K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the connections, the reasoned, logical connections that make mathematics manageable. As a result, implementation of Georgia's Performance Standards places a greater emphasis on problem solving, reasoning, representation, connections, and communication.

Georgia Mathematics Performance Standards Kindergarten

By the end of kindergarten, students will understand small numbers, quantities, and simple shapes in their everyday environment. They will also count, compare, describe and sort objects, and develop a sense of properties and patterns. Students will begin to understand measurement through the direct comparison of objects, money by making fair trades with coins and the concept of time by experiencing a daily schedule.

Instruction and assessment should include the use of manipulatives and appropriate technology. Topics should be represented in multiple ways including concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used in the context of real world phenomena.

NUMBER AND OPERATIONS

Students will correctly represent the number and order of objects using numbers and understand them.

MKN1. Students will connect numerals to the quantities they represent.

- a. Count a number of objects up to 30.
- b. Produce models for number words through ten.
- c. Write numerals through 20 to label sets.
- d. Sequence and identify using ordinal numbers (1st-10th).
- e. Compare two or more sets of objects (1-10) and identify which set is equal to, more than, or less than the other.

- f. Estimate quantities using five and ten as a benchmark. (e.g. 9 is one five and four more. It is closer to 10, which can be represented as one ten or two fives, than it is to five.)
- g. Use informal strategies to share objects equally (divide) between two to three people or sets.
- h. Identify coins by name and value (penny, nickel, dime, and quarter).
- i. Count out pennies to buy items that together cost less than 30 cents.
- j. Make fair trades *using* combinations *involving* pennies and nickels *and* pennies and dimes.

MKN2. Students will use representations to model addition and subtraction.

- a. Use counting strategies to find out how many items are in two sets when they are combined, *separated*, *or compared*.
- b. Build number combinations up to 10 (e.g., 4 and 1, 2 and 3, 3 and 2, 4 and 1 for five) and for doubles to 10 (3 and 3 for six).
- c. Use objects, pictures, numbers, or words to create, solve and explain story problems (*combining*, *separating*, *or comparing*) for two numbers that are each less than 10.

MEASUREMENT

Students will explore quantitative situations involving, *length, capacity, weight, and height by direct comparison. Students will explore time through calendars and schedules*.

MKM1. Students will group objects according to common properties such as

longer/shorter, more/less, taller/shorter, and heavier/lighter.

- a. Compare and order objects on the basis of length.
- b. Compare and order objects on the basis of capacity.
- c. Compare and order objects on the basis of height.
- d. Compare and order objects on the basis of weight.

MKM2. Students will understand the measurement of calendar time.

- a. Know the names of the days of the week, *as well as understand yesterday, today and tomorrow*.
- b. Know the months of the year.
- c. Know the four seasons.

MKM3. Students will understand time as it relates to a daily schedule.

- a. Order daily events.
- b. Tell the time when daily events occur, such as morning, afternoon, and evening.
- c. Know the name of the day of the week when weekly events occur in class

GEOMETRY

Students will recognize and name basic geometric shapes and spatial relationships.

MKG1. Students will correctly name simple two and three-dimensional figures, and recognize them in the environment.

- a. Recognize and name the following basic two-dimensional figures: triangles, *quadrilaterals* (rectangles, squares) and circles.
- b. Recognize and name the following three-dimensional *figures: spheres* and cubes.
- c. Observe concrete objects in the environment and *represent the objects using basic shapes*.
- d. Combine basic *figures to form other basic and complex figures into basic figures; decompose basic and complex figures into basic figures.*
- e. Compare geometric shapes and identify similarities and differences of the following two and three-dimensional shapes: triangles, rectangles, squares, circles, spheres, and cubes.

MKG2. Students will understand basic spatial relationships.

- a. Identify when an object is beside another object, above another object, or below another object.
- b. Identify when an object is in front of another object, behind another object, inside another object, or outside it.

MKG3. Students will identify, create, extend, and transfer patterns from one representation to another using actions, objects, and geometric shapes.

- a. Identify missing elements within *a given pattern*.
- b. Extend a given pattern and recognize similarities in different patterns.
- c. Create a pattern in a different context with attributes similar to a given pattern.

DATA ANALYSIS AND PROBABILITY

Students will pose questions and gather data about themselves and their surroundings.

MKD1. Students will pose information questions, collect data, organize, and *display* results using objects, pictures, and picture graphs.

