

12.1 The Arithmetic of Equations

Connecting to Your World

Silk is one of the most beautiful and luxurious of all fabrics. It is spun from the cocoons of silkworms. Silk manufacturers know from experience that to produce enough silk to make just one elegant Japanese kimono they will need over 3000 cocoons. In a similar fashion, chemists need to know how much reactant is needed to make a certain amount of product. The answer lies in chemical equations. From a balanced chemical equation, you can determine the quantities of reactants and products in a reaction.



Using Everyday Equations

When you bake cookies, you probably use a recipe. A cookie recipe tells you the precise amounts of ingredients to mix to make a certain number of cookies, as shown in Figure 12.1. If you need a larger number of cookies than the recipe provides, you can double or triple the amounts of ingredients. **Key Concepts** A balanced chemical equation provides the same kind of quantitative information that a recipe does. In a cookie recipe, you can think of the ingredients as the reactants, and the cookies as the products.

Here is another example. Imagine you are in charge of manufacturing for the Tiny Tyke Tricycle Company. The business plan for Tiny Tyke requires the production of 640 custom-made tricycles each week. One of your responsibilities is to be sure that there are enough parts available at the start of each workweek to make these tricycles. How can you determine the number of parts you need per week?



Guide for Reading

Key Concepts

- How is a balanced equation like a recipe?
- How do chemists use balanced chemical equations?
- In terms of what quantities can you interpret a balanced chemical equation?
- What quantities are conserved in every chemical reaction?

Vocabulary

stoichiometry

Reading Strategy

Using Prior Knowledge Before you read, jot down three things you know about balanced chemical equations. When you have read the section, explain how what you already knew helped you learn something new.

Figure 12.1 A cookie recipe tells you the number of cookies that you can expect to make from the listed amounts of ingredients. **Using Models** How can you express a cookie recipe as a balanced equation?

Section 12.1 The Arithmetic of Equations 353

12.1

1 FOCUS

Objectives

- 12.1.1 Explain** how balanced equations apply to both chemistry and everyday situations.
- 12.1.2 Interpret** balanced chemical equations in terms of moles, representative particles, mass, and gas volume at STP.
- 12.1.3 Identify** the quantities that are always conserved in chemical reactions.

Guide for Reading

Build Vocabulary

L2

Paraphrase Introduce the term stoichiometry in your own words. Stress that stoichiometry allows students to calculate the amounts of chemical substances involved in chemical reactions using information obtained from balanced chemical equations.

Reading Strategy

L2

Relate Text and Visuals Have students construct tables similar to Figure 12.3 to interpret balanced chemical equations for stoichiometric problems. Students need to use only those levels of interpretation—particles, molecules, moles, mass, or gas volumes—appropriate to the problem.

2 INSTRUCT

Connecting to Your World

Have students study the photograph and read the text that opens the section. Ask, **How many cocoons would be required to produce enough silk for two Japanese kimonos?** (*twice as many, or 6000 cocoons*) **How did you calculate the number of cocoons?** (*by multiplying the number needed for one kimono by two*)

Answers to...

Figure 12.1 Acceptable answers will list ingredients as reactants and cookies as the product.



Section Resources

Print

- **Guided Reading and Study Workbook**, Section 12.1
- **Core Teaching Resources**, Section 12.1 Review
- **Transparencies**, T122–T125
- **Laboratory Manual**, Lab 19

Technology

- **Interactive Textbook with ChemASAP**, Problem Solving 12.1, 12.4, Assessment 12.1
- **Go Online**, Section 12.1

Using Everyday Equations

Discuss



Write this statement on the board: “A frame, a seat, wheels, a handlebar, and pedals are needed to assemble a complete tricycle.” Have students look at the visual representation of how a tricycle is assembled and ask, **Does this statement correctly describe the process?** (no) **What is the correct statement?** (A frame, a seat, three wheels, a handlebar, and two pedals are needed to assemble a complete tricycle.)

Using Balanced Chemical Equations

Discuss



Remind students that during a chemical reaction, atoms are rearranged into new combinations and groupings. It is somewhat similar to changing silk cocoons into a kimono. Just as a chemical reaction has reactants and products, the cocoon is the reactant and the kimono is the product. Much like having the proper supply of silk cocoons, chemists must have an adequate supply of reactants for a chemical reaction.

Word Origins



The term *spectrometry* means “the study of light or spectrum.”

