# Chapter 10
# Circles

## Section 10.5
## Segment Lengths in Circles

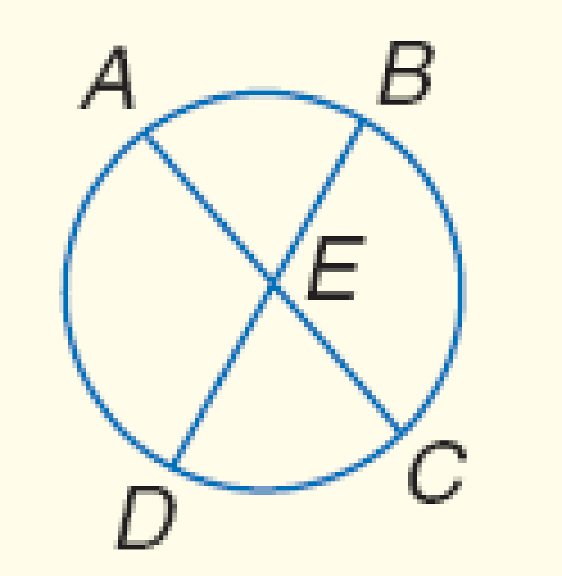
GOAL 1 **Find the lengths of segments of chords.**

GOAL 2 **Find the lengths of segments of tangents and secants.**

GOAL 1 **Find the lengths of segments of chords.**

## Theorem 10.15

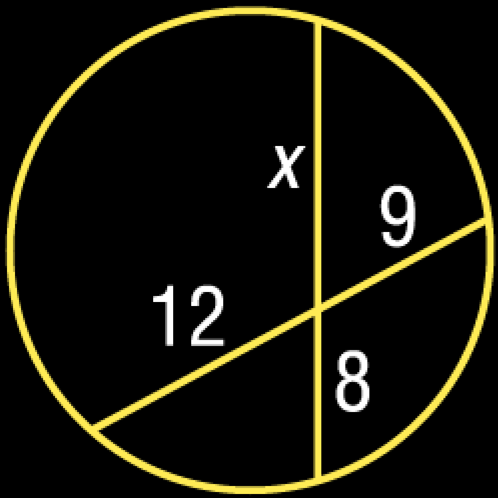
If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.




Example: $AE \cdot EC = BE \cdot ED$

## Example 1

**Find *x*.**




| | |
|---|---|
| $x \cdot 8 = 12 \cdot 9$ | Theorem 10.15 |
| $8x = 108$ | Multiply. |
| $x = 13.5$ | Divide each side by 8. |

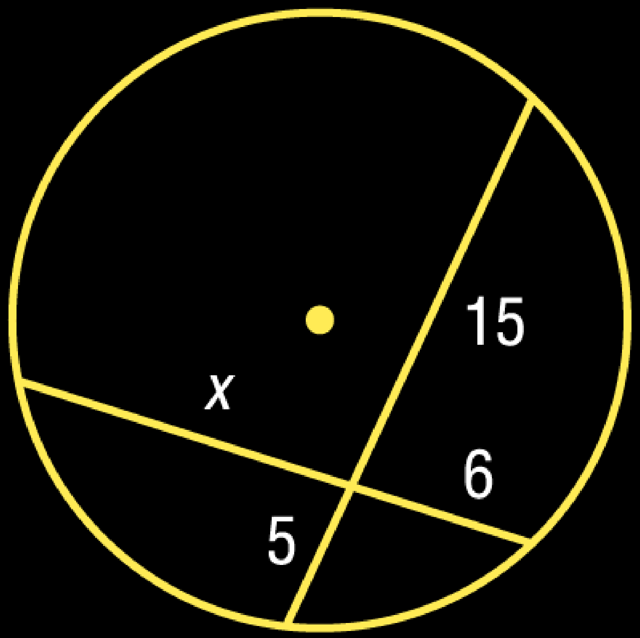
**Answer:** 13.5

End of slide

GOAL 1 Find the lengths of segments of chords.

