Unit: Accentuate the Negative

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School: Campus Community School

Grade/Course: 7th grade Math

Rationale: Integers are a subset of rational numbers. They are defined as whole numbers and their opposites including zero. Knowing how to add, subtract, multiply and divide integers will give the students the knowledge to extend this understanding to the *real number system.* Students can use the generalizations made about integers to extend their work to negative and positive rational numbers.

CONTENT

Unit Focus Questions:
What is the relationship between integers, rational numbers and real numbers?
How do we add, subtract, multiple, and divide these numbers?

Content Summary:

Rational numbers and irrational numbers make up the set of all real numbers. Rational numbers have the form of a/b where a and b are both integers and when converted to a decimal are either repeating (i.e 1/3 = .33333…) or terminating (i.e 1/(-4) = -.25). As mentioned in the rational, integers are defined as whole numbers and their opposites, thus whole numbers, including zero and integers are also rational numbers since -3/1 = -3.0 and 0/5 = 0.0.

When adding, subtracting, multiplying and dividing rational numbers, care must be taken to determine the sign of solutions correctly. In addition, if the temperature is +30° (meaning 30° above zero) and then increases another +15°the final temperature will be °45. +30 + (+15) = 45. Likewise if the temperature is -5°(meaning 5°below zero) and decreases another 5 degrees (-5°), then the final temperature will be -10°. -5°+ (-5) = -10. If the temperature is +30°(meaning 30°above zero) and decreases 15°(-15°), the final temperature will be 15°. +30 + (-15) = 15. But if the temperature is +30°(meaning 30°above zero) and it decreases 35°(-35°) the final temperature will be -5 °. Using examples and number lines (both horizontal and vertical) leads students to develop the algorithms or rules used to add and subtract rational numbers. These rules are a "positive plus another positive is positive", a "negative plus a negative is negative", a "positive plus a negative" and a "negative plus a positive" will either be negative or positive depending on whether you have more negatives or positives to begin with. A "negative plus a negative is negative" can also be thought of as a negative minus a positive since you are moving the same direction on the number line.

Algorithms for multiplying and dividing, which can also be modeled using number lines or chip diagrams, can be determined using the understanding that multiplication is repeated and addition and division is repeated subtraction and the understanding of fact families. For example, 5 x -3 is five sets of – 3 (-3 + -3 + -3 + -3 + -3) added together or -15. The same for (-15)/(-3), how many -3s make up -15? -15= (-3) + (-3) + (-3) + (-3) + (-3) which is 5 sets of (-3). Using fact families for 5 • -3 = 15, algorithms can be determined for multiplication and division. A "positive multiplied/divided by a positive is a positive", a "negative multiplied/divided by a positive is a negative", a "positive multiplied/divided by a negative, is negative" and a "negative multiplied/divided by a negative is positive".

Properties of numbers and operations for whole numbers also apply and hold true for rational numbers. The Commutative Property, the Distributive Property, the Order of Operations and the Additive/Multiplicative Inverses are used to simplify expressions and equations, and to solve contextual problems.

Content Standards:

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line. (*Investigation 1, 2, and 4)*

7.NS.A.1a Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. (*Investigation 1 and 2)*

7.NS.A.1b Understand $p+q$ as a number located a distance $|q|$ from p, in a positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of zero. Interpret sums of rational numbers by describing real-world contexts. (*Investigation 1 and 2)*

7.NS.A.1c Understand subtraction of rational numbers as adding the inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (*Investigation 1 and 2)*

7.NS.A.1d Apply properties of operations as strategies to add or subtract rational numbers. (*Investigation 2 and 4)*

7.NS.A.2 Apply and extend previous understandings of multiplication and division of fractions to divide rational numbers. (*Investigation 3 and 4)*

7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (*Investigation 3 and 4)*

7.NS.A.2b Understand that integers can be divided provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q)=(-p)/q=p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. (*Investigation 3)*

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers. (*Investigation 3 and 4)*

7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. *Investigation 3*

7.NS.A.3 Solve real-world problems involving the four operations with rational numbers. (*Investigation 1, 2, 3, and 4)*

7.EE.B.3 Solve multi-step and real-life mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. (*Investigation 2, 3, and 4)*

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (*Investigation 1)*

7.EE.B.4b Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p,

q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *(Investigation 1)*

Mathematical Practices:
Students identify and record their personal experiences with the Standards for Mathematical Practice during the Mathematical Reflections at the end of each Investigation but some investigations focus on specific practices as listed below.
MP.1 Make sense of problems and persevere in solving them.
Students are engaged every day in solving problems and, over time, learn to persevere in solving them. Students build understanding by reflecting, connecting, and communicating. These student-centered problem situations engage students in articulating the "knowns" in a problem situation and determining a logical solution pathway. The student-student and student-teacher dialogues help students not only to make sense of the problems, but also to persevere in finding appropriate strategies to solve them.
MP.2 Reason abstractly and quantitatively.
Students reason abstractly and quantitatively when they determine whether the product of two or more rational numbers is positive or negative in Problem 3.2 and when they use the Distributive Property to compare and verify multiple solution methods in Problem 4.3.
MP.3 Construct viable arguments.
In Problem 1.1, students find the difference in points scored for two teams. They may justify their answers by finding each team's point difference from zero and then adding.
MP.4 Model with mathematics.
Students use multiplication number sentences to model a relay race in Problem 3.1. They use positive and negative numbers to represent running speeds to the right and to the left. They also use positive and negative numbers to represent times in the future and in the past.
MP.5 Use appropriate tools strategically.
In Problem 1.3, students use number lines to explore sums of positive and negative numbers in the familiar context of temperature changes. Students mark a starting temperature on a number line and move to the right for a temperature increase and move to the left for a temperature decrease. Students use this tool to transition from adding and subtracting concrete temperatures to adding and subtracting abstract integers.
MP.6 Attend to precision.
Students attend to precision when they work with the Order of Operations in Problem 4.1. They use parentheses in different places within expressions to make the greatest and least possible values. As well as any time they are communicating with one another and the class about the mathematics being used.
MP.7 Look for and make use of structure.
In Problem 2.4, students examine the structure of fact families as they rewrite addition sentences as subtraction sentences and subtraction sentences as addition sentences. Students then determine which number sentence within a fact family makes it easiest to find the value of a missing number.
MP.8 Look for and express regularity in repeated reasoning.
Students observe patterns in Problem 2.1 when they categorize groups of addition sentences. They may categorize a group by the signs of the addends or by the method they used to find the sums.

INSTRUCTION

Learning Progression:
Projected Length of Time: 23 days, 80 minute block schedule, meeting every day, all year.

Investigation	Lesson Focus Questions	Key Concepts / Vocabulary	Formative Assessment	Summative Assessment
Investigation 1: Extending the Number System **Timeframe: 5 days**	**How do we use models to add and subtract both positive and negative numbers?**	**Integers, negative number, number sentence, opposites, positive number, rational numbers**	**Problems 1.1, 1.2, and 1.3; ACE problems; Mathematical Reflection**	**Investigation 1 Quiz**
Investigation 2: Adding and Subtracting Rational Numbers **Timeframe: 6 days**	**How do we add and subtract rational numbers?**	**Absolute value, additive identity, additive inverses, algorithm, commutative property**	**Problems 2.1, 2.2, 2.3, 2.4; ACE problems; Mathematical Reflection**	**Investigation 2 Quiz**
Investigation 3: Multiplying and Dividing Rational Numbers **Timeframe: 6 days**	**How do we multiply and divide rational numbers?**	**Multiplicative identity, multiplicative inverses**	**Problems 3.1, 3.2, 3.3; ACE problems; Mathematical Reflection**	**Investigation 3 Quiz**
Investigation 4: Properties of Operations **Timeframe: 6 days**	**What properties of can be used to simplify expressions and solve equations and contextual problems?**	**Distributive Property, expanded form, factored form, number sentence, and order of operations**	**Problems 4.1 and 4.2; ACE problems; Mathematical Reflection**	

Culminating Unit Assessment:

Unit Test-1 day

Investigation 1: Extending the Number System

Lesson Focus Questions How do we use models to add and subtract both positive and negative numbers?