Process Standards

Each topic studied in this course should be developed with careful thought toward helping every student achieve the following process standards.

MKP1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

MKP2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

MKP3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

MKP4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

MKP5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions are for teacher reference only and are not intended to be memorized by students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

Terms/Symbols:

numbers through 30, *number words through ten*, set, longer, shorter, heavier, lighter, morning, afternoon, evening, yesterday, today, tomorrow, days of the week, months of the year, seasons, triangle, *quadrilateral*, rectangle, square, circle, sphere, cube, beside, above, below, in front of, behind, inside, outside, more, less, equal, *ordinal numbers*, *picture graph*

By the end of grade one, students will understand and use the concept of ones and tens in the place value number system. The students will add and subtract small numbers with ease. They will represent quantity with numbers, models, diagrams, and number sentences. They will begin to use tools for measuring and observe, create, and decompose geometric shapes and solve simple problems including those involving spatial relationships. The students will pose questions, record data, and interpret simple charts and picture graphs.

Instruction and assessment should include the use of manipulatives and appropriate technology. Topics should be represented in multiple ways including symbolic, verbal/written, numeric/data-based, graphical, and concrete/pictorial. Concepts should be introduced and used in the context of real world phenomena.

Concepts/Skill to Maintain

Number words

Ordinal numbers

Equivalence

Basic 2-Dimensional and 3-Dimensional geometric shapes

Spatial relationships – positional words

Calendar time and daily schedule

Estimating-using 5 and 10 as benchmarks

Name and value of coins

Measurement - comparing and ordering by direct comparison

NUMBER AND OPERATIONS

Students will understand how to represent numbers, and be able to add and subtract small numbers.

M1N1. Students will estimate, model, compare, order, and represent whole numbers up to 100.

- a. Represent numbers *up to 100* using a variety of models, diagrams, and number sentences. Represent numbers larger than 10 in terms of tens and ones *using manipulatives* and pictures.
- b. Correctly count and represent the number of objects in a set using numerals.
- c. Compare small sets using the terms greater than, less than, and equal to.
- d. Understand the magnitude and order of numbers up to 100 by making ordered sequences and representing them on a number line.

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- e. Exchange equivalent quantities of coins by making fair trades involving combinations of pennies, nickels, dimes, and quarters *up to one dollar*; count out a combination *of coins* needed to purchase items *up to one dollar*.
- f. Identify bills (\$1, \$5, \$10, \$20) by name and value and exchange equivalent quantities by making fair trades involving combinations of bills; count out a combination of bills needed to purchase items *that total up to* twenty dollars.

M1N2. Students will understand place value notation for the numbers 1 to 99. (Discussions may allude to 3-digit numbers to assist in understanding place value.)

- a. Determine to which ten a given number is closest using tools such as a sequential number line or chart.
- b. Represent collections of less than 30 objects with 2-digit numbers and *understand the meaning of place value*.
- c. Decompose numbers from 10 to 99 as the appropriate number of tens and ones.

M1N3. Students will add and subtract numbers less than 100, as well as understand and use the inverse relationship between addition and subtraction.

- a. Identify one more than, one less than, 10 more than, and 10 less than a given number.
- b. Skip-count by 2s, 5s, and 10s, forward and backwards; to and from numbers up to 100.
- c. Compose/decompose numbers up to 10 (e. g. 3+5=8, 8=5+2+1).
- d. Understand a variety of situations to which subtraction may apply: taking away from a set, comparing two sets, and determining how many more or how many less.
- e. Understand addition and subtraction number combinations using strategies such as counting on, counting back, doubles and making tens.
- f. Know the single-digit addition facts to 18 and corresponding subtraction facts with understanding and fluency. (Use strategies such as relating to facts already known, applying the commutative property, and grouping facts into families.)
- g. Apply addition and subtraction to 2 digit numbers without regrouping (e.g. 15 +4,80-60,56+10,100-30,52+5).
- h. Solve and create word problems involving addition and subtraction to 100 without regrouping. Use words, pictures and concrete models to interpret story problems and reflect the combining of sets as addition and taking away or comparing elements of sets as subtraction.

M1N4. Students will count collections of up to 100 objects by dividing them into equal parts and represent the results using words, pictures, or diagrams.