**Your Turn**

**Find *x*.**



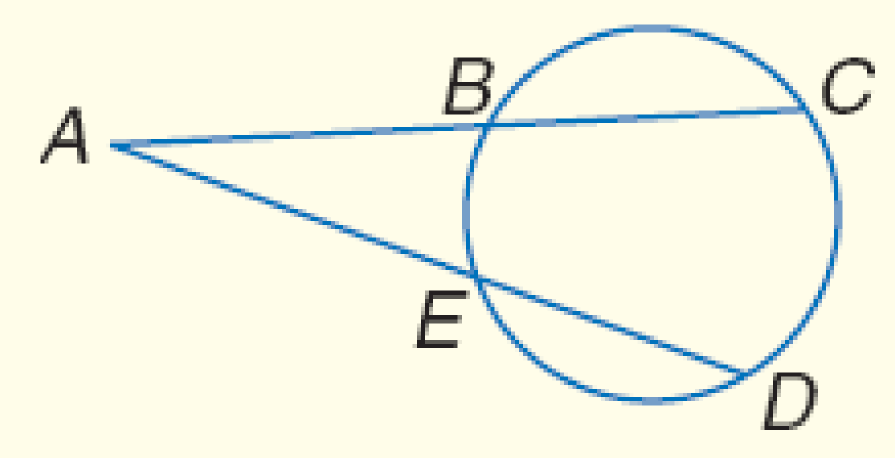

**Answer:** 12.5

End of slide

GOAL 2 Find the lengths of segments of tangents and secants.

## Theorem 10.16

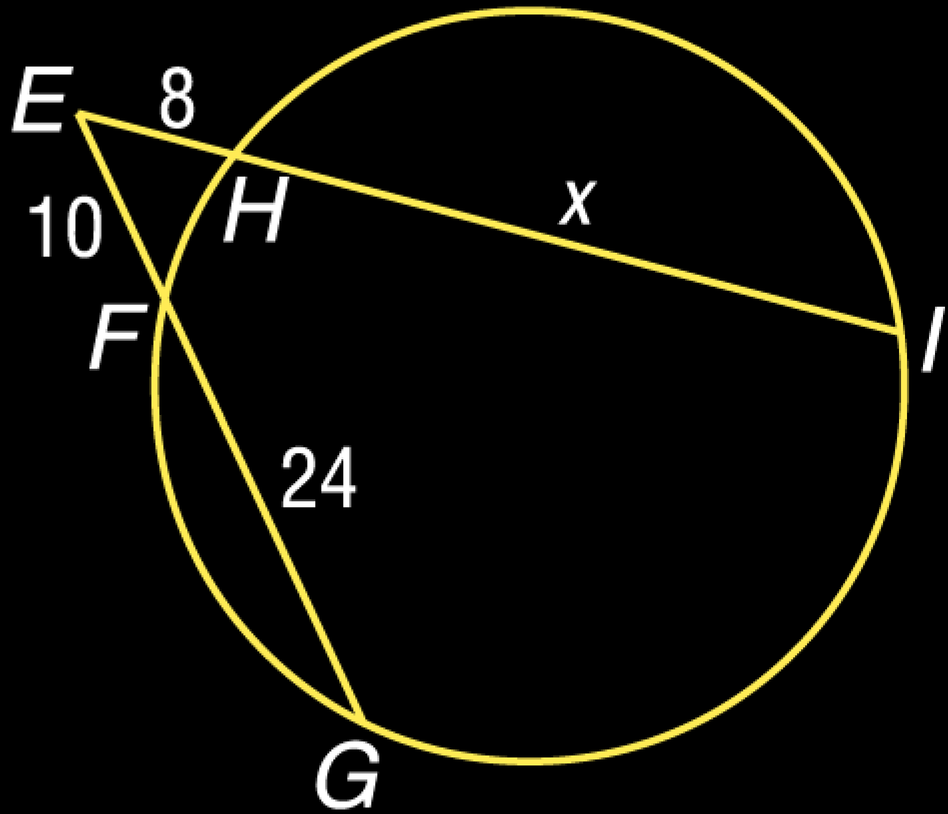
If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.




Example: $AB \cdot AC = AE \cdot AD$

## Example 3

**Find *x* if $EF = 10$, $EH = 8$, and $FG = 24$.**

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GOAL 2 **Find the lengths of segments of tangents and secants.**

**Example 3**

| | |
|---|---|
| $EH \cdot EI = EF \cdot EG$ | Secant Segment Products |
| $8 \cdot (8 + x) = 10 \cdot (10 + 24)$ | Substitution |
| $64 + 8x = 340$ | Distributive Property |
| $8x = 276$ | Subtract 64 from each side. |
| $x = 34.5$ | Divide each side by 8. |

