14.3

1 FOCUS

Objectives

14.3.1 Compute the value of an unknown using the ideal gas law.

14.3.2 Compare and contrast real and ideal gases.

Guide for Reading

Build Vocabulary L2

Word Parts Have students look up the meanings of *ideal* and *real*. Have them write definitions that include the terms *theoretical* and *actual*.

Reading Strategy L2

Make Inferences Have students use their understanding of the terms *real* and *ideal* to infer the differences between real and ideal gases. As the students read the section, have them evaluate and revise their inference.

2 INSTRUCT

Connecting to Your World

Have students study the photograph and read the text that opens the section. Ask, **How does the existence of dry ice violate the assumptions of the kinetic theory?** *(The kinetic theory assumes that there are no attractions among the particles in a gas. But there must be attractions between carbon dioxide molecules for the gas to solidify.)*

Ideal Gas Law

Discuss L2

Sketch two containers with identical volumes on the board. Tell students one container is filled with neon gas and the other with helium at the same temperature and pressure. Ask students to use the ideal gas law to find the equation for the number of moles in each container. Students should determine that $n_{He} = n_{Ne}$.

14.3 Ideal Gases

Guide for Reading

Key Concepts

- What is needed to calculate the amount of gas in a sample at given conditions of volume, temperature, and pressure?
- Under what conditions are real gases most likely to differ from ideal gases?

Vocabulary

ideal gas constant
ideal gas law

Reading Strategy

Building Vocabulary After you read this section, explain the difference between *ideal* and *real* as these terms are applied to gases.

Connecting to Your World Solid carbon dioxide, or dry ice, is used to protect products that need to be kept cold during shipping. The adjective *dry* refers to a key advantage of shipping with dry ice. Dry ice doesn't melt. It sublimes. Dry ice can exist because gases don't obey the assumptions of kinetic theory at all conditions. In this section, you will learn how real gases differ from the ideal gases on which the gas laws are based.

Ideal Gas Law

With the combined gas law, you can solve problems with three variables: pressure, volume, and temperature. The combined gas law assumes that the amount of gas does not vary. You cannot use the combined gas law to calculate the number of moles of a gas in a fixed volume at a known temperature and pressure. **To calculate the number of moles of a contained gas requires an expression that contains the variable *n*.** The combined gas law can be modified to include the number of moles.

The number of moles of gas is directly proportional to the number of particles. The volume occupied by a gas at a specified temperature and pressure also must depend on the number of particles. So moles must be directly proportional to volume as well. You can introduce moles into the combined gas law by dividing each side of the equation by n.

$$\frac{P_1 \times V_1}{T_1 \times n_1} = \frac{P_2 \times V_2}{T_2 \times n_2}$$

This equation shows that $(P \times V)/(T \times n)$ is a constant. This constant holds for ideal gases—gases that conform to the gas laws.

If you know the values for P, V, T, and n for one set of conditions, you can calculate a value for the constant. Recall that 1 mol of every gas occupies 22.4 L at STP (101.3 kPa and 273 K). You can use these values to find the value of the constant, which has the symbol R and is called the ideal gas constant. Insert the values of P, V, T, and n into $(P \times V)/(T \times n)$.

$$R = \frac{P \times V}{T \times n} = \frac{101.3\ \text{kPa} \times 22.4\ \text{L}}{273\ \text{K} \times 1\ \text{mol}} = 8.31\ (\text{L·kPa})/(\text{K·mol})$$

The **ideal gas constant** (R) has the value 8.31 (L·kPa)/(K·mol). The gas law that includes all four variables—P, V, T, and n—is called the **ideal gas law**. It is usually written as follows.

$$P \times V = n \times R \times T \quad \text{or} \quad PV = nRT$$

Section Resources

Print

- **Guided Reading and Study Workbook,** Section 14.3
- **Core Teaching Resources,** Section 14.3 Review, Interpreting Graphics
- **Transparencies,** T156–T157
- **Probeware Laboratory Manual,** Section 14.3

Technology

- **Interactive Textbook with ChemASAP,** Problem Solving 14.24, Assessment 14.3
- **Virtual Chemistry Labs,** 13, 14

SAMPLE PROBLEM 14.5

Using the Ideal Gas Law to Find the Amount of a Gas

A deep underground cavern contains 2.24×10^6 L of methane gas (CH_4) at a pressure of 1.50×10^3 kPa and a temperature of 315 K. How many kilograms of CH_4 does the cavern contain?

