- The solutions of $ax^2 + bx + c = 0$, given by the quadratic formula are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
  - If $b^2 - 4ac > 0$, the equation will have two distinct real roots.
  - If $b^2 - 4ac = 0$, the equation will have two equal real roots.
  - If $b^2 - 4ac < 0$, the equation will have no real roots.

Developing inquiry skills

In the opening scenario for this chapter you looked at how crates of emergency supplies were dropped from a plane. The functions $h(t)$ and $g(t)$ give the height of the crate:

During free fall: $h(t) = -4.9t^2 + 720$

With parachute open: $g(t) = -5t + 670$

How realistic is this model in representing this real-life situation? State at least two advantages of the model, and give at least one criticism.

Suppose a heavier crate was dropped, but its parachute was the same. How would the model be different for this heavier crate?

Suggest suitable functions to model the path of a javelin through the air, or the path of a basketball as it leaves a player's hands and passes through a hoop. What do you need to consider in finding a model for each? How could each model help to make predictions in a real-life scenario?

Chapter review

Click here for a mixed review exercise

1 Sketch each line, marking on your sketch the axial intercepts:

a $y = 2x + 4$

b $y - 3 = -4(x + 2)$

c $2x + 3y - 6 = 0$

2 Find an equation for each line:

a the line which passes through the points $(-4, 2)$ and $(8, -1)$

b the line which is parallel to the line $y = \frac{1}{2}x + 3$ and has y-intercept $(0, -5)$

c the line which is perpendicular to $y = -\frac{2}{3}x + 7$ and passing through the point $(2, 4)$

d the line which is passes through $(-3, -4)$ with gradient 0

3 Consider the function $f(x) = \begin{cases} 2x + 1, & -2 \le x < 1 \\ 3, & 1 \le x \le 3 \end{cases}$

a Find $f(1)$ and $f(2)$. **b** Graph $y = f(x)$.

4 Describe the series of transformations of the graph of $y = f(x)$ that lead to the graph of the given functions.

a $y = 2f(x - 3)$ **b** $y = \frac{1}{2}f(x) + 5$

c $y = -f(x + 2) - 1$ **d** $y = f(3x)$

e $y = f(-x) + 6$

5 In each case, find the indicated features of the graph of $y = f(x)$.

a $f(x) = 2(x - 3)(x + 7)$; x-intercepts, equation of the axis of symmetry

b $f(x) = -3(x - 4)^2 + 2$; equation of the axis of symmetry, vertex

c $f(x) = -x^2 - 4x + 6$; equation of the axis of symmetry, y-intercept

6 The function $f(x) = 3x^2 + 18x + 20$ can be written in the form $f(x) = a(x - h)^2 + k$.

a Find the values of:

i a **ii** h **iii** k

b Write down the vertex of the graph of $y = f(x)$.

c The graph of $y = g(x)$ is a translation of the graph of $y = f(x)$ right 5 units and down 3 units. Find the vertex of the graph of $y = g(x)$.

7 Solve each equation:

a $(x - 3)^2 = 64$ **b** $(x + 2)^2 = 7$

c $x^2 + 14x + 49 = 0$ **d** $x^2 + x - 12 = 0$

e $3x^2 + 4x - 7 = 0$

8 The equation $-x^2 + 3kx - 4 = 0$ has two equal real roots. Find the possible values of k.

9 The y-intercept of the graph of a quadratic function is $(0, -16)$ and the x-intercepts are $(-4,0)$ and $(2,0)$. Find the equation of the function in the form $f(x) = ax^2 + bx + c$ where a, b and c are constants.

10 Solve each equation. Give your answers correct to 3 significant figures.

a $2x^2 - 6x - 5 = 0$ **b** $-x^2 - 3x = 0.5x - 7$

11 The height, h metres above the water, of a stone thrown from a bridge is modelled by the function $h(t) = 18 + 13t - 4.9t^2$, where t is the time in seconds after the stone is thrown.

a Find the initial height from which the stone is thrown.

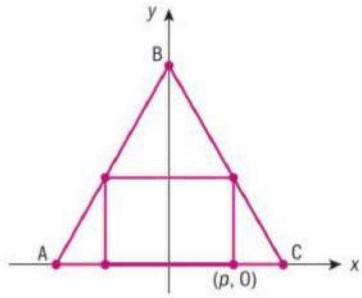
b Find the maximum height reached by the stone.

c Find the amount of time it takes for the stone to hit the water below the bridge.

d Write down the domain of the function h in the context of this real-life scenario.

e Find the length of time for which the height of the stone is greater than 23 m.