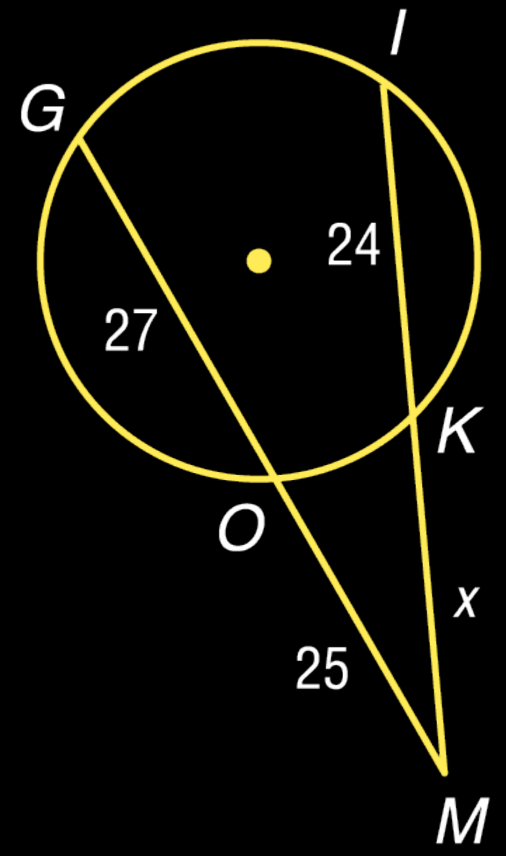
**Answer:** 34.5

STOP! End of slide

GOAL 2 Find the lengths of segments of tangents and secants.

**Your Turn**

**Find $x$ if $GO = 27$, $OM = 25$, and $IK = 24$.**



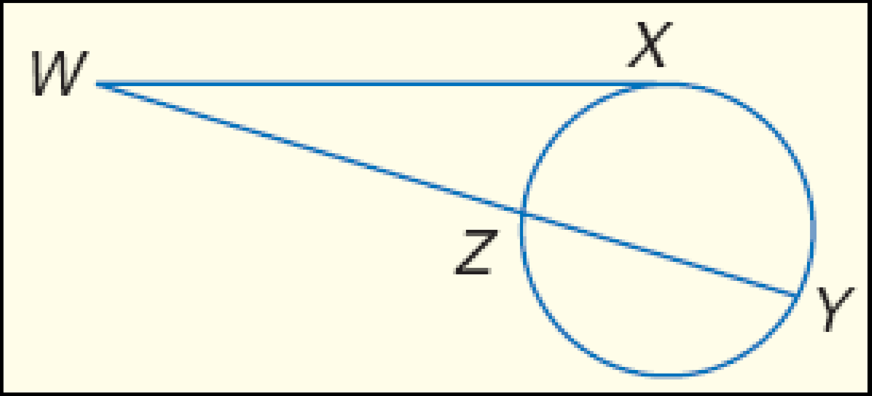

**Answer:** 26

STOP! End of slide

**GOAL 2** Find the lengths of segments of tangents and secants.

## Theorem 10.17

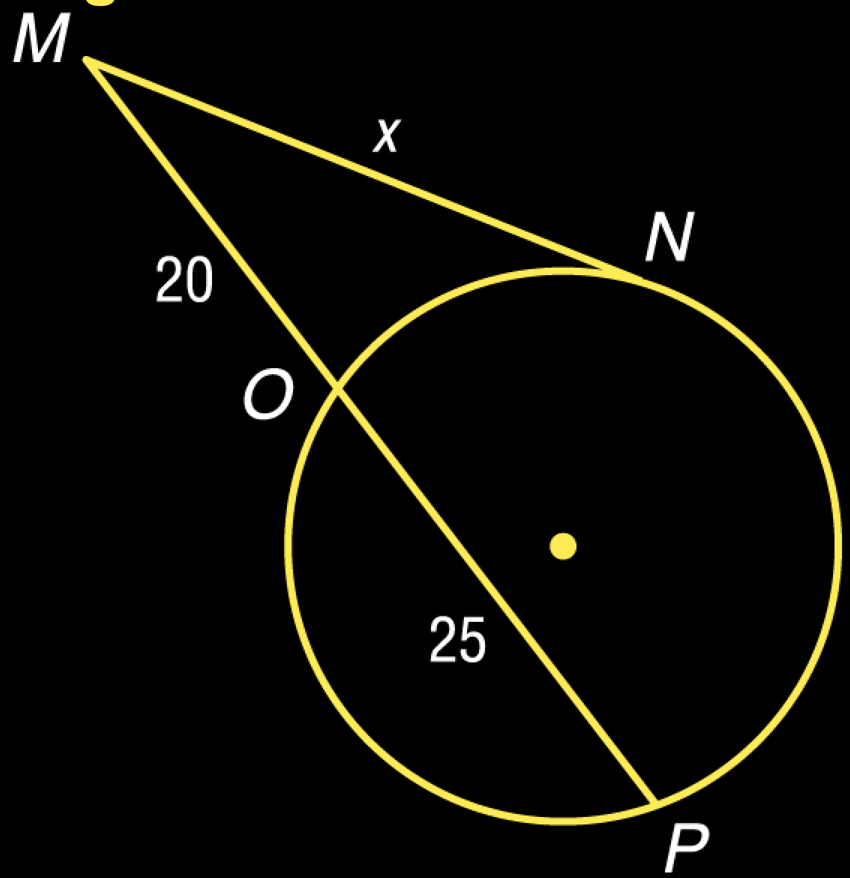
If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.




