CHAPTER

2 Modeling a Business

Business, more than any other occupation, is a continual dealing with the future; it is a continual calculation, an instinctive exercise in foresight.

Henry R. Luce, Publisher and Philanthropist

What do you think Henry Luce meant in this quote?

If you do an Internet search for the word *model*, you will find it used in a variety of contexts. There are computer models, medical models, working models, scientific models, business models, data models, and more. In each of these cases, a model is a representation. It is a well-formulated plan that can be used to represent a situation. Often, the model is a simplified version of the actual scenario. In this chapter, you will learn how to use mathematics to model a new business venture. You will use variables and examine relationships among those variables as you explore the business concepts of expense, revenue, and profit. Statistics play an important role in the creation and use of a business model. You will represent real-world situations with equations, analyze the data that has been derived from those equations, use data to make predictions, and generate and interpret graphs in order to maximize profit. As you work through this chapter, you will realize the importance of Henry Luce's words. The models you explore will examine the future success or failure of a product and will help you make decisions that will mold and shape its future.

Really?

Think of all the successful brand name products you use on a daily basis. Imagine all of the brain power and creative energy that went into inventing, perfecting, advertising, and selling these items. With all the resources major corporations have to make sure a product will sell, can you ever imagine a successful corporation producing a product that is a failure?

The business world is full of products that have failed. Some of the failures are from very well-known companies, such as The Coca-Cola Company, Ford Motor Company, and Sony Corporation of America. Below are three famous failures.

- **New Coke** In April 1985, The Coca-Cola Company changed the recipe of Coca-Cola, and the public was outraged. By July 1985, the company decided to bring back the original formula for Coca-Cola.
- **Edsel** The Ford Motor Company unveiled its new 1958 Edsel in the fall of 1957. The car had a radically different look and was often described as ugly. The buying public did not like the sound of the name, and the advertising campaign had flaws. By 1959, production ceased on the Edsel, and the term is still used as a synonym for a failed design.
- **Sony Betamax** In the late 1970s, Sony offered the first home video tape recorder, called the Betamax. Other manufacturers soon offered their own units. The units offered by other manufacturers all used the VHS tape format, and movies could be played on any brand's machine, except Sony's machine. Eventually, Sony had to abandon Betamax and issued its own VHS format tape recorders.

You can do online research to find more details on these and other famous product failures.

Really!

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To guess is cheap. To guess wrongly is expensive.
Ancient Chinese Proverb

2-1 Interpret Scatterplots

Objectives

- Graph bivariate data.
- Interpret trends based on scatterplots.
- Draw lines and curves of best fit.

Key Terms

- data
- univariate data
- bivariate data
- scatterplot
- trend
- correlation
- positive correlation
- negative correlation
- causal relationship
- explanatory variable
- response variable

How do scatterplots display trends?

Any set of numbers is called a set of **data**. A single set of numbers is called **univariate data**. When a business owner keeps a list of monthly sales amounts, the data in the list is univariate data. Data that lists pairs of numbers and shows a relationship between the paired numbers is called **bivariate data**. If a business owner keeps records of the number of units sold each month and the monthly sales amount, the set is bivariate data.

A **scatterplot** is a graph that shows bivariate data using points on a graph. Scatterplots may show a general pattern, or **trend**, within the data. A trend means a relationship exists between the two variables.

A trend may show a **correlation**, or *association*, between two variables. A **positive correlation** exists if the value of one variable increases when the value of the other increases. A **negative correlation** exists if the value of one variable decreases when the value of the other variable increases.


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A trend may also show a **causal relationship**, which means one variable *caused* a change in the other variable. The variable which causes the change in the other variable is the **explanatory variable**. The affected variable is the **response variable**. While a trend may indicate a correlation or a causal relationship, it does not have to. If two variables are correlated, it does not mean that one caused the other.

Skills and Strategies

You can graph a scatterplot by hand. You can also graph a scatterplot on a graphing calculator.

EXAMPLE 1

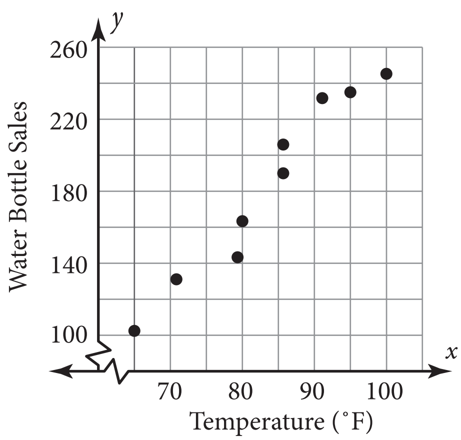
Rachael runs a concession stand at the park, where she sells water bottles. She keeps a list of each day's high temperature and the number of water bottles she sells each day. Rachael is looking for trends that relate the daily high temperature to the number of water bottles she sells each day. She thinks these two variables might be related and wants to investigate possible trends using a scatterplot. Below is the list of her ordered pairs.

(65, 102), (71, 133), (79, 144), (80, 161), (86, 191),

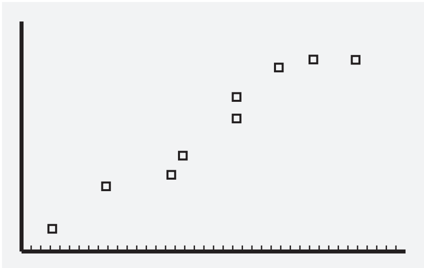
(86, 207), (91, 235), (95, 237), (100, 243)

Construct a scatterplot by hand on graph paper. Then enter the data in a graphing calculator to create a scatterplot.

SOLUTION In each ordered pair, the first number is the high temperature for the day in degrees Fahrenheit. The second number is the number of water bottles sold. Think of these as the *x*- and *y*-coordinates. The scatterplot is drawn by plotting the points with the given coordinates.



Choose a scale for each axis that allows the scatterplot to fit in the required space. To choose the scale, look at the greatest and least numbers that must be plotted for each variable. Label the axes accordingly. Then plot each point with a dot. Notice that you do not connect the dots in a scatterplot.



Use the statistics features on your graphing calculator to graph the scatterplot. Your display should look similar to the one shown.

■ CHECK YOUR UNDERSTANDING

If the temperature reaches 68 degrees Fahrenheit tomorrow, about how many water bottles do you predict will be sold? Explain.

EXAMPLE 2

Rachael wants to interpret the trend shown in the scatterplot. What do you notice about the relationship between temperature and water bottle sales? Is there an explanatory variable and a response variable?

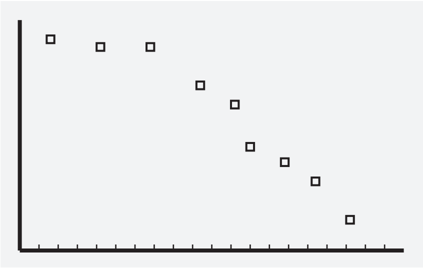
SOLUTION As the temperature rises, the water bottle sales generally increase. So, there is a correlation between the data. Because the *y*-values increase when the *x*-values increase, the correlation is *positive*. Additionally, the rise in temperature *caused* the increase in the number of bottles sold. Therefore, the temperature is the *explanatory variable* and the number of bottles sold is the *response variable*.

■ CHECK YOUR UNDERSTANDING

A local coffee shop sells hot chocolate. The manager keeps track of the temperature for the entire year and the hot chocolate sales. A scatterplot is graphed with temperature on the horizontal axis and hot chocolate sales on the vertical axis. Do you think the scatterplot shows a positive or negative correlation? Is there causation? Explain.

EXAMPLE 3

Determine if the following scatterplot depicts a positive correlation or a negative correlation.



SOLUTION As the *x*-values increase, the *y*-values decrease. Therefore, this scatterplot shows a negative correlation between the two variables.

■ CHECK YOUR UNDERSTANDING

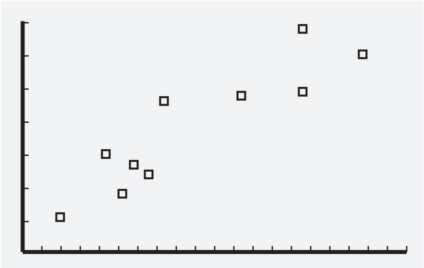
A local medical school is studying growth of students in grades 1–12. The height of each student in inches and the length of each student's foot in centimeters is recorded, and a scatterplot is constructed. Do you think the scatterplot shows a positive correlation or a negative correlation? Is there causation?

EXAMPLE 4

An elementary school principal compiled the following data about ten students at Compsett Elementary School. The first number represents a student's height in inches. The second number is the student's reading level. Create a scatterplot of the data. Do you think a person's height causes a higher reading level?

Height (inches)	Reading Level
48	5.8
63	9.2
49	5.5
43	4.1
46	6.1
55	7.6
59	8.1
60	10.0
47	4.9
50	7.7

SOLUTION The scatterplot shows a positive correlation.



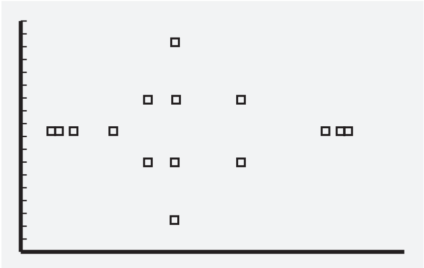
A person's height does not cause a higher reading level. Most likely, both height and reading level for elementary school students increase with age. Keep in mind that if two variables are correlated, they are associated in some way. The student's height does not *cause* the reading levels to be a certain value.

CHECK YOUR UNDERSTANDING

Think of an example of data that might have a negative correlation but there is no causation.

EXAMPLE 5

The scatterplot at the right shows the relationship between the number of text messages made by each of ten juniors while studying for Mr. Galati's chemistry test last week and their scores on the test. Describe the trends you see in the data.



SOLUTION As the number of text messages increases, test grades do not increase, so there is no positive correlation. As the number of text messages increases, test grades do not decrease, so there is no negative correlation. There is no trend in the data, so there is no correlation.

CHECK YOUR UNDERSTANDING

Students in a biology class measure the circumference and diameter of every tree on the school property. The students create a table of ordered pairs and plan to draw a scatterplot. Should there be a positive correlation, a negative correlation, or no correlation?

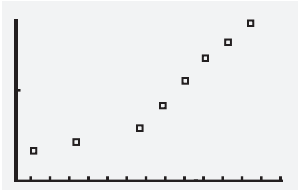
Applications

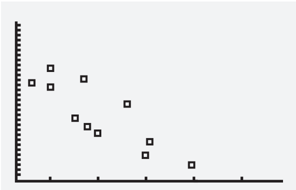
To guess is cheap. To guess wrongly is expensive.
Ancient Chinese Proverb

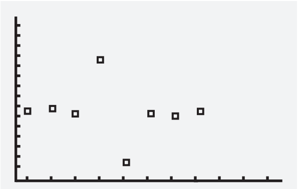
1. Use what you learned in this lesson to explain how the quote can be interpreted by a business person.

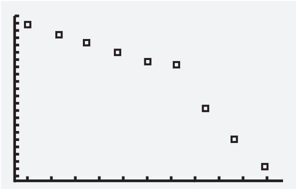
2. A scatterplot shows the number of days that have passed and the number of days left in a month. The explanatory variable is the number of days that have passed. The response variable is the number of days left. Is there a positive or negative correlation? Explain.

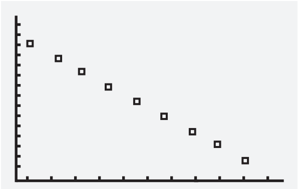
3. Examine each scatterplot. Identify each as showing a positive correlation, a negative correlation, or no correlation.

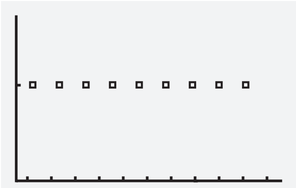
a. 

b. 

c. 

d. 

e. 

f. 

4. In a–d, each set of bivariate data has a causal relationship. Determine the explanatory and response variables for each set of data.
a. height and weight of a student
b. grade on a math test and number of hours the student studied
c. number of hours worked and paycheck amount
d. number of gallons of gas consumed and weight of a car

5. The table shows the personal income per capita (per person) in the United States for seven selected years.
a. Draw a scatterplot for the data.
b. Describe the correlation.

Year	Per Capita Income in Dollars
2002	30,838
2003	31,530
2004	33,157
2005	34,690
2006	36,794
2007	38,615
2008	39,751

6. The following set of ordered pairs gives the results of a science experiment. Twelve people were given different daily doses of vitamin C, in milligrams, for a year. This is the *x*-value. They reported the number of colds they got during the year. This is the *y*-value.

(100, 4), (100, 4), (100, 3), (250, 3), (250, 2), (250, 2),

(500, 1), (500, 2), (500, 1), (1,000, 1), (1,000, 2), (1,000, 1)

a. Construct a scatterplot.
b. Describe the correlation.
c. Should the scientists label the vitamin C intake the explanatory variable and the number of colds the response variable? Explain.

7. The enrollment at North Shore High School is given in the table. In each year, the number of students on the baseball team was 19.

Year	Enrollment
2006	801
2007	834
2008	844
2009	897
2010	922

a. If *x* represents the year and *y* represents the enrollment, draw a scatterplot to depict the data.
b. Describe the correlation from the scatterplot.
c. If *x* represents the enrollment and *y* represents the number of students on the baseball team, draw a scatterplot to depict the data.
d. Describe the correlation from the scatterplot.

8. The MyTunes Song Service sells music downloads. Over the past few years, the service has lowered its prices. The table shows the price per song and the number of songs downloaded per day at that price.

Price per Song	Number of Downloads (in thousands)
\$0.89	1,212
\$0.79	1,704
\$0.69	1,760
\$0.59	1,877
\$0.49	1,944
\$0.39	2,011

a. Examine the data without drawing a scatterplot. Describe any trends you see.
b. Draw a scatterplot. Describe the correlation.
c. Approximate the number of downloads at a price of \$0.54 per song. Explain your reasoning.

9. Perform an online search to answer the questions below.
a. Find your state's population for each of the last ten years.
b. Create a table of bivariate data. Let *x* represent the year, and let *y* represent the population.
c. Create a scatterplot for the data.
d. Describe the correlation between the year and your state's population.

The only useful function of a statistician is to make predictions, and thus provide a basis for action.
William Edwards Deming, Professor and Statistician

2-2 Linear Regression

Objectives

- Be able to fit a regression line to a scatterplot.
- Find and interpret correlation coefficients.
- Make predictions based on lines of best fit.

