Chapter 5 Chapter Review

Probability is used to describe the long-term regularity that results from many repetitions of the same chance process. To calculate the probability of a particular outcome without any formulas, we can use random digits to simulate many repetitions. Section 5.1 discussed randomness and simulation.

Probability models assign probabilities to outcomes. Any such model must obey the rules of probability. Section 5.2 presented these rules, along with two useful tools for describing outcomes of chance behavior—Venn diagrams and two-way tables.

In some cases, we have partial information about the result of a chance process. This leads to the study of conditional probability in Section 5.3. Tree diagrams help organize our thinking about conditional probability in situations that involve a sequence of outcomes.

Chapter Review Exercises

These exercises are designed to help you review the important ideas and methods of the chapter. Relevant learning objectives are provided in bulleted form before the relevant exercise(s).

- Interpret probability as a long-run relative frequency.

R5.1. Rainy days The TV weatherman says, "There's a 30% chance of rain tomorrow." Explain what this statement means.

- Use simulation to model chance behavior.

R5.2. Click it or else From police records, it has been determined that 15% of drivers stopped for routine license checks are not wearing seat belts. If a police officer stops 10 vehicles, how likely is it that two consecutive drivers won't be wearing their seat belts?

(a) Describe the design of a simulation to estimate this probability. *Explain clearly* how you will use the partial table of random digits below to carry out your simulation.

(b) Carry out three repetitions of the simulation. Copy the random digits below onto your paper. Then mark on or directly above the table to show your results.

29077	14863	61683	47052	62224	51025
95052	90908	73592	75186	87136	95761
27102	56027	55892	33063	41842	81868
43367	49497	72719	96758	27611	91596

R5.3. Sandblasters Eight teams of the world's best sand sculptors gathered for an "extreme" competition. At five points during the competition, a randomly selected team's sculpture was blown up. That team was then forced to build a new sculpture from scratch. No team could be selected for destruction more than once. Last year's winning team did not have a sculpture blown up either last year or this year. Other teams were suspicious. Should they be? Design and carry out a simulation to answer this question. (Team Sanding Ovation won the contest with their "zipperhead" sculpture.)

- Describe a probability model for a chance process.

R5.4. Weird dice Nonstandard dice can produce interesting distributions of outcomes. Suppose you have two balanced, six-sided dice. Die A has faces with 2, 2, 2, 2, 6, and 6 spots. Die B has three faces with 5 spots and three faces with 1 spot. Imagine that you roll both dice at the same time.

(a) Find a probability model for the difference (Die A − Die B) in the total number of spots on the up-faces.

(b) Which die is more likely to roll a higher number? Justify your answer.

- Use basic probability rules, including the complement rule and the addition rule for mutually exclusive events.

R5.5. Race and ethnicity The Census Bureau allows each person to choose from a long list of races. That is, in the eyes of the Census Bureau, you belong to whatever race you say you belong to. Hispanic (also called Latino) is a separate category. Hispanics may be of any race. If we choose a resident of the United States at random, the Census Bureau gives these probabilities:[25]

	Hispanic	Not Hispanic
Asian	0.001	0.044
Black	0.006	0.124
White	0.139	0.674
Other	0.003	0.009

(a) Verify that this is a legitimate assignment of probabilities.

(b) What is the probability that a randomly chosen American is Hispanic?

(c) Non-Hispanic whites are the historical majority in the United States. What is the probability that a randomly chosen American is *not* a member of this group?

(d) Explain why $P(\text{white or Hispanic}) \neq P(\text{white}) + P(\text{Hispanic})$. Then find $P(\text{white or Hispanic})$.

- Use a Venn diagram to model a chance process involving two events.
- Use the general addition rule to calculate $P(A \cup B)$.

R5.6. Getting into college Ramon has applied to both Princeton and Stanford. According to his counselor, the probability that Princeton will admit him is 0.4, the probability that Stanford will admit him is 0.5, and the probability that both will admit him is 0.2.

(a) Make a Venn diagram to model this chance process.

(b) What is the probability that neither university admits Ramon?

(c) What's the probability that he gets into at least one of the two schools? Use the general addition rule to confirm that your answer is correct.

- When appropriate, use a tree diagram to describe chance behavior.
- Use the general multiplication rule to solve probability questions.
- Compute conditional probabilities.

R5.7. Steroids A company has developed a drug test to detect steroid use by athletes. The test is accurate 95% of the time when an athlete has taken steroids. It is 97% accurate when an athlete hasn't taken steroids. Suppose that the drug test will be used in a population of athletes in which 10% have actually taken steroids. Let's choose an athlete at random and administer the drug test.

(a) Make a tree diagram showing the sample space of this chance process.

(b) What's the probability that the randomly selected athlete tests positive? Show your work.