**Example:** $WX \cdot WX = WZ \cdot WY$

**Your Turn**

**Find $x$. Assume that segments that appear to be tangent are tangent.**




**Answer:** 30

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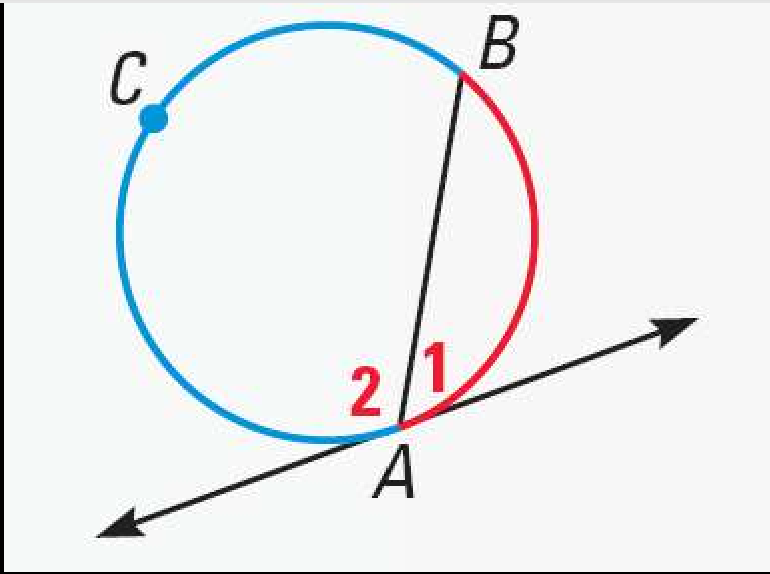
GOAL 1 **Use angles formed by tangents and chords**

THEOREM

## THEOREM 10.12

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

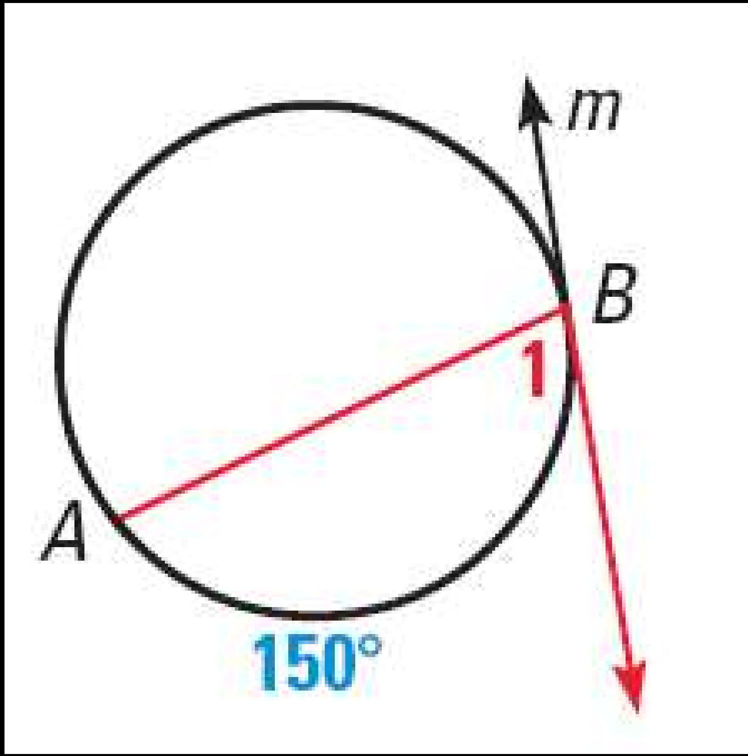



$$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$$

$$m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$$

## Example 1a Finding Angle and Arc Measures

Line $m$ is tangent to the circle. Find the measure of the red angle or arc.