Key Terms

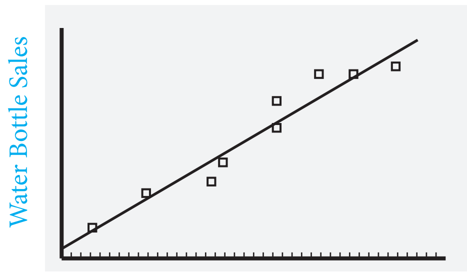
- line of best fit
- linear regression line
- least squares line
- domain
- range
- interpolation
- extrapolation
- correlation coefficient
- strong correlation
- weak correlation
- moderate correlation

How can the past predict the future?

Many scatterplot points can be approximated by a single line that best fits the scattered points. This line may be called a: **line of best fit**, **linear regression line**, or **least squares line**. This line can be used to display a trend and predict corresponding variables for different situations. It is more efficient to rely on the single line rather than the scatterplot points because the line can be represented by an equation.

Recall that the **domain** is a set of first elements of the ordered pairs, and the **range** is the set of corresponding second elements. **Interpolation** means to predict corresponding y-values, given an x-value within the domain. **Extrapolation** means to predict corresponding y-values outside of the domain.

The scatterplot shown is from Example 1 in the previous lesson. The line shown is a line of best fit because it closely follows the trend of the data points. The blue labels are included to identify the axes, but will not be shown on a calculator display. Generally, the distance the points lie from the line of best fit determines how good a predictor the line is. If most of the points lie close to the line, the line is a better predictor of the trend of the data than if the points lie far from the line. If the points lie far from the line, the line is not good for predicting a trend.



The **correlation coefficient**, r, is a number between -1 and 1 inclusive that is used to judge how closely the line fits the data. Negative correlation coefficients show negative correlations, and positive correlation coefficients show positive correlations. If the correlation coefficient is near 0, there is little or no correlation. Correlation coefficients with an absolute value greater than 0.75 are **strong correlations**. Correlation coefficients with an absolute value less than 0.3 are **weak correlations**. Any other correlation is a **moderate correlation**.

Skills and Strategies

The line of best fit and the correlation coefficient can be found using a graphing calculator.

EXAMPLE 1

Find the equation of the linear regression line for Rachael's scatterplot in Example 1 from Lesson 2-1. Round the slope and y-intercept to the nearest hundredth. The points are given below.

(65, 102), (71, 133), (79, 144), (80, 161), (86, 191),

(86, 207), (91, 235), (95, 237), (100, 243)

SOLUTION Although it is possible to find the linear regression equation using paper and pencil, it is a lengthy process. Using the linear regression feature on a graphing calculator produces more accurate results.

Enter the ordered pairs into your calculator. Then use the statistics menu to calculate the linear regression equation. The equation is of the form $y = mx + b$, where m is the slope and b is the y-intercept. Rounding the slope and y-intercept to the nearest hundredth, the equation of the regression line is $y = 4.44x - 187.67$.

Note that calculators may use different letters to represent the slope or the y-intercept. Remember that the coefficient of x is the slope.

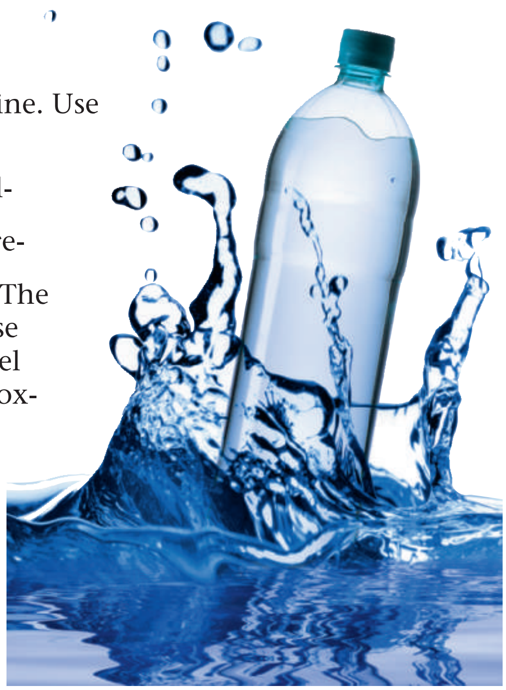
CHECK YOUR UNDERSTANDING

Find the equation of the linear regression line of the scatterplot defined by these points: (1, 56), (2, 45), (4, 20), (3, 30), and (5, 9). Round the slope and y-intercept to the nearest hundredth.

EXAMPLE 2

Interpret the slope as a rate for Rachael's linear regression line. Use the equation from Example 1.

SOLUTION The formula for slope is $m = \dfrac{\Delta y}{\Delta x}$. The range values, y, represent bottles sold and the domain values, x, represent temperatures. The slope is a rate of bottles per degree. The slope is 4.44, which means that for each one-degree increase in temperature, 4.44 more water bottles will be sold. Rachael cannot sell a fraction of a water bottle, so she will sell approximately 4 more bottles for each degree the temperature rises.



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CHECK YOUR UNDERSTANDING

Approximately how many more water bottles will Rachael sell if the temperature increases 2 degrees?

EXAMPLE 3

Rachael is stocking her concession stand for a day in which the temperature is expected to reach 106 degrees Fahrenheit. How many water bottles should she pack?

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SOLUTION The linear regression equation tells Rachael the approximate number of bottles she should sell given a specific temperature. Substitute 106 for x in the equation, and compute y, the number of water bottles she should expect to sell.

Equation of the regression line	$y = 4.44x - 187.67$
Substitute 106 for x.	$y = 4.44(106) - 187.67$
Simplify.	$y = 282.97$

If the trend continues and the temperature reaches 106 degrees Fahrenheit, Rachael should expect to sell approximately 283 water bottles. She should stock 283 bottles. This is an example of extrapolation because 106 degrees Fahrenheit was not between the high and low x-values of the original domain.

CHECK YOUR UNDERSTANDING

How many water bottles should Rachael pack if the temperature forecasted were 83 degrees? Is this an example of interpolation or extrapolation? Round to the nearest integer.

EXAMPLE 4

Find the correlation coefficient to the nearest hundredth for the linear regression for Rachael's data. Interpret the correlation coefficient.

SOLUTION Use a graphing calculator to find the correlation coefficient.

Round r to the nearest hundredth. $r = 0.97$

Because 0.97 is positive and greater than 0.75, there is a strong positive correlation between the high temperature and the number of water bottles sold.

CHECK YOUR UNDERSTANDING

Find the correlation coefficient to the thousandth for the linear regression for the data in Check Your Understanding for Example 1. Interpret the correlation coefficient.

EXTEND YOUR UNDERSTANDING

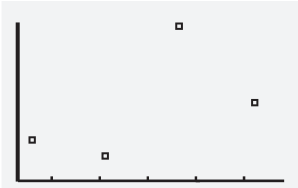
Carlos entered data into his calculator and found a correlation coefficient of -0.28. Interpret this correlation coefficient.

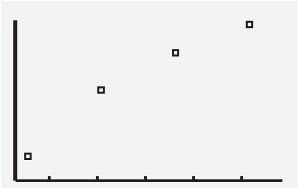
Applications

The only useful function of a statistician is to make predictions, and thus provide a basis for action.
William Edwards Deming, Professor and Statistician

1. Apply what you have learned in this lesson to give an interpretation of the quote.

2. Over the past four years, Reggie noticed that as the price of a slice of pizza increased, her college tuition also increased. She found the correlation coefficient was $r = 0.49$. Which of the following scatterplots most accurately displays Reggie's data? Explain.

a. 

b. 

3. In Exercise 2, would the price of a slice of pizza be labeled as the explanatory variable and the tuition as the response variable? Explain.

4. The table gives enrollments at North Shore High School.
   a. Find the equation of the regression line. Round the slope and y-intercept to the nearest hundredth.
   b. What is the slope of the linear regression line?
   c. What are the units of the slope expressed as a rate?
   d. Based on the linear regression line, how many students will be enrolled in the year 2016? Round to the nearest integer.
   e. Is your answer to part d an example of interpolation or extrapolation? Explain.
   f. Find the correlation coefficient to the nearest hundredth.
   g. Describe the correlation.

Year	Enrollment
2006	801
2007	834
2008	844
2009	897
2010	922

5. Examine the data from Exercise 4.
   a. Find the mean (arithmetic average) of the five years.
   b. Find the mean of the five enrollment figures.
   c. Create an ordered pair whose x-value is the mean of the years and whose y-value is the mean of the enrollments.
   d. Show that the ordered pair satisfies the linear regression equation. What does this mean regarding the regression line?

6. Describe each of the following correlation coefficients using the terms strong, moderate, or weak and positive or negative.
   a. $r = 0.21$
   b. $r = -0.87$
   c. $r = 0.55$
   d. $r = -0.099$
   e. $r = 0.99$
   f. $r = -0.49$

7. The table gives the number of songs downloaded from MyTunes at different prices per song.

Price per Song	Number of Downloads (in thousands)
$0.89	1,212
$0.79	1,704
$0.69	1,760
$0.59	1,877
$0.49	1,944
$0.39	2,011

a. Find the equation of the linear regression line. Round the slope and *y*-intercept to the nearest hundredth.

b. What is the slope of the linear regression line?

c. What are the units of the slope when it is expressed as a rate?

d. Based on the linear regression line, how many thousands of downloads would MyTunes expect if the price was changed to $0.45? Round to the nearest integer.

e. Is your answer to part d an example of interpolation or extrapolation?

f. Find the correlation coefficient to the nearest hundredth.

g. Describe the correlation.

8. Julie is a waitress. On the left is a log of her tips for yesterday's shift.

Amount of Restaurant Bill ($)	Tip Amount ($)
45.55	7.00
52.00	15.00
66.00	6.00
24.44	6.00
57.90	15.00
89.75	23.00
33.00	8.00

a. Find the equation of the linear regression line. Round the slope and *y*-intercept to the nearest hundredth.

b. What is the slope of the linear regression line?

c. What are the units of the slope when it is expressed as a rate?

d. Based on the linear regression line, what tip would Julie receive if the restaurant bill were $120? Round to the nearest dollar.

e. Is your answer to part d an example of interpolation or extrapolation? Explain.

f. Find the correlation coefficient for this data. Round to the nearest hundredth.

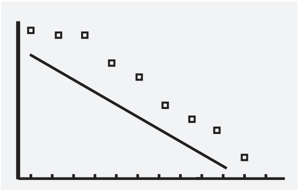
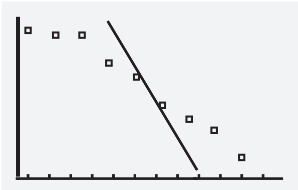
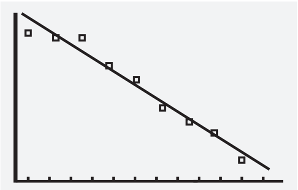
g. Describe the correlation.

h. Based on the linear regression line, Julie creates a spreadsheet to compute predicted tips for any restaurant bill amount. Write the formula that can be used to compute the predicted tips in column B.

	A	B
1	Restaurant Bill	Predicted Tip
2		
3		
4		
5		

9. Explain why the sign of the slope of a regression line must be the same as the sign of the correlation coefficient.

10. Which of the following scatterplots shows a correct line of best fit?

a.  **b.**  **c.** 

11. Is it possible for a linear regression line to go through every point on the scatterplot? Is it possible for a linear regression line to not go through any point on the scatterplot?

> *Teach a parrot the terms "supply and demand" and you've got an economist.*
>
> **Thomas Carlyle,** Philosopher

Supply and Demand 2-3

Key Terms

- widget
- function
- demand function
- demand
- supply
- wholesale price
- markup
- retail price
- equilibrium
- shift

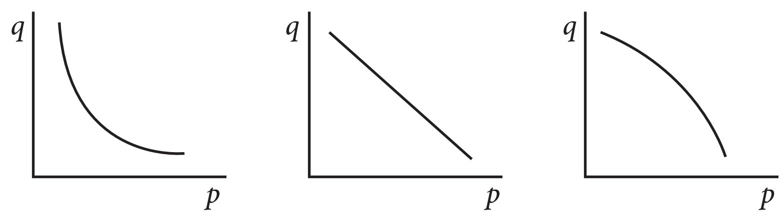
Objectives

- Understand the slopes of the supply and demand curves.
- Find points of equilibrium.

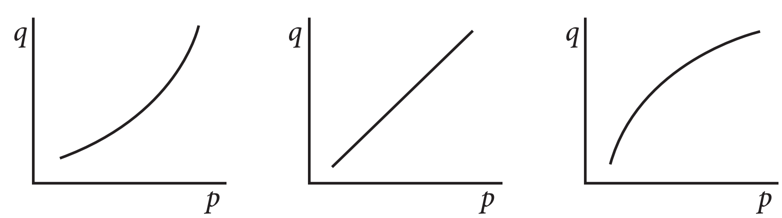
How do manufacturers decide the quantity of a product they will produce?

Economists often call a new, unnamed product a **widget**. If a business develops a new product, the number of items they need to manufacture is a key question they need to address. Graphs may be used to help answer this question. Such graphs compare the price of an item, p, and the quantity sold, q. The horizontal axis is labeled p and the vertical axis is labeled q. The set of p values on the horizontal axis is the *domain*. The set of q values on the vertical axis is the *range*. A **function** is a rule that assigns a unique member of the range to each element of the domain.

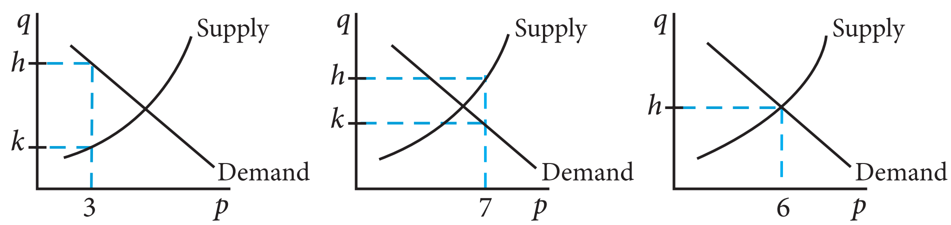
Graphs using the p- and q- axes can be used to model a trend in consumers' interest in a product. A **demand function** relates the quantity of a product to its price. If a widget has a low price, many people may want it and will be able to afford it, so a large quantity may be sold. If it has a high price, fewer widgets will be sold. As the price increases, **demand** (the quantity consumers want) is likely to decrease, and as price decreases, demand increases. The graph of the demand function has a negative slope. However, its curvature varies.