1 Analyze ***List the knowns and the unknown.***

Knowns

- $P = 1.50 \times 10^3$ kPa
- $V = 2.24 \times 10^6$ L
- $T = 315$ K
- $R = 8.31$ (L·kPa)/(K·mol)
- molar mass$_{CH_4}$ = 16.0 g

Unknown

- ? kg CH_4

Calculate the number of moles (n) using the ideal gas law. Use the molar mass to convert moles to grams. Then convert grams to kilograms.

2 Calculate ***Solve for the unknown.***

Rearrange the equation for the ideal gas law to isolate n.

$$n = \frac{P \times V}{R \times T}$$

Substitute the known quantities into the equation to find the number of moles of methane.

$$n = \frac{(1.50 \times 10^3\ \cancel{\text{kPa}}) \times (2.24 \times 10^6\ \cancel{\text{L}})}{8.31\ \frac{\cancel{\text{L}} \times \cancel{\text{kPa}}}{\cancel{\text{K}} \times \text{mol}} \times 315\ \cancel{\text{K}}} = 1.28 \times 10^6 \text{ mol } CH_4$$

A mole-mass conversion gives the number of grams of methane.

$$1.28 \times 10^6\ \cancel{\text{mol } CH_4} \times \frac{16.0 \text{ g } CH_4}{1\ \cancel{\text{mol } CH_4}} = 20.5 \times 10^6 \text{ g } CH_4$$
$$= 2.05 \times 10^7 \text{ g } CH_4$$

Convert this answer to kilograms.

$$2.05 \times 10^7\ \cancel{\text{g}}\ CH_4 \times \frac{1 \text{ kg}}{10^3\ \cancel{\text{g}}} = 2.05 \times 10^4 \text{ kg } CH_4$$

3 Evaluate ***Does the result make sense?***

Although the methane is compressed, its volume is still very large. So it is reasonable that the cavern contains a large mass of methane.

Practice Problems

23. When the temperature of a rigid hollow sphere containing 685 L of helium gas is held at 621 K, the pressure of the gas is 1.89×10^3 kPa. How many moles of helium does the sphere contain?

24. A child's lungs can hold 2.20 L. How many grams of air do her lungs hold at a pressure of 102 kPa and a body temperature of 37°C? Use a molar mass of 29 g for air, which is about 20% O_2 (32 g/mol) and 80% N_2 (28 g/mol).

Engineers use drilling rods to explore for natural gas in the crust below the ocean floor.

Math Handbook

For help with conversion problems, go to page R66.

Interactive Textbook

Problem Solving 14.24
Solve Problem 24 with the help of an interactive guided tutorial.

with ChemASAP

Sample Problem 14.5

Answers

23. 251 or 2.51×10^2 mol He(*g*)
24. 2.5 g air

Practice Problems Plus L2

At 34.0°C, the pressure inside a nitrogen-filled tennis ball with a volume of 148 cm^3 is 212 kPa. **How many moles of N_2 are in the tennis ball?** (*1.23×10^{-2} mol N_2*)

A helium-filled balloon contains 0.16 mol He at 101 kPa and a temperature of 23°C. **What is the volume of the gas in the balloon?** (*3.9 L*)

Math Handbook

For a math refresher and practice, direct students to conversion factors, page R66.

Discuss L2

Explain that Avogadro's hypothesis makes it possible to relate the molar quantity of a gas to its temperature, volume, and pressure. The hypothesis assumes that as long as the particles are not tightly packed, equal volumes of gases at the same temperature and pressure contain equal numbers of particles. Emphasize that Avogadro's hypothesis works only for gases because a large portion of their volume is empty space. At low pressure, the volumes of individual molecules are negligible compared to the volume of the container holding the gas. The volume of a gas depends on the number of particles present, not their size. Ask, **How would you determine the mass of a balloon full of helium gas at STP without making any mass measurements?** (According to Avogadro's hypothesis, one mole of a gas has a volume of 22.4 L at STP. Calculate the volume of the balloon in liters at STP. Then, convert from liters to moles. To find the mass, multiply the number of moles by the molar mass of He.)