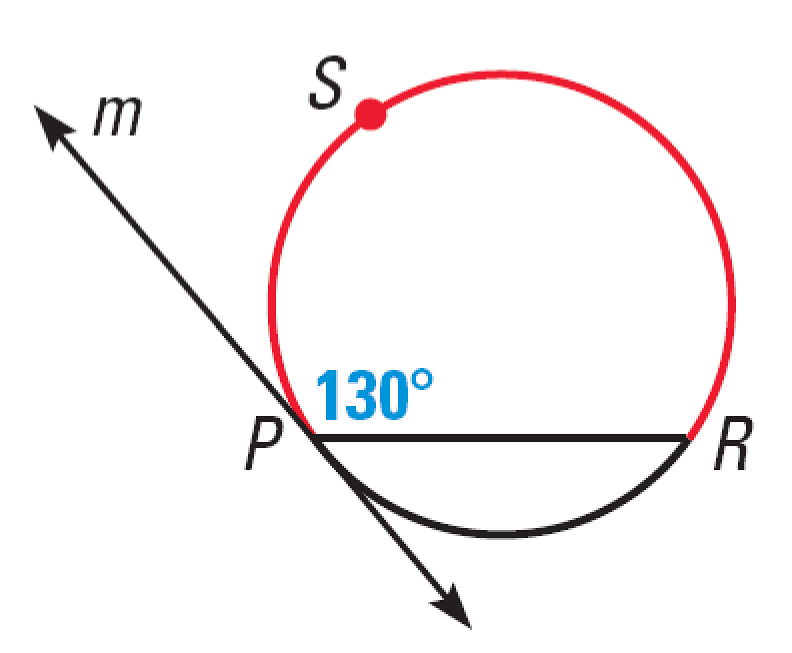



**SOLUTION**

$$m\angle 1 = \frac{1}{2}(150^\circ) = 75^\circ$$

## Example 1b Finding Angle and Arc Measures

Line $m$ is tangent to the circle. Find the measure of the red angle or arc.



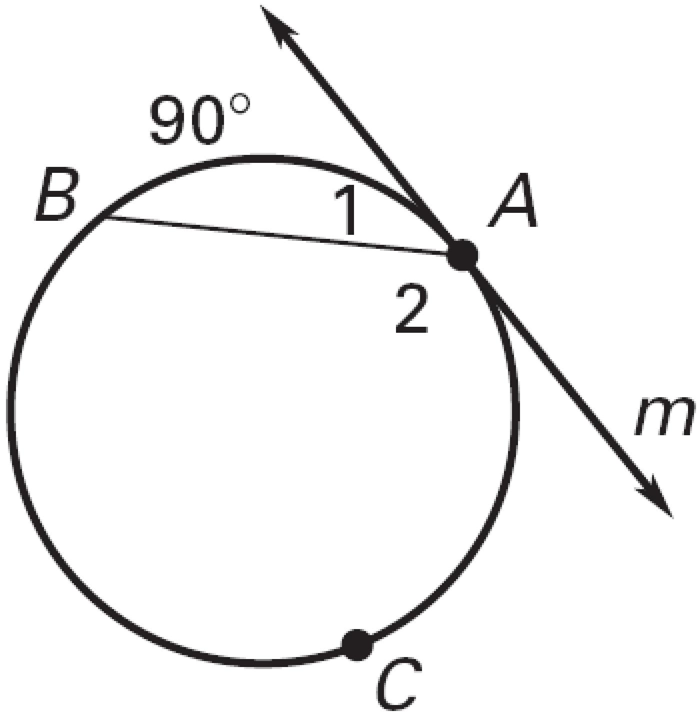

**SOLUTION**

$$m\overset{\frown}{RSP} = 2(130°) = 260°$$

## GOAL 1 Use angles formed by tangents and chords

### Your Turn — Finding Angle and Arc Measures

Line $m$ is tangent to the circle.




**a.** Find $m\angle 1$

$$m\angle 1 = \frac{1}{2}(90^\circ) = 45^\circ$$

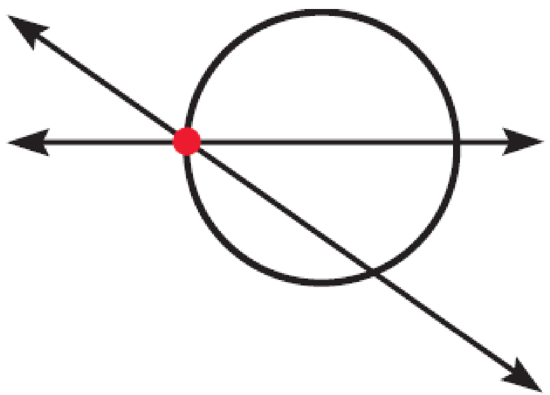
**b.** $m\overset{\frown}{ACB}$

$$m\angle 2 = 180^\circ - m\angle 1 = 180^\circ - 45^\circ = 135^\circ$$

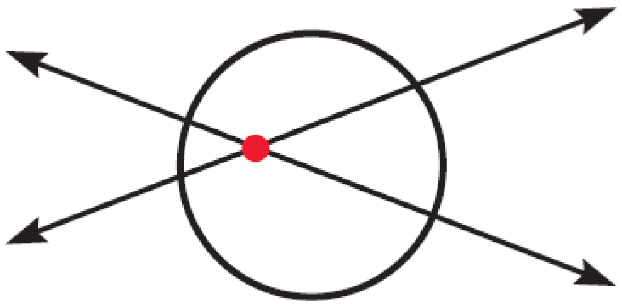
$$m\overset{\frown}{ACB} = 2(135^\circ) = 270^\circ$$