Graphs using p- and q- axes can also be used to model a trend in the manufacturing of a product. *Producers* provide **supply** (the quantity of items available to be sold). If a widget sells for a high price, the manufacturer may be willing to produce many items to maximize profit. If the widget sells for a lower price, the manufacturer may produce less. As price increases, supply increases. The graph of the supply function has a positive slope. Its curvature also varies.



Economic decisions require research, and knowledge of the laws of supply and demand. To examine the law of supply and demand, graph both functions on the same axes. Examine what happens as the manufacturer sets different prices. Keep in mind that the manufacturer sells the items to retailers, such as stores, and not directly to the general public. The price the manufacturer charges the retailer is the **wholesale price**. Retailers increase the price a certain amount, called **markup**, so the retailer can make a profit. The price the retailer sells the item to the public for is the **retail price**.



Look at the graph on the left. If the price is set at $3.00, consumers will demand *h* widgets, and manufacturers will be willing to supply *k* widgets. There will be a shortage of widgets. High demand and low supply often create a rise in price. The manufacturer knows that people want widgets, yet there are not enough of them. If the price rises, demand will fall.

Look at the center graph. If the price is set at $7.00, consumers will demand *k* widgets, and manufacturers will be willing to supply *h* widgets. There will be too much supply, and manufacturers will have to lower the price to try and sell the high inventory of widgets.

Look at the graph on the right. At a price of $6.00, the number of widgets demanded by consumers is equal to the number of widgets manufacturers will supply. Where the functions of supply and demand intersect, the market is in **equilibrium**.

Skills and Strategies

Supply depends on the expenses involved in producing a widget and the price for which it can be sold. The factors of supply will be outlined in Lesson 2-4. This lesson concentrates on demand.

EXAMPLE 1

The Wacky Widget Company sells widgets for $2.00 each wholesale. A local store has a markup of $1.59. What is the retail price?

SOLUTION Add the markup to the wholesale price.

Markup + Wholesale price = Retail price $\quad 2.00 + 1.59 = 3.59$

The retail price is $3.59.

CHECK YOUR UNDERSTANDING

The wholesale price of an item is *x* dollars. The retail price is *r* dollars. Express the markup algebraically.

EXAMPLE 2

The Robear Corporation sells teddy bears at a wholesale price of \$23.00. If a store marks this up 110%, what is the retail price?

SOLUTION Compute the markup amount. Then calculate the retail price.

Markup rate × Wholesale price = Markup $\quad 1.10 \times \$23 = \$25.30$

Wholesale price + Markup = Retail price $\quad \$23.00 + \$25.30 = \$48.30$

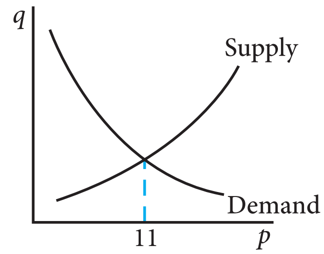
The retail price is \$48.30.

CHECK YOUR UNDERSTANDING

A banner company sells 5-foot banners to retailers for x dollars. The St. James Sign Shop marks them up 90%. Express the retail price at the St. James store algebraically.

EXAMPLE 3

The graph shows the supply and demand curves for a widget. Explain what happens if the price is set at \$9.00.



SOLUTION Because \$9.00 is less than the equilibrium price, demand will exceed supply. Suppliers will attempt to sell the widget at a higher price.

CHECK YOUR UNDERSTANDING

Use the graph to explain what happens if the price is set at \$15.00.

EXAMPLE 4

A company wants to base the price of its product on demand for the product, as well as on expenses. It takes a poll of several of its current retailers to find out how many widgets they would buy at different wholesale prices. The results are shown in the table. The company wants to use linear regression to create a demand function. What is the equation of the demand function? Round the slope and y-intercept to the nearest hundredth.

Wholesale Price (\$) (p)	Quantity Retailers Would Purchase (in thousands) (q)
15.25	3,456
15.50	3,005
15.75	2,546
16.00	2,188
16.25	1,678
16.50	1,290
16.75	889
17.00	310

SOLUTION Use the linear regression feature on your graphing calculator. The equation is $q = -1{,}756.19p + 30{,}238.82$. This represents the demand function.

CHECK YOUR UNDERSTANDING

Explain why it makes sense that the demand function has a negative slope.

Applications

Teach a parrot the terms "supply and demand" and you've got an economist.

Thomas Carlyle, Philosopher

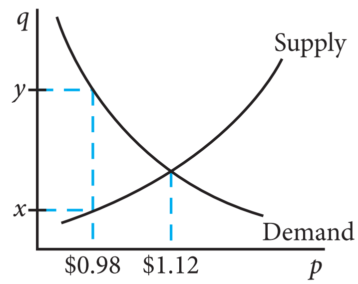
1. Interpret the quote in the context of what you learned in this lesson.

2. An automobile GPS system is sold to stores at a wholesale price of \$97. A popular store sells them for \$179.99. What is the store's markup?

3. A CD storage rack is sold to stores at a wholesale price of \$18.
  a. If a store has a \$13 markup, what is the retail price of the CD rack?
  b. Find the percent increase of the markup to the nearest percent.

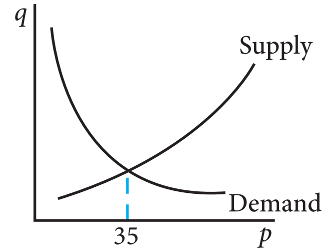
4. A bicycle sells for a retail price of b dollars from an online store. The wholesale price of the bicycle is w.
  a. Express the markup algebraically.
  b. Express the percent increase of the markup algebraically.

5. The graph shows supply and demand curves for the newest SuperWidget.



  a. What is the equilibrium price?
  b. What will happen if the price is set at \$0.98?
  c. How many SuperWidgets are demanded at a price of \$0.98?
  d. How many SuperWidgets are supplied at a price of \$0.98?
  e. What will happen if the price is set at \$1.22?

6. The graph below shows supply and demand curves for a new mp3 player accessory.
  a. What is the equilibrium price?
  b. Describe the relationship of supply and demand if the item were sold for \$20.
  c. Describe the relationship of supply and demand if the item were sold for \$40.
  d. Name the domain that will increase demand.
  e. Name the domain that will increase supply.



7. The supply and demand curves for a new widget are shown in the graph. Notice there are two demand curves. The original demand curve is d_1. Months after the product was introduced, there was a possible health concern over use of the product, and demand dropped to the new demand curve, d_2. The movement of the demand curve is called a **shift**.

a. What was the equilibrium price before the demand shift?

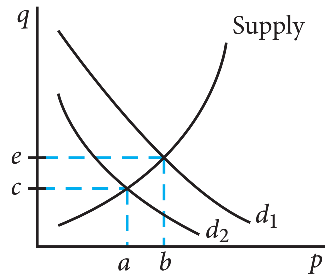
b. What was the equilibrium quantity before the demand shift?

c. What was the equilibrium price after the demand shift?

d. What was the equilibrium quantity after the demand shift?

e. Express algebraically the difference in quantity demanded at price *b* before and after the shift.

f. Copy a rough sketch of the graph into your notebook. Label the curves. Where would the demand curve have shifted if a health benefit of the new widget was reported?



8. Debbie is president of a company that produces garbage cans. The company has developed a new type of garbage can that is animal-proof, and Debbie wants to use the demand function to help set a price. She surveys ten retailers to get an approximation of how many garbage cans would be demanded at each price, and creates a table.

a. Find the equation of the linear regression line. Round the slope and *y*-intercept to the nearest hundredth.

b. Give the slope of the regression line and interpret the slope as a rate.

c. Find the correlation coefficient and interpret it. Round to the nearest hundredth.

d. Based on the linear regression line, how many garbage cans would be demanded at a wholesale price of $18.00? Round to the nearest hundred garbage cans.

e. Was your answer to part d an example of extrapolation or interpolation? Explain.

f. Look at your answer to part d. If the company sold that many garbage cans at $18.00, how much money would the company receive from the garbage can sales?

Wholesale Price ($) ($p$)	Quantity Demanded By Retailers (in hundreds) (q)
13.00	744
13.50	690
14.00	630
14.50	554
15.00	511
15.50	456
16.00	400
16.50	300
17.00	207
17.50	113

9. A company that produces widgets has found its demand function to be $q = -1{,}500p + 90{,}000$.

a. For each dollar increase in the wholesale price, how many fewer widgets are demanded?

b. How many widgets would be demanded at a price of $20?

c. How many widgets would be demanded at a price of $21?

d. What is the difference in quantity demanded caused by the $1 increase in wholesale price?

e. The company sets a price of $22.50. How many widgets will be demanded?

f. How much will all of the widgets cost the store to purchase at a price of $22.50?

g. If the store marks up the widgets that cost $22.50 at a rate of 50%, what is the retail price of each widget?

> *An economist is an expert who will know tomorrow why the things he predicted yesterday didn't happen today.*
>
> **Laurence J. Peter,** Professor

2-4 Fixed and Variable Expenses

Objectives

- Understand the difference between fixed and variable expenses.
- Create an expense equation based on fixed and variable expenses.

Key Terms

- variable expenses
- fixed expenses
- expense equation
- revenue
- revenue equation
- profit
- loss
- breakeven point

What expenses are involved in the manufacturing process?

A group of art school students have decided to start a business producing hand-painted jeans. They made a list of expenses for running the business. Some of these items must be purchased while others may be rented.

- factory space
- furniture
- delivery trucks
- electricity
- telephone
- computer
- office supplies
- jeans
- paint
- packaging
- postage
- labor

All of these expenses fall into one of two categories—variable expenses or fixed expenses. Expenses that depend on the number of items produced are **variable expenses**. Examples of variable expenses are costs for raw materials such as jeans, paint, office supplies, and labor expenses, because these costs change based on the quantity produced.

Some expenses do not change based on the quantity produced. These expenses are **fixed expenses**. Examples of fixed expenses are the cost of the furniture and the computer. The cost of having the lights on is the same regardless of how many items are produced, so this is also a fixed expense.

The *total expenses* is the sum of the fixed and variable expenses. The **expense equation** is

$E = V + F$ where E represents total expenses, V represents variable expenses, and F represents fixed expenses

The income a business receives from selling its product is **revenue**. Revenue is the price for which each was sold times the number of products sold. The **revenue equation** is

$R = pq$ where R represents revenue, p represents the price of the product, and q represents the quantity of products sold

The difference obtained when expenses are subtracted from revenue is a **profit** when positive and a **loss** when negative. When the expenses and the revenue are equal, there is no profit or loss. This is the **breakeven point**.

Skills and Strategies

Business owners use mathematical models to analyze expenses and revenue to determine profitability of a product.

EXAMPLE 1

The art students have researched all of their potential expenses. The fixed expenses are $17,600. The labor and materials required for each pair of painted jeans produced cost $7.53. Represent the total expenses as a function of the quantity produced, q.

SOLUTION If q units are produced at a cost of $7.53 per unit, the variable expenses can be represented by:

$$V = 7.53q$$

The fixed expenses, $17,600, do not depend on the quantity produced. The total expenses, E, are the sum of the variable and the fixed expenses.

$$E = V + F$$
$$E = 7.53q + 17{,}600$$

The total expenses, E, are a function of the quantity produced, q.

CHECK YOUR UNDERSTANDING

A widget manufacturer's expense function is $E = 6.00q + 11{,}000$. What are the variable costs to produce one widget?

EXAMPLE 2

Kivetsky Ski Supply manufactures hand warmers for skiers. The expense function is $E = 1.18q + 12{,}000$. Find the average cost of producing one pair of hand warmers if 50,000 hand warmers are produced.

SOLUTION Find the total cost of producing 50,000 hand warmers by substituting 50,000 for q in the expense function.

$$E = 1.18q + 12{,}000$$

Substitute. $E = 1.18(50{,}000) + 12{,}000$

Multiply. $E = 59{,}000 + 12{,}000$

Simplify. $E = 71{,}000$

To find the average cost, divide the total cost by the number produced.

Divide by 50,000. $\dfrac{71{,}000}{50{,}000} = 1.42$

The average cost to produce one hand warmer is $1.42 when 50,000 are produced.

CHECK YOUR UNDERSTANDING

The expense function for a certain product is $E = 3.40q + 189{,}000$. Express the average cost of producing q items algebraically.

EXAMPLE 3

Willie's Widgets has created a demand function for its widgets, where q is the quantity demanded and p is the price of one widget.

$$q = -112p + 4{,}500$$

Its expense function is $E = 3.00q + 18{,}000$. Express the expense function as a function in terms of p.

SOLUTION Because E is a function of q, and q is a function of p, express E as a function of p using substitution.

$$E = 3.00q + 18{,}000$$

Substitute for q. $\quad E = 3.00(-112p + 4{,}500) + 17{,}600$

Distribute. $\quad E = -336p + 13{,}500 + 17{,}600$

Simplify. $\quad E = -336p + 31{,}100$

The expense function in terms of the price, p, is $E = -336p + 31{,}100$.

CHECK YOUR UNDERSTANDING

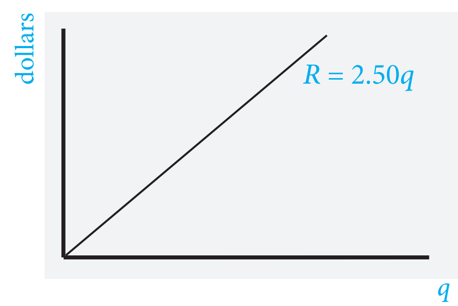
A corporation's expense function is $E = 4.00q + 78{,}000$. The demand function was determined to be $q = -4p + 3{,}000$. Express E in terms of p.

EXAMPLE 4

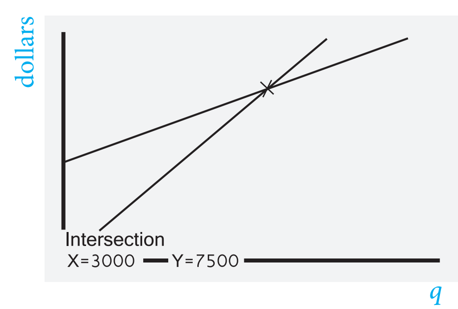
Wally's Widget World created a monthly expense equation, $E = 1.10q + 4{,}200$. Wally's Widget World plans to sell its widgets to retailers at a wholesale price of \$2.50 each. How many widgets must be sold to reach the breakeven point?

