



Chapter review

Click here for a mixed review exercise



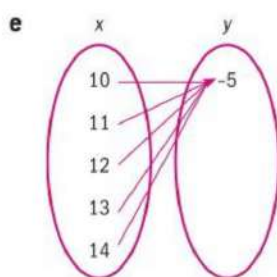
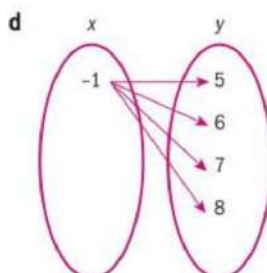
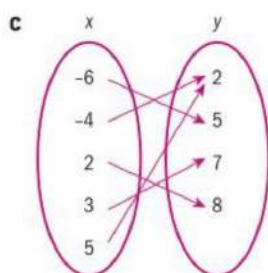
- 1 Decide whether the following relations are functions or not.

a

x	-9	-5	0	1	3	5
y	-8	-6	-9	-1	0	5

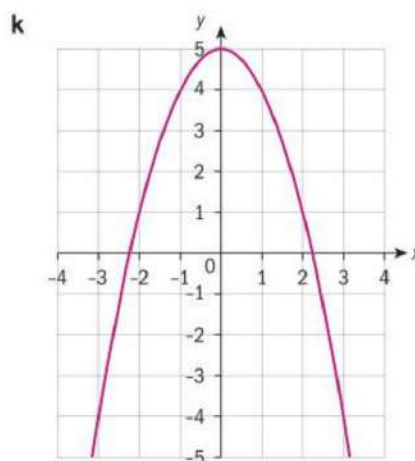
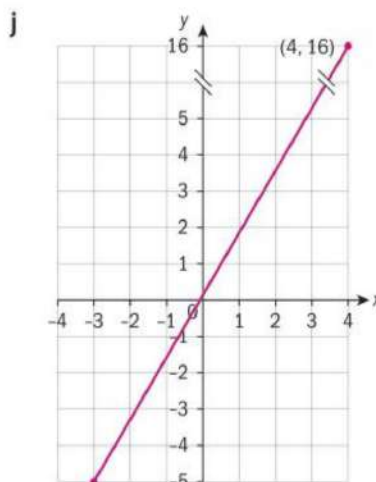
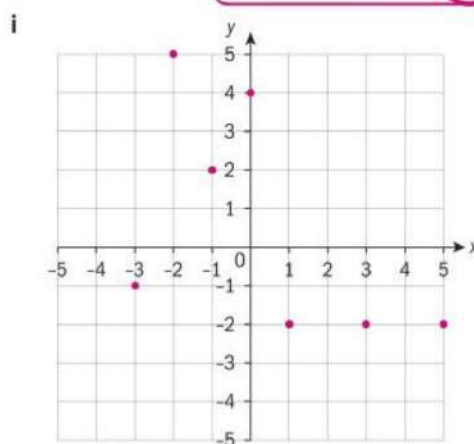
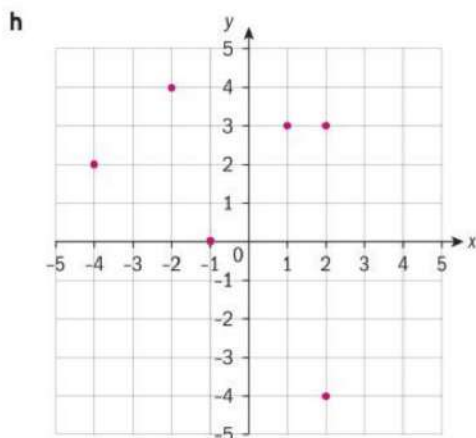
b

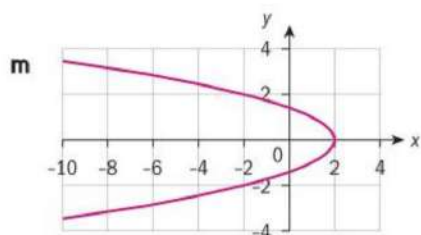
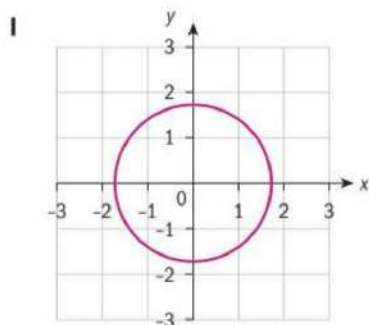
x	-11	-1	-1	11
y	5	7	0	8