- a. Use informal strategies to share objects equally between two to five people.
- b. Build number patterns, including concepts of even and odd, using various concrete representations. (Examples of concrete representations include a hundreds chart, ten grid frame, place value chart, number line, counters, or other objects.)
- c. Identify, label, and relate fractions (halves, fourths) as equal parts of a *collection of objects or* a whole using pictures and models.
- d. *Understand halves and fourths as representations of equal parts of a whole.*

MEASUREMENT

Students will measure basic quantitative attributes of concrete objects.

M1M1. Students will compare and/or order the length, *height*, weight, or capacity of two or more objects by using direct comparison or a nonstandard unit.

- a. Directly compare *and/or order* length, *height*, weight, and capacity of concrete objects.
- b. Estimate and measure using a non-standard unit that is smaller than the object to be measured.
- c. Measure with a tool by creating a "ruled" stick, tape, or container by marking off ten segments of the repeated single unit.

M1M2. Students will develop an understanding of the measurement of time.

- a. Tell time to the nearest hour and half hour and understand the movement of the minute hand and how it relates to the hour hand.
- b. Begin to understand the relationship of calendar time by knowing the number of days in a week and months in a year.
- c. Compare and/or order the sequence or duration of events (e.g., shorter/longer and before/after).

GEOMETRY

Students will understand the concepts of basic geometric shapes and spatial relationships of concrete objects.

- M1G1. Students will study and create various two and three-dimensional figures and identify basic figures (squares, circles, triangles, and rectangles) within them.
 - a. Build, draw, name, and describe triangles, rectangles, pentagons, and hexagons.
 - b. Build, represent, name, and describe cylinders, cones, and *rectangular prisms*.
 - c. Create pictures and designs using shapes, including overlapping shapes.
- M1G2. Students will compare, contrast, and/or classify geometric shapes by the common attributes of position, shape, size, number of sides, and number of corners.
- M1G3. Students will arrange and describe objects in space by proximity, position, and direction (near, far, below, above, up, down, behind, in front of, next to, and left or right of).

DATA ANALYSIS AND PROBABILITY

Students will pose questions, collect, organize and interpret data about themselves and their surroundings.

M1D1. Students will create simple tables and graphs and interpret them.

- a. Interpret tally marks, picture graphs, and bar graphs.
- b. *Pose questions, collect, sort*, organize and record data using objects, pictures, tally marks, picture graphs, *and bar graphs*.

Process Standards

Each topic studied in this course should be developed with careful thought toward helping every student achieves the following process standards.

M1P1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.

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- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

M1P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

M1P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

M1P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

M1P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

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Terms/Symbols:

place value: ones, tens, greater than, less than, equal to, fewer than, more than, equivalent, sum/add, difference/subtract, coins: penny, nickel, dime, quarter, bills, fair trade, compare/contrast, length, height, weight, estimate, hexagon, cylinder, cone, rectangular prism, corner, vertex, =, +, -, even, odd, tally mark, bar graph, 1/2, 1/4, skip counting

By the end of grade two, students will understand place value and number relationships in addition and subtraction and use simple concepts of multiplication. They will measure length with appropriate units and determine perimeter. Students will classify shapes and see relationships among them by recognizing their geometric attributes. They will know the relationships of time and count back change. The students will collect, analyze, and interpret data using bar graphs and Venn diagrams.

Instruction and assessment should include the use of manipulatives and appropriate technology. Topics should be represented in multiple ways including symbolic, verbal/written, numeric/data-based, graphical, and concrete/pictorial. Concepts should be introduced and used in the context of real world phenomena.

Concepts/Skill to Maintain

Fluency with single digit addition/subtraction facts to 18

Fair trades with coins or bills

Duration and sequence of events

Number patterns-skip count, odd/even

Fact families

Fractions: halves, fourths

Tally marks Picture graphs

Estimation: rounding to nearest ten

Telling time

Measurement – estimating, comparing, and ordering Basic geometric figures and spatial relationships

NUMBER AND OPERATIONS

Students will further develop their understanding of numbers (including fractions) and how to represent them. The students will understand and apply addition, subtraction and multiplication through concrete manipulation and perform basic calculations.

M2N1. Students will use multiple representations of numbers to connect symbols to auantities.

a. Represent numbers using a variety of models, diagrams, and number sentences (e.g., 4703 represented as 4,000 + 700 + 3, and units, 47 hundreds + 3, or 4,500 + 203).