SOLUTION Use a graphing calculator. Graph the revenue function, $R = 2.50q$, using R as revenue and q as the quantity sold. Notice that q is the independent variable, so the horizontal axis is labeled q. The labels on the graph below will not be shown on the calculator display.

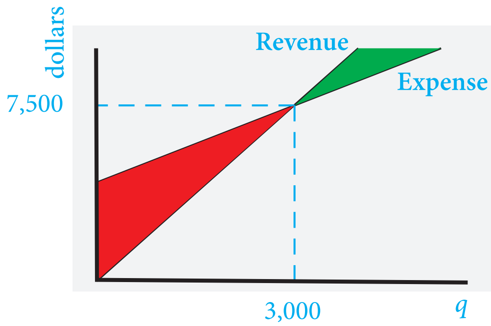
Graph the revenue function on a coordinate grid.



Graph the expense function on the same coordinate grid. Use your calculator's graph intersection feature to find the point of intersection.



It will cost \$7,500 to manufacture 3,000 widgets. If 3,000 widgets are sold to stores for \$2.50 each, the company will receive \$7,500. Notice that no profit is made—the revenue equals the expense. The point (3,000, 7,500) is the breakeven point. Look at sections of the graph before and after the breakeven point to see if you understand what happens.



If more than 3,000 widgets are produced, the company will be operating in the green area of the graph, where $R > E$. A profit is made when revenue is greater than expenses.

If less than 3,000 widgets are produced, the company will be operating in the red area of the graph, where $E > R$. The company is not making enough revenue to pay its expenses.

CHECK YOUR UNDERSTANDING

If the company sells 2,900 widgets, is Wally's Widget World operating above or below the breakeven point? What is the difference between revenue and expense?

EXAMPLE 5

Find the solution to Example 4 algebraically.

SOLUTION To find the breakeven point, set the revenue and expense equations equal to each other.

$$R = E$$

Substitute. $\quad 2.50q = 1.10q + 4{,}200$

Subtract $1.10q$ from each side. $\quad 1.40q = 4{,}200$

Divide each side by 1.40. $\quad \dfrac{1.40q}{1.40} = \dfrac{4{,}200}{1.40}$

$$q = 3{,}000$$

The breakeven point occurs when the quantity produced, q, equals 3,000.

CHECK YOUR UNDERSTANDING

Find the breakeven point for the expense function, $E = 5.00q + 60{,}000$, and the revenue function, $R = 7.00q$.

Applications

An economist is an expert who will know tomorrow why the things he predicted yesterday didn't happen today.

Laurence J. Peter, Professor

1. Interpret the quote according to what you have learned in this chapter.
2. The Gidget Widget Corporation produces widgets. The fixed expenses are \$65,210, and the variable expenses are \$4.22 per widget. Express the expense function algebraically.
3. A corporation produces mini-widgets. The variable expenses are \$1.24 per mini-widget, and the fixed expenses are \$142,900.
   - **a.** How much does it cost to produce 1 mini-widget?
   - **b.** How much does it cost to produce 20,000 mini-widgets?
   - **c.** Express the expense function algebraically.
   - **d.** What is the slope of the expense function?
   - **e.** If the slope is interpreted as a rate, give the units that would be used.
4. The expense function for the Wonder Widget is $E = 4.14q + 55{,}789$.
   - **a.** What is the fixed cost in the expense function?
   - **b.** What is the cost of producing 500 Wonder Widgets?
   - **c.** What is the average cost per widget of producing 500 Wonder Widgets? Round to the nearest cent.
   - **d.** What is the total cost of producing 600 Wonder Widgets?
   - **e.** What is the average cost per widget of producing 600 Wonder Widgets? Round to the nearest cent.
   - **f.** As the number of widgets increased from 500 to 600, did the average expense per widget increase or decrease?
   - **g.** What is the average cost per widget of producing 10,000 Wonder Widgets? Round to the nearest cent.
5. The Royal Ranch Pool Supply Corporation manufactures chlorine test kits. The kits have an expense equation of $E = 5.15q + 23{,}500$. What is the average cost per kit of producing 3,000 test kits? Round to the nearest cent.
6. The fixed costs of producing a Wild Widget are \$34,000. The variable costs are \$5.00 per widget. What is the average cost per widget of producing 7,000 Wild Widgets? Round to the nearest cent.
7. Wanda's Widgets used market surveys and linear regression to develop a demand function based on the wholesale price. The demand function is $q = -140p + 9{,}000$. The expense function is $E = 2.00q + 16{,}000$.
   - **a.** Express the expense function in terms of p.
   - **b.** At a price of \$10.00, how many widgets are demanded?
   - **c.** How much does it cost to produce the number of widgets from part b?

8. Wind Up Corporation manufactures widgets. The monthly expense equation is $E = 3.20q + 56{,}000$. They plan to sell the widgets to retailers at a wholesale price of \$6.00 each. How many widgets must be sold to reach the breakeven point?

9. The Lerneg Corporation computed its monthly expense equation as $E = 11.00q + 76{,}000$. Its products will be sold to retailers at a wholesale price of \$20.00 each. How many items must be sold to reach the breakeven point? Round to the nearest integer.

10. Solve Exercise 9 using the graph intersection feature on your graphing calculator. Are the answers equivalent?

11. Variable costs of producing widgets account for the cost of gas required to deliver the widgets to retailers. A widget producer finds the average cost of gas per widget. The expense equation was recently adjusted from $E = 4.55q + 69{,}000$ to $E = 4.98q + 69{,}000$ in response to the increase in gas prices.

a. Find the increase in the average cost per widget.

b. If the widgets are sold to retailers for \$8.00 each, find the breakeven point prior to the adjustment in the expense function.

c. After the gas increase, the company raised its wholesale cost from \$8 to \$8.50. Find the breakeven point after the adjustment in the expense function. Round to the nearest integer.

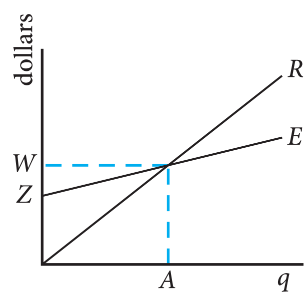
12. Examine the graph of expense and revenue.

a. What is the breakeven point?

b. If quantity C is sold and $C < A$, is there a profit or a loss? Explain.

c. If quantity D is sold and $D > A$, is there a profit or a loss? Explain.

d. The y-intercept of the expense function is Z. Interpret what the company is doing if it operates at the point $(0, Z)$.



13. Billy invented an innovative baseball batting glove he named the Nokee and made his own TV infomercial to sell it. The expense function for the Nokee is $E = 6.21q + 125{,}000$. The Nokee sells for \$19.95.

a. Represent the average expense A for one Nokee algebraically.

b. Set your calculator viewing window to show x-values between 0 and 1,000, and y-values from 0 to 2,000. Let x represent q and let y represent A. Graph the average expense function.

c. Is the average expense function linear?

d. Is the average expense function increasing or decreasing as q increases?

e. If only one Nokee is produced, what is the average cost per Nokee to the nearest cent?

f. If 100,000 Nokees are produced, what is the average cost per Nokee to the nearest cent?

14. Lorne has determined the fixed cost of producing his new invention is N dollars. The variable cost is \$10.75 per item. What is the average cost per item of producing W items?

> *Money often costs too much.*
>
> **Ralph Waldo Emerson,** Poet

2-5 Graphs of Expense and Revenue Functions

Objectives

- Write, graph, and interpret the expense function.
- Write, graph, and interpret the revenue function.
- Identify the points of intersection of the expense and revenue functions.
- Identify breakeven points, and explain them in the context of the problem.

Key Terms

- nonlinear function
- second-degree equation
- quadratic equation
- parabola
- leading coefficient
- maximum value
- vertex of a parabola
- axis of symmetry

How can expense and revenue be graphed?

The total expense for the production of a certain item is the amount of money it costs to manufacture and place it on the market. One contributor to consumer demand is the price at which an item is sold. Expense relies on the quantity produced and demand relies on price, so the expense function can be written in terms of price.

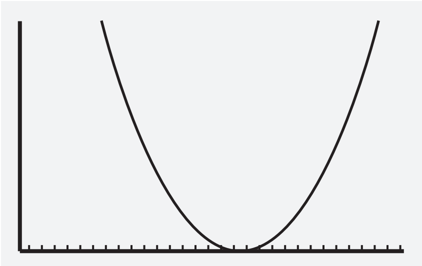
Recall that revenue is the total amount a company collects from the sale of a product or service. Revenue depends on the demand for a product, which is a function of the price of the product. The relationship between price, demand, expense, and revenue can be better understood when the functions are graphed on a coordinate plane.

Both the demand and expense functions are linear. However, as you will see, revenue is a **nonlinear function** when it is expressed in terms of price. That is, the graph of the revenue function is not a straight line. The revenue function has a variable raised to an exponent of 2, so it is a **second-degree equation** and is known as a **quadratic equation**. A quadratic equation can be written in the form

$$y = ax^2 + bx + c \text{ where } a, b, \text{ and } c \text{ are real numbers and } a \neq 0$$

The graph of a quadratic equation is called a **parabola**. In the revenue graph pictured, the horizontal axis represents the price of an item, and the vertical axis represents the revenue. If the **leading coefficient**, a, is *positive*, then the parabola opens upward.

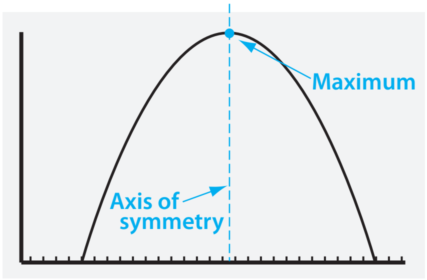
Parabola with a Positive Leading Coefficient



If the leading coefficient is *negative*, then the parabola opens downward. The downward parabola models the revenue function. In the graph, the parabola reaches a **maximum value** at its peak. This point is the **vertex of the parabola** and yields the price at which revenue can be maximized.

The **axis of symmetry** is a vertical line that can be drawn through the *vertex of the parabola* so that the dissected parts of the parabola are mirror images of each other. The point on the horizontal axis through which the *axis of symmetry* passes is determined by calculating $-\frac{b}{2a}$.

Parabola with a Negative Leading Coefficient



Skills and Strategies

The graphical relationship between revenue and expense reveals much information helpful to a business.

EXAMPLE 1

A particular item in the Picasso Paints product line costs \$7.00 each to manufacture. The fixed costs are \$28,000. The demand function is $q = -500p + 30{,}000$ where q is the quantity the public will buy given the price, p. Graph the expense function in terms of price on the coordinate plane.

SOLUTION The expense function is $E = 7.00q + 28{,}000$. Substitute for q to find the expense function in terms of price.

Expense function.	$E = 7.00q + 28{,}000$
Substitute.	$E = 7.00(-500p + 30{,}000) + 28{,}000$
Use the Distributive property.	$E = -3{,}500p + 210{,}000 + 28{,}000$
Simplify.	$E = -3{,}500p + 238{,}000$

The horizontal axis represents price, and the vertical axis represents expense. Both variables must be greater than 0, so the graph is in the first quadrant.

To determine a viewing window, find the points where the expense function intersects the vertical and horizontal axes. Neither p nor E can be 0 because both a price of 0 and an expense of 0 would be meaningless in this situation. But, you can use $p = 0$ and $E = 0$ to determine an appropriate viewing window.

To find the vertical axis intercept, let $p = 0$ and solve for E.

$$E = -3{,}500p + 238{,}000$$

Let $p = 0$.

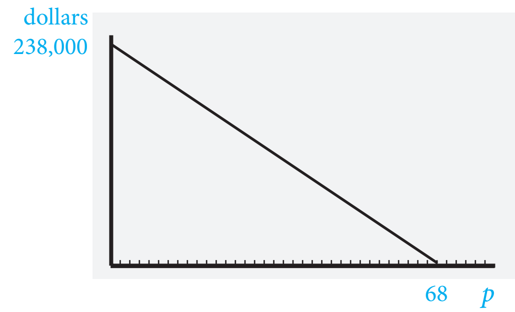
$$E = -3{,}500(0) + 238{,}000$$
$$E = 238{,}000$$

To find the horizontal axis intercept, let $E = 0$ and solve for p.

Let $E = 0$.

$$0 = -3{,}500p + 238{,}000$$
$$3{,}500p = 238{,}000$$
$$p = 68$$

Use the intercepts to determine the size of the viewing window. In order to get a full picture, use window values that are slightly greater than the intercepts. Choose an appropriate scale for each axis. In this case, set the horizontal axis at 0 to 70 with a scale of 10 and the vertical axis at 0 to 240,000 with the scale at 50,000. Enter the function and graph.



CHECK YOUR UNDERSTANDING

An electronics company manufactures earphones for portable music devices. Each earphone costs \$5 to manufacture. Fixed costs are \$20,000. The demand function is $q = -200p + 40{,}000$. Write the expense function in terms of q and determine a suitable viewing window for that function. Graph the expense function.

EXAMPLE 2

What is the revenue equation for the Picasso Paints product? Write the revenue equation in terms of the price.

SOLUTION Revenue is the product of the price and quantity, or $R = pq$. The quantity, q, is expressed in terms of the price, so the expression can be substituted into the revenue equation.

Revenue equation	$R = pq$
Substitute.	$R = p(-500p + 30{,}000)$
Distribute.	$R = p(-500p) + p(30{,}000)$
Simplify.	$R = -500p^2 + 30{,}000p$

The revenue function is a quadratic equation in the form $y = ax^2 + bx + c$ where $a = -500$, $b = 30{,}000$, and $c = 0$. Because the leading coefficient is negative, the graph is a parabola that opens downward.

CHECK YOUR UNDERSTANDING

Determine the revenue if the price per item is set at \$25.00.

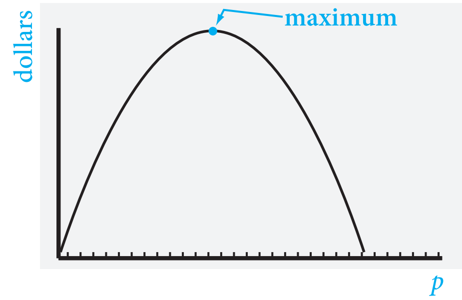
EXAMPLE 3

Graph the revenue equation on a coordinate plane.