Ideal Gases and Real Gases

Quick LAB

Carbon Dioxide from Antacid Tablets L2

Objective After completing this activity, students will be able to:
- measure the amount of carbon dioxide gas given off when antacid tablets dissolve in water.

Skills Focus Observing, Calculating, Measuring

Prep Time 10 minutes
Class Time 40 minutes

Safety If you use latex balloons, check to see if any students are allergic to latex.

Expected Outcome The volumes of CO_2 produced will reflect the amount of antacid each balloon contains.

Analyze and Conclude

1. The volume of the balloon is directly proportional to the number of tablets.
2. Answers will vary, but the masses and numbers of moles should be in ratios of 1:2:3 for the three balloons.
3. Possible answer: 2.0 g of $NaHCO_3$ (molar mass = 84.01 g) should yield about 1.2×10^{-2} mol of CO_2.

For Enrichment L3

Have students use a similar procedure to compare different brands of effervescent antacids instead of different amounts of the same antacid.

Quick LAB

Carbon Dioxide from Antacid Tablets

Purpose
To measure the amount of carbon dioxide gas given off when antacid tablets dissolve in water.

Materials
- 6 effervescent antacid tablets
- 3 rubber balloons (spherical)
- plastic medicine dropper
- water
- clock or watch
- metric tape measure
- graph paper
- pressure sensor (optional)

Procedure

Sensor version available in the Probeware Lab Manual.

1. Break six antacid tablets into small pieces. Keep the pieces from each tablet in a separate pile. Put the pieces from one tablet into the first balloon. Put the pieces from two tablets into a second balloon. Put the pieces from three tablets into a third balloon. **CAUTION** *If you are allergic to latex, do not handle the balloons.*
2. After you use the medicine dropper to squirt about 5 mL of cold water into each balloon, immediately tie off each balloon.
3. Shake the balloons to mix the contents. Allow the contents to warm to room temperature.
4. Measure and record the circumference of each balloon several times during the next 20 minutes.
5. Use the maximum circumference of each balloon to calculate its volume. (*Hint:* Volume of a sphere $= \frac{4\pi r^3}{3}$ and r = circumference/2π.)

Analyze and Conclude
1. Make a graph of volume versus number of tablets. Use your graph to describe the relationship between the number of tablets used and the volume of the balloon.
2. Assume that the balloon is filled with carbon dioxide gas at 20°C and standard pressure. Calculate the mass and the number of moles of CO_2 in each balloon at maximum inflation.
3. If a typical antacid tablet contains 2.0 g of sodium hydrogen carbonate, how many moles of CO_2 should one tablet yield? Compare this theoretical value with your results.

Ideal Gases and Real Gases

An ideal gas is one that follows the gas laws at all conditions of pressure and temperature. Such a gas would have to conform precisely to the assumptions of kinetic theory. Its particles could have no volume, and there could be no attraction between particles in the gas. As you probably suspect, there is no gas for which these assumptions are true. So an ideal gas does not exist. Nevertheless, at many conditions of temperature and pressure, real gases behave very much like an ideal gas.

The particles in a real gas do have volume, and there are attractions between the particles. Because of these attractions, a gas can condense, or even solidify, when it is compressed or cooled. For example, if water vapor is cooled below 100°C at standard atmospheric pressure, it condenses to a liquid. The behavior of other real gases is similar, although lower temperatures and greater pressures may be required. Such conditions are required to produce the liquid nitrogen in Figure 14.14. **Real gases differ most from an ideal gas at low temperatures and high pressures.**

Figure 14.14 In this flask used to store liquid nitrogen, there are two walls with a vacuum in between.