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- b. Understand the relative magnitudes of numbers using 10 as a unit, 100 as a unit, or 1000 as a unit. Represent 2-digit numbers with drawings of tens and ones and 3-digit numbers with drawings of hundreds, tens, and ones.
- c. Use money as a medium of exchange. *Make change* and use decimal notation and the dollar and cent symbols to represent the collection of coins and currency.

M2N2. Students will build fluency with multi-digit addition and subtraction.

- a. Correctly add and subtract two whole numbers up to three digits each with regrouping.
- b. Understand and use the inverse relation between addition and subtraction to solve problems and check solutions.
- c. Use mental math strategies such as benchmark numbers to solve problems.
- d. Use basic properties of addition (commutative, associative, and identity) to simplify problems (e.g. 98 + 17 by taking two from 17 and adding it to the 98 to make 100 and replacing the original problem by the sum 100 + 15).
- e. Estimate to determine if solutions are reasonable for addition and subtraction.

M2N3. Students will understand multiplication, multiply numbers, and verify results.

- a. Understand multiplication as repeated addition.
- b. Use repeated addition, arrays, and counting by multiples (skip counting) to correctly multiply 1-digit numbers and construct the multiplication table.
- c. Use the multiplication table (grid) to determine a product of two numbers.
- d. Use repeated subtraction, equal sharing, and forming equal groups to divide large collections of objects and determine factors for multiplication.

M2N4. Students will understand and compare fractions.

- a. Model, identify, label, and compare fractions (thirds, sixths, eighths, tenths) as a representation of equal parts of a whole or of a set.
- b. Know that when all fractional parts are included, such as three thirds, the result is equal to the whole.

M2N5. Students will represent and interpret quantities and relationships using mathematical expressions including equality and inequality signs $(=, >, <, \neq)$.

- a. Include the use of boxes or to represent a missing value.
- b. Represent problem solving situations where addition, subtraction or multiplication may be applied using mathematical expressions.

MEASUREMENT

Students will understand length, time, and temperature and choose an appropriate tool to measure them.

- M2M1. Students will know the standard units of inch, foot, yard, and metric units of centimeter and meter and measure length to the nearest inch or centimeter.
 - a. Compare the relationship of one unit to another by measuring objects twice using different units each time.
 - b. Estimate lengths, and then measure to determine if estimations were reasonable.
 - c. Determine an appropriate tool and unit for measuring.
- M2M2. Students will tell time to the nearest five minutes and know relationships of time such as the number of *seconds in a minute*, minutes in an hour and hours in a day.
- M2M3. Students will explore temperature.
 - a. Determine a reasonable temperature for a given situation.
 - b. Read a thermometer.

GEOMETRY

Students will understand basic and compound geometric shapes together with the elements from which they are composed.

- M2G1. Students will describe and classify plane figures (triangles, square, rectangle, trapezoid, quadrilateral, pentagon, hexagon, and irregular polygonal shapes) according to the number *of sides* and vertices and the sizes of angles (right angle, obtuse, acute).
- M2G2. Students will describe and classify solid geometric figures (prisms, *pyramids*, cylinders, cones, and spheres) according to such things as the number of edges and vertices and the number and shape of faces and angles.
 - a. Recognize the (plane) shapes of the faces of a geometric solid and count the number of faces of each type.
 - b. Recognize the shape of an angle as a right angle, an obtuse, or acute angle.
- M2G3. Students will describe the change in attributes as two and three-dimensional shapes are cut and rearranged.

DATA ANALYSIS AND PROBABILITY

Students will pose questions, collect, organize, and interpret data about themselves and their surroundings.

M2D1. Students will create simple tables and graphs and interpret their meaning.

- a. Create, organize and display data using pictographs, Venn diagrams, bar graphs, picture graphs, simple charts, and tables to record results with scales of 1, 2 and 5.
- b. Know how to interpret picture graphs, Venn diagrams, and bar graphs.

Process Standards

Each topic studied in this course should be developed with careful thought toward helping every student achieves the following process standards.

M2P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

M2P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

M2P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

M2P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

M2P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

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Terms/Symbols:

place value: thousands, sum, difference, product, *factor, multiple*, multiply, regroup, array, numerator, denominator, inch, foot, yard, centimeter, meter, *polygon*, right angle, obtuse, acute, edge, face, vertex/vertices, prism, plane, >, <, =, \neq , +, -, x, minute, hour, Venn diagram, *pictograph, scale, symbol for equality, symbol for inequality*