## GOAL 2 Use angles formed by lines that intersect a circle

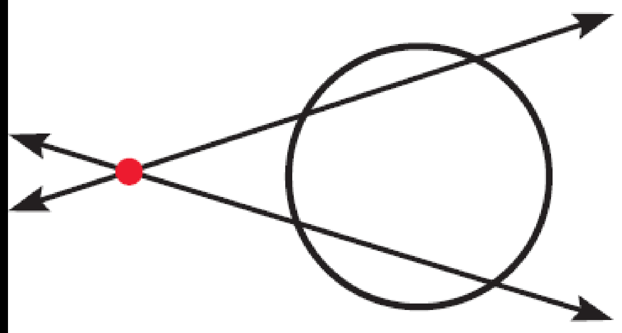
If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle



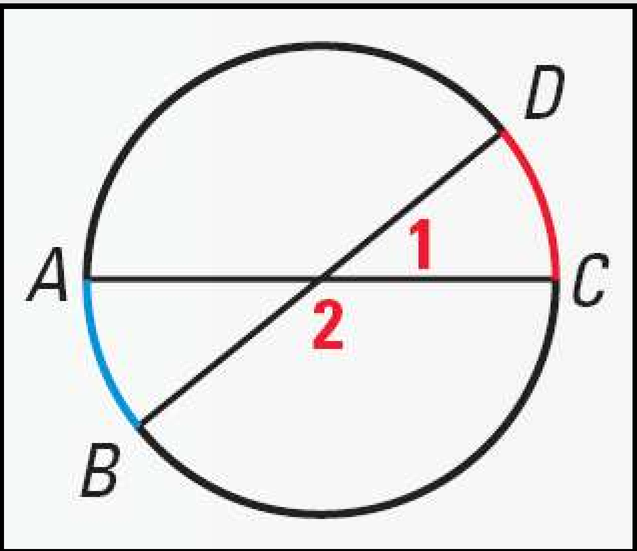
outside the circle

GOAL 2 Use angles formed by lines that intersect a circle

THEOREM

**THEOREM 10.13**

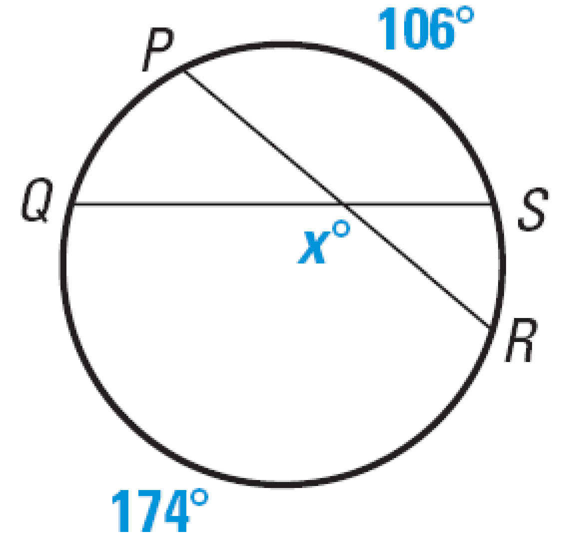
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.




$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$$

$$m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$$

## Example 3 Finding the Measure of an Angle Formed by Two Chords




Find the value of $x$.