Standards: 7.NS.A.1, 7.NS.A.1a, 7.NS.A.1b, 7.NS.A.1c, 7.NS.A.3 ,7.EE.B.4, 7.EE.B.4b, MP.3, MP.5

Learner Outcomes (KUD):
Students will:
Use appropriate notation to indicate positive and negative numbers.
Compare and order positive and negative numbers and locate them on a number line.
Relate direction and distance to the number line.
Use models and rational numbers to represent and solve problems.
Add, subtract, multiply and divide integers using models.
Justify their answers.
Use appropriate tools strategically.

Pre-Requisites: Understanding that there are other numbers besides whole numbers. How to use a number line. How to add and subtract numbers.

Key Concepts/Vocabulary: Integer, negative number, number sentence, opposites, positive number, rational numbers

Activating Strategy: Teacher asks, "*Where do numbers such as fractions, decimals and negative numbers fall on the number line? Let's see what you think.*" Working in pairs, students will be given number lines and a variety of numbers from different number sets and asked to determine where the numbers lie on the number line. Students will then share and justify their reasoning to their partners.

Learner Activities: (5 days)

Problem 1.1 Playing Math Fever – Using Positive and Negative Numbers (Day 1): .

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Students read the introduction to Investigation 1 - Extending the Number System on pg. 7 and the top of page 8 of their student books.
3. As a class discuss the answers to the following question:
   What is the relationship between -0.6 and 0.6? Which number is greater, -2.3 or 1.2? How do you know? How can you use a number line to help you list -2.3, -3.5 and 1.7 in order?
4. Teacher will have a student read problem "1.1 Playing Math Fever using Positive and Negative Numbers". This problem involves a game with positive and negative scores.
5. The teacher begins discussing Question A with the whole class by displaying the scores of the three teams (Super Brains: -300, Rocket Scientists: 150, and Know-It-Alls: -500).
6. Have students answer the following questions. Thinking about these ideas should give the class a good understanding of the context and help them to work through parts B-F:
   Which team has the highest score? (Rocket Scientists) *Which team has the lowest*

score? (Know-It-Alls) *How did you decide?* (The Rocket Scientists is the only team with a positive score, which is a score greater than 0. The Know-It-Alls have the lowest score, because their score of --500 is more negative than the Super Brains' score of -300)
How *many pairs of two teams are there to compare?* [Record the list of the needed comparisons (Super Brains vs. Know-It-Alls, Super Brains vs. Rocket Scientists, Know-It-Alls vs. Rocket Scientists). This list will help students to answer Question A part (2) when they work in groups.] *The Super Brains have a score of -300 points. How did they reach that score?* (Possible answer: The Super Brains may have answered a 200-point question correctly and then missed two 250-point questions. This can be written: 200−250−250= - 300.) *Does anyone have a different idea?*

7. Students complete Parts B-F in groups of 2. As students work through the problem the teacher should prompt them to think about what operations they use to keep track of the scores and notice how the score goes higher or lower depending on whether the team answers correctly or incorrectly.
8. Discuss answers to B-F as a class.
   For Question B, teacher will ask students to explain how they found each team's final score. Once the class agrees on the final scores, discuss the remainder of Question B.
   For Questions C–E, ask students to share their strategies for finding the missing numbers in the number sentences. Focus particular attention on strategies for finding missing addends. For Question F, see whether or not students understand that the two methods provide equivalent expressions. Ask students which method they prefer.
9. Students will do an assortment of problems from the ACE problems and finish whatever is not done for homework: Applications #1-8, Connections #56 – 58, Extensions #78

Problem 1.2 Extending the Number Line (Day 2): This Problem uses temperature measurement to extend number lines to include negative numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. As a class, discuss the Student Edition visuals on page 11 in order to learn how comfortable students are with placing negative integers and rational numbers on a number line.
3. Teacher poses the following questions to engage the students in the tasks of Problem 1.2. *On a horizontal number line, where are positive and negative numbers located in relation to 0?* (Positive numbers are to the right of the 0 mark, and negative numbers are to the left of the 0 mark.) *Where is the opposite of 12 on a vertical number line?* (-12; 12 equally spaced marks below zero) *Where is the opposite of -9 on a vertical number line?* (9; 9 equally spaced marks above zero)
4. Students begin working on Part A individually and then move to working in pairs on Parts B-C.
5. As students are working, teacher circulates and has students explain how they are

determining their solutions. Encourage students to use sketches of number lines to complete the problems and show their thinking using both vertical and horizontal number lines.

6. After most students have completed through Part B, teacher will pose the following questions: *What happens to the values of numbers as you move from left to right (bottom to top) on the number line? What happens to the values of numbers as you move from right to left (top to bottom)?*
7. Teacher will lead a discussion of students' solutions and strategies for the Problem.
8. Student complete Exit Ticket Problem 1.2 (attached) and complete the following ACE Problems for homework: Applications:#9-35, Connections:#59-75, Extensions:#79-87 (attached)

Problem 1.3 From Sauna to Snowbank Using a Number Line (Day 3): In this problem students write number sentences and sketch number lines, both vertical and horizontal, to compare temperatures. They encounter changing temperatures and determine the resulting temperature.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Teacher shows the Launch video that represents the opening text of Problem 1.3. Engage students in a discussion of experiences with temperature changes.
4. Teacher refers students to pg. 14 in their Accentuate the Negative Student Edition to examine the graphics of thermometers and poses the following questions: *What unit of measurement is used in the thermometers?* (degrees Fahrenheit or Centigrade) *How do you read the thermometers shown?* (by reading the top line of the red fill) *How is the horizontal number line comparing the Outside and Sauna temperatures different from the vertical thermometers? How is it the same?*
5. Teacher discusses locations of countries and states by referencing a map or a globe. Encourage students to make connections between a place's location and the hot or cold temperatures people might experience living in those places (e.g., Alaska or Finland vs. California or Mexico). Suggested questions: *Where have you experienced the hottest and coldest temperatures?* (Answers will vary.) *What were the temperatures in those places?* (Answers will vary.) *How do you write temperatures that are below zero?* (Use a negative sign to show that a temperature is below zero, or write the number of degrees "below zero.") *You can think of a thermometer as a vertical number line. On a thermometer, where are the positive numbers located in relation to* 0°F*?* (above the 0 mark) *Where are the negative numbers located in relation to 0°F?* (below the 0 mark) *Suppose the temperature is* +5°F. *Where is* +5°F *located on this vertical number line in relation to* 0°F*?* (5 units above the 0°F mark) *Suppose the temperature is -5*°F. *Where is -5*°F *located on this vertical number line in relation to 0?* (5 units below the 0°F mark)
6. Teacher refers students to the questions at the bottom of pg 14 and pg 15. Students discuss with a partner and then share out their answers as a class.
7. Students work independently on Problem 1.3. As the teacher circulates, have

students explain how they are determining their solutions. Encourage students to use sketches on number lines to show their thinking and to write addition and/or subtraction number sentences to describe their work. Discuss as class showing numbers sentences to represent the problems.

8. Student complete ACE Problems for homework if necessary: Application #36-48, Connections # 76-77.

Summary and Reflection (Day 4):

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems and review/discuss any misconceptions/misunderstanding from Problems1.1-1.3.
3. Students will complete Math Reflections 1 individually and then share with other students. Finally discuss as a whole class.

Quiz (Day 5):

1. Students will take Accentuate the Negative Investigation 1 Quiz.

Summarizing Strategy:
Math Reflections 1 (Pg. 28 – 29) from Accentuate the Negative

Formative Assessment: Problems 1.1, 1.2, and 1.3; Problem 1.2 Exit Ticket; ACE problems; Mathematical Reflections 1	**Summative Assessment:** Accentuate the Negative Investigation 1 Quiz

Teaching Strategies/Tips:
Use CMP lesson plans to make sure all necessary materials are available.
Have students explain how they are determining their solutions. Encourage students to use sketches on number lines to show their thinking and to write addition and/or subtraction number sentences to describe their work.