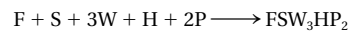
Word Origins

Stoichiometry comes from the combination of the Greek words *stoikheion*, meaning “element,” and *metron*, meaning “to measure.” Stoichiometry is the calculation of amounts of substances involved in chemical reactions. **What do think the term *spectrometry* means?**

To simplify this discussion, assume that the major components of the tricycle are the frame (F), the seat (S), the wheels (W), the handlebars (H), and the pedals (P), in other words, your reactants. The figure below illustrates how an equation can represent the manufacturing of a single tricycle.



The finished tricycle, your product, has a “formula” of FSW_3HP_2 . The balanced equation for making a single tricycle is



This balanced equation is a “recipe” to make a single tricycle: Making a tricycle requires assembling one frame, one seat, three wheels, one handlebar, and two pedals. Now look at Sample Problem 12.1. It shows you how to use the balanced equation to calculate the number of parts needed to manufacture a given number of tricycles.

Using Balanced Chemical Equations

Nearly everything you use is manufactured from chemicals—soaps, shampoos and conditioners, CDs, cosmetics, medicines, and clothes. In manufacturing such items, the cost of making them cannot be greater than the price they are sold at. Otherwise, the manufacturer will not make a profit. Therefore, the chemical processes used in manufacturing must be carried out economically. This is where balanced equations help.

A balanced chemical equation tells you what amounts of reactants to mix and what amounts of product to expect. **Chemists use balanced chemical equations as a basis to calculate how much reactant is needed or product is formed in a reaction.** When you know the quantity of one substance in a reaction, you can calculate the quantity of any other substance consumed or created in the reaction. Quantity usually means the amount of a substance expressed in grams or moles. However, quantity could just as well be in liters, tons, or molecules.

The calculation of quantities in chemical reactions is a subject of chemistry called **stoichiometry**. Calculations using balanced equations are called stoichiometric calculations. For chemists, stoichiometry is a form of bookkeeping. For example, accountants can track income, expenditures, and profits for a small business by tallying each in dollars and cents. Chemists can track reactants and products in a reaction by stoichiometry. It allows chemists to tally the amounts of reactants and products using ratios of moles or representative particles.



Checkpoint How is stoichiometry similar to bookkeeping?



SAMPLE PROBLEM 12.1

Using a Balanced Equation as a Recipe

In a five-day workweek, Tiny Tyke is scheduled to make 640 tricycles. How many wheels should be in the plant on Monday morning to make these tricycles?

1 Analyze List the knowns and the unknown.

Knowns

- number of tricycles = 640 tricycles = 640 FSW_3HP_2
- $F + S + 3W + H + 2P \longrightarrow FSW_3HP_2$

Unknown

- number of wheels = ? wheels

The desired conversion is tricycles (FSW_3HP_2) \longrightarrow wheels (W). The balanced equation tells you that each tricycle has three wheels, or $1 FSW_3HP_2 = 3 W$. The problem can be solved by using the proper conversion factor derived from this expression.

2 Calculate Solve for the unknown.

You can write two conversion factors relating wheels to tricycles.

$$\frac{3 W}{1 FSW_3HP_2} \quad \text{and} \quad \frac{1 FSW_3HP_2}{3 W}$$

The desired unit is W, so use the conversion factor on the left—the one that has W in the numerator. Multiply the number of tricycles by the conversion factor.

$$640 FSW_3HP_2 \times \frac{3W}{1 FSW_3HP_2} = 1920 W$$

3 Evaluate Does the result make sense?

If three wheels are required for each tricycle, and a total of more than 600 tricycles are being made, then a number of wheels in excess of 1800 is a logical answer. The unit of the known cancels with the unit in the denominator of the conversion factor, and the answer is in the unit of the unknown.

Practice Problems

1. Tiny Tike has decided to make 288 tricycles each day. How many tricycle seats, wheels, and pedals are needed?
2. Write an equation that gives your own “recipe” for making a skateboard.



Math Handbook

For help with dimensional analysis, go to page R66.

Interactive Textbook

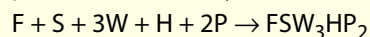
Problem Solving 12.1 Solve Problem 1 with the help of an interactive guided tutorial.

with **ChemASAP**

Discuss

L2

Write the equation that represents the production of a tricycle.



Point out that the balanced equation contains information that not only relates reactant to products but relates one reactants to another.