SOLUTION The graph opens downward. It has a maximum height at the vertex. Here $b = 30{,}000$ and $a = -500$.

$$\frac{-b}{2a} = \frac{-30{,}000}{2(-500)} = \frac{-30{,}000}{-1{,}000} = 30$$

The axis of symmetry passes through 30 on the horizontal axis. To determine the height of the parabola at 30, evaluate the parabola when $p = 30$.



Revenue equation $\quad R = -500p^2 + 30{,}000p$

Substitute 30 for p. $\quad R = -500(30)^2 + 30{,}000(30)$

Simplify. $\quad R = 450{,}000$

When the price is set at \$30, the maximum revenue is \$450,000. The vertical axis must be as high as the maximum value. For the viewing window, use a number slightly greater than the maximum value, such as 500,000, so you can get the entire picture of the graph. Use the same horizontal values on the axis as used for the expense function.

CHECK YOUR UNDERSTANDING

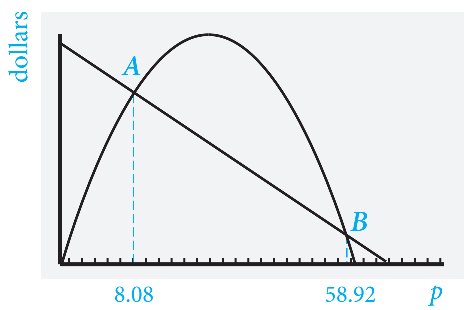
Use the graph in Example 3. Which price would yield the higher revenue, \$28 or \$40?

EXAMPLE 4

The revenue and expense functions are graphed on the same set of axes. The points of intersection are labeled A and B. Explain what is happening at those two points.

SOLUTION Where the revenue and the expense functions intersect, revenue and expenses are equal. These are breakeven points. Notice there are two such points.

Before intersection point A, the expenses are greater than the revenue. The first breakeven price is approximately at \$8.08. The second is approximately at \$58.92. From point A to point B, the revenue is greater than the expenses. To the right of point B, the expenses again are greater than the revenue.



CHECK YOUR UNDERSTANDING

Why is using the prices of \$7.50 and \$61.00 not in the best interest of the company?

Applications

Money often costs too much.

Ralph Waldo Emerson, Poet

1. How might the quote apply to what you have learned?

2. Rich and Betsy Cuik started a small business. They manufacture a microwavable coffee-to-go cup called Cuik Cuppa Coffee. It contains spring water and ground coffee beans in a tea-bag-like pouch. Each cup costs the company \$1.00 to manufacture. The fixed costs for this product line are \$1,500. Rich and Betsy have determined the demand function to be $q = -1{,}000p + 8{,}500$, where p is the price for each cup.
   a. Write the expense equation in terms of the demand, q.
   b. Express the expense equation found in part a in terms of the price, p.
   c. Determine a viewing window on a graphing calculator for the expense function. Justify your answer.
   d. Draw and label the graph of the expense function.
   e. Write the revenue function in terms of the price.
   f. Graph the revenue function in a suitable viewing window. What price will yield the maximum revenue? What is the revenue at that price? Round both answers to the nearest cent.
   g. Graph the revenue and expense functions on the same coordinate plane. Identify the points of intersection using a graphing calculator. Round your answers to the nearest cent. Identify the price at the breakeven points.

3. Orange-U-Happy is an orange-scented cleaning product that is manufactured in disposable cloth pads. Each box of 100 pads costs \$5 to manufacture. The fixed costs for Orange-U-Happy are \$40,000. The research development group of the company has determined the demand function to be $q = -500p + 20{,}000$, where p is the price for each box.
   a. Write the expense equation in terms of the demand, q.
   b. Express the expense function in terms of the price, p.
   c. Determine a viewing window on a graphing calculator for the expense function. Justify your answer.
   d. Draw and label the graph of the expense function.
   e. Write the revenue function in terms of the price.
   f. Graph the revenue function in a suitable viewing window. What price will yield the maximum revenue? What is the revenue at that price? Round answers to the nearest cent.
   g. Graph the revenue and expense functions on the same coordinate plane. Identify the points of intersection using a graphing calculator, and name the breakeven points. Round to the nearest cent. Identify the price at the breakeven points.

Risk comes from not knowing what you're doing.
Warren Buffet, Businessman

Breakeven Analysis 2-6

Key Terms

- zero net difference
- quadratic formula

Objectives

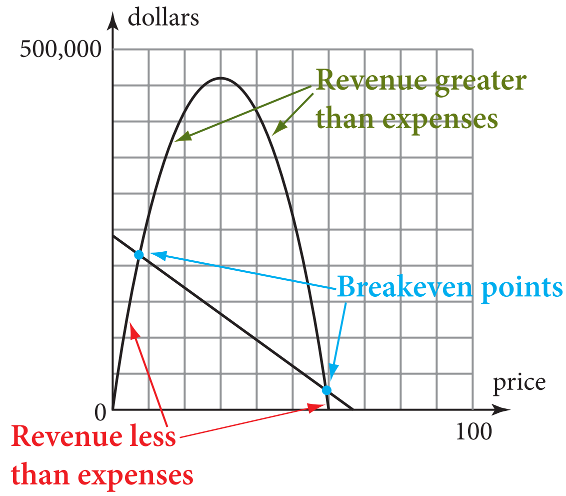
- Determine the breakeven prices and amounts using technology or algebra.

What happens when revenue equals expense?

When the revenue function is a quadratic equation, you can determine the values at which the expense and revenue functions are equal. Graph the revenue and expense functions on the same coordinate plane.

The functions intersect at the values at which the expense and revenue functions are equal (blue points), or the breakeven points. The difference between expense and revenue equals zero, which is a **zero net difference**.

Notice that the prices on the revenue function between the breakeven points result in revenue greater than expense. Prices beyond the breakeven points result in revenue less than expense. Decision makers must examine the breakeven points carefully to set appropriate prices to yield maximum revenue.



Breakeven points may be found algebraically or by using a graphing calculator. To find breakeven points algebraically, set the expense and revenue functions equal to each other. Rewrite the resulting equation with 0 on one side of the equal sign and all the remaining terms on the other side.

This gives a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$. The **quadratic formula** can then be used to solve this quadratic equation. The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ where $\pm$ (read *plus or minus*) indicates the two solutions

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Skills and Strategies

Breakeven analysis can be used to make product price-setting decisions. Calculation and interpretation must be done during breakeven analysis.

EXAMPLE 1

Determine the prices at the breakeven points for the Picasso Paints product in Lesson 2-5. The expense function is $E = -3{,}500p + 238{,}000$, and the revenue function is $R = -500p^2 + 30{,}000p$.

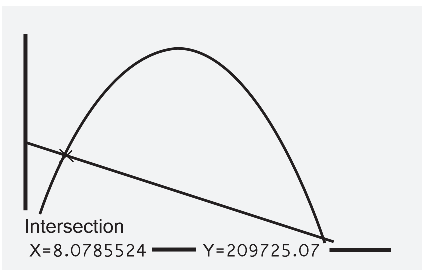
SOLUTION The breakeven point occurs when the expense and revenue functions intersect. There are two ways to determine the breakeven point—using technology or algebra.

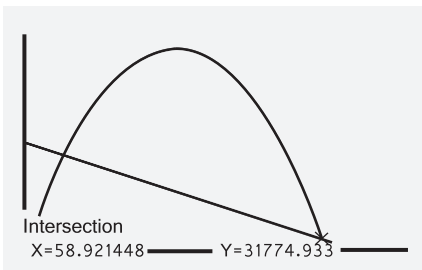
If a graphing calculator is used, let x represent the price, and express the equations in terms of x. Enter each function into the calculator, and graph them both using a suitable viewing window. Find the intersection points.

Because the calculator will display answers to many decimal places, round the intersection points to the nearest cent. The figures below show two graphing calculator screens after the Intersect feature has been used.

Shown below on the left is the leftmost breakeven point; the price is at $8.08.

On the right is the rightmost breakeven point; the price is $58.92.





The breakeven point can also be found algebraically. Set the expense and revenue functions equal to each other. Rewrite the resulting equation with 0 on one side and all the remaining terms on the other side. The quadratic equation is usually written with the leading coefficient positive.

$$E = R$$

Set the expressions equal. $\quad -3{,}500p + 238{,}000 = -500p^2 + 30{,}000p$

Rewrite to set equal to 0. $\quad +500p^2 - 30{,}000p = +500p^2 - 30{,}000p$

$$500p^2 - 33{,}500p + 238{,}000 = 0$$

In the quadratic equation $500p^2 - 33{,}500p + 238{,}000 = 0$, $a = 500$, $b = -33{,}500$, and $c = 238{,}000$. Because the equation is written in terms of p, use p instead of x in the quadratic formula. Substitute values for a, b, and c, and calculate.

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Use + in the formula.

$$p = \frac{-(-33{,}500) + \sqrt{(-33{,}500)^2 - 4(500)(238{,}000)}}{2(500)} \approx 58.92$$

Use – in the formula.

$$p = \frac{-(-33{,}500) - \sqrt{(-33{,}500)^2 - 4(500)(238{,}000)}}{2(500)} \approx 8.08$$

The prices at the breakeven points are \$8.08 and \$58.92.

CHECK YOUR UNDERSTANDING

The expense function for a particular product is $E = -2{,}000p + 125{,}000$. The revenue function for that product is $R = -600p^2 + 18{,}000p$. Determine the prices at the breakeven points for this product both algebraically and graphically.

EXTEND YOUR UNDERSTANDING

Knowing that the two breakeven prices have been rounded to the nearest cent, what would you expect when each is substituted into the expense and revenue equations?

EXAMPLE 2

Determine the revenue and expense for the Picasso Paints product at the breakeven points found in Example 1.

SOLUTION The y-value of the breakeven point will be both the revenue and expense values. These values can be determined by substituting the values of p into the expense and revenue equations.

Substitute 8.08 for p. $E = -3{,}500(8.08) + 238{,}000 = 209{,}720$

$R = -500(8.08)^2 + 30{,}000(8.08) = 209{,}756.80$

Substitute 58.92 for p. $E = -3{,}500(58.92) + 238{,}000 = 31{,}780$

$R = -500(58.92)^2 + 30{,}000(58.92) = 31{,}816.80$

When the breakeven price is approximately \$8.08, the expense and revenue values are each close to \$209,750.

When the breakeven price is approximately \$58.92, the expense and revenue values are each approximately \$31,800.

Because of rounding, the expense and revenue are not equivalent. They are, however, approximately equal.

CHECK YOUR UNDERSTANDING

How could you have improved on the error when calculating the expense and revenue values?

EXAMPLE 3

Use a spreadsheet to determine the breakeven price for the Picasso Paints product.

SOLUTION Develop general expense and revenue equations so the spreadsheet can calculate the breakeven prices regardless of the situations. The expense function is $E = Vp + F$, where p equals price, V equals variable costs, and F equals fixed costs. The revenue function is $R = Ap^2 + Bp$ where p equals price and A and B are values specific to the particular situation. Set the expense equation equal to the revenue equation.

$$E = R$$

Subtract $Vp + F$ from both sides.

$$Vp + F = Ap^2 + Bp$$
$$-Vp - F = -Vp - F$$
$$0 = Ap^2 + Bp - Vp - F$$

Combine like terms.

$$0 = Ap^2 + (B - V)p - F$$

This is a quadratic equation where $a = A$, $b = (B - V)$, and $c = -F$. Depending on the spreadsheet software, cell formulas may vary. The Picasso Paints functions are $E = -3{,}500p + 238{,}000$ and $R = -500p^2 + 30{,}000$.

The values of V and F from the expense function are entered into cells B3 and B4. The values of A and B from the revenue function are entered into cells B7 and B8. Cells B15 and B16 are the breakeven prices calculated using the quadratic formula.

For B15 the formula is =(–B12+SQRT(B12^2–4*B11*B13))/(2*B11).

For B16 the formula is =(–B12–SQRT(B12^2–4*B11*B13))/(2*B11).

	A	B
1	**Breakeven Calculator**	
2	The expense equation has the form $Vp + F$.	$-3{,}500p + 238{,}000$
3	Enter the value of V in cell B3.	–3,500
4	Enter the value of F in cell B4.	238,000
5		
6	The revenue equation has the form $Ap^2 + Bp$.	$-500p^2 + 30{,}000p$
7	Enter the value of A in cell B7.	–500
8	Enter the value of B in cell B8.	30,000
9		
10	Solve the quadratic equation.	$-500p^2 + 33{,}500p +$ $-238{,}000$
11	where $a =$	–500
12	$b =$	33,500
13	$c =$	–238,000
14		
15	The price at the first breakeven point is	$8.08
16	The price at the second breakeven point is	$58.92

CHECK YOUR UNDERSTANDING

What cell formulas were used to identify the a, b, and c values shown in B11, B12, and B13?

Applications

Risk comes from not knowing what you're doing.
Warren Buffet, Businessman

1. How might the quote apply to what you have learned?

2. A manufacturer has determined that the combined fixed and variable expenses for the production and sale of 500,000 items are $10,000,000. What is the price at the breakeven point for this item?

3. A supplier of school kits has determined that the combined fixed and variable expenses to market and sell G kits is W.

   a. What expression models the price of a school kit at the breakeven point?

   b. Suppose a new marketing manager joined the company and determined that the combined fixed and variable expenses would only be 80% of the cost if the supplier sold twice as many kits. Write an expression for the price of a kit at the breakeven point using the new marketing manager's business model.

4. A jewelry manufacturer has determined the expense equation for necklaces to be $E = 1{,}250q + 800{,}000$, where q is the quantity demanded. At a particular price, the breakeven revenue is $2,600,000.

   a. What is the quantity demanded at the breakeven point?

   b. If the breakeven revenue changes to 3.5 million, will the quantity demanded have increased or decreased? Explain.

5. A manufacturer determines that a product will reach the breakeven point if sold at either $80 or $150. At $80, the expense and revenue values are both $300,000. At $150, the expense and revenue values are both $100,000.

On graph paper, graph possible revenue and expense functions that depict this situation. Circle the breakeven points.

6. iSports is considering producing a line of baseball caps with wireless cellphone earpieces attached. The breakeven point occurs when the price of a cap is $170 or $350. At $170, the expense and revenue values are both $2,600,000. At $350, the expense and revenue values are both $900,000.

On graph paper, graph possible revenue and expense functions that depict this situation. Circle the breakeven points.