Differentiation (content/process/product):
Throughout this Investigation, Labsheets 1ACE are available to use for scaffolding. Shorten assignments and provide notes as necessary. For extensions provide ACE problems that are related to the days problem and give the students about 3-6 problems per day extra.

Resources: Pearson Connected Mathematics Project (CMP3), Math-Drills.com (fluency), http://www.mathdashboard.com/cmp3/

Attachments (assessments, rubrics, graphic organizers, projects, etc.):
ACE Problems, Problem 1.2 Exit Ticket, Problem 1.2 Exit Ticket Possible Answers, Number Lines (2/student), Labsheets 1ACE, Mathematical Reflections 1 (Pg. 28 – 29) from Accentuate The Negative Student Edition, Accentuate the Negative Investigation 1 Quiz, Accentuate the Negative Investigation 1 Quiz – KEY, ACE Problems

Name: _____ Date: _____
Class: _____

Exit Ticket Problem 1.2

1. Draw a number line from -5 to 5. What would happen to the values of the numbers if we continue the number line to the left?

2. Which number is less, $^{-}999$ or $^{-}1{,}000$? Explain why your answer makes sense.

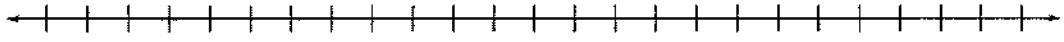
3. How do you find the opposites of a number on the number line? Provide evidence of your thinking.

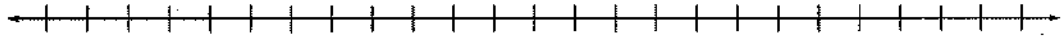
Exit Ticket Problem 1.2 Possible Answers

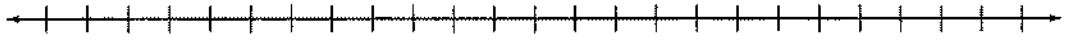
1. What would happen to the values of the numbers if we continue the shown number line to the left? *(The numbers continue to decrease in value.)*
2. Which number is less, $^{-}999$ or $^{-}1{,}000$? ($^{-}1{,}000$, because it is further to the left.)
3. How do you find the opposite of a number? *(Change the sign before the number. If a number is positive, its opposite is negative. If a number is negative, its opposite is positive.)*
4. How are a number and its opposite related? *(They are the same distance from 0 on the number line.)*

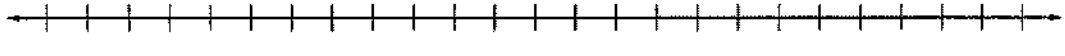
Name Date Class

Number Lines









Name ..... Date ..... Class .....

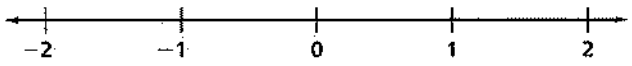
Labsheet 1ACE

Exercises 9 and 10

For each set of rational numbers in Exercises 9 and 10, draw a number line and locate the points. Remember to draw your number line with an appropriate *scale* and then plot the points.

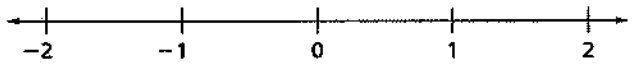
HINT: It might help you think about a scale for your number lines if you wrote all numbers in either fraction or decimal form.

9. $\frac{-2}{8}, \frac{1}{4}, -1.5, 1\frac{3}{4}$



HINT: What would be a good way to scale the number line for Exercise 9—halves or fourths?

10. $-1.25, \frac{-1}{3}, 1.5, \frac{-1}{6}$



HINT: What would be a good way to scale the number line for Exercise 10—halves, thirds, fourths, sixths, or twelfths?

Name _____ Date _____ Class _____

Labsheet 1ACE Exercise 48

48. Find the value for each labeled point on the number line. Then use the values to calculate each change.

a. *A* to *B*

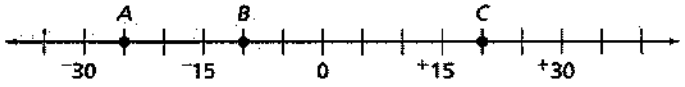
b. *A* to *C*

c. *B* to *C*

d. *C* to *A*

e. *B* to *A*

f. *C* to *B*



Mathematical Reflections

In this Investigation, you learned ways to order and operate with positive and negative numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **How** do you decide which of two numbers is greater when
   - **a.** both numbers are positive?
   - **b.** both numbers are negative?
   - **c.** one number is positive and one number is negative?
2. **How** does a number line help you compare numbers?
3. **When** you add a positive number and a negative number, how do you determine the sign of the answer?
4. If you are doing a subtraction problem on a chip board, and the board does not have enough chips of the color you wish to subtract, **what** can you do to make the subtraction possible?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Nick described his thoughts in the following way:

We used the number line to determine the temperature in Problem 1.3, Question E. We started at −5°F. Since the temperature rose 20°F during the day, we moved 20 tick marks to the right, which put us at 15°F. Since the temperature fell 25°F during the night, we moved 25 tick marks to the left and landed on −10°F. Then we moved 40 tick marks to the right to 30°F because of the heat wave. The temperature fell 70°F, so we moved to the left 70 tick marks. We are now at −40°F. We think it is unusual for the temperature to drop this much overnight and wonder where Sally lives.

Common Core Standards for Mathematical Practice

MP5 Use appropriate tools strategically.

- What other Mathematical Practices can you identify in Nick's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Applications

For Exercises 1–4, describe a sequence of five correct or incorrect answers that would produce each Math Fever score. Write a number sentence for each score.

1. Super Brains: 300
2. Rocket Scientists: $^-200$
3. Know-It-Alls: $^-250$
4. Teacher's Pets: 0

5. **Multiple Choice** Which numbers are listed from least to greatest?

   A. 300, 0, $^-200$, $^-250$

   B. $^-250$, $^-200$, 0, 300

   C. 0, $^-200$, $^-250$, 300

   D. $^-200$, $^-250$, 300, 0

For Exercises 6–8, find each Math Fever team's score. Write a number sentence for each team. Assume that each team starts with 0 points.

6. **Protons**

Point Value	Answer
250	Correct
100	Correct
200	Correct
150	Incorrect
200	Incorrect

7. **Neutrons**

Point Value	Answer
200	Incorrect
50	Correct
250	Correct
150	Incorrect
50	Incorrect

8. **Electrons**

Point Value	Answer
50	Incorrect
200	Incorrect
100	Correct
200	Correct
150	Incorrect

For each set of rational numbers in Exercises 9 and 10, draw a number line and locate the points. Remember to choose an appropriate scale.

9. $^-\frac{2}{8}, \frac{1}{4}, ^-1.5, 1\frac{3}{4}$
10. $^-1.25, ^-\frac{1}{3}, 1.5, ^-\frac{1}{6}$

11. Order the numbers from least to greatest.

$$23.6 \quad ^-45.2 \quad 50 \quad ^-0.5 \quad 0.3 \quad \frac{3}{5} \quad -\frac{4}{5}$$

Copy each pair of numbers in Exercises 12–19. Then insert $<$, $>$, or $=$ to make each a true statement.

12. $3 \blacksquare 0$

13. $^{-}23.4 \blacksquare 23.4$

14. $46 \blacksquare {}^{-}79$

15. $^{-}75 \blacksquare {}^{-}90$

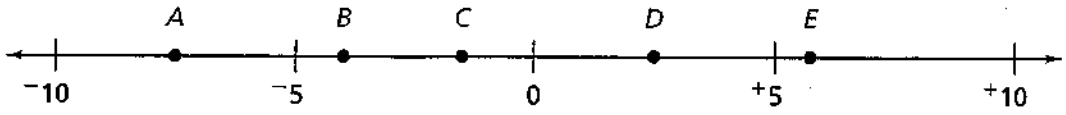
16. $^{-}300 \blacksquare 100$

17. $^{-}1{,}000 \blacksquare {}^{-}999$

18. $^{-}1.73 \blacksquare {}^{-}1.730$

19. $^{-}4.3 \blacksquare {}^{-}4.03$

20. a. Estimate values for points *A*–*E*.



b. On a copy of the number line, graph the following numbers.

$$^{-}9 \quad 10.5 \quad \frac{1}{2} \quad -\frac{5}{2}$$

c. Describe the location of a number and its opposite on the number line.