Ask, **How many handlebars are needed to complete tricycles if 24 pedals are available?** (12) Show students how to set up the proportions to solve this problem.

$$? H = 24 P \times (1 H / 2 P) = 12 H$$

Ask, **How many handlebars are needed to complete tricycles if 24 seats are available?** (24) **24 wheels?** (8)

Sample Problem 12.1

Answers

1. 288 seats, 864 wheels, 576 pedals
2. Answers will vary but should include the correct number of “parts” to make the product.

Practice Problems Plus

L2

Chapter 21 Assessment problem 57 is related to Sample Problem 12.1.

Math Handbook

For a math refresher and preview, direct students to dimensional analysis, page R66.

Answers to...



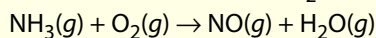
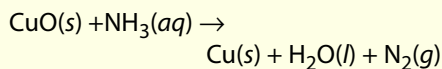
Checkpoint In stoichiometry, the amount of product formed by a chemical reaction can be calculated based on the amounts of the reactants. In bookkeeping, the profits of a business can be calculated based on income and expenditures.

Interpreting Chemical Equations

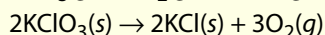
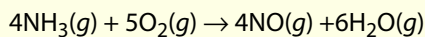
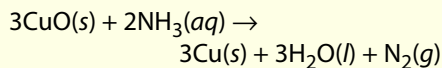
Discuss

L2

Review balancing chemical reactions by writing several unbalanced equations on the board. For example:



Have students balance the equations as shown.



Ask, **Why is it not correct to balance an equation by changing the subscripts in one or more of the formulas?** (*Changing the subscripts in a formula changes the chemical identity of the substance.*)

Mass Conservation in Chemical Reactions

Use Visuals

L1

Figure 12.3 Remind students that the term STP represents “standard temperature and pressure.” Ask, **What are the values of STP?** (0°C and 101.3 kPa)

Why is the volume of a gas usually measured at STP? (*The volume of a gas is usually measured at STP because its volume varies with temperature and pressure.*) **What is the molar volume of any gas at STP?** (22.4 L/mol) **How many particles does it contain?** (22.4 L of any ideal gas at STP contains 6.02×10^{23} particles of that gas.)

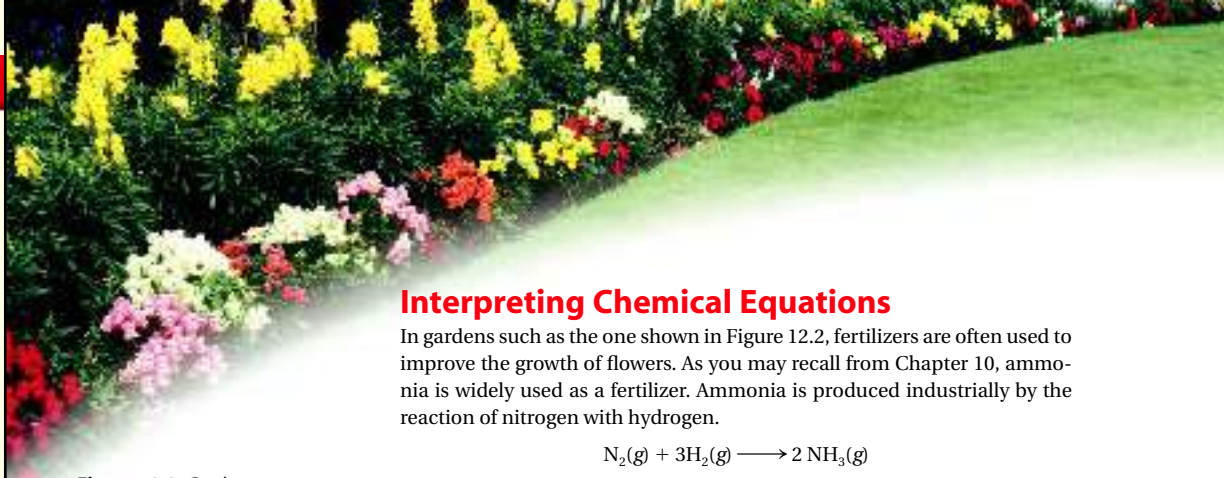
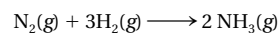


Figure 12.2 Gardeners use ammonium salts as fertilizer. The nitrogen in these salts is essential to plant growth.

Interpreting Chemical Equations

In gardens such as the one shown in Figure 12.2, fertilizers are often used to improve the growth of flowers. As you may recall from Chapter 10, ammonia is widely used as a fertilizer. Ammonia is produced industrially by the reaction of nitrogen with hydrogen.