**SOLUTION**

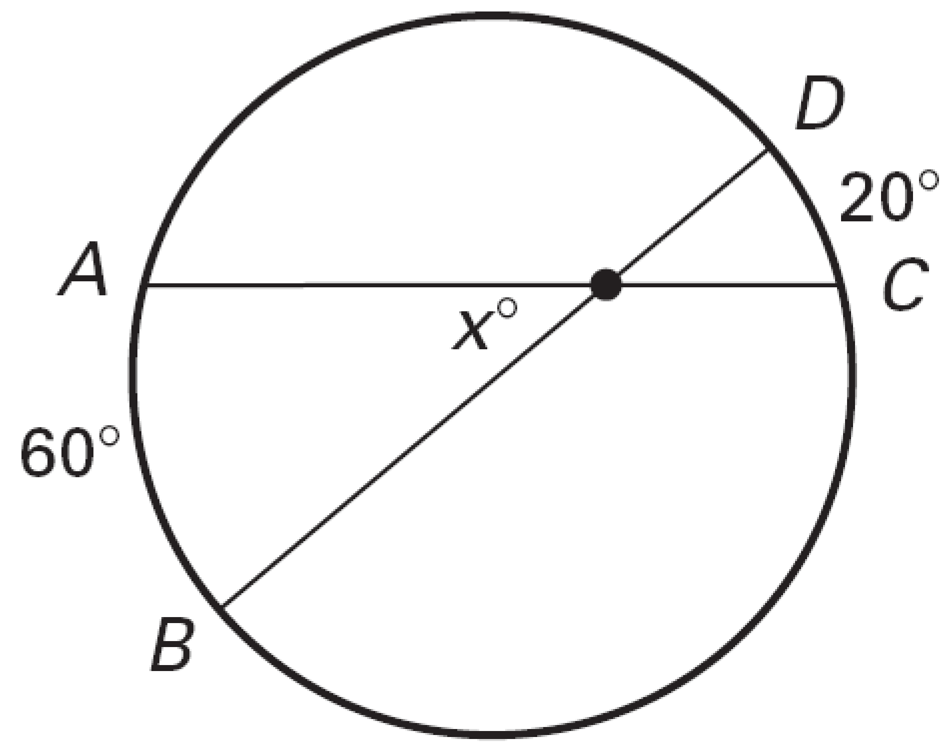
$x° = \frac{1}{2}(m\overset{\frown}{PS} + m\overset{\frown}{RQ})$ Apply Theorem 10.13.

$x° = \frac{1}{2}(106° + 174°)$ Substitute.

$x = 140$ Simplify.

## Your Turn: Finding the Measure of an Angle Formed by Two Chords

Find the value of $x$.




$$x° = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$$

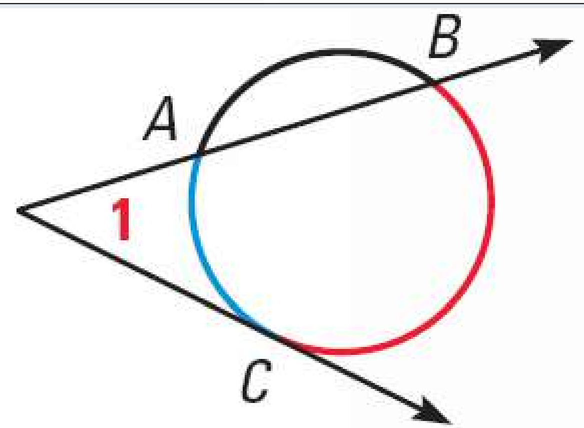
$$x° = \frac{1}{2}(60° + 20°)$$

$$x = 40$$

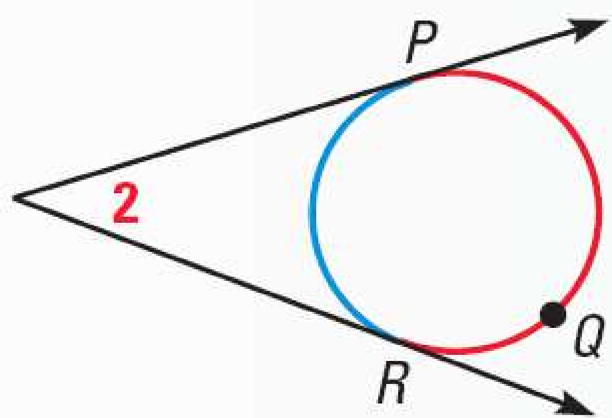
## THEOREM

### THEOREM 10.14

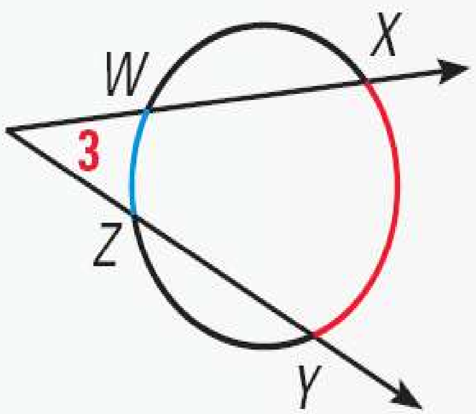
If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.




$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



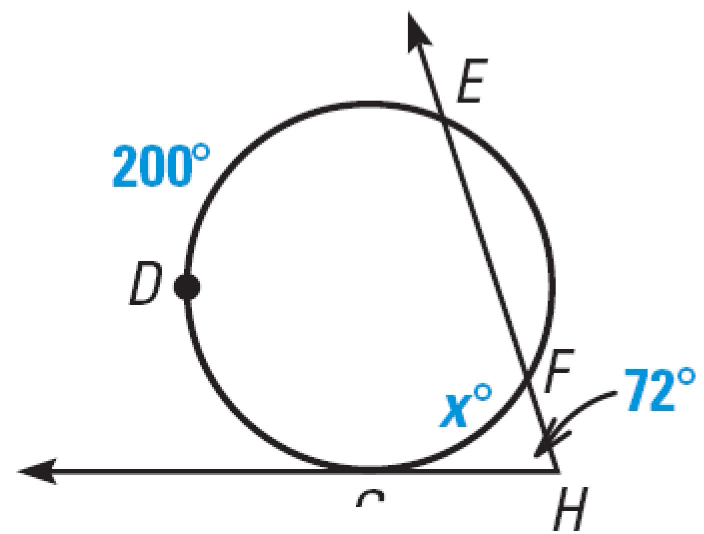

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$




$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

## Example 4a Using Theorem 10.14

Find the value of $x$.