21. For each pair of numbers, identify which number is farther from $^{+}1$. Explain your reasoning.

a. $^{-}7$ or $^{+}3$

b. $^{-}10$ or $^{+}7$

22. Identify the temperature that is halfway between each pair of temperatures.

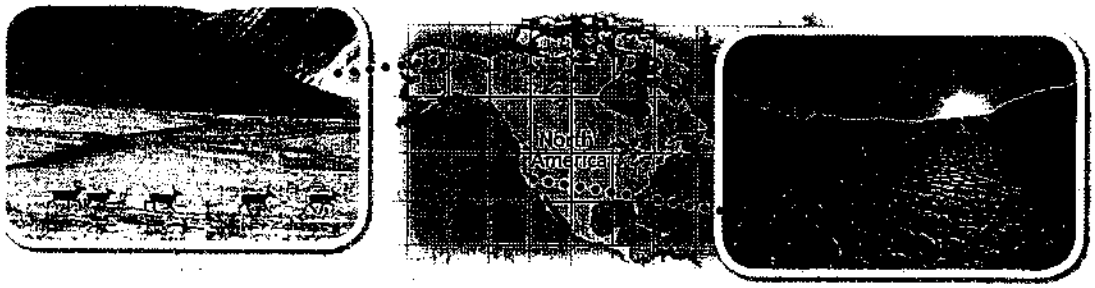
a. $^{-}23°F$ and $^{+}23°F$

b. $^{-}20°F$ and $^{+}10°F$

c. $^{+}20°F$ and $^{-}10°F$

Did You Know?

The record high and low temperatures in the United States are 134°F in Death Valley, California and $^{-}80°F$ in Prospect Creek in the Endicott Mountains of Alaska. Imagine going from 134°F to $^{-}80°F$ in an instant!



For Exercises 23–30, graph each statement on a number line.

23. x is less than 7.

24. x is greater than or equal to $^-7$.

25. $x < {}^-2$

26. $x \geq {}^-1$

27. $x \leq 8$

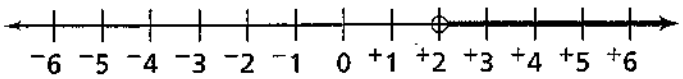
28. $x < 5$

29. ${}^-3 < x < 5$

30. $x > {}^-6$

For Exercises 31–34, write an inequality for each set of numbers on the number line.

31.



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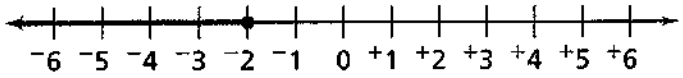

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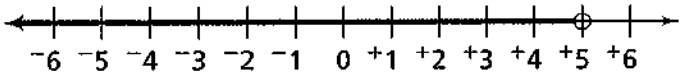

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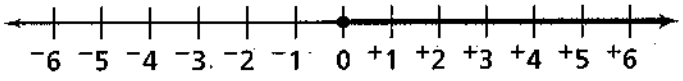
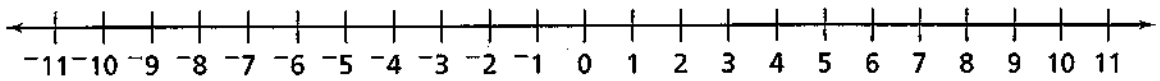


35. The school cafeteria can hold at most 150 people.

a. Write a number sentence to represent the number of people that can be in the cafeteria at any time during the day.

b. Graph your answer to part (a) on a number line.

For Exercises 36–45, follow the steps using the number line. What is the final position?



36. Start at 8. Add $^-7$.

37. Start at $^-8$. Add 10.

38. Start at $^-3$. Add $^-5$.

39. Start at 7. Add $^-7$.

40. Start at $^-2$. Add 12.

41. Start at 3. Subtract 5.

42. Start at $^-2$. Subtract 2.

43. Start at 4. Subtract 7.

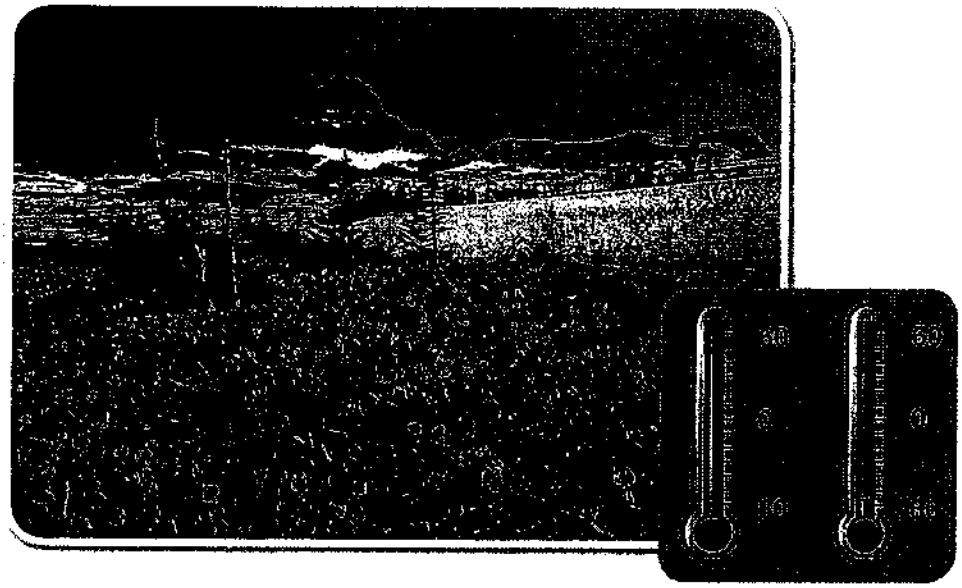
44. Start at 0. Subtract 5.

45. Start at $^-8$. Subtract 3.

46. **a.** What are the opposites of 3, 7.5, and $^{-}2\frac{2}{3}$?

b. For each number in part (a), find the sum of that number and its opposite.

47. The greatest recorded one-day temperature change occurred in Browning, Montana (bordering Glacier National Park), from January 23–24, 1916. The temperature fell from 44°F to $^{-}$56°F in less than 24 hours.

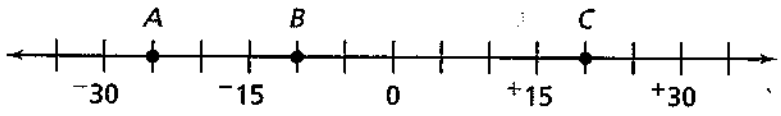


a. What was the temperature change that day?

b. Write a number sentence to represent the change.

c. Show the temperature change on a number line.

48. Find the value for each labeled point on the number line. Then use the values to calculate each change.



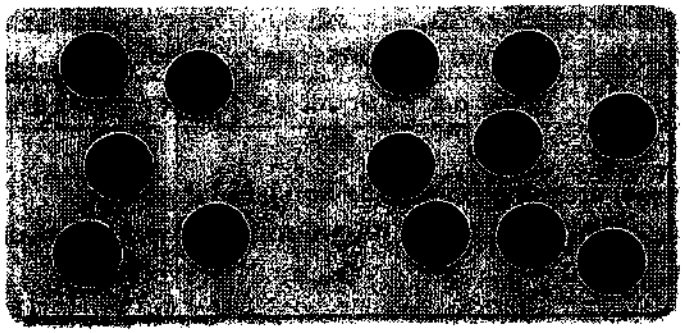
a. A to B **b.** A to C **c.** B to C

d. C to A **e.** B to A **f.** C to B

For Exercises 49–52, find the missing part for each chip problem. Write a number sentence for each problem.

	Start With	Rule	End With	Number Sentence
49.	●●●	Add 5 ●	■	■
50.	●●●	Subtract 3 ●	■	■
51.	●●●●●	■	●●	■
52.	■	Subtract 3 ●	●●●●	■

53. Write a story problem for this situation. Find the value represented by the chips on the board.



For Exercises 54 and 55, use the chip board from Exercise 53.