7. SeaShade produces beach umbrellas. The expense function is $E = -19{,}000p + 6{,}300{,}000$ and the revenue function is $R = -1{,}000p^2 + 155{,}000p$.

   a. Graph the expense and revenue functions. Label the maximum and minimum values for each axis. Circle the breakeven points.

   b. Determine the prices at the breakeven points.

   c. Determine the revenue and expense amounts for each of the breakeven points.

8. Where-R-U produces global positioning systems (GPS) that can be used in a car. The expense equation is $E = -5{,}000p + \$8{,}300{,}000$, and the revenue equation is $R = -100p^2 + 55{,}500p$.
   a. Graph the expense and revenue functions. Circle the breakeven points.
   b. Determine the prices at the breakeven points. Round to the nearest cent.
   c. Determine the revenue and expense amounts for each of the breakeven points. Round to the nearest cent.

9. The student government at State College is selling inexpensive bookcases for dorm rooms to raise money for school activities. The expense function is $E = -200p + 10{,}000$ and the revenue function is $R = -18p^2 + 800p$.
   a. At what price would the maximum revenue be reached? What would that maximum revenue be? Round to the nearest cent.
   b. Graph the expense and revenue functions. Circle the breakeven points.
   c. Determine the prices at the breakeven points. Round to the nearest cent.
   d. Determine the revenue and expense amounts for each of the breakeven points. Round to the nearest cent.

10. An electronics store is selling car chargers for cell phones. The expense function is $E = -300p + 13{,}000$ and the revenue function is $R = -32p^2 + 1{,}200p$.
   a. At what price would the maximum revenue be reached?
   b. What would that maximum revenue be? Round to the nearest cent.
   c. Graph the expense and revenue functions. Circle the breakeven points.
   d. Determine the prices at the breakeven points. Round to the nearest cent.
   e. Determine the revenue and expense amounts for each of the breakeven points. Round to the nearest cent.

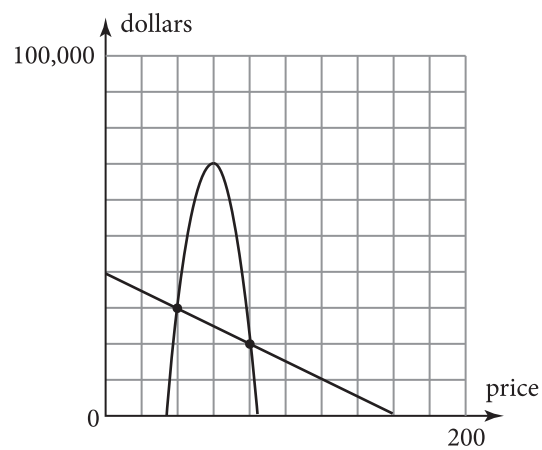
Use the graph to answer Exercises 11–14.

11. At what price is the maximum profit reached?

12. What are the breakeven prices?

13. Name two prices where the revenue is greater than the expenses.

14. Name two prices where the revenue is less than the expenses.



Nobody ever lost money taking a profit.

Bernard Baruch, Businessman

The Profit Equation 2-7

Key Terms

- profit
- maximum profit

Objectives

- Determine a profit equation given the expense and revenue equations.
- Determine the maximum profit and the price at which that maximum is attained.

How do revenue and expenses contribute to profit calculation?

In the business world, two companies may produce and sell the same product. It is possible that both companies will have the same maximum revenue based on their individual revenue functions. However, other aspects of production may not be identical.

Revenue is the amount of money a company makes from the sale of goods or services. While the maximum revenue may be the same for each product, the revenue and expense equations may be different. Figure A displays the graph of the revenue function for the product. Figure B displays the graph of the revenue function of a similar product produced by a different company. The graphs of the revenue functions yield the same maximum values but model different situations.

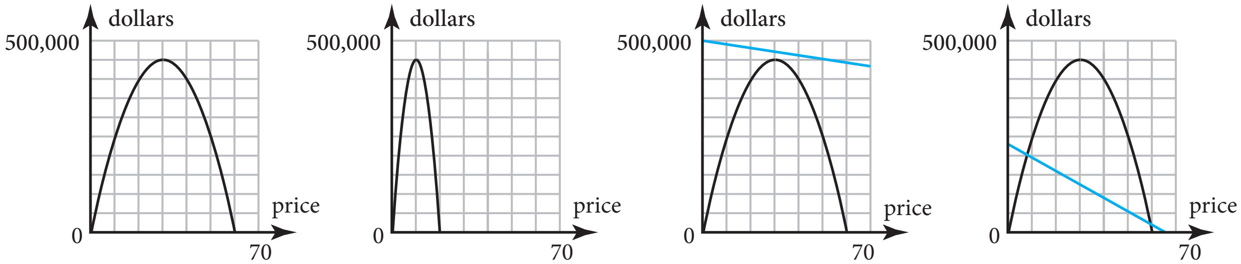


Figure A Figure B Figure C Figure D

Figures C and D depict the same revenue functions but different expense functions. Figure C shows an expense function that is always greater than the revenue graph. Therefore, the company makes no money on the manufacture and sale of the product. The company cannot even reach a breakeven point because there are no breakeven points. In Figure D, the expense function is greater than the revenue function at certain prices, but there are also prices at which the opposite is true. A company would prefer Figure D to model the sale of its product because there are prices at which money coming in is greater than money spent. The money a company makes after expenses have been deducted from revenue is **profit**. In Figure D, there are prices at which the company will make a profit. In Figure C, no matter the price, the revenue is never enough to offset the expenses.

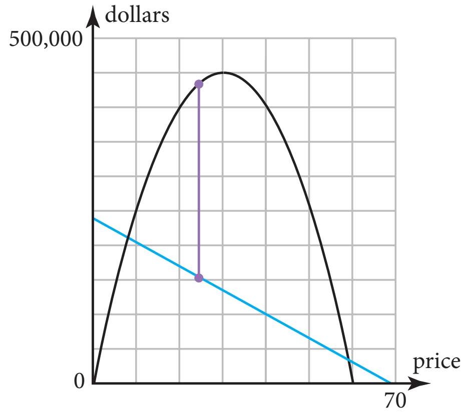


Figure E

Graphically, *profit* is the vertical distance between the revenue and expense functions. In Figure E, the top of the vertical line segment (in purple) hits the revenue graph at \$437,500 when the price is about \$25.

The bottom of the vertical line segment hits the expense graph at \$150,500 at the same price. The vertical length of this segment is 437,500 − 150,500 = 287,000 and is the *profit* the company makes when price is about \$25.

$P = R - E$ where P is profit, R is revenue, and E is expenses

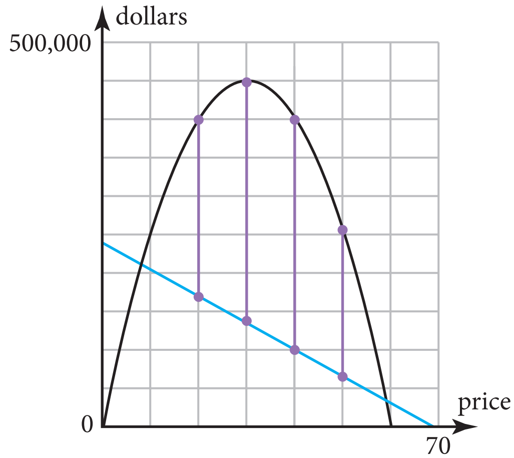


Figure F

In Figure F, several profit line segments have been drawn for different prices. The longest segment would represent the greatest difference between revenue and expense at a given price. The greatest difference between revenue and expense denotes **maximum profit**.

It is difficult to make a visual determination as to where the maximum profit line might be drawn. Algebra and graphing must be used to make a more precise determination.

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Skills and Strategies

The revenue and expense functions are both functions of price, p, so, you can create a profit function in terms of p.

EXAMPLE 1

Determine the profit equation for the Picasso Paints product in Lesson 2-5. The revenue and expense functions were

$$R = -500p^2 + 30{,}000p$$
$$E = -3{,}500p + 238{,}000$$

SOLUTION Profit is the difference between revenue and expense.

$$P = R - E$$

Substitute for R and E. $P = -500p^2 + 30{,}000p - (-3{,}500p + 238{,}000)$

Distribute. $P = -500p^2 + 30{,}000p + 3{,}500p - 238{,}000$

Combine like terms. $P = -500p^2 + 33{,}500p - 238{,}000$

The profit equation is $P = -500p^2 + 33{,}500p - 238{,}000$. It is a downward parabola because the leading coefficient is negative.

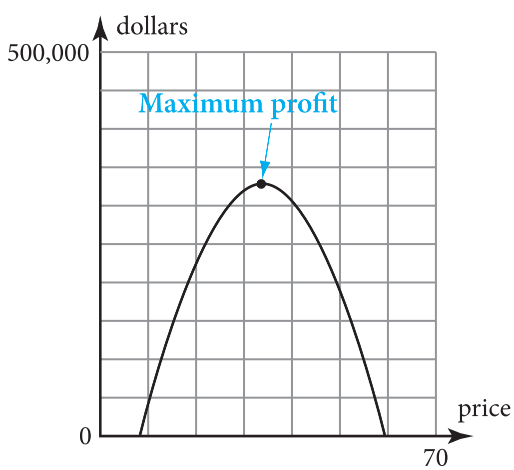
CHECK YOUR UNDERSTANDING

Suppose that the revenue and expense functions are $R = -350p^2 + 18{,}000p$ and $E = -1{,}500p + 199{,}000$. Write the profit equation.

EXAMPLE 2

Use a graphing calculator to draw the graph of the profit equation from Example 1. What is the maximum profit?

SOLUTION Enter the profit equation $P = -500p^2 + 33{,}500p - 238{,}000$ into a graphing calculator. Use the same viewing window you used for the graphs of the revenue and expense functions in Lesson 2-5. The maximum profit is at the vertex of the parabola. Graphing calculators have a maximum feature that determines the value of the maximum point of a function. Let x represent the price, p. The maximum is 323,125 when $x = 33.50$.



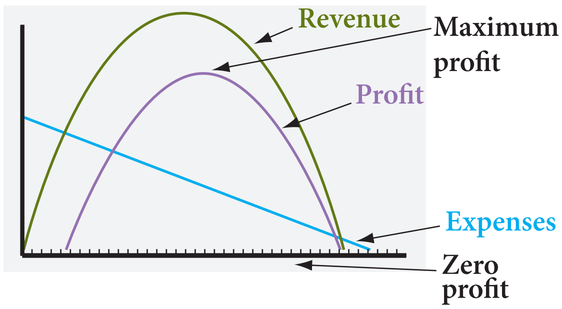
CHECK YOUR UNDERSTANDING

Sketch the graph. Identify the points where zero profit is made. Explain your reasoning.

EXAMPLE 3

Graph the revenue, expense, and profit functions on the same coordinate plane. Interpret the zero-profit points, the maximum profit, and how the functions relate to each other.

SOLUTION The maximum profit is at the vertex of the profit graph. The price at this point yields the greatest difference between the revenue and expense functions. There are two zero-profit points on the graph of the profit function. No profit is made at the selling price where expense is equal to revenue. Visually, this is the price at which the revenue and expense functions intersect.



■ CHECK YOUR UNDERSTANDING

Must maximum profit occur at the same price as the maximum revenue?

EXAMPLE 4

Algebraically, determine the price of the Picasso Paints product that yields the maximum profit.

SOLUTION To determine the maximum profit algebraically, recall that the maximum value occurs on the axis of symmetry. For the profit function $P = -500x^2 + 33{,}500x - 238{,}000$, $a = -500$, $b = 33{,}500$, and $c = -238{,}000$. The x- intercept of the axis of symmetry is determined by calculating $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-33{,}500}{2(-500)} = \frac{-33{,}500}{-1{,}000} = 33.5$$

The axis of symmetry intercepts the horizontal axis at 33.5. To determine the height of the parabola at 33.5, evaluate the function when $p = 33.5$.

Quadratic equation	$P = -500p^2 + 33{,}500p - 238{,}000$
Substitute 33.5 for *p*.	$P = -500(33.5)^2 + 33{,}500(33.5) - 238{,}000$
Simplify.	$P = 323{,}125$

At \$33.50, a maximum profit of \$323,125 is attained.

■ CHECK YOUR UNDERSTANDING

Use the profit function from Example 1 Check Your Understanding. Determine the price to the nearest cent that yields the maximum profit.

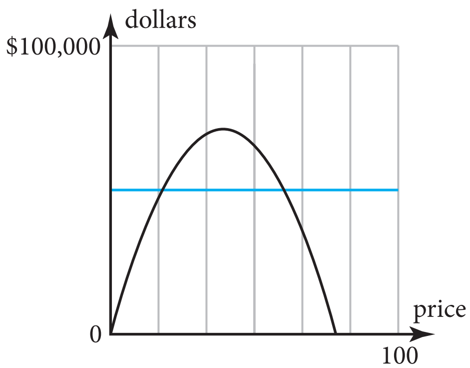
Applications

Nobody ever lost money taking a profit.
Bernard Baruch, Businessman

1. How might the quote apply to what has been outlined in this lesson?

Examine each of the graphs in Exercises 2–5. In each case, the blue graph represents the expense function and the black graph represents the revenue function. Describe the profit situation in terms of the expense and revenue functions.

2.



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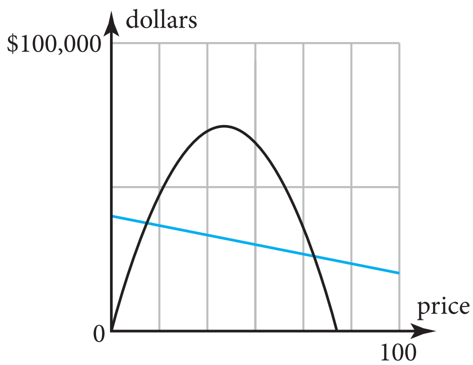
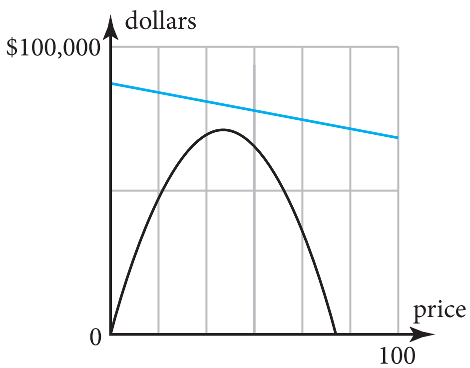


4.