f $y = 3x + 2$

g $y = \sqrt{x} + 4$

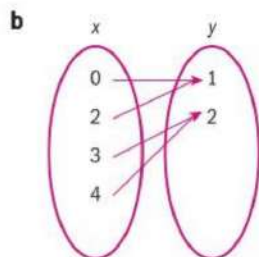




2 State the domain and range for the functions below:

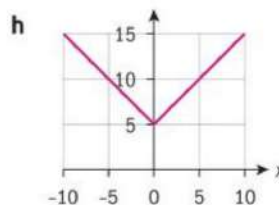
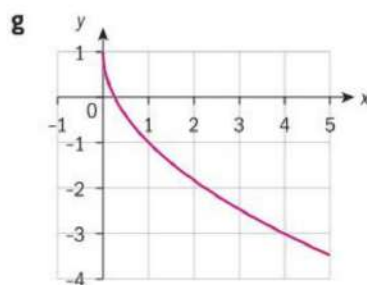
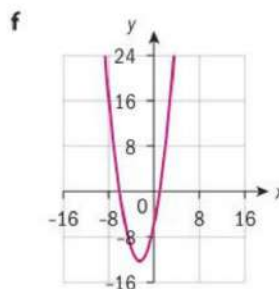
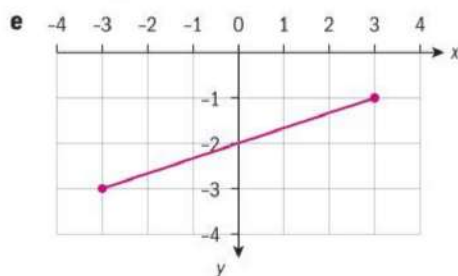
a

x	-5	-1	0	1	4	9
y	-8	0	6	-1	9	1



c $\{(-8, 2), (-5, 3), (0, 2), (1, -2)\}$

d $y = -\frac{1}{2}x + 1$



3 Use the functions $f(x) = x^2 - 6$, $g(x) = -2x$ and $h(x) = -4$ to evaluate:

- a** $f(3)$ **b** $f(-2)$ **c** $g(-6)$
d $f(1) + h(2)$ **e** $2f(0) - 2g(-1)$
f $h(0) \times f(-1)$ **g** $g^{-1}(-3)$ **h** $f(g(x))$
i $f \circ g^{-1}(x)$

4 Use your GDC to help you sketch the graphs of the following functions and state their domain and range:

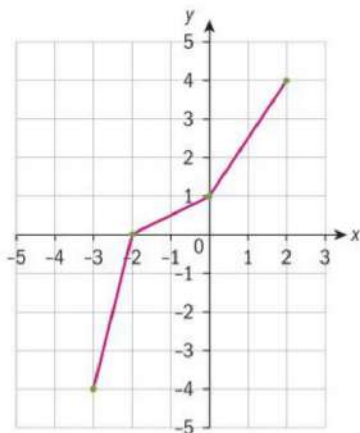
- a** $y = |x^3| - 2$ **b** $y = 2x^4 - 5x^3 + x - 2$

5 For the following pairs of functions, determine algebraically if they are inverses:

- a** $f(x) = -4x + 2$, $g(x) = -\frac{x-2}{4}$
b $f(x) = \frac{1}{2}x - 4$, $g(x) = -\frac{x-2}{4}$
c $f(x) = \frac{1}{2}x^2 + 4$, $g(x) = 2x + \frac{1}{4}$
d $f(x) = \frac{2x+3}{3x-1}$, $g(x) = \frac{3+x}{3x-2}$



- 6 The graph below shows $y = f(x)$ for $-3 \leq x \leq 2$.



- a i Write down the value of $f(-3)$.
 ii Write down the value of $f(2)$.
 b Find the domain of f^{-1} .
 c Sketch the graph of f^{-1} .
- 7 Let $f(x) = (x+2)^3$. Let g be a function such that $(f \circ g)(x) = -8x^6$.
- 8 Let $f(x) = 2\sqrt{x} + x^2$. Let h be a function such that $h(16) = -2$. Find $(f \circ h^{-1})(-2)$.
- 9 Show that $f(x) = -\frac{3}{x}$ is a self-inverse function.

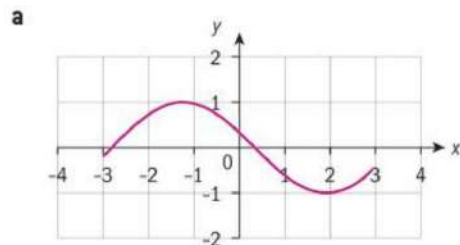


- 12 P1: A function is given by $f(x) = 128x - 15$, $-3 < x < 15$.