54. What is the new overall value of the board when you

a. remove 3 red chips?

b. then add 3 black chips?

c. then add 200 black chips and 195 red chips?

55. Describe three different ways to change the numbers of black and red chips on the original board, but leave the value of the board unchanged.

Connections

56. In a football game, one team makes seven plays in the first quarter. The results of those plays are (in order): gain of 7 yards, gain of 2 yards, loss of 5 yards, loss of 12 yards, gain of 16 yards, gain of 8 yards, loss of 8 yards.

a. What is the overall gain (or loss) from all seven plays?

b. What is the average gain (or loss) per play?

For Exercises 57 and 58, find the total number of strokes over or under par for each golf player. Write number sentences with positive and negative integers to show each result.

	Player	Round 1	Round 2	Round 3	Round 4
57.	Elijah Sparks	4 over par	6 under par	3 under par	1 over par
58.	Keiko Aida	2 under par	1 under par	5 over par	5 under par

For Exercises 59–64, draw a number line and label it with an appropriate scale. Graph and name the two numbers described on the number line.

59. two fractions between 0 and 1

60. two fractions between 2 and 3

61. two fractions between $^-1$ and 0

62. two decimals between $^-3$ and $^-2$

63. two decimals between 4 and 5

64. two decimals between $^-4$ and $^-3$

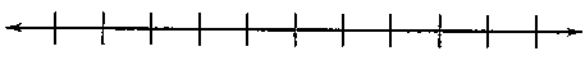
There is always a rational number between two other rational numbers. For Exercises 65–67, graph the two numbers on a number line. Then graph and label a point between the two numbers.

65. 1.4 and 1.5

66. $^-1.42$ and $^-1.4$

67. $^-5\frac{1}{2}$ and $^-5\frac{1}{4}$

For Exercises 68 and 69, copy the number line below.

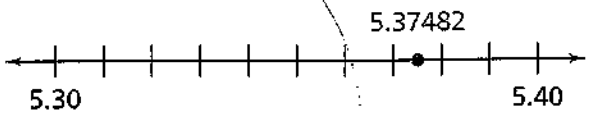


68. Label the first tick mark 28.36 and the last tick mark 28.37. Label the appropriate tick mark for 28.369. Then label the remaining tick marks.

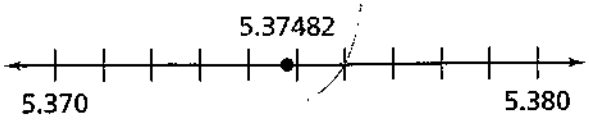
69. Label the first tick mark $^-7.7$ and the last tick mark $^-7.6$. Label the appropriate tick mark for $^-7.65$. Then label the remaining tick marks.

For Exercises 70 and 71, label the tick marks on each number line. Explain why you labeled them that way.

70.



71.



For Exercises 72–75, order the numbers from least to greatest.

72. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{5}{9}$, $\frac{9}{25}$

73. 20.33, 2.505, 23.30, 23

74. 1.52, $1\frac{4}{7}$, 2, $\frac{9}{6}$

75. 3, $\frac{19}{6}$, $2\frac{8}{9}$, 2.95

For Exercises 76 and 77, use the following. The highest point on earth is the top of Mount Everest. It is 29,035 feet above sea level. The lowest exposed land is the shore of the Dead Sea. It is 1,310 feet below sea level.

76. Multiple Choice What is the change in elevation from the top of Everest to the shore of the Dead Sea?

F. $^{-}$30,345 feet

G. $^{-}$27,725 feet

H. 27,725 feet

J. 30,345 feet

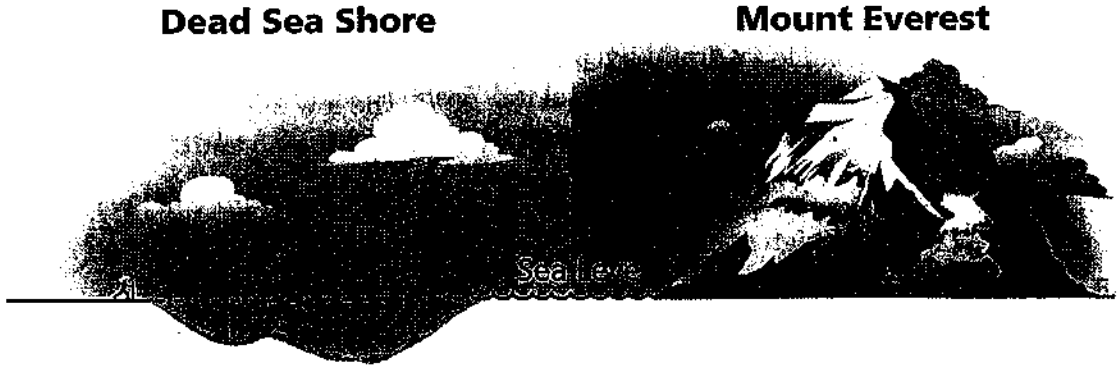
77. Multiple Choice What is the change in elevation from the shore of the Dead Sea to the top of Everest?

A. $^{-}$30,345 feet

B. $^{-}$27,725 feet

C. 27,725 feet

D. 30,345 feet



Extensions

78. At the start of December, Kenji had a balance of $595.50 in his checking account. The following is a list of transactions he made during the month.

Date	Transaction	Balance
December 1		$595.50
December 5	Writes a check for $19.95	
December 12	Writes a check for $280.88	
December 15	Deposits $257.00	
December 17	Writes a check for $58.12	
December 21	Withdraws $50.00	
December 24	Writes checks for $17.50, $41.37, and $65.15	
December 26	Deposits $100.00	
December 31	Withdraws $50.00	

a. Copy and complete the table.

b. What was Kenji's balance at the end of December?

c. When was his balance the greatest? When was his balance the least?

For Exercises 79–84, find all the values of x that satisfy the statement. Then sketch the solution on a number line.

79. $x + 2$ is negative.

80. $x - 5$ is greater than 0.

81. $x + 3 < 1$

82. $x + 3 \geq 2$

83. $3 - x < 0$

84. $6 \leq x - 4$

For Exercises 85–87, find the missing temperature in each situation.

85. On Monday, the high temperature was 20°C. The low temperature was $^-15$°C. What temperature is halfway between the high and the low?

86. On Tuesday, the low temperature was $^-8$°C. The temperature halfway between the high and the low is 5°C. What was the high temperature?

87. On Wednesday, the high temperature was $^-10$°C. The low temperature was $^-15$°C. What temperature is halfway between the high and the low?

Find values for A and B that make each mathematical sentence true.

88. $^+A + {}^-B = {}^-1$

89. $^-A + {}^+B = 0$

90. $^-A - {}^-B = {}^-2$

Accentuate the Negative Investigation 1 Quiz

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

1) Match the correct vocabulary word with the correct phrase to make a complete sentence. (7.NS.A.1a)

Word Bank
Positive numbers
Negative numbers
Integers
Opposites
Rational numbers

_____ numbers to the left of the zero.

_____ numbers the same distance from zero on a number line, whose sum is zero.

_____ numbers to the right of the zero.

_____numbers expressed as one integer over another integer.

_____ whole numbers, both positive and negative, including zero.

2. Use a number line to show how you would add the following numbers, then write a number sentence to represent your model. (7.NS.A.1b)

a. -3, 4

b. -12, 7

c. 9, -6

d. -3, -1

3. Use a number line to show how you would subtract the following numbers, then write a number sentence to represent your model. (7.NS.A.1c)

a. -3, 4

b. -12, 7

c) 9, -6

d) -3, -1

4. Write the opposites of the following numbers next to the numbers. Explain how you know they are opposites. (7.NS.A.1b)

2	_____	4	_____	-121	_____	-70	_____
89	_____	$-\frac{3}{4}$	_____	$\frac{7}{13}$	_____	-64	_____