The balanced chemical equation tells you the relative amounts of reactants and product in the reaction. However, your interpretation of the equation depends on how you quantify the reactants and products. **A balanced chemical equation can be interpreted in terms of different quantities, including numbers of atoms, molecules, or moles; mass; and volume.** As you study stoichiometry, you will learn how to read a chemical equation in terms of any of these quantities.

Number of Atoms At the atomic level, a balanced equation indicates that the number and type of each atom that makes up each reactant also makes up each product. Thus, both the number and types of atoms are not changed in a reaction. In the synthesis of ammonia, the reactants are composed of two atoms of nitrogen and six atoms of hydrogen. These eight atoms are recombined in the product.

Number of Molecules The balanced equation indicates that one molecule of nitrogen reacts with three molecules of hydrogen. Nitrogen and hydrogen will always react to form ammonia in a 1:3:2 ratio of molecules. If you could make 10 molecules of nitrogen react with 30 molecules of hydrogen, you would expect to get 20 molecules of ammonia. Of course, it is not practical to count such small numbers of molecules and allow them to react. You could, however, take Avogadro's number of nitrogen molecules and make them react with three times Avogadro's number of hydrogen molecules. This would be the same 1:3 ratio of molecules of reactants. The reaction would form two times Avogadro's number of ammonia molecules.

Moles A balanced chemical equation also tells you the number of moles of reactants and products. The coefficients of a balanced chemical equation indicate the relative numbers of moles of reactants and products in a chemical reaction. This is the most important information that a balanced chemical equation provides. Using this information, you can calculate the amounts of reactants and products. In the synthesis of ammonia, one mole of nitrogen molecules reacts with three moles of hydrogen molecules to form two moles of ammonia molecules. As you can see from this reaction, the total number of moles of reactants does not equal the total number of moles of product.



Checkpoint What do the coefficients of a balanced chemical equation indicate?

Facts and Figures

Ammonia in the Nitrogen Cycle

Earth's atmosphere contains 0.01 parts per million of ammonia, and small amounts of ammonia occur in volcanic gases. Most ammonia cycles through the living world without returning to the atmosphere. Ammonia plays a role in several stages of the nitrogen cycle. Nitrogen-fixing bacteria form nodules, or swellings, on the roots of plants in the legume family, such as beans and

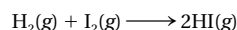
clover plants. These bacteria change atmospheric nitrogen into ammonia molecules or ammonium ions. Other bacteria break down the nitrogenous material in dead plants and animals into ammonia molecules. Certain soil bacteria oxidize these molecules into nitrate ions, the form readily absorbed by plant roots. When a plant dies, this cycle begins again.

Mass A balanced chemical equation obeys the law of conservation of mass. This law states that mass can be neither created nor destroyed in an ordinary chemical or physical process. As you recall, the number and type of atoms does not change in a chemical reaction. Therefore, the total mass of the atoms in the reaction does not change. Using the mole relationship, you can relate mass to the number of atoms in the chemical equation. The mass of 1 mol of N_2 (28.0 g) plus the mass of 3 mol of H_2 (6.0 g) equals the mass of 2 mol of NH_3 (34 g). Although the number of moles of reactants does not equal the number of moles of product, the total number of grams of reactants does equal the total number of grams of product.

Volume If you assume standard temperature and pressure, the equation also tells you about the volumes of gases. Recall that 1 mol of any gas at STP occupies a volume of 22.4 L. The equation indicates that 22.4 L of N_2 reacts with 67.2 L (3×22.4 L) of H_2 . This reaction forms 44.8 L (2×22.4 L) of NH_3 .

Mass Conservation in Chemical Reactions

Figure 12.3 summarizes the information derived from the balanced chemical equation for the formation of ammonia. As you can see, the mass of the reactants equals the mass of the products. In addition, the number of atoms of each type in the reactants equals the number atoms of each type in the product. **Mass and atoms are conserved in every chemical reaction.** However, molecules, moles, formula units, moles, and volumes are not necessarily conserved—although they may be. Consider, for example, the formation of hydrogen iodide,



In this reaction, molecules, moles, and volume are all conserved. But in the majority of chemical reactions, they are not.






