + 20 + m\angle LMN = 180$ — *Substitute the given values.*

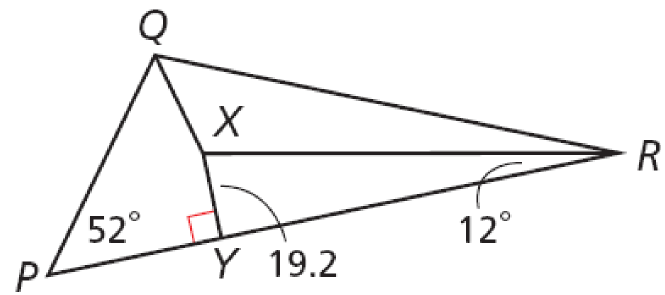
$m\angle LMN = 60°$ — *Subtract 120° from both sides.*

$m\angle PMN = \frac{1}{2} m\angle LMN$ — $\overline{PM}$ *is the bisector of* $\angle LMN$.

$m\angle PMN = \frac{1}{2}(60°)$ — *Substitute 60° for* $m\angle LMN$.

Check It Out! Example 3a

$\overline{QX}$ and $\overline{RX}$ are angle bisectors of Δ*PQR*. Find the distance from *X* to $\overline{PQ}$.

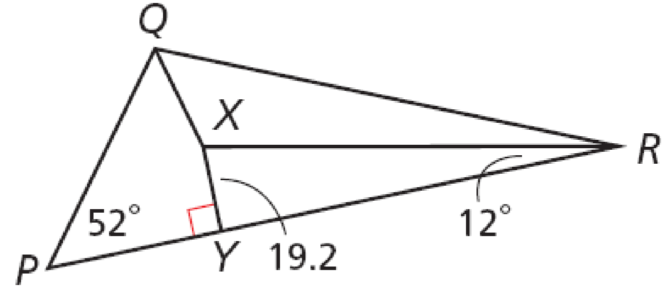


X is the incenter of Δ*PQR*. By the Incenter Theorem, *X* is equidistant from the sides of Δ*PQR*.

The distance from *X* to $\overline{PR}$ is 19.2. So the distance from *X* to $\overline{PQ}$ is also 19.2.

Check It Out! Example 3b

$\overline{QX}$ and $\overline{RX}$ are angle bisectors of ΔPQR. Find $m\angle PQX$.



$m\angle QRY = 2m\angle XRY$ — $\overline{XR}$ is the bisector of $\angle QRY$.

$m\angle QRY = 2(12°) = 24°$ — Substitute 12° for $m\angle XRY$.

$m\angle PQR + m\angle QRP + m\angle RPQ = 180°$ — Δ Sum Thm.

$m\angle PQR + 24 + 52 = 180$ — Substitute the given values.

$m\angle PQR = 104°$ — Subtract 76° from both sides.

$m\angle PQX = \frac{1}{2} m\angle PQR$ — $\overline{QX}$ is the bisector of $\angle PQR$.

$m\angle PQX = \frac{1}{2}(104°)$ — Substitute 104° for $m\angle PQR$.

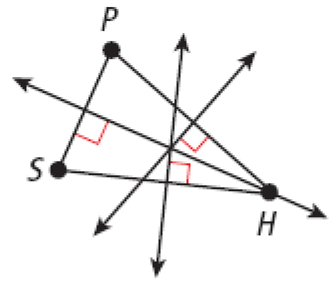
$m\angle PQX = 52°$

Example 4: Community Application

A city planner wants to build a new library between a school, a post office, and a hospital. Draw a sketch to show where the library should be placed so it is the same distance from all three buildings.

Let the three towns be vertices of a triangle. By the Circumcenter Theorem, the circumcenter of the triangle is equidistant from the vertices.

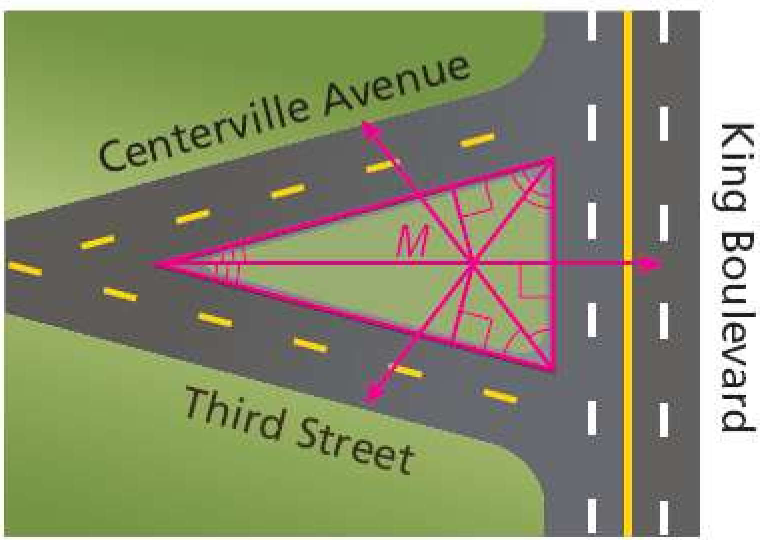
Draw the triangle formed by the three buildings. To find the circumcenter, find the perpendicular bisectors of each side. The position for the library is the circumcenter.



Check It Out! Example 4

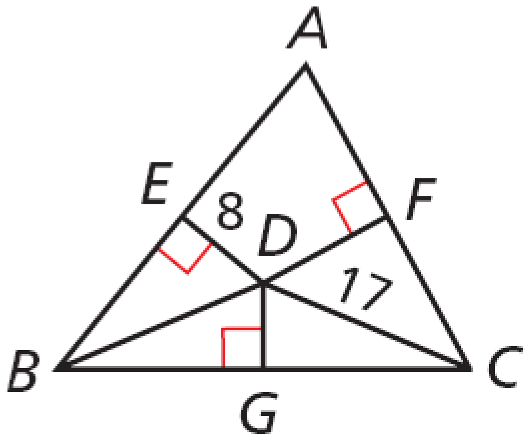
A city plans to build a firefighters' monument in the park between three streets. Draw a sketch to show where the city should place the monument so that it is the same distance from all three streets. Justify your sketch.

By the Incenter Thm., the incenter of a Δ is equidistant from the sides of the Δ. Draw the Δ formed by the streets and draw the ∠ bisectors to find the incenter, point *M*. The city should place the monument at point *M*.



Lesson Quiz: Part I

1. $\overline{ED}$, $\overline{FD}$, and $\overline{GD}$ are the perpendicular bisectors of ΔABC. Find BD.



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2. $\overline{JP}$, $\overline{KP}$, and $\overline{HP}$ are angle bisectors of ΔHJK. Find the distance from P to HK.

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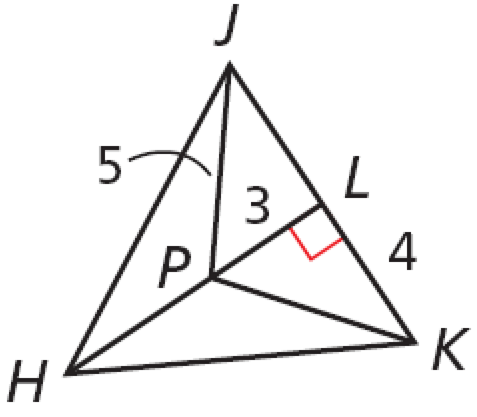

Lesson Quiz: Part II

3. Lee's job requires him to travel to *X*, *Y*, and *Z*. Draw a sketch to show where he should buy a home so it is the same distance from all three places.