5. The **record high and low temperatures** in the United States are 134° F in Death Valley, California and -80° F in Prospect Creek in the Endicott Mountains of Alaska. What is the temperature difference between these to records. **Use a model and number sentence to show your work.** (7.NS.A.3)

Accentuate the Negative Investigation 1 Quiz - KEY

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

1. Match the correct vocabulary word with the correct phrase to make a complete sentence. (7.NS.A.1a)

Word Bank

Positive numbers
Negative numbers
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Rational numbers

_____ numbers to the left of the zero.

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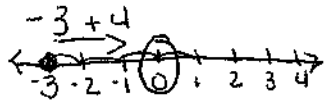
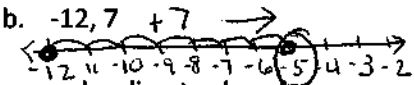
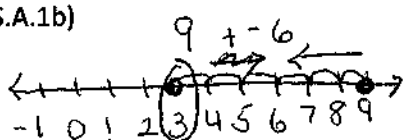
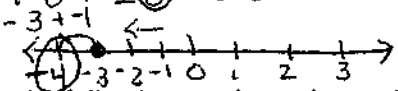
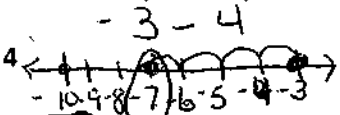
a. -3, 4 

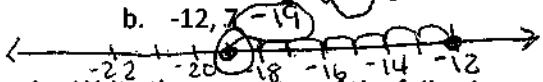
b. -12, 7 

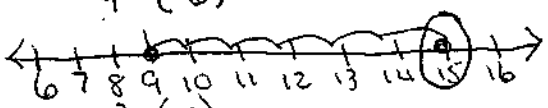
c. 9, -6 

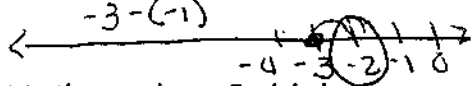
d. -3, -1 

3. Use a number line to show how you would subtract the following numbers, then write a number sentence to represent your model. (7.NS.A.1c)

a. -3, 4 

b. -12, 7 

c) 9, -6 

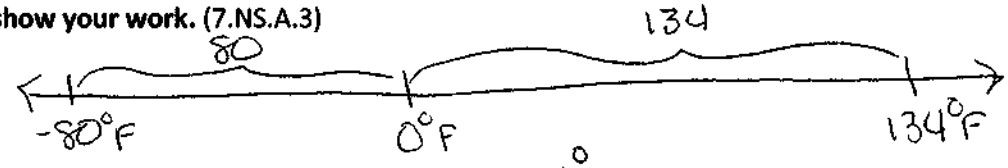
d) -3, -1 

4. Write the opposites of the following numbers next to the numbers. Explain how you know they are opposites. (7.NS.A.1b)

2	-2	4	-4	-121	121	-70	70
89	-89	$-\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{13}$	$-\frac{7}{13}$	-64	64

If they are added together the sum is zero.

5. The **record high and low temperatures** in the United States are 134°F in Death Valley, California and -80°F in Prospect Creek in the Endicott Mountains of Alaska. What is the temperature difference between these to records. **Use a model and number sentence to show your work.** (7.NS.A.3)



80 + 134 = 214°

Investigation 2: Adding and Subtracting Rational Numbers

Lesson Focus Questions: How do we add and subtract rational numbers?

Standards: 7.NS.A.1b, 7.NS.A.1c, 7.NS.A.1d, 7.NS.A.3 ,7.EE.B.3, MP.7, MP.8

Learner Outcomes (KUD):
Students will:
Develop algorithms for adding, subtracting, multiplying and dividing integers.
Interpret and write mathematical sentences to show relationships and solve problems.
Demonstrate an understanding of and use the Commutative Property for addition of integers/rational numbers.
Demonstrate an understanding of and use Fact Families for addition and subtraction of integers/rational numbers.
Look for and make use of structure.
Look for and express regularity in repeated reasoning.

Pre-Requisites: Understanding of number lines, addition, and subtraction and previous models for integers.

Key Concepts/Vocabulary: Absolute value, additive identity, additive inverses, algorithm, Commutative Property

Activating Strategy: "Who's The Winner?" Teacher will give students the following directions:
Each player starts with 20 singles. Player 1 rolls the dice. If the number is even player 1 gives player 2 the amount shown on the dice. If the number is odd player 1 receives that amount from player 2. Then Player 2 rolls the dice. Both players are to record each transaction since at some point they may owe money they do not have.

Let students play for 5 minutes. Teacher will ask each group of 2, "Who won?, How do you know who won?, What happened if you owed your partner money and then rolled and owed them more? What happened if you owed your partner money and then rolled and you received money from them?"

Learner Activities: (6 days)
Problems 2.1 Extending Addition to Rational Numbers (Day 1): In this problem students are introduced to the idea of an algorithm and challenged to develop an algorithm for addition of rational numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Teacher displays Teaching Aid 2.1A: Linda's Chip Model and presents the following problem: "Linda has 8 video games and her friend has 5. How many do they have all together?" Have students describe how the Chip Model is used to find the solution.
   Teacher asks, What other model can you use to represent this problem?" *with a number line*. Ask for a student to show what that model would look like.
3. Now teacher displays Teaching Aid 2.1B: Weather Station Number Line Model and asks "How can you model this problem using a chip model? What number sentence can you write to represent this model? How is this problem different from the first? Discuss.
4. Teacher gives students a few minutes to read the introduction to Investigation 2 –

Adding and Subtracting Rational Numbers on page 30-31 in their student books silently and asks for a thumbs up when they are done. When most students have finished ask, "What is an algorithm?" "How can you predict whether the sum of two integers is 0, positive or negative?"

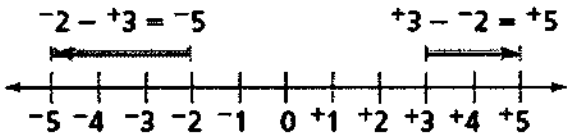
5. Students working in pairs begin working on Problem 2.1 Part A-B. Students should use what they know about the two models to help them come up with the sums in each group. Tell students that the groups in Question A of Problem 2.1 have something in common that will help them develop an algorithm for addition with integers. Have them find the sums for each group and think about what the examples have in common.
6. Teacher displays Teaching Aid 2.1C: Addition Grouping and have different groups present their solutions. Use the following questions to help summarize what they have discovered. How do you solve addition problems in which both signs of the numbers are the same? How do you solve addition problems in which the signs of the numbers are different? For what types of expressions did you get a negative number? A positive number? How can we write our observations as rules or algorithms?
7. Students continue working on Parts C-G in pairs. After students work on Part C for a while, have them check their work for parts 1-3 with other groups of students. Teacher will call on students and groups to discuss whether or not their algorithm worked and how thy came to agreement with one another. Revise algorithm is necessary.
8. As a whole class discuss solutions and stories to Parts C-G. In Parts E,F, andG, students will encounter additive inverses. The teacher may introduce the definition of this term during the discussion or at the beginning of Problem 2.2.
9. Students will do an assortment of problems from the ACE problems for homework. Applications #1-17, Connections #60, Extensions # 64.

Problem 2.2 Extending Subtraction to Rational Numbers (Day 2): The students explore addition and subtraction of rational numbers using chip models and number line models. They develop algorithms for these operations.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Using the text on pg. 34, the teacher introduces students to the terms *additive inverses* and *additive identity* and provides additional examples to reinforce these concepts. Ask is 5 + 2 + – 2 the same as 5? Why or why not?
4. Teacher displays Teaching Aid 2.2A: Kim's Chip Model separate from the text. This is a simple and obvious example. However, the goal in discussing it is to focus on the actions of the problem. Show students how chips can be used to model these actions and lead to a solution. This helps emphasize the kind of problem for which chips are useful.
5. Teacher displays Teaching Aid 2.2B: Otis's Chip Model and models the actions of this example with the students. This situation, +5– +7, is more complex. In this situation, students need to think about a way to represent +5so that +7can be removed. This means showing +5 as some combination of black and red chips calling attention to the use of additive inverses and additive identity in solving the

example. Make sure to write mathematical sentences that reflect the changes made to the board.