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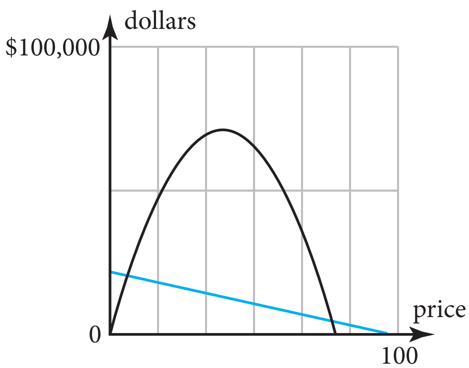


In Exercises 6–9, write the profit function for the given expense and revenue functions.

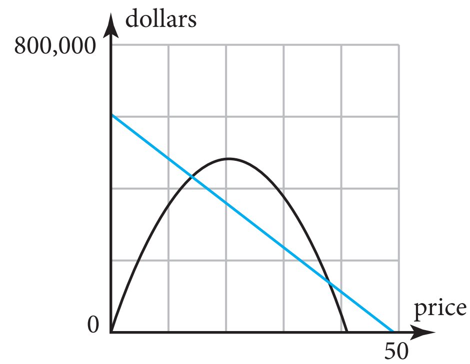
6. $E = -20{,}000p + 90{,}000$
$R = -2{,}170p^2 + 87{,}000p$

7. $E = -6{,}500p + 300{,}000$
$R = -720p^2 + 19{,}000p$

8. $E = -2{,}500p + 80{,}000$
$R = -330p^2 + 9{,}000p$

9. $E = -12{,}500p + 78{,}000$
$R = -1{,}450p^2 + 55{,}000p$

10. Examine the revenue (black) and expense (blue) functions. Estimate the price at the maximum profit. Explain your reasoning.



11. The expense and revenue functions yield a profit function, but the equation can represent no profit made for any price. One of the profit functions in Exercises 6–9 models such a situation.

a. Determine which profit function models a no profit situation.

b. What does a profit function look like when no profit can be made?

12. Determine the maximum profit and the price that would yield the maximum profit for each.

a. $P = -400p^2 + 12{,}400p - 50{,}000$

b. $P = -370p^2 + 8{,}800p - 25{,}000$

c. $P = -170p^2 + 88{,}800p - 55{,}000$

13. Greenyard's manufactures and sells yard furniture made out of recycled materials. It is considering making a lawn chair from recycled aluminum and fabric products. The expense and revenue functions are $E = -1{,}850p + 800{,}000$ and $R = -100p^2 + 20{,}000p$.

a. Determine the profit function.

b. Determine the price, to the nearest cent, that yields the maximum profit.

c. Determine the maximum profit, to the nearest cent.

14. Mountaineer Products Incorporated manufactures mountain-bike accessories. It is considering making a new type of reflector for night biking. The expense and revenue functions are $E = -450p + 90{,}000$ and $R = -185p^2 + 9{,}000p$.

a. Determine the profit function.

b. Determine the price, to the nearest cent, that yields the maximum profit.

c. Determine the maximum profit, to the nearest cent.

15. Business Bargains manufactures office supplies. It is considering selling sticky-notes in the shape of the state in which they will be sold. The expense and revenue functions are $E = -250p + 50{,}000$ and $R = -225p^2 + 7{,}200p$.

a. Determine the profit function.

b. Determine the price, to the nearest cent, that yields the maximum profit.

c. Determine the maximum profit, to the nearest cent.

16. FlipFlops manufactures beach sandals. Their expense and revenue functions are $E = -300p + 32{,}000$ and $R = -275p^2 + 6{,}500p$.

a. Determine the profit function.

b. Determine the price, to the nearest cent, that yields the maximum profit.

c. Determine the maximum profit, to the nearest cent.

All models are wrong. Some models are useful.
George Box, Statistician and Quality Control Pioneer

Mathematically Modeling a Business

2-8

Key Terms

- dependence
- transitive property of dependence

Objectives

- Recognize the transitive property of dependence as it is used in a business model.
- Use multiple pieces of information, equations, and methodologies to model a new business.

How can you mathematically model a start-up business?

Statistics are necessary in making business decisions. The relationship between supply and demand; expense, revenue, and profit; and breakeven points must be analyzed. All of the factors may be modeled together to assess business situations.

Dependence is used in many contexts.

- In sports, baseball fans *depend* on the manager of the team to lead the team to victory. In turn, the manager of the team *depends* on the players to work hard to succeed.
- In politics, voters *depend* on their local elected officials to represent them. Local elected officials *depend* on state government officials to give them the support they need to represent the voters.
- When starting a business venture, expenses *depend* on the demanded quantity of the product. Demand *depends* on the price of the product.

These are a few examples of dependence in daily life. In the first example, if the fans depend on the manager and the manager depends on the players, the fans depend on the players as well. In the second example, if the voters depend on the local elected officials and the local officials depend on the state officials, the voters depend upon the state officials, too. Finally, in the last example, if expenses depend on quantity and quantity depends on price, expenses also depend on price. These are examples of the **transitive property of dependence**.

If x *depends on* y and y *depends on* z, it follows that x *depends on* z.

The determination of the price that yields the maximum profit depends on a number of factors that precede it. Mathematical modeling using algebra is an illustration of the use of the transitive property in business.

Skills and Strategies

Mathematically modeling a business situation helps you better understand the relationships between and among variables.

EXAMPLE 1

Determine the expense, E, for production of an item when the price, p, is \$60 given $E = 50q + 80{,}000$ and $q = 80p + 100{,}000$.

SOLUTION E depends on q, and q depends on p. To find the value of E at a particular price, p, use substitution or express the expense equation directly in terms of price. Both methods illustrate the transitive property of dependence.

The first method uses the price given to find a value for q that can be substituted into the equation for E.

Method 1

Use the equation for q.	$q = 80p + 100{,}000$
Substitute 60 for p.	$q = 80(60) + 100{,}000$
Simplify.	$q = 104{,}800$
Use the equation for E.	$E = 50q + 80{,}000$
Substitute 104,800 for q.	$E = 50(104{,}800) + 80{,}000$
Simplify.	$E = 5{,}320{,}000$

The second method substitutes the value of q in terms of p into the expense equation, and then uses the price given to find the value of E.

Method 2

Use the equation for E.	$E = 50q + 80{,}000$
Substitute $80p + 100{,}000$ for q.	$E = 50(80p + 100{,}000) + 80{,}000$
Distribute.	$E = 4{,}000p + 5{,}000{,}000 + 80{,}000$
Calculate.	$E = 4{,}000p + 5{,}080{,}000$
Substitute 60 for p.	$E = 4{,}000(60) + 5{,}080{,}000$
Simplify.	$E = 5{,}320{,}000$

The expenses total \$5,320,000.

CHECK YOUR UNDERSTANDING

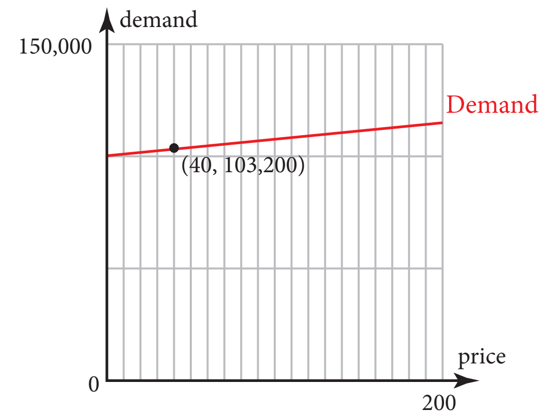
Determine the expense, E, for production of an item when the price, p, is \$42 given $E = 50q + 80{,}000$ and $q = 80p + 100{,}000$.

EXTEND YOUR UNDERSTANDING

Suppose $A = 20x + 30$, $x = 30y + 40$, and $y = 40z + 50$. Describe how the value of A depends on the value of z.

EXAMPLE 2

A business model uses a summary analysis of the situation in terms of dependent variables. Examine the three graphs of a business situation for the production of widgets. The graphs depict numerical information that is needed to complete the summary analysis.

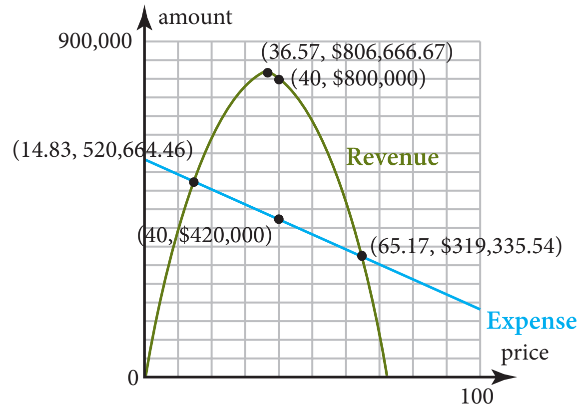


Write the summary analysis in terms of the data presented in the graphs. The summary analysis should have the following format.

In summary, to start this business, __?__ widgets should be manufactured. Each should be sold for $__?__.

The breakeven point is reached at a price of $__?__ or $__?__, but a profit is made at any price between those prices.

At the selling price, there is revenue of $__?__ and expenses of $__?__, resulting in a profit of $__?__.



SOLUTION In summary, to start this business, 103,200 widgets should be manufactured. Each should be sold for $40.

This is asking the price that will yield the maximum profit and the quantity demanded at that price.

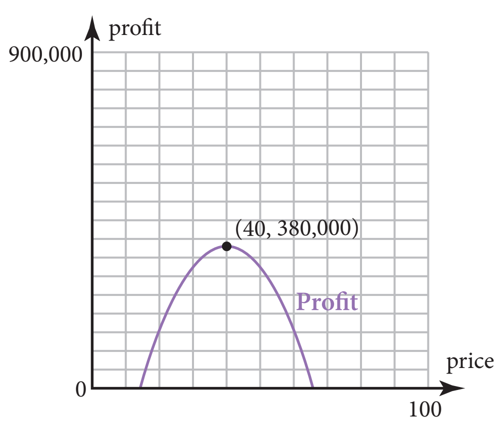
The breakeven point is reached at a price of $14.83 or $65.17, but a profit is made at any price between those prices. These amounts can be found at the intersection points of the expense and revenue functions.

At the selling price, there is revenue of $800,000 and expenses of $420,000, resulting in a profit of $380,000.

Use the price that yields the maximum profit, $40, in both the expense and profit functions.

CHECK YOUR UNDERSTANDING

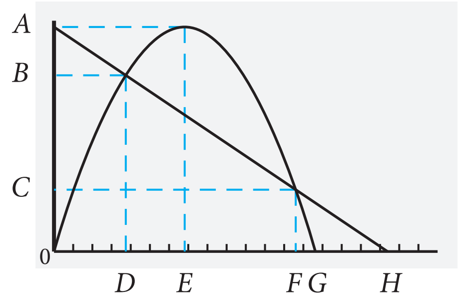
Use the points labeled on the graphs to show that the maximum profit at the selling price is the difference between the revenue and expense values at that price.



Applications

All models are wrong. Some models are useful.
George Box, Statistician and Quality Control Pioneer

1. How might those words apply to what has been outlined in this lesson?
2. Use the following graph to answer the questions below. Let Y_1 = cost function and Y_2 = revenue function.



   a. Explain the significance of point (D, B).
   b. Explain the significance of point (E, A).
   c. Explain the significance of point (F, C).
   d. Explain the significance of point (G, 0).
   e. Explain the significance of point (H, 0).
   f. Where do you think the maximum profit might occur?

3. Draw a graph of cost function Y_1 and revenue function Y_2 that meet the following criteria.

   Maximum revenue: $125,000

   Breakeven points: $40 and $160

   Price at which no revenue is made: $200

   Maximum expenses: $105,000

Use the following situation to answer Exercises 4–20.
A company produces a security device known as Toejack. Toejack is a computer chip that parents attach between the toes of a child, so parents can track the child's location at any time using an online system. The company has entered into an agreement with an Internet service provider, so the price of the chip will be low. Set up a demand function—a schedule of how many Toejacks would be demanded by the public at different prices.

4. As the price increases, what is expected to happen to the quantity demanded?
5. The horizontal axis represents price, and the vertical axis represents quantity. Does the demand function have a positive or negative slope? Explain.

6. The company decides to conduct a market research survey to determine the best price for the device. The variable p represents price, and q represents quantity demanded. The points are listed as (p, q).

(14, 8,200), (11, 9,100), (16, 7,750), (16, 8,300), (14, 8,900),
(17, 7,100), (13, 8,955), (11, 9,875), (11, 9,425), (18, 5,825)

Make a scatterplot of the data. Does the data look like it has a linear form?

7. Find the regression equation. Remember the quantity demanded, q, is the dependent variable. Round the slope and y–intercept to the nearest hundredth.

8. Is the linear regression line a good predictor? Explain.

9. Examine the data to see if there is any relationship between the price and the quantity demanded. Determine the correlation coefficient between price and demand, rounded to nearest hundredth. Explain the significance of the correlation coefficient.

10. Fixed costs are \$24,500, and variable costs are \$6.12 per Toejack. Express expenses, E, as a function of q, the quantity produced.

11. Express the revenue, R, in terms of p and q.

12. Express the revenue, R, in terms of p.

13. Use the transitive property of dependence to express expense, E, in terms of p. Round to the nearest hundredth.

14. Graph the expense and revenue functions.
   a. Determine an appropriate maximum horizontal-axis value.
   b. Determine an appropriate maximum vertical-axis value.
   c. Sketch the graphs of the expense and revenue functions.

15. Determine the coordinates of the maximum point on the revenue graph. Round the coordinates to the nearest hundredth.

16. Determine the breakeven points. Round to the nearest hundredth.

17. Express the profit, P, in terms of p.

18. Graph the profit function. Determine the coordinates of the maximum point of the profit graph. At what price, p, is profit maximized? Round to the nearest cent. This will be the price at which Toejack will sell!

19. Write the business summary statement by filling in the blanks.

In summary, to start this business, __**a.**__ Toejacks should be manufactured. Each should be sold at \$__**b.**__. The breakeven point is reached at a price of \$__**c.**__ or \$__**d.**__, but a profit is made at any price between those prices. At the selling price, there is a revenue of \$__**e.**__ and expenses of \$__**f.**__, resulting in a profit of \$__**g.**__.

20. If shares of stock are sold with an initial value of \$5 each, how many shares must be sold to get enough money to start the business?