12 A rectangle is inscribed in isosceles triangle ABC as shown in the diagram.



The altitude of triangle ABC from B to side AC is 7 cm and AC = 8 cm. The coordinates of one of the vertices of the inscribed rectangle are $(p, 0)$.

a Write down the coordinates of points A, B and C.

b Find the equation of the line passing through points B and C.

c Find the dimensions of the rectangle inscribed in the triangle, in terms of p.

d Write down an expression for the area of the inscribed rectangle in terms of p.

e Find the dimensions of the rectangle with maximum possible area.

f Find the maximum possible area of the inscribed rectangle.

Exam-style questions

13 **P2:** A line has equation $-7x - 12y + 168 = 0$

a Write down the equation of the line in the form $y = mx + c$. (2 marks)

b Given that the line intersects the x-axis at point A and the y-axis at point B, find the coordinates of A and B. (2 marks)

c Calculate the area of triangle OAB. (2 marks)

14 **P2:** **a** Using your GDC, sketch the curve of $y = -2.9x^2 + 4.1x + 5.9$ for $-1 \le x \le 2$. (2 marks)

b Write down the coordinates of the points where the curve intersects the x- or y-axis. (2 marks)

c Write down the range of y. (2 marks)

15 **P2: a** Show that the solutions to the equation $x^2 - 6x - 43 = 0$ may be written in the form $x = p \pm q\sqrt{13}$, where p and q are positive integers. (4 marks)

b Hence, or otherwise, solve the inequality $x^2 - 6x - 43 \leq 0$ (2 marks)

16 **P1:** A function f is defined by $f(x) = 3(x-1)^2 - 18,\ x \in \mathbb{R}$.

a Write $f(x)$ in the form $ax^2 + bx + c$, where a, b and c are constants. (2 marks)

b Find the coordinates of the vertex of the graph of f. (1 mark)

c Find the equation of the axis of symmetry of the graph of f. (1 mark)

d State the range of f. (2 marks)

e The graph of f is translated through the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ to form a curve representing a new function $g(x)$. Find $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. (3 marks)

17 **P1: a** Solve the equation $8x^2 + 6x - 5 = 0$ by factorization. (4 marks)

b Determine the range of values of k for which $8x^2 + 6x - 5 = k$ has no real solutions. (3 marks)

18 **P1:** Consider the function $f(x) = x^2 - 10x + 27,\ x \in \mathbb{R}$.

a Show that the function f can be expressed in the form $f(x) = a(x-h)^2 + k$, where a, h and k are constants. (3 marks)

b Hence write down the coordinates of the vertex of the graph of $y = f(x)$. (1 mark)

c Hence write down the equation of the line of symmetry of the graph of $y = f(x)$. (1 mark)

19 **P1:** The quadratic curve $y = x^2 + bx + c$ intersects the x-axis at $(10, 0)$ and has equation of line of symmetry $x = \frac{5}{2}$.

a Find the values of b and c. (4 marks)

b Hence, or otherwise, find the other two coordinates where the curve intersects the coordinate axes. (2 marks)

20 **P1:** Consider the function $f(x) = 2x^2 - 4x - 8,\ x \in \mathbb{R}$.

a Show that the function f can be expressed in the form $f(x) = a(x-h)^2 + k$, where a, h and k are constants. (3 marks)

b The function $f(x)$ may be obtained through a sequence of transformations of $g(x) = x^2$. Describe each transformation in turn. (3 marks)

21 **P1:** Consider the equation $f(x) = 2kx^2 + 6x + k,\ x \in \mathbb{R}$.

a In the case that the equation $f(x) = 0$ has two equal real roots, find the possible values of k. (4 marks)

b In the case that the equation of the line of symmetry of the curve $y = f(x)$ is $x + 1 = 0$, find the value of k.

c Solve the equation $f(x) = 0$ when $k = 2$. (3 marks)

22 **P1:** A curve $y = f(x)$ passes through the points with coordinates $A(-12,10)$, $B(0,-16)$, $C(2,9)$, and $D(14, -10)$

a Write down the coordinates of each point after the curve has been transformed by $f(x) \mapsto f(2x)$. (4 marks)

b Write down the coordinates of each point after the curve has been transformed by $f(x) \mapsto f(-x) + 3$. (4 marks)

Functions