Warm Up

1. What is the name of the point where the angle bisectors of a triangle intersect?

incenter

Find the midpoint of the segment with the given endpoints.

2. (–1, 6) and (3, 0)

(1, 3)

3. (–7, 2) and (–3, –8)

(–5, –3)

4. Write an equation of the line containing the points **(**3, 1**)** and **(**2, 10**)** in point-slope form.

$y - 1 = -9(x - 3)$

Objectives

Apply properties of medians of a triangle.

Apply properties of altitudes of a triangle.

Vocabulary

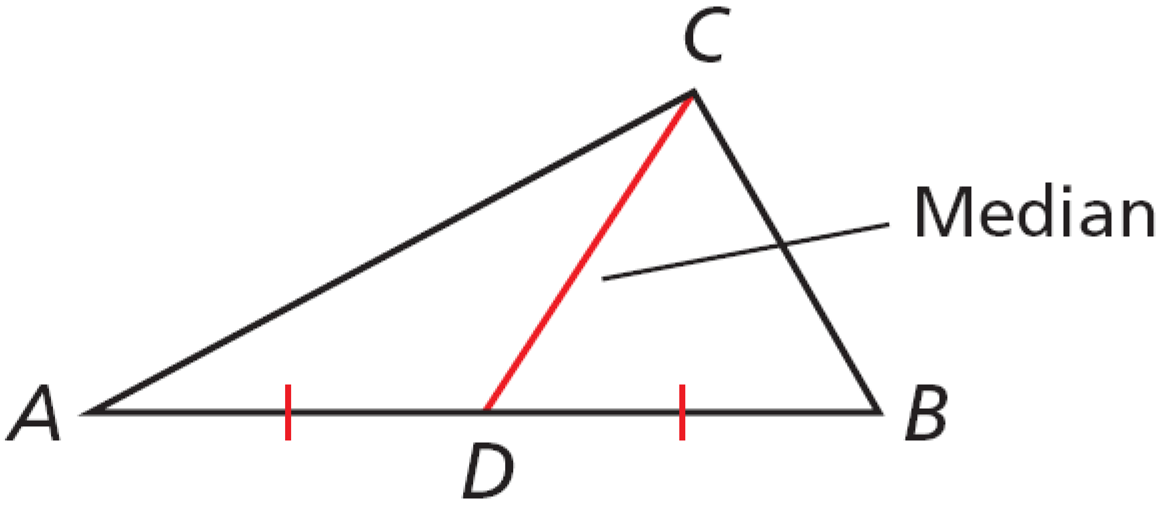
median of a triangle

centroid of a triangle

altitude of a triangle

orthocenter of a triangle

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



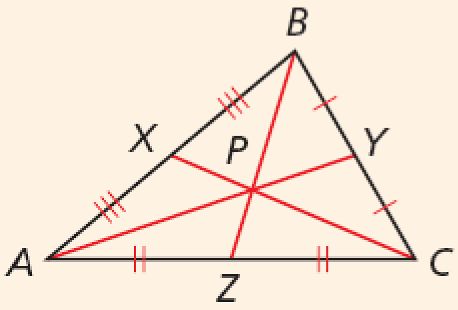
Every triangle has three medians, and the medians are concurrent.

The point of concurrency of the medians of a triangle is the **centroid of the triangle** . The centroid is always inside the triangle. The centroid is also called the *center of gravity* because it is the point where a triangular region will balance.

Theorem 5-3-1 Centroid Theorem

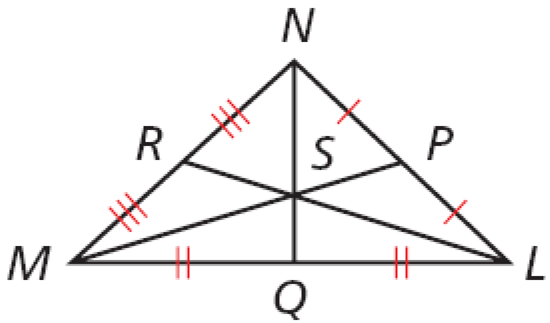
The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

$AP = \frac{2}{3}AY$ $\quad$ $BP = \frac{2}{3}BZ$ $\quad$ $CP = \frac{2}{3}CX$



Example 1A: Using the Centroid to Find Segment Lengths

In Δ*LMN*, *RL* = 21 and *SQ* =4. Find *LS*.