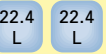
$N_2(g)$	+	$3H_2(g)$	\longrightarrow	$2NH_3(g)$
	+		\longrightarrow	
2 atoms N	+	6 atoms H	\longrightarrow	2 atoms N and 6 atoms H
1 molecule N_2	+	3 molecules H_2	\longrightarrow	2 molecules NH_3
10 molecules N_2	+	30 molecules H_2	\longrightarrow	20 molecules NH_3
$1 \times \left(6.02 \times 10^{23} \right)$ molecules N_2	+	$3 \times \left(6.02 \times 10^{23} \right)$ molecules H_2	\longrightarrow	$2 \times \left(6.02 \times 10^{23} \right)$ molecules NH_3
1 mol N_2	+	3 mol H_2	\longrightarrow	2 mol NH_3
28 g N_2	+	3×2 g H_2	\longrightarrow	2×17 g NH_3
34 g reactants			\longrightarrow	34 g products
Assume STP				
	+		\longrightarrow	
22.4 L N_2		67.2 L H_2		44.8 L NH_3

Figure 12.3 The balanced chemical equation for the formation of ammonia can be interpreted in several ways. **Predicting** How many molecules of NH_3 could be made from 5 molecules of N_2 and 15 molecules of H_2 ?

Section 12.1 The Arithmetic of Equations 357

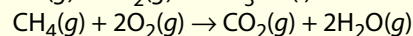
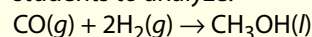
Differentiated Instruction

Less Proficient Readers



Have students construct a table like the one shown in Figure 12.3 for the reaction of hydrogen gas with oxygen gas to form water. Students should begin by writing the balanced equation. Encourage students to draw pictures as shown in Figure 12.3 to represent reactants and products. Have students use

the completed table to answer questions such as, "How many moles of water are produced by reacting 4 moles of hydrogen gas with excess oxygen?" Other reactions for students to analyze:



For: Links on Conservation of Mass
Visit: www.SciLinks.org
Web Code: cdn-1121

Download a worksheet on **Conservation of Mass** for students to complete and find additional support for NSTA SciLinks.

TEACHER Demo

Interpreting a Chemical Equation

Purpose Students interpret a balanced equation of the reaction of magnesium and hydrochloric acid.

Materials 2.5–3.5-cm strip of magnesium, 50 mL 1M HCl(aq) in a 100-mL beaker, baking soda

Safety Wear safety glasses and an apron. Neutralize remaining HCl(aq) with baking soda before flushing down the drain.

Procedure Identify the two reactants as magnesium and hydrochloric acid. Have students observe the reaction as you carefully add the magnesium strip to the acid. Ask students to write a balanced chemical equation for the reaction of magnesium and hydrochloric acid. $[Mg(s) + 2HCl(aq) \rightarrow MgCl_2(aq) + H_2(g)]$ Have students interpret the equation in terms of particles, moles, and molar masses.

Expected Outcome Students should express the balanced equation at the particle level as one atom of magnesium reacts with two molecules of hydrogen chloride to produce one formula unit of magnesium chloride and one molecule of hydrogen gas. Similarly, one mole of magnesium reacts with two moles of hydrogen chloride to produce one mole of magnesium chloride and one mole of hydrogen gas. Finally, 24.31 g Mg + 72.92 g HCl produces 95.21 g $MgCl_2$ + 2.02 g H_2 .

Answers to...

Figure 12.3 10 molecules NH_3



the relative numbers of representative particles, the relative numbers of moles, and, for gases, the relative volumes

Section 12.1 (continued)

CONCEPTUAL PROBLEM 12.1

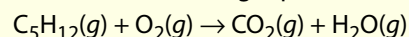
Answers

3. 2 molecules H_2 + 1 molecule $\text{O}_2 \rightarrow$
2 molecules H_2O
2 mol H_2 + 1 mol $\text{O}_2 \rightarrow$ 2 mol H_2O
44.8 L H_2 + 22.4 L $\text{O}_2 \rightarrow$ 44.8 L H_2O
4. 2 mol C_2H_2 + 5 mol $\text{O}_2 \rightarrow$
4 mol CO_2 + 2 mol H_2O
44.8 L C_2H_2 + 112 L $\text{O}_2 \rightarrow$
89.6 L CO_2 + 44.8 L H_2O
212 g reactants \rightarrow 212 g products

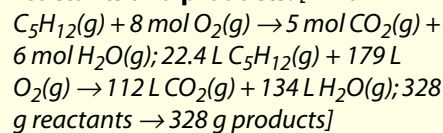
Practice Problems Plus

L2

Balance the following equation:



Interpret the balanced equation in terms of relative number of moles, volumes of gas at STP, and masses of reactants and products. [1 mol



ASSESS

Evaluate Understanding

L2

Have pairs of students write a balanced chemical equation for a simple reaction. Have pairs exchange equations and write quantitative relationships between reactants and products in terms of mass, moles, particles, and, where appropriate, volumes.