**SOLUTION**

$m\angle GHF = \frac{1}{2}(m\widehat{EDG} - m\widehat{GF})$ Apply Theorem 10.14.

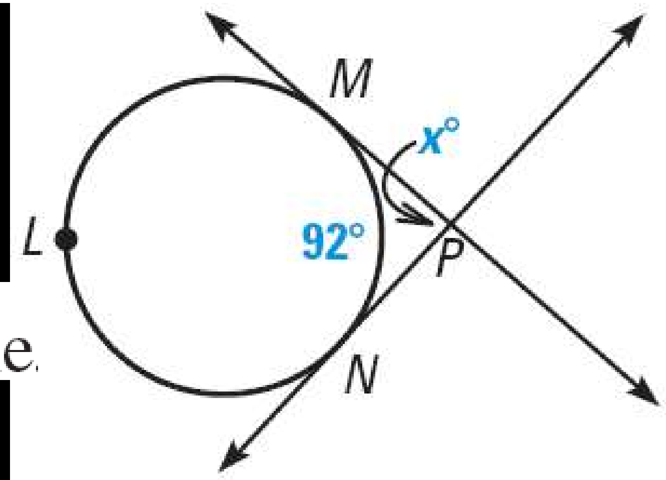
$72° = \frac{1}{2}(200° - x°)$ Substitute.

$144 = 200 - x$ Multiply each side by 2.

$x = 56$ Solve for $x$.

# Example 4b Using Theorem 10.14

Find the value of $x$.




**SOLUTION**

Because $\overset{\frown}{MN}$ and $\overset{\frown}{MLN}$ make a whole circle,

$$m\overset{\frown}{MLN} = 360° - 92° = 268°$$

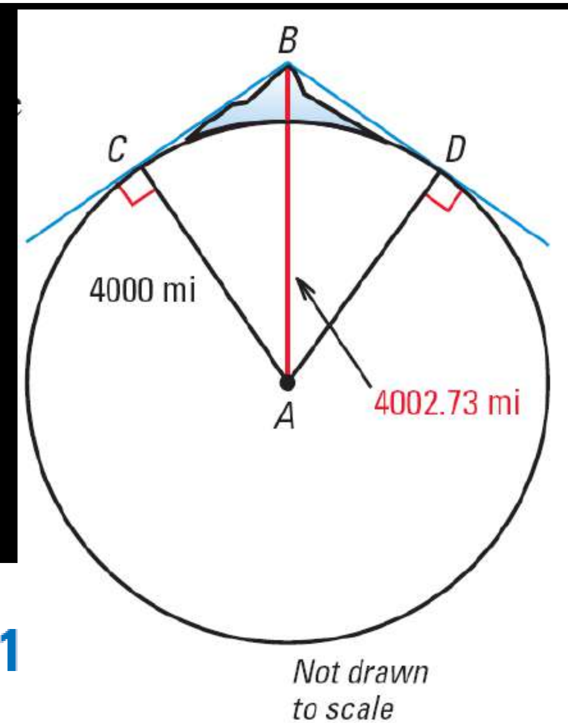
$$x = \frac{1}{2}(m\overset{\frown}{MLN} - m\overset{\frown}{MN})$$ Apply Theorem 10.14.

$$= \frac{1}{2}(268 - 92)$$ Substitute.

$$= \frac{1}{2}(176)$$ Subtract.

$$= 88$$ Multiply.

## Example 5 Describing the View from Mount Rainier




**SOLUTION**

$\overrightarrow{BC}$ and $\overrightarrow{BD}$ are tangent to Earth. You can solve right $\triangle BCA$ to see that $m\angle CBA \approx 87.9°$. So, $m\angle CBD \approx 175.8°$. Let $m\overparen{CD} = x°$.

$175.8 \approx \frac{1}{2}[(360 - x) - x]$ Apply Theorem 1

$175.8 \approx \frac{1}{2}(360 - 2x)$ Simplify.

$175.8 \approx 180 - x$ Distributive property

$x \approx 4.2$ Solve for $x$.