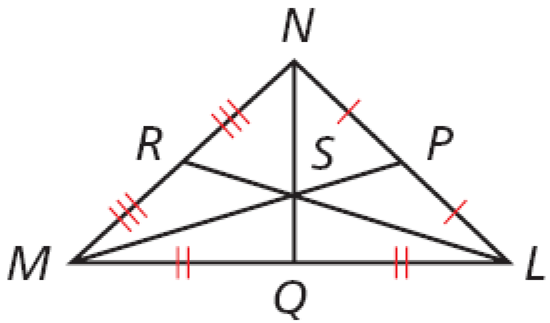
$$LS = \frac{2}{3}RL$$ *Centroid Thm.*

$$LS = \frac{2}{3}(21)$$ *Substitute 21 for RL.*

$$LS = 14$$ *Simplify.*

Example 1B: Using the Centroid to Find Segment Lengths

In ΔLMN, $RL = 21$ and $SQ = 4$. Find NQ.



$NS = \frac{2}{3}NQ$ *Centroid Thm.*

$NS + SQ = NQ$ *Seg. Add. Post.*

$\frac{2}{3}NQ + SQ = NQ$ *Substitute $\frac{2}{3}$ NQ for NS.*

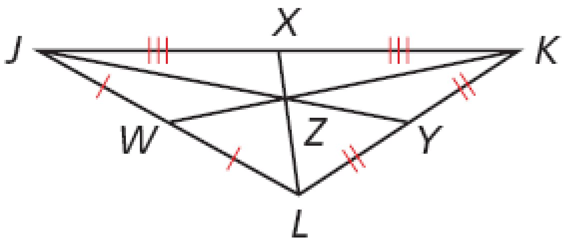
$SQ = \frac{1}{3}NQ$ *Subtract $\frac{2}{3}$ from both sides.*

$4 = \frac{1}{3}NQ$ *Substitute 4 for SQ.*

$12 = NQ$ *Multiply both sides by 3.*

Check It Out! Example 1a

In ΔJKL, ZW = 7, and LX = 8.1. Find KW.



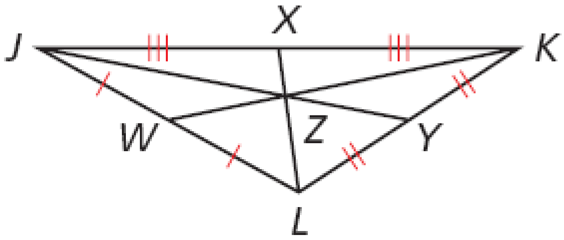
$$ZW = \frac{2}{3}KW$$ *Centroid Thm.*

$$7 = \frac{2}{3}KW$$ *Substitute 7 for ZW.*

$$KW = 21$$ *Multiply both sides by 3.*

Check It Out! Example 1b

In Δ*JKL*, *ZW* = 7, and *LX* = 8.1. Find *LZ*.