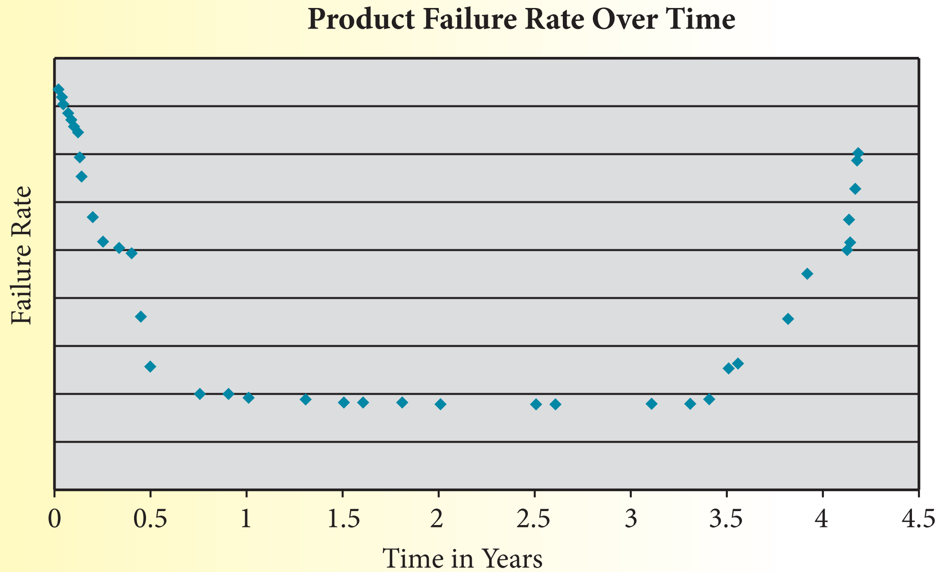
CHAPTER 2

Assessment

Real Numbers

You Write the Story!!

Examine the scatterplot below. It depicts the rate of failures reported for a single product. A product failure is when a product fails to do what it was manufactured to do. The diagram below is called a "bathtub curve." Write a short newspaper-type article centered on this graph, based on a hypothetical situation you create. An electronic copy of the graph can be found at school.cengage.com/math/financialalgebra. Copy and paste it into your article.



Reality Check

1. Corporations pride themselves on the recognition of their logos. This project involves creating a game. Use the library or the Internet to find logos of twelve popular brands. Look only for logos that do not include the corporate name. Paste the logos on a sheet of paper. With teacher approval, give the class the sheet of logos, and ask students to write the brand name next to each logo.

2. Use the Internet or library to find more details on the failures of the three products mentioned at the beginning of the chapter: New Coke, the Edsel, and the Sony Betamax. Write a report on the specific reasons for each failure.

3. Find an example of a product not mentioned in this chapter that failed. Prepare a report detailing the story behind the product's invention and its failure.

4. Interview a local businessperson. Ask for examples of fixed and variable expenses. Do not ask for amounts as that information is private. Make a comprehensive list. Also ask about the history of the business and how he or she became involved in it.

5. Trace the history of the portable, hand held calculator from inception to the present. Include brand names, sales figures, features, pictures, model names, etc. Prepare the findings in a report.

6. On January 1, 1962, the Beatles, looking for their first recording contract, had an audition with Decca records. They were turned down, and legend has it that Decca said " . . . guitar groups are on the way out." Use the library and/or the Internet to research the international sales of Beatles recordings over the past 5 decades and any other financially-related Beatles facts. Compare Decca's decision to the product failures discussed in this chapter. What are the similarities and differences? Prepare the findings in a report.

7. How prevalent are brand names in society? With teacher approval, get two copies of the same newspaper or magazine. Divide the class into two teams. First, give the number of pages in that publication. Next, have each team predict the number of times a brand name will be used. Have each team read through the entire publication (teams may split up the reading) and count every brand name that is written in the publication. If a name is written multiple times, count every time it is written. Write in the newspapers or magazines to easily keep track. Compare findings, and if they differ, find out where they differ. How did the findings compare to the predictions?

8. Search the Internet for three different websites that offer breakeven calculators. Compare and contrast them. What can one do that the other(s) can't? Write up a recommendation for the use of one of the three calculators and justify the recommendation.

9. Look either in the Business section of a newspaper or online to identify three companies that claim to have made a profit over a period of time. Research how they each define *profit,* and state the monetary amounts that each is quoting as their profit for the time period.

10. Many children set up lemonade stands in front of their homes when they were younger. What would the fixed costs of a lemonade stand be? What would be the variable costs? At what price would a glass of lemonade be sold? Explain how to decide the price. Estimate revenues and profit over a one-week period.

Dollars and Sense — Your Financial News Update

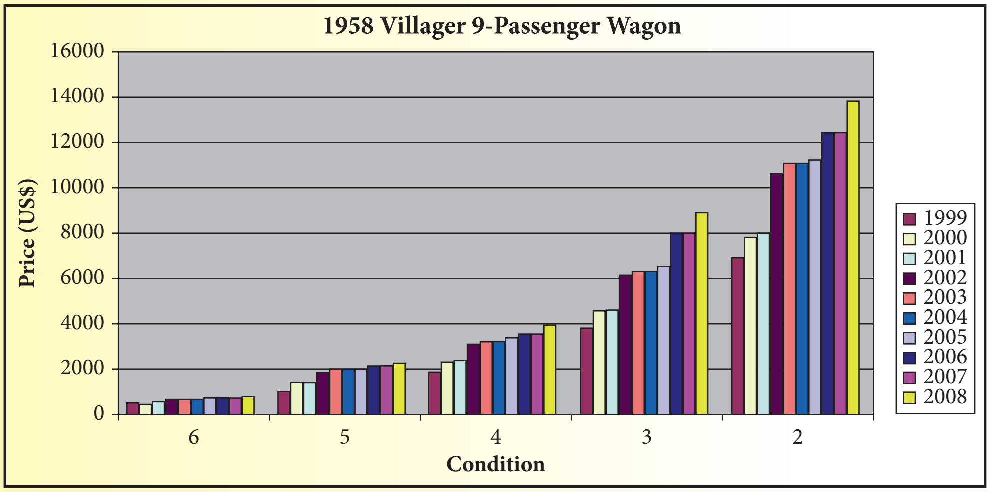
Go to www.cengage.com/school/math/financialalgebra where you will find a link to a website containing current issues about modeling a business.

Really? Really! REVISITED

Ford Motor Company predicted 1958 Edsel sales to exceed 200,000 cars. Only 63,107 of the 1958 models were sold. The 1960 model saw only 2,846 cars sold. During the three years of production, approximately 110,847 Edsels were manufactured.

Today, less than 6,000 of them remain. As a rare curiosity, they are prized by many car collectors and have grown in value.

The following bar graph gives the values of a 1958 Edsel Villager 9-Passenger Station Wagon during the years 1999–2008. Because the value depends on the condition of the car, car appraisers rate the condition of a classic car. On this graph, Condition 2 is the best condition and Condition 6 is the poorest condition.



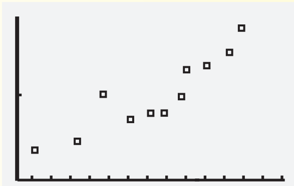
Source: Edsel.com, http://www.edsel.com/charts/e58vil9p.jpg

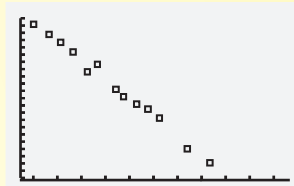
1. Give an approximate value for the 1958 Villager in a condition of 6 in 2008.
2. Give an approximate value for the 1958 Villager in a condition of 2 in 2008.
3. Approximate the difference between a 1958 Villager in a condition of 6 and in a condition of 2 in 2008.
4. Approximate the difference between a 1958 Villager in a condition of 2 in 1999 and in 2008.

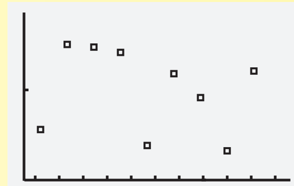
Look at the general trend of the graph. If a failure is kept for long enough, it may prove to be a good decision! Go online and look for pictures of the original 1958 Edsel. Also find photos of the 1958 Chevrolet, the 1958 Chrysler, and the 1958 Buick. Does the Edsel look very different from the other cars? Consumers thought it did! And that's reality!

Applications

1. Examine each scatterplot. Identify each as showing a positive correlation, a negative correlation, or no correlation.

a.


b.


c.


2. If each set of bivariate data has a causal relationship, determine the explanatory and response variables for each set of data.

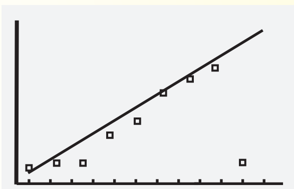
a. number of hours spent reading and page number on which you are reading

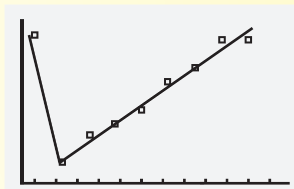
b. calories burned and number of minutes of exercising

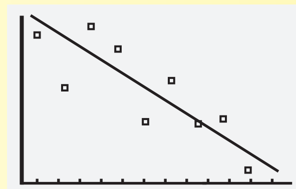
c. amount paid as income tax and the amount of a paycheck

d. pounds of hamburger used to make a meatloaf and number of people that can be fed from the meatloaf

3. Which of the following scatterplots does not show a line of best fit?

a.


b.


c.


4. Describe each of the following correlation coefficients using the terms strong, moderate, or weak and positive or negative.

a. $r = 0.17$

b. $r = -0.62$

c. $r = -0.88$

d. $r = 0.33$

e. $r = 0.49$

f. $r = -0.25$

g. $r = 0.91$

5. The graph shows supply and demand curves for the newest game controller for the VVV video game system.

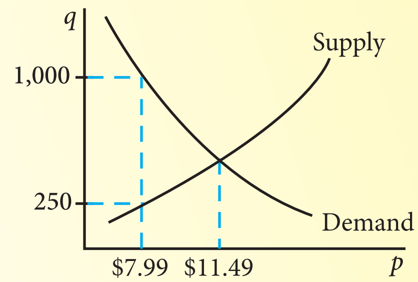
a. What is the equilibrium price?

b. What will happen if the price is set at $7.99?

c. How many game controllers are demanded at a price of $7.99?

d. How many game controllers are supplied at a price of $7.99?

e. What will happen if the price is set at $12.99?



6. The demand function for a certain product is $q = -300p + 10{,}000$. The fixed expenses are \$500,000 and the variable expenses are \$2 per item produced.
   a. Express the expense function in terms of q.
   b. Use substitution to express the expense function in terms of p.
   c. If the price is set at \$20, what quantity will be demanded?
   d. If price is set at \$25, find the demand. Use these numbers in a complete sentence that explains what they mean.
   e. If $q = 1{,}000$ widgets, find E, the cost (expense) of producing them. Use both numbers in a complete sentence to explain what they mean.
   f. If the price is set at $p = \$15$, how much will it cost to produce the correct number of widgets? Use these numbers in a complete sentence to explain what they mean.

7. Let the expense function for a particular item be defined as $y = -1{,}950x + 53{,}000$. Let the revenue function be defined as $y = -450x^2 + 10{,}000x$, where x represents price in each equation. Use this window.
   a. Graph the two functions.
   b. Determine the x- and y-coordinates of the first point where the two graphs intersect. Round those values to the nearest whole number.
   c. Explain the significance of this point in the context of expense, revenue, and price.

```
WINDOW
  Xmin=0
  Xmax=30
  Xscl=1
  Ymin=0
  Ymax=70000
  Yscl=1
  Xres=1
```

8. Express the revenue equation in terms of price given the demand function.
   a. $q = -900p + 120{,}000$
   b. $q = -88{,}000p + 234{,}000{,}000$

9. The expense function for a widget is $E = -3{,}000p + 250{,}000$. The revenue function is $R = -600p^2 + 25{,}000p$.
   a. Write the profit equation in simplified form.
   b. Use the axis of symmetry formula to determine the maximum profit price and the maximum profit.

Use this situation to answer Exercises 10–25.

A company is interested in producing and selling a new device called an eyePOD (eyewear personal optical device). The eyePOD is an MP3 and video player built into a pair of sunglasses. The user can listen to music from the small earphones and watch videos projected on the screen behind the glasses.

10. As the price of the eyePOD increases, what is expected to happen to the quantity demanded?

11. The horizontal axis represents price, and the vertical axis represents quantity. Does the demand function have a positive or negative slope? Explain.

12. The market research department conducted consumer surveys at college campuses and reported its results. In these ordered pairs, the first number represents price, p, and the second number represents the quantity demanded, q. The points are listed as (p, q).

(300, 10,000), (325, 8,900), (350, 8,800), (375, 8,650), (400, 6,700), (425, 6,500), (450, 5,000), (475, 4,500), (500, 4,450), (525, 3,000)

Make a scatterplot of the data.

13. What is the correlation coefficient? Round it to the nearest hundredth. Is this line a good predictor? Explain.

14. Write the regression equation. Remember that the demanded quantity, q, is the dependent variable. Round the slope and y-intercept to the nearest hundredth.

15. The accounting department has calculated that this could be the biggest product to hit the market in years. It anticipates the fixed costs to be \$160,000 and the variable cost to be \$150 per eyePOD. Express expenses, E, as a function of q, the quantity produced.

16. Express the revenue, R, in terms of p and q.

17. Express the revenue, R, in terms of p.

18. Recall the transitive property of dependence. Express expenses, E, in terms of p. Round to the nearest hundredth.

19. Graph the expense and revenue functions.
a. Determine an appropriate maximum horizontal-axis value.
b. Determine an appropriate maximum vertical-axis value.
c. Sketch the graphs of the expense and revenue functions.

20. Determine the coordinates of the maximum point on the revenue graph. Round to the nearest hundredth.

21. Determine the breakeven points. Round to the nearest hundredth.

22. Express the profit, P, in terms of p.

23. Graph the profit function. Determine the coordinates of the maximum point of the profit graph. At what price, p, is profit maximized? Round to the nearest cent. This will be the price at which one eyePOD will sell!

24. Write the business summary statement by filling in the blanks.

In summary, to start this business, __**a.**__ eyePODS should be manufactured. Each should be sold at \$__**b.**__. The breakeven point is reached at a price of \$__**c.**__ or \$__**d.**__, but a profit is made at any price between those prices. At the selling price, there is a revenue of \$__**e.**__ and expenses of \$__**f.**__, resulting in a profit of \$__**g.**__.

25. If shares of stock are sold with an initial value of \$10 each, how many shares must be sold to get enough money to start the business?