(c) Suppose that the chosen athlete tests positive. What's the probability that he or she actually used steroids? Show your work.

- Determine whether two events are independent.

R5.8. Mike's pizza You work at Mike's pizza shop. You have the following information about the 7 pizzas in the oven: 3 of the 7 have thick crust, and of these 1 has only sausage and 2 have only mushrooms. The remaining 4 pizzas have regular crust, and of these 2 have only sausage and 2 have only mushrooms. Choose a pizza at random from the oven.

(a) Are the events {getting a thick-crust pizza} and {getting a pizza with mushrooms} independent? Explain.

(b) You add an eighth pizza to the oven. This pizza has thick crust with only cheese. Now are the events {getting a thick-crust pizza} and {getting a pizza with mushrooms} independent? Explain.

- Find the probability that an event occurs using a two-way table.

R5.9. Deer and pine seedlings As suburban gardeners know, deer will eat almost anything green. In a study of pine seedlings at an environmental center in Ohio, researchers noted how deer damage varied with how much of the seedling was covered by thorny undergrowth:[26]

	Deer Damage	
Thorny Cover	**Yes**	**No**
None	60	151
<1/3	76	158
1/3 to 2/3	44	177
>2/3	29	176

(a) What is the probability that a randomly selected seedling was damaged by deer?

(b) What are the conditional probabilities that a randomly selected seedling was damaged, given each level of cover?

(c) Does knowing about the amount of thorny cover on a seedling change the probability of deer damage? Justify your answer.

- When appropriate, use the multiplication rule for independent events to compute probabilities.

R5.10. A random walk on Wall Street? The "random walk" theory of stock prices holds that price movements in disjoint time periods are independent of each other. Suppose that we record only whether the price is up or down each year, and that the probability that our portfolio rises in price in any one year is 0.65. (This probability is approximately correct for a portfolio containing equal dollar amounts of all common stocks listed on the New York Stock Exchange.)

(a) What is the probability that our portfolio goes up for three consecutive years?

(b) What is the probability that the portfolio's value moves in the same direction (either up or down) for three consecutive years?

R5.11. Blood types Each of us has an ABO blood type, which describes whether two characteristics called A and B are present. Every human being has two blood type alleles (gene forms), one inherited from our mother and one from our father. Each of these alleles can be A, B, or O. Which two we inherit determines our blood type. The table shows what our blood type is for each combination of two alleles. We inherit each of a parent's two alleles with probability 0.5. We inherit independently from our mother and father.

Alleles inherited	Blood type
A and A	A
A and B	AB
A and O	A
B and B	B
B and O	B
O and O	O

(a) Hannah and Jacob both have alleles A and B. Diagram the sample space that shows the alleles that their next child could receive. Then give the possible blood types that this child could have, along with the probability for each blood type.

(b) Jennifer has alleles A and O. Jose has alleles A and B. They have two children. What is the probability that at least one of the two children has blood type B? Show your method.

Chapter 5 AP Statistics Practice Test

Section I: Multiple Choice *Select the best answer for each question.*

T5.1. Dr. Stats plans to toss a fair coin 10,000 times in the hope that it will lead him to a deeper understanding of the laws of probability. Which of the following statements is true?

(a) It is unlikely that Dr. Stats will get more than 5000 heads.

(b) Whenever Dr. Stats gets a string of 15 tails in a row, it becomes more likely that the next toss will be a head.

(c) The fraction of tosses resulting in heads should be close to 1/2.

(d) The chance that the 100th toss will be a head depends somewhat on the results of the first 99 tosses.

(e) All of the above statements are true.

T5.2. China has 1.2 billion people. Marketers want to know which international brands they have heard of. A large study showed that 62% of all Chinese adults have heard of Coca-Cola. You want to simulate choosing a Chinese at random and asking if he or she has heard of Coca-Cola. One correct way to assign random digits to simulate the answer is:

(a) One digit simulates one person's answer; odd means "Yes" and even means "No."

(b) One digit simulates one person's answer; 0 to 6 mean "Yes" and 7 to 9 mean "No."

(c) One digit simulates the result; 0 to 9 tells how many in the sample said "Yes."

(d) Two digits simulate one person's answer; 00 to 61 mean "Yes" and 62 to 99 mean "No."

(e) Two digits simulate one person's answer; 00 to 62 mean "Yes" and 63 to 99 mean "No."

T5.3. Choose an American household at random and record the number of vehicles they own. Here is the probability model if we ignore the few households that own more than 5 cars:

Number of cars:	0	1	2	3	4	5
Probability:	0.09	0.36	0.35	0.13	0.05	0.02

A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?