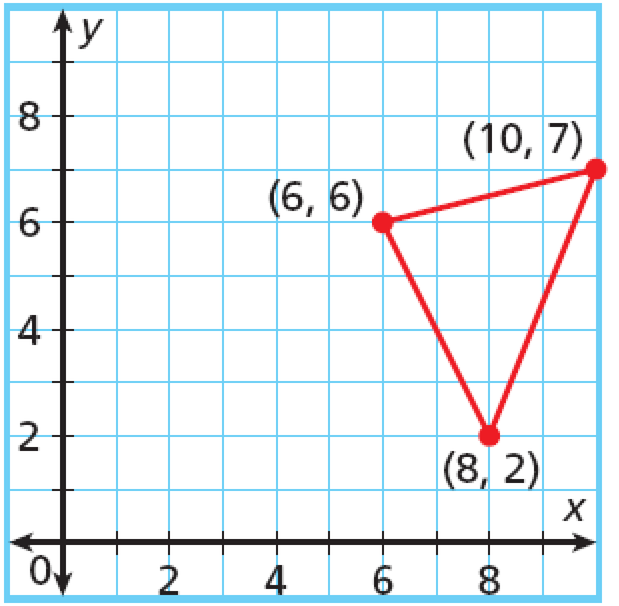
$$LZ = \frac{2}{3}LX$$ *Centroid Thm.*

$$LZ = \frac{2}{3}(8.1)$$ *Substitute 8.1 for LX.*

$$LZ = 5.4$$ *Simplify.*

Example 2: Problem-Solving Application

A sculptor is shaping a triangular piece of iron that will balance on the point of a cone. At what coordinates will the triangular region balance?



Example 2 Continued

1 Understand the Problem

The **answer** will be the coordinates of the centroid of the triangle. The **important information** is the location of the vertices, $A(6, 6)$, $B(10, 7)$, and $C(8, 2)$.

2 Make a Plan

The centroid of the triangle is the point of intersection of the three medians. So write the equations for two medians and find their point of intersection.

Example 2 Continued

3 Solve

Let M be the midpoint of $\overline{AB}$ and N be the midpoint of $\overline{AC}$.

$$M = \left(\frac{6+10}{2}, \frac{6+7}{2}\right) = \left(8, 6\frac{1}{2}\right) \qquad N = \left(\frac{6+8}{2}, \frac{6+2}{2}\right) = (7, 4)$$

$\overline{CM}$ is vertical. Its equation is $x = 8$. $\overline{BN}$ has slope 1. Its equation is $y = x - 3$. The coordinates of the centroid are D(8, 5).

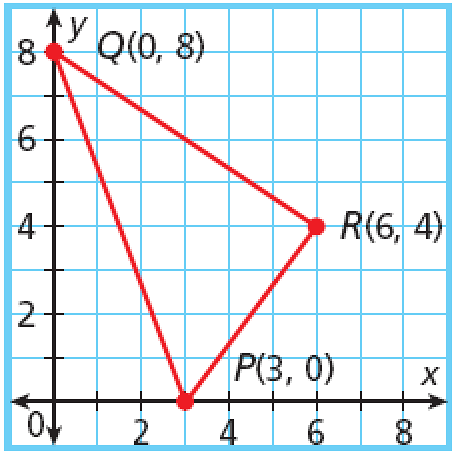
Example 2 Continued

4 Look Back

Let *L* be the midpoint of *BC*. The equation ~~for *AL*~~ is $y = -\frac{1}{2}x + 9$, which intersect~~s *x*~~ = 8 at *D*(8, 5).

Check It Out! Example 2

Find the average of the *x*-coordinates and the average of the *y*-coordinates of the vertices of ΔPQR. Make a conjecture about the centroid of a triangle.



Check It Out! **Example 2 Continued**

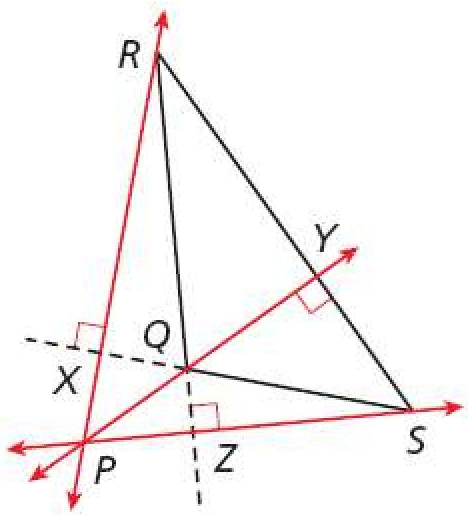
The *x*-coordinates are 0, 6 and 3. The average is 3.

The *y*-coordinates are 8, 4 and 0. The average is 4.