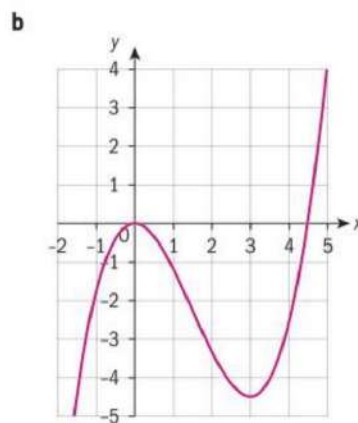
- a Determine the value of $f\left(\frac{3}{2}\right)$. (2 marks)
 b Determine the range of the function f . (4 marks)
 c Determine the value of a such that $f(a) = 1162.6$ (2 marks)



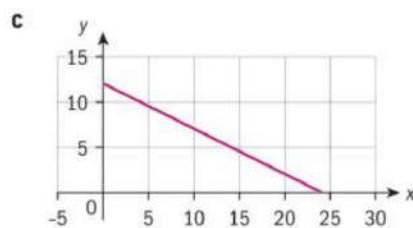
- 13 P1: State i the domain, and ii the range for each of the following functions.



(2 marks)



(2 marks)



(2 marks)

Exam-style questions

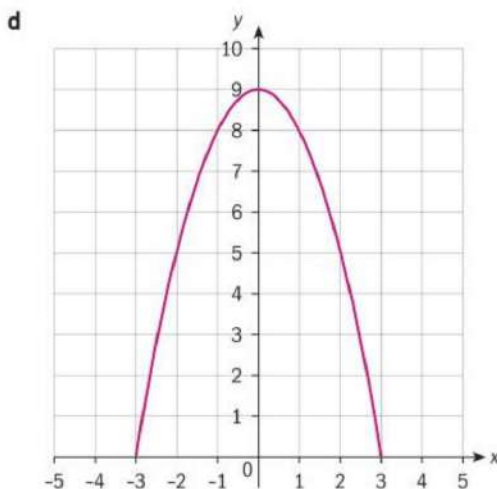


- 10 P1: Find the range of the following functions.

- a $f(x) = 5x + 1$, domain $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$ (2 marks)
 b $f(x) = 4 - 2x$, domain $\{x = -1, 0, 1, 2, 3, 4\}$ (2 marks)
 c $f(x) = x^2$, domain $\{x \in \mathbb{R}, 0 \leq x \leq 10\}$ (2 marks)
 d $f(x) = 250 - 12.5x$, domain $\{x \in \mathbb{R}, 0 \leq x \leq 10\}$ (2 marks)

- 11 P2: $f(x) = 4x - 2$, $x \in \mathbb{R}$ and $g(x) = x^2 - 8x + 15$, $x \in \mathbb{R}$.

- a Find $f(-2)$. (2 marks)
 b Find $g(-2)$. (2 marks)



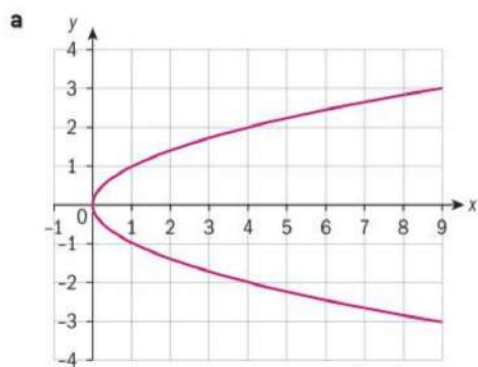
(2 marks)

14 P2: A function $f(x)$ is defined as $f(x) = 3x - 10$

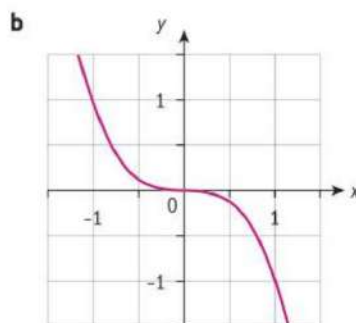
- Given that the range of $f(x)$ is $5 < f(x) < 50$, find the domain of $f(x)$. (4 marks)
- Find $ff(10)$. (3 marks)
- Find the inverse function, $f^{-1}(x)$. (2 marks)
- State the range of the inverse function. (2 marks)



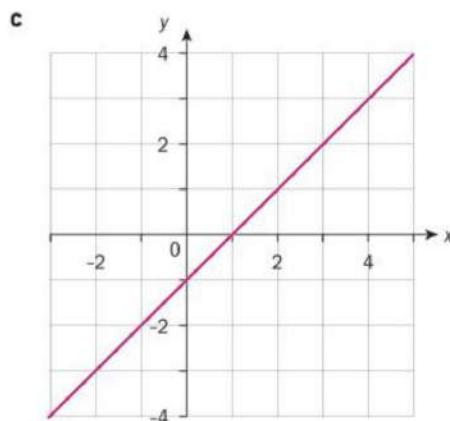
15 P1: State which of the following graphs represent functions, giving reasons for your answers.