6. As a class begin Problem 2.2. For Question A and B, teacher displays Teaching Aid 2.2C: Number Line Model for Subtraction. Have students focus on the number line model providing both distance (absolute value) and direction. Ask students to explain how the number line in Question A relates to the number line in the introduction. For Question C, display Teaching Aid 2.2D: Subtraction Grouping. Have different teams of students present their solutions. Ask: How do you compute subtraction problems in which both signs are the same? *(When both numbers have the same sign, subtract their absolute values. If the first number is greater, the answer is positive. If the second number is greater, the answer is negative.)* How do you compute subtraction problems in which the signs of the numbers are different? *(When the numbers have opposite signs, add their absolute values. If the first number is greater, the answer is positive. If the second number is greater, the answer is negative.)* Discuss the similarities and differences between the expressions in each group in Question C to help students formulate an algorithm. Ask: How do you know if the difference is positive or negative? When you are subtracting with integers, must you always think of the operation as subtraction? Why or why not? For Question D, have students check their work with other students and other groups to see if they can determine whether their algorithms work with rational numbers. Have them justify their responses. Question F, has students focus on the Commutative Property. Looking back at the introduction, they can see that addition is commutative. However, as they experiment with subtraction examples, they will see that this property does not extend to subtraction. Suggest that a number line representation can help students visualize the property and the fact that it is not valid for subtraction.



These are not the same.

7. To check for understanding have students consider the following pairs of expressions.
   $^-7 + {}^-8$ and $^-7 - {}^+8$
   $^-12 - {}^-4$ and $^-4 - {}^-12$
   Tell students that without computing any of the answers, predict how the expressions within each pair compare to each other. Will the two answers in each pair be the same or different? If they are different, which will be greater? Why?
8. Students will do an assortment of problems from the ACE problems for homework. Applications #18–37, Connections #61–62, Extensions #65–66.

Problem 2.3 The +/- Connection (Day 3): This Problem promotes informal experiences with addition and subtraction of signed numbers. Students connect the operations of

addition and subtraction (including the relationships between these two operations) to actions on chip board displays.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Teacher uses Teaching Aid 2.3: Chip Model +/− and Chip Board with either two color chips or two color markers (red, black) to engage students with the idea that two different actions on the chip board (one addition and one subtraction) can both result in the same value. This is designed to help students build the +/− connection. Present the following problems to the class and ask students to show how each should be represented using the chip board. Students should recognize that restating subtraction of a positive as addition of a negative or restating subtraction of a negative as addition of a positive produces the same results and can simplify calculations.

+7 + ⁻ 6 = +1 *(begin with 7 black chips and add 6 red chips, make zero pairs (+,−) to show 1 black chip left*

+7 − + 6 = +1 *(begin with 7 black chips, remove 6 black chips from the board to show 1 black chip left)*

+7 + +6 = +13 *(begin with 7 black chips, add 6 black chips to the board to make 13)*

+7 − ⁻6= +13 *(begin with 7 black chips, in order to subtract ⁻6, add 6 zero pairs (+6, ⁻6) and remove 6 red chips to show 13 black chips)*

4. Students will read page 39 in their student books with a partner and then begin the exercises in Problem 2.3. As students work encourage them to share and discuss answers with other groups making sure to justify their reasoning.
5. As a class discuss all exercises from Problem 2.3. Teacher asks the following questions to help students summarize their thinking:
   What do you know about the relationship between addition and subtraction of integers? *You can change a subtraction expression to addition by adding the opposite, and you can change an addition expression to subtraction by subtracting the opposite. The two forms of each expression are equal in value.*
6. Students will do the following ACE problems for homework. Applications # 38–49 Connections # 63 Extensions # 67–68.

Problem 2.4 Fact Families (Day 4): This problem connects whole-number fact families to addition and subtraction of signed numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Teacher shows students the launch video for Problem 2.4 . This engaging video gives students a visual of how numbers relate to one another within a fact family. The animation shows the terms within a fact family changing position but not value. This video can help students recall how to form fact families before they extend their knowledge of number relationships to rational numbers.
4. Teacher writes 4 + 6 = 10 on the board and asks and says, "What are the other three number sentences that make a fact family with 4+6=10? Look carefully at the relationships in the fact family. Why are these relationships true?" (The two addition sentences are true because addition is commutative. *The two subtraction sentences are true because either addend can be found by subtracting the other addend from the sum, 10.)* " Do the relationships in fact families with rational

numbers always hold true?" *(Yes. The addition sentences hold true because addition is commutative for all rational numbers. If you apply the Subtraction Property of Equality to the addition sentences, you get the two subtraction sentences. So the subtraction sentences hold true as well.)*

5. Teacher distributes Labsheet 2.4: Fact Family Table as they begin Problem 2.4. As teacher circulates, ask students to explain their reasoning. If a student or pair is struggling, you might want to observe and ask questions to help them develop a fact family with negative numbers. Here is an example:
   $^{-}7+2=^{-}5 \qquad 2+(^{-}7)=^{-}5 \qquad ^{-}5-2=^{-}7 \qquad ^{-}5-(^{-}7)=2$
   Students need to see that these ideas are just extensions of what they already know from whole numbers. Make sure students are using the fact families to answer the questions in parts B–E. Question E may be difficult for some students because of the variable *n*. In this case, the teacher might suggest that they replace *n* with a question mark and say in words what they are trying to find. If students are still confused, have them substitute 40 for *n* and write the fact family. Then have them rewrite the fact family with *n* in the place of 40.Be sure to show students how to read equations such as the ones below. Equations with many negative signs in different contexts can be difficult for students to interpret. For example, students should read the second equation below as "The opposite of n equals negative eight."
   If $-n=8$, *what does* n *equal?* (–8)
   If $-n=-8$, *what does* n *equal?* (8)
   If students finish the problem quickly, you can have them repeat Question A using decimals and mixed numbers for the values of *a* and *b* to generalize their conjectures from integers to rational numbers.
6. To summarize student experiences with this problem, have students share their results for each number sentence. Highlight Questions D and E, asking students how they might use what they know about fact families to solve for an unknown number in addition and subtraction sentences.
7. Students will do the following ACE problems for homework. Applications # 50–59 Extensions # 69.

Review (Day 5)

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems
3. Have students individually complete Mathematical Reflections 2 (Pg. 52) from Accentuate The Negative Student Edition and then discuss as a class to review for the quiz.

Quiz (Day 6)

1. Students will take Accentuate the Negative Investigation 2 Quiz.

Summarizing Strategy: Have students complete Mathematical Reflections 2 (Pg. 52-53) from Accentuate The Negative Student Edition individually and then discuss as a class to review for the quiz.

Formative Assessment: Problems 2.1, 2.2, 2.3, 2.4; ACE problems; Mathematical Reflection	**Summative Assessment:** Investigation 2 Quiz

Teaching Strategies/Tips: Use CMP lesson plans to make sure all necessary materials are available. Have students explain how they are determining their solutions. Encourage students to use sketches on number lines to show their thinking and to write addition and/or subtraction number sentences to describe their work.

Differentiation (content/process/product
Throughout this Investigation, Labsheet 2ACE is available to use for scaffolding. Shorten assignments and provide notes as necessary.
For extensions provide ACE problems that are related to the days problem and give the students about 3-6 extra problems per day.