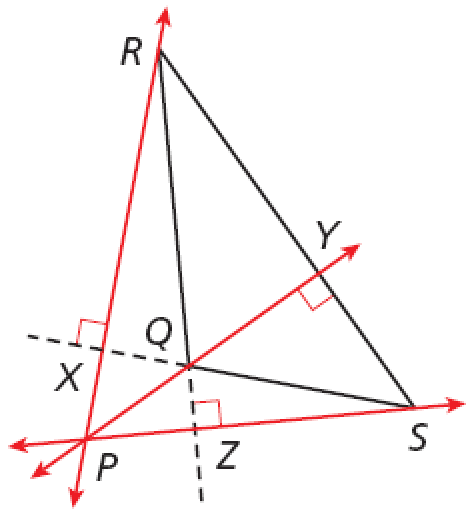
The *x*-coordinate of the centroid is the average of the *x*-coordinates of the vertices of the Δ, and the *y*-coordinate of the centroid is the average of the *y*-coordinates of the vertices of the Δ.

An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.

Every triangle has three altitudes. An altitude can be inside, outside, or on the triangle.



In Δ*QRS*, altitude *QY* is inside the triangle, but *RX* and *SZ* are not. Notice that the lines containing the altitudes are concurrent at *P*. This point of concurrency is the **orthocenter of the triangle**.



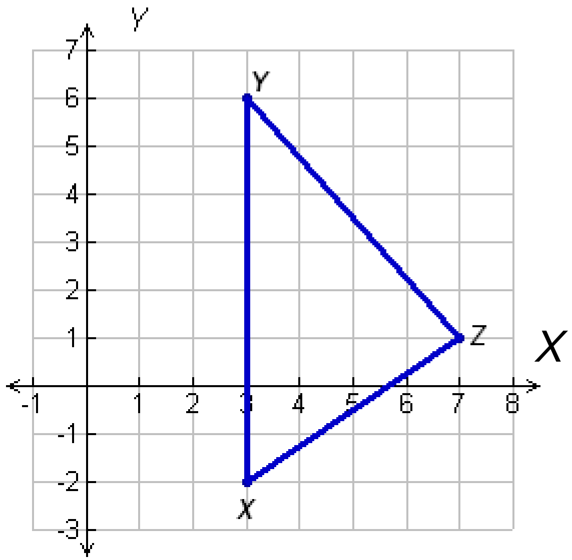
Helpful Hint

The height of a triangle is the length of an altitude.

Example 3: Finding the Orthocenter

Find the orthocenter of ΔXYZ with vertices *X*(3, –2), *Y*(3, 6), and *Z*(7, 1).

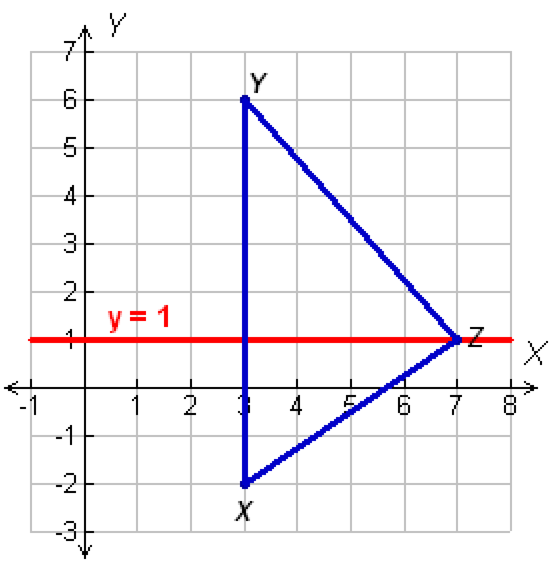
Step 1 Graph the triangle.



Example 3 Continued

Step 2 Find an equation of the ~~line~~ containing the altitude from *Z* to *XY*.

Since $\overleftrightarrow{XY}$ is vertical, the altitude is horizontal. The line containing it must pass through *Z*(7, 1) so the equation of the line is $y = 1$.