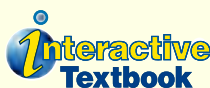
Reteach

L1

Remind students that the coefficients in a balanced chemical equation state the relationships among substances involved in the reaction.

Writing Activity

Moles of reactants and products are conserved for some reactions, but this is generally not the case.



If your class subscribes to the Interactive Textbook, use it to review key concepts in Section 12.1.

with ChemASAP

CONCEPTUAL PROBLEM 12.1

Interpreting a Balanced Chemical Equation

Hydrogen sulfide, which smells like rotten eggs, is found in volcanic gases. The balanced equation for the burning of hydrogen sulfide is:



Interpret this equation in terms of

- numbers of representative particles and moles.
- masses of reactants and products.

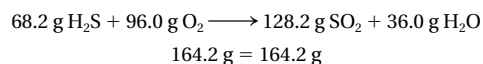
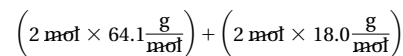
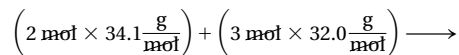
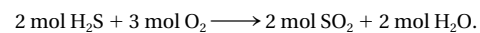
1 Analyze Identify the relevant concepts.

- The coefficients in the balanced equation give the relative number of molecules or moles of reactants and products.
- A balanced chemical equation obeys the law of conservation of mass.

2 Solve Apply concepts to this situation.

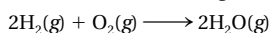
- 2 molecules H_2S + 3 molecules $\text{O}_2 \rightarrow$
2 molecules SO_2 + 2 molecules H_2O
2 mol H_2S + 3 mol $\text{O}_2 \rightarrow$
2 mol SO_2 + 2 mol H_2O

- Multiply the number of moles of each reactant and product by its molar mass:



Practice Problems

3. Interpret the equation for the formation of water from its elements in terms of numbers of molecules and moles, and volumes of gases at STP.



4. Balance the following equation.



Interpret the balanced equation in terms of

relative numbers of moles, volumes of gas at STP, and masses of reactants and products.



Problem-Solving 12.4 Solve Problem 4 with the help of an interactive guided tutorial.

with ChemASAP

12.1 Section Assessment

- Key Concept** How is a balanced equation similar to a recipe?
- Key Concept** How do chemists use balanced equations?
- Key Concept** Chemical reactions can be described in terms of what quantities?
- Key Concept** What quantities are always conserved in chemical reactions?
- Interpret the given equation in terms of relative numbers of representative particles, numbers of moles, and masses of reactants and products.
$$2\text{K}(\text{s}) + 2\text{H}_2\text{O}(\text{l}) \rightarrow 2\text{KOH}(\text{aq}) + \text{H}_2(\text{g})$$
- Balance this equation: $\text{C}_2\text{H}_5\text{OH}(\text{l}) + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{g})$. Show that the balanced equation obeys the law of conservation of mass.

Writing Activity

Explanatory Paragraph Explain this statement: "Mass and atoms are conserved in every chemical reaction, but moles are not necessarily conserved."



Assessment 12.1 Test yourself on the concepts in Section 12.1.

with ChemASAP

Section 12.1 Assessment

- Both a balanced equation and a recipe give quantitative information about the starting and end materials.
- as a basis to calculate how much reactant is needed or product is formed in a reaction
- numbers of atoms, molecules, or moles; mass; and volumes
- mass and atoms
- 2 atoms K + 2 molecules $\text{H}_2\text{O} \rightarrow$
2 formula units KOH + 1 molecule H_2
2 mol K + 2 mol $\text{H}_2\text{O} \rightarrow$
2 mol KOH + 1 mol H_2
78.2 g K + 36.0 g $\text{H}_2\text{O} \rightarrow$
112.2 g KOH + 2.0 g H_2
- $\text{C}_2\text{H}_5\text{OH} + 3\text{O}_2 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}$
46.0 g $\text{C}_2\text{H}_5\text{OH}$ + 96.0 g $\text{O}_2 \rightarrow$
88.0 g CO_2 + 54.0 g H_2O
142.0 g reactants \rightarrow 142.0 g products