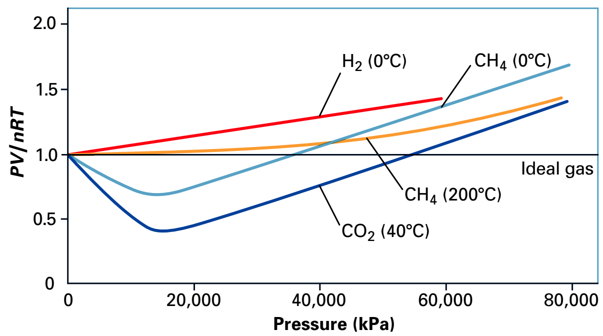


Figure 14.15 This graph shows how real gases deviate from the ideal gas law at high pressures.

INTERPRETING GRAPHS

a. Observing What are the values of $(PV)/(nRT)$ for an ideal gas at 20,000 and 60,000 kPa?
b. Comparing What variable is responsible for the differences between the two (CH_4) curves?
c. Making Generalizations How does an increase in pressure affect the $(PV)/(nRT)$ ratio for real gases?

Figure 14.15 shows how the value of the ratio (PV/nRT) changes as pressure increases. For an ideal gas, the result is a horizontal line because the ratio is always equal to 1. For real gases at high pressure, the ratio may deviate, or depart, from the ideal. When the ratio is greater than 1, the curve rises above the ideal gas line. When the ratio is less than 1, the curve drops below the line. The deviations can be explained by two factors. As attractive forces reduce the distance between particles, a gas occupies less volume than expected, causing the ratio to be less than 1. But the actual volume of the molecules causes the ratio to be greater than 1.

In portions of the curves below the line, intermolecular attractions dominate. In portions of the curves above the line, molecular volume dominates. Look at the curves for methane (CH_4) at 0°C and at 200°C. At 200°C, the molecules have more kinetic energy to overcome intermolecular attractions. So the curve for CH_4 at 200°C never drops below the line.

14.3 Section Assessment

25. **Key Concept** What do you need to calculate the amount of gas in a sample at given conditions of temperature, pressure, and volume?
26. **Key Concept** Under what conditions do real gases deviate most from ideal behavior?
27. What is an ideal gas?
28. Determine the volume occupied by 0.582 mol of a gas at 15°C if the pressure is 81.8 kPa.
29. What pressure is exerted by 0.450 mol of a gas at 25°C if the gas is in a 0.650-L container?
30. Use the kinetic theory of gases to explain this statement: No gas exhibits ideal behavior at all temperatures and pressures.

Connecting Concepts

Polarity At standard pressure, ammonia will condense at −33.3°C. At the same pressure, nitrogen does not condense until −195.79°C. Use what you learned about intermolecular attractions and polarity in Section 8.3 to explain this difference.

Interactive Textbook

Assessment 14.3 Test yourself on the concepts in Section 14.3.

with ChemASAP

Interpreting Graphs L2

a. 1.0 in both cases
b. temperature
c. The ratio increases.

Enrichment Question L3

How would a curve for CO_2 at 200°C differ from the curve at 40°C? *(At a higher temperature, the behavior of a gas is closer to the ideal; so the curve might not dip below the straight line.)*

Use Visuals L1

Figure 14.15 Point out that the straight line represents ideal conditions. For an ideal gas, by definition $PV = nRT$, and the ratio of PV to nRT is 1. When the volume of a gas is greater than expected, the ratio tends to be greater than 1. When the volume is less than expected, the ratio tends to be less than 1. The main factors that affect the volume are intermolecular attractions and the actual volume of the particles.

3 ASSESS

Evaluate Understanding L2

Have students apply the ideal gas law to a sample of gas in a balloon. Ask them to explain why in a balloon n and R are constants and P, V, and T are variables.

Reteach L1

Point out to students that one advantage of the ideal gas law is that it enables them to find the number of moles of a gas by measuring its temperature, pressure, and volume.

Connecting Concepts

The nitrogen molecule is nonpolar and the ammonia molecule is polar. So there are stronger intermolecular attractions in ammonia.