Example 3 Continued

Step 3 Find an equation of the line containing the altitude from *Y* to *XZ*.

$$\text{slope of } \overleftrightarrow{XZ} = \frac{1-(-2)}{7-3} = \frac{3}{4}$$

The slope of a line perpendicular to *XZ* is $-\frac{4}{3}$. This line must pass throu(3, 6).

$$y - y_1 = m(x - x_1)$$ *Point-slope form.*

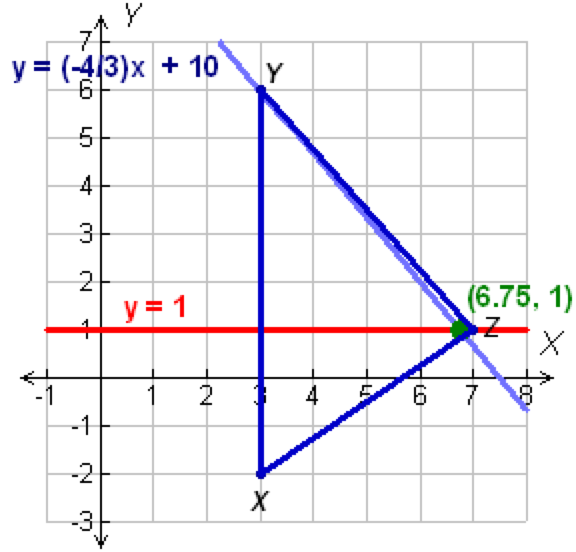
$$y - 6 = -\frac{4}{3}(x - 3)$$ *Substitute 6 for y_1, $-\frac{4}{3}$ for m, and 3 for x_1.*

$$y - 6 = -\frac{4}{3}x + 4$$ *Distribute $-\frac{4}{3}$.*

$$y = -\frac{4}{3}x + 10$$ *Add 6 to both sides.*

Example 3 Continued

Step 4 Solve the system to find the coordinates of the orthocenter.



$$\begin{cases} y = 1 \\ y = -\frac{4}{3}x + 10 \end{cases}$$

$1 = -\frac{4}{3}x + 10$ *Substitute 1 for y.*

$-9 = -\frac{4}{3}x$ *Subtract 10 from both sides.*

$6.75 = x$ *Multiply both sides by* $-\frac{3}{4}$.

The coordinates of the orthocenter are (6.75, 1).

Check It Out! Example 3

Show that the altitude to $\overline{JK}$ passes through the orthocenter of Δ*JKL*.

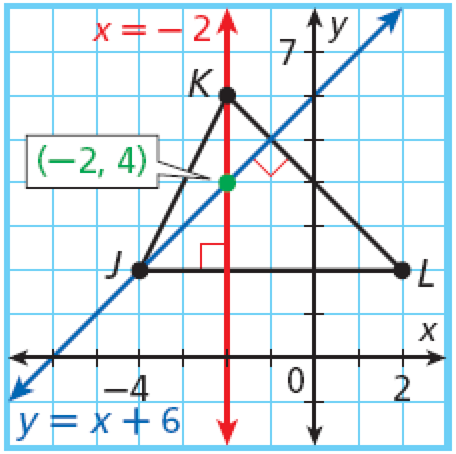
An equation of the altitude to $\overline{JK}$ is

$$y = -\frac{1}{2}x + 3.$$

$$4 = -\frac{1}{2}(-2) + 3.$$

$$4 = 1 + 3$$

$$4 = 4 \checkmark$$



Therefore, this altitude passes through the orthocenter.

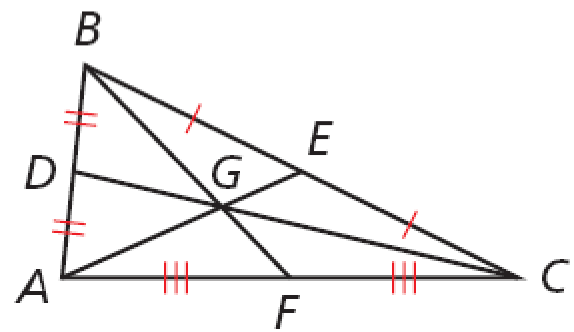
Lesson Quiz

Use the figure for Items 1–3. In Δ*ABC*, *AE* = 12, *DG* = 7, and *BG* = 9. Find each length.

1. *AG* 8

2. *GC* 14

3. *GF* 13.5



For Items 4 and 5, use Δ*MNP* with vertices *M* (–4, –2), *N* (6, –2) , and *P* (–2, 10). Find the coordinates of each point.

4. the centroid (0, 2)

5. the orthocenter $\left(-2, -\frac{2}{3}\right)$