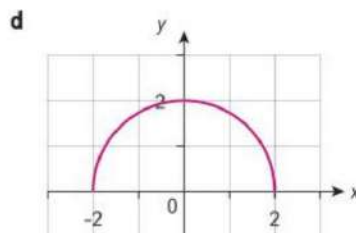
(2 marks)



(2 marks)



(2 marks)



(2 marks)

16 P1: Consider the function $f(x) = \frac{k}{x-1} + 1$, $x > 1$, $x \in \mathbb{R}$, $k \in \mathbb{R}$.

- Show that $f(x)$ is a self-inverse function. (3 marks)
- State the range of f . (2 marks)
- Sketch the graph of $y = f(x)$. (2 marks)

17 P1: Consider the function $f(x) = \frac{1-2x}{3x+6}$, $x \neq -2$, $x \in \mathbb{R}$.

- State the range of f . (1 mark)



b Find the inverse function $f^{-1}(x)$.
(3 marks)

c State the domain and the range of $f^{-1}(x)$.
(2 marks)



18 P1: Consider the functions $f(x) = x^2$, $x \in \mathbb{R}$; and $g(x) = 2x - 1$, $x \in \mathbb{R}$.

a Solve the equation $f(x) = g(x)$.
(3 marks)

b Solve the equation $fg(x) = gf(x)$.
(5 marks)

19 P2: Katie organises a party for her work colleagues. She has a maximum budget of \$1000.

The cost to rent a local hall is \$430 for the evening.

She also has to budget for food, which will cost approximately \$14.50 per person.

a Write down a formula connecting the total cost of the party (\$C) with the number of people attending the party (p).
(2 marks)

b Explain why $C = f(p)$ is a function.
(1 mark)

c Derive an expression for p in terms of C .
(2 marks)

d Hence, calculate the greatest number of people Katie is able to invite.
(2 marks)

e Given that only 16 people attend the party, calculate how much each guest should be charged so that Katie covers her costs.
(3 marks)



20 P1: The function $h(x)$ is defined as $h(x) = \frac{x}{3} + 2$, $x \geq 0$, $x \in \mathbb{R}$.

a State the range of $h(x)$.
(1 mark)

b Derive an expression for the inverse function, $h^{-1}(x)$.
(3 marks)

c Find an expression for $hh(x)$ in the form $hh(x) = ax + b$, where a and b are constants.
(3 marks)

d Solve the equation $h(x) = h^{-1}(x)$.
(2 marks)

e Explain why the equation $h(x) = h^{-1}(x)$ has the same solutions as the equation $h(x) = x$.
(1 mark)

21 P1: The function $p(x)$ is defined by $p(x) = x^2 + 4x - 11$, $x \in \mathbb{R}$.

Given that $p(x) = fgh(x)$ and $f(x) \neq x$, $g(x) \neq x$, $h(x) \neq x$, find possible functions for $f(x)$, $g(x)$, and $h(x)$.
(5 marks)



22 P1: Consider the functions $f(x) = x^2 - 4$, $g(x) = \frac{1}{x+1}$, $h(x) = 2^x$, $x \in \mathbb{R}$.

a Find the range of $f(x)$.
(1 mark)

b Find the range of $g(x)$.
(1 mark)

c Find the range of $h(x)$.
(1 mark)

d Find an expression for $gf(x)$.
(2 marks)

e Solve the equation $gf(x) = 9$.
(2 marks)

f Solve the inequality $gh(x) > \frac{1}{17}$.
(5 marks)



23 P1: Consider the function $p(x) = x^3$, $-2 \leq x \leq 2$, $x \in \mathbb{R}$.

a Find the range of $p(x)$.
(2 marks)

b Find an expression for the inverse function $p^{-1}(x)$.
(2 marks)

c Find all the solutions to the equation $p(x) = p^{-1}(x)$.
(2 marks)

d Sketch the graphs of $y = p(x)$ and $y = p^{-1}(x)$ on the same axes.
(2 marks)



24 P1: a Show that $r(x) = \frac{3x+5}{4x-3}$, $x \in \mathbb{R}$, $x \neq \frac{3}{4}$ is a self-inverse function.
(3 marks)

b Hence determine the value of $r(5)$.
(2 marks)



25 P1: The function $f(x)$ is one-to-one and defined such that $f(x) = x^2 - 6x + 13$, $x \geq k$, $x \in \mathbb{R}$, $k \in \mathbb{R}$.

a Find the least possible value for k .
(3 marks)

b Find an expression for the inverse function $f^{-1}(x)$.
(3 marks)

c State the domain and the range of $f^{-1}(x)$.
(2 marks)



26 P1: Given that $f(x) = x - 3$ and $gf(x) = 2x^2 + 18$, derive an expression for the function $g(x)$.
(4 marks)