Section 14.3 Assessment

25. an expression that contains the variable n
26. Real gases deviate from ideal behavior at low temperatures and high pressures.
27. An ideal gas is a gas that follows the gas laws at all conditions of pressure and temperature.
28. 17.0 L
29. 1.71×10^3 kPa
30. In real gases, there are attractions between molecules, and the molecules have volume. At low temperatures, attractions between molecules pull them together and reduce the volume. At high pressures, the volume occupied by the molecules is a significant part of the total volume.

Interactive Textbook

If your class subscribes to the Interactive Textbook, use it to review key concepts in Section 14.3.

with ChemASAP

Technology & Society

Diving In

Decompression sickness is an application of Henry's law (which is discussed in Chapter 16): At a given temperature, the solubility of a gas in a liquid is directly proportional to the pressure of the gas. Dissolved nitrogen is more problematic than dissolved oxygen because oxygen released during decompression can be removed from the blood and used by the cells. Nitrogen is not used up by the cells and must be excreted through the lungs. Varying the composition of the compressed gas and using dive charts are two strategies for combating decompression sickness.

Discuss L2

Discuss the content of the article in the context of Dalton's law of partial pressures. Remind students that, according to Dalton's law of partial pressures, the total pressure exerted by a mixture of gases is equal to the sum of the partial pressures exerted by each of the different gases in the mixture. The fractional contribution to pressure exerted by each gas does not change as the temperature, pressure, or volume changes as long as the composition of the mixture is constant. Ask students to use Dalton's law of partial pressures to explain how changing the mixture of gases in the tanks used by divers can help prevent problems.