(a) 7% (b) 13% (c) 20% (d) 45% (e) 55%

T5.4. Computer voice recognition software is getting better. Some companies claim that their software correctly recognizes 98% of all words spoken by a trained user. To simulate recognizing a single word when the probability of being correct is 0.98, let two digits simulate one word; 00 to 97 mean "correct." The program recognizes words (or not) independently. To simulate the program's performance on 10 words, use these random digits:

60970 70024 17868 29843 61790 90656 87964 18883

The number of words recognized correctly out of the 10 is

(a) 10 (b) 9 (c) 8 (d) 7 (e) 6

Questions T5.5 to T5.7 refer to the following setting. One thousand students at a city high school were classified according to both GPA and whether or not they consistently skipped classes. The two-way table below summarizes the data.

	GPA		
Skipped Classes	**<2.0**	**2.0–3.0**	**>3.0**
Many	80	25	5
Few	175	450	265

T5.5. What is the probability that a student has a GPA under 2.0?

(a) 0.227 (b) 0.255 (c) 0.450 (d) 0.475 (e) 0.506

T5.6. What is the probability that a student has a GPA under 2.0 or has skipped many classes?

(a) 0.080 (b) 0.281 (c) 0.285 (d) 0.365 (e) 0.727

T5.7. What is the probability that a student has a GPA under 2.0 given that he or she has skipped many classes?

(a) 0.080 (b) 0.281 (c) 0.285 (d) 0.314 (e) 0.727

T5.8. For events A and B related to the same chance process, which of the following statements is true?

(a) If A and B are mutually exclusive, then they must be independent.

(b) If A and B are independent, then they must be mutually exclusive.

(c) If A and B are not mutually exclusive, then they must be independent.

(d) If A and B are not independent, then they must be mutually exclusive.

(e) If A and B are independent, then they cannot be mutually exclusive.

T5.9. Choose an American adult at random. The probability that you choose a woman is 0.52. The probability that the person you choose has never married is 0.25. The probability that you choose a woman who has never married is 0.11. The probability that the person you choose is either a woman or has never been married (or both) is therefore about

(a) 0.77. (b) 0.66. (c) 0.44. (d) 0.38. (e) 0.13.

T5.10. A deck of playing cards has 52 cards, of which 12 are face cards. If you shuffle the deck well and turn over the top 3 cards, one after the other, what's the probability that all 3 are face cards?

(a) 0.001 (b) 0.005 (c) 0.010 (d) 0.012 (e) 0.02

Section II: Free Response *Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

T5.11. Your teacher has invented a "fair" dice game to play. Here's how it works. Your teacher will roll one fair *eight-sided* die, and you will roll a fair *six-sided* die. Each player rolls once, and the winner is the person with the higher number. In case of a tie, neither player wins. The table shows the sample space of this chance process.

	Teacher Rolls							
You Roll	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								

(a) Let A be the event "your teacher wins." Find $P(A)$.

(b) Let B be the event "you get a 3 on your first roll." Find $P(A \cup B)$.

(c) Are events A and B independent? Justify your answer.

T5.12. Three machines—A, B, and C—are used to produce a large quantity of identical parts at a factory. Machine A produces 60% of the parts, while Machines B and C produce 30% and 10% of the parts, respectively. Historical records indicate that 10% of the parts produced by Machine A are defective, compared with 30% for Machine B and 40% for Machine C.

(a) Draw a tree diagram to represent this chance process.

(b) If we choose a part produced by one of these three machines, what's the probability that it's defective? Show your work.

(c) If a part is inspected and found to be defective, which machine is most likely to have produced it? Give appropriate evidence to support your answer.

T5.13. Researchers are interested in the relationship between cigarette smoking and lung cancer. Suppose an adult male is randomly selected from a particular population. The following table shows the probabilities of some events related to this chance process:

Event	Probability
Smokes	0.25
Smokes and gets cancer	0.08
Does not smoke and does not get cancer	0.71

(a) Find the probability that the individual gets cancer given that he is a smoker. Show your work.

(b) Find the probability that the individual smokes or gets cancer. Show your work.

(c) Two adult males are selected at random. Find the probability that at least one of the two gets cancer. Show your work.

T5.14. Based on previous records, 17% of the vehicles passing through a tollbooth have out-of-state plates. A bored tollbooth worker decides to pass the time by counting how many vehicles pass through until he sees two with out-of-state plates.[27]

(a) Describe the design of a simulation to estimate the average number of vehicles it takes to find two with out-of-state plates. Explain clearly how you will use the partial table of random digits below to carry out your simulation.

(b) Perform three repetitions of the simulation you described in part (a). Copy the random digits below onto your paper. Then mark on or directly above the table to show your results.

41050	92031	06449	05059	59884	31880
53115	84469	94868	57967	05811	84514
84177	06757	17613	15582	51506	81435
75011	13006	63395	55041	15866	06589