CLASS Activity

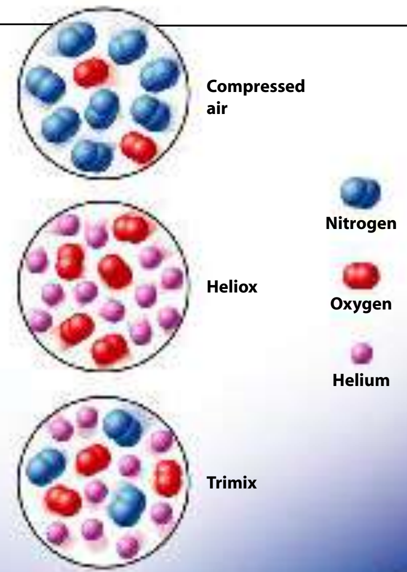
Effect of Depth on Partial Pressures L2

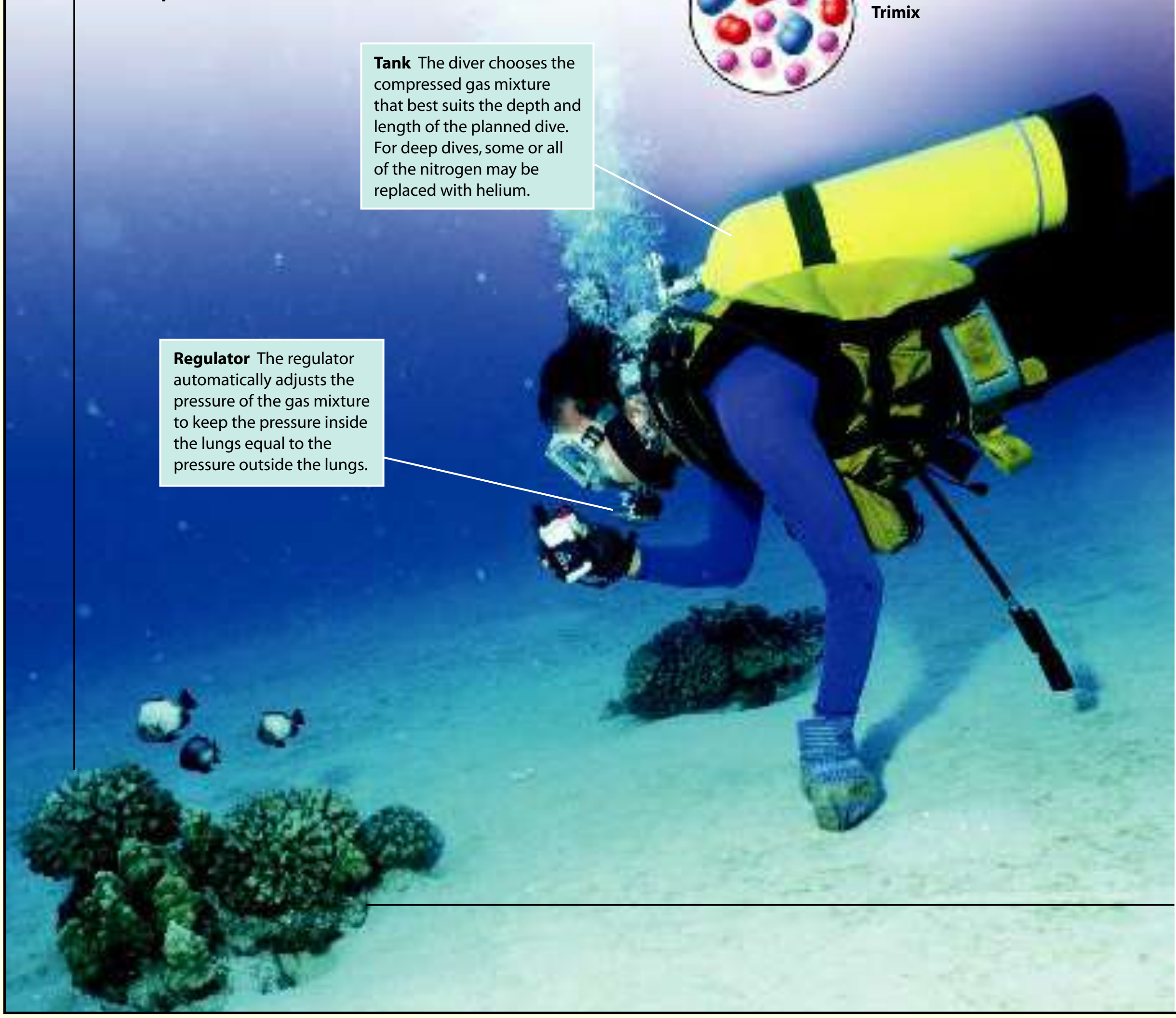
With Table 14.1 displayed on an overhead projector, point out that the air at sea level is about 21% oxygen and 78% nitrogen. The values for the partial pressures given in Table 14.1 reflect conditions at 1 atm or 101.3 kPa. Have students calculate the partial pressures of each gas that a scuba diver, breathing the same air mixture, would experience at depths of 100 feet (approximately 4 atm) and 300 feet (approximately 10 atm).

Technology & Society

Diving In

Divers who depend on the air in their lungs can stay under water for only a few minutes. But with tanks of compressed air, scuba divers can stay under water for hours. They can descend to great depths to explore a coral reef or salvage a sunken ship. Compressed air allows engineers to build, inspect, and repair ships, bridges, and oil platforms. **Interpreting Diagrams** ***Which of the two main components of air must be in a diver's tank?***





Facts and Figures

Decompression Sickness

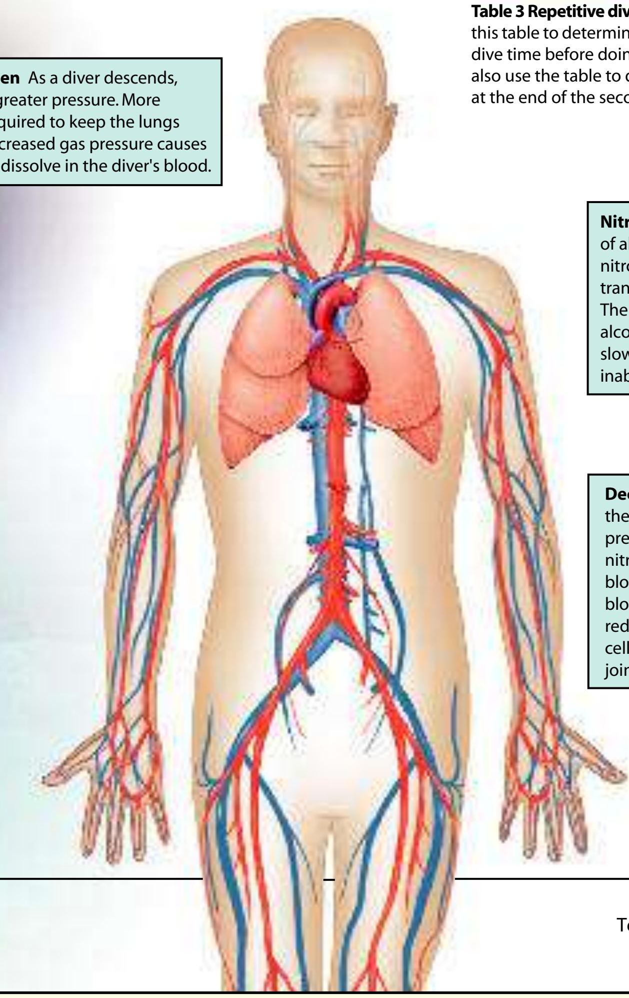
The invention of compressed air in the 1840s allowed people to work under water, with one major drawback. Decompression sickness was initially called caisson disease because workers constructing the Brooklyn Bridge worked in water-tight containers called caissons. The pressure of the air in the caissons needed to be greater than atmospheric pressure to withstand the pressure of the surrounding water. The condition was also called "the bends." In 1878, Paul Bert stated that workers could avoid the bends if they ascended gradually to the surface. Bert referred to the work of Robert Boyle. In 1667, Boyle observed a bubble form in the eye of a viper that was placed in a compressed atmosphere and then removed. (Boyle reported that the viper appeared distressed by the experience.)

Dive tables help a diver avoid decompression sickness. Divers use the data to control the length and frequency of their dives, and the rate at which they return to the surface.

Table 1 End-of-dive lettergroup At the end of a dive, the diver matches the time and depth of the dive to a letter group. Letter A represents the least amount of nitrogen left in the blood after the dive. The circled numbers show the maximum time a diver can spend at a given depth without having to make a decompression stop during the ascent.

Table 2 Surface interval time The longer a diver spends at the surface, the more nitrogen is excreted through the lungs. A diver can move from Group H to Group A after 8 hours at the surface.

Table 3 Repetitive dive timetable Divers use this table to determine an adjusted maximum dive time before doing a second dive. They also use the table to determine a letter group at the end of the second dive.



Dissolved nitrogen As a diver descends, the water exerts greater pressure. More gas pressure is required to keep the lungs expanded. This increased gas pressure causes more nitrogen to dissolve in the diver's blood.

Nitrogen narcosis Below a depth of about 30 meters, dissolved nitrogen interferes with the transmission of nerve impulses. The effects are similar to those of alcohol and include dizziness, slowed reaction time, and an inability to think clearly.

Decompression sickness As the diver returns to the surface, pressure decreases and dissolved nitrogen is released from the blood. Bubbles of nitrogen can block small blood vessels and reduce the supply of oxygen to cells, causing severe pain in the joints, dizziness, and vomiting.

Use Visuals L1

Discuss the purpose of dive tables and explain how to use them. Further explanation of the tables is provided at the NAUI website. Have students use the indicated table to answer the following questions.

Table 1

1. How long can a diver stay at 24 m without needing to make a decompression stop? *(35 minutes)*

2. A diver stays at a depth of 33 m for 20 minutes. **Will she have to make a decompression stop when returning to the surface? Explain.** *(Yes, the maximum time with no decompression at this depth is 15 minutes.)*

Table 2

Explain that after a dive, a diver is assigned a letter based on the depth and time of the dive. This letter determines how soon the diver can dive again. The higher the letter, the longer it takes for nitrogen to be removed from the blood.

1. A diver assigned to Group G has been out of the water for 4 hours. **What group is he now in?** *(Group C)*
2. **After how many hours has all nitrogen that might be harmful been eliminated from the blood?** *(24 hours)*

Differentiated Instruction

Gifted and Talented L3

Have students find out how to use the repetitive dive table (Table 3). Have them teach other students how to use the table and provide problems for the students to solve.

Answers to...

Interpreting Diagrams Oxygen is essential; nitrogen is optional.