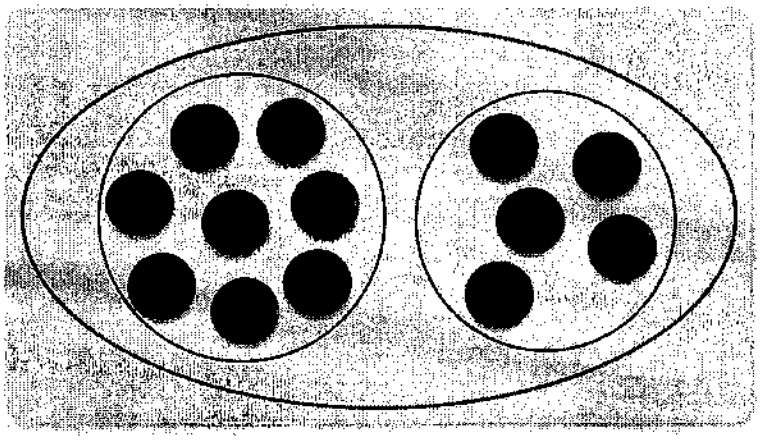
Resources: Pearson Connected Mathematics Project (CMP3), Math-Drills.com (fluency), http://www.mathdashboard.com/cmp3/

Attachments (assessments, rubrics, graphic organizers, projects, etc.):
Teaching Aid 2.1A:Linda's Chip Model, Teaching Aid 2.1B:Weather Station Number Line Model, Teaching Aid 2.1C: Addition Grouping, ACE Problems, Teaching Aid 2.2A: Kim's Chip Model, Teaching Aid 2.2B: Otis's Chip Model, Teaching Aid 2.2C: Number Line Model for Subtraction, Teaching Aid 2.2D: Subtraction Grouping, Labsheet 2ACE, Teaching Aid 2.3: Chip Model +/−, Chip Board, Fact Family Table, ACE Problems, Mathematical Reflections 2 (Pg. 52) from Accentuate The Negative Student Edition, Investigation 2 Quiz

Problem 2.1 Linda's Chip Model

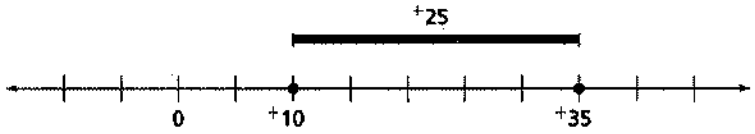
Linda has 8 video games, and her friend has 5. Together they have $8 + 5 = 13$ games.

8 + 5 = 13

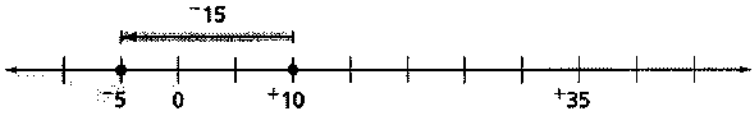


Problem 2.1 Weather Station Number Line Model

At a desert weather station, the temperature at sunrise was 10°C. It rose 25°C by mid-day. The temperature at noon was 10°C + 25°C = 35°C.



Suppose, instead of rising 25°C, the temperature fell 15°C.



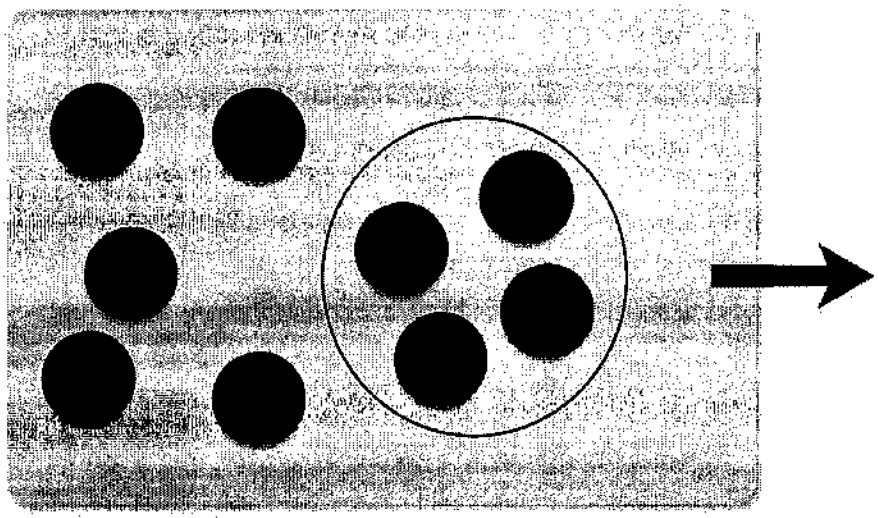
Problem 2.1 Addition Grouping

Group 1	Group 2
$^{+}2 + {}^{+}8$	$^{+}2 + {}^{-}8$
$^{-}2 + {}^{-}8$	$^{-}2 + {}^{+}8$
$^{+}8 + {}^{+}12$	$^{+}8 + {}^{-}12$
$^{-}8 + {}^{-}12$	$^{-}8 + {}^{+}12$

Problem 2.2 Kim's Chip Model

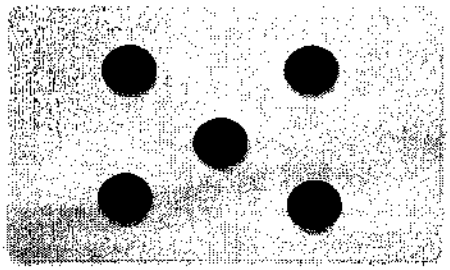
Kim had 9 DVDs. She sold 4 at a yard sale. She now has 9 − 4 = 5 of those DVDs left.

9 − 4 = 5



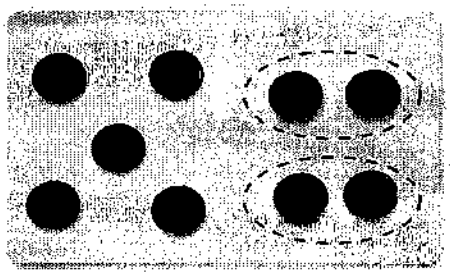
Problem 2.2 Otis's Chip Model

Otis earned $5 raking leaves. He wants to buy a used bike that costs $7. His older sister puts 5 black chips on the table to represent the money Otis has.



What is the value of Otis's board?

Otis's sister asks, "How much more money do you need?" Otis replies, "I could find out by taking away $7. But I can't take away $7 because there aren't seven black chips on the board!" His sister adds two black chips and two red chips.

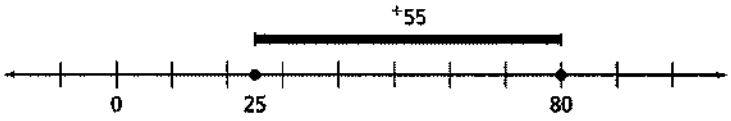


Is the value of the board the same with the new chips added? Explain.

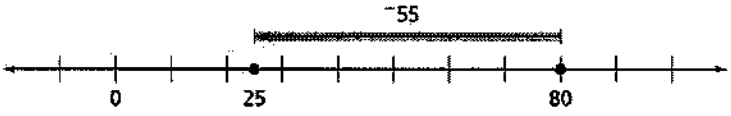
How does this help Otis find how much more he needs?

Problem 2.2 Number Line Model for Subtraction

$$80 - 25 = 55$$



$$25 - 80 = {}^{-}55$$



Problem 2.2 Subtraction Grouping

Group 1
$^{+}12 - {}^{+}8$
$^{-}5 - {}^{-}7$
$^{-}4 - {}^{-}2$
$^{+}2 - {}^{+}4$

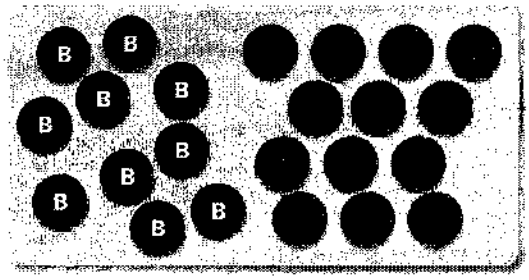
Group 2
$^{+}12 - {}^{-}8$
$^{-}5 - {}^{+}7$
$^{-}4 - {}^{+}2$
$^{+}2 - {}^{-}4$

Name ..... Date ..... Class .....

Labsheet 2ACE

Exercises 15 and 16

The chip board below has 10 black and 13 red chips.



HINT: Remember black equals a negative and red equals a positive.

15. What is the value shown on the board?

16. Write a number sentence to represent each situation. Then find the new value of the chip board.

a. Remove 5 red chips from the original board.

What will be the value of the chip board?

Write a number sentence to show your work.

b. Then add 5 black chips.

What will be the value of the chip board?

Write a number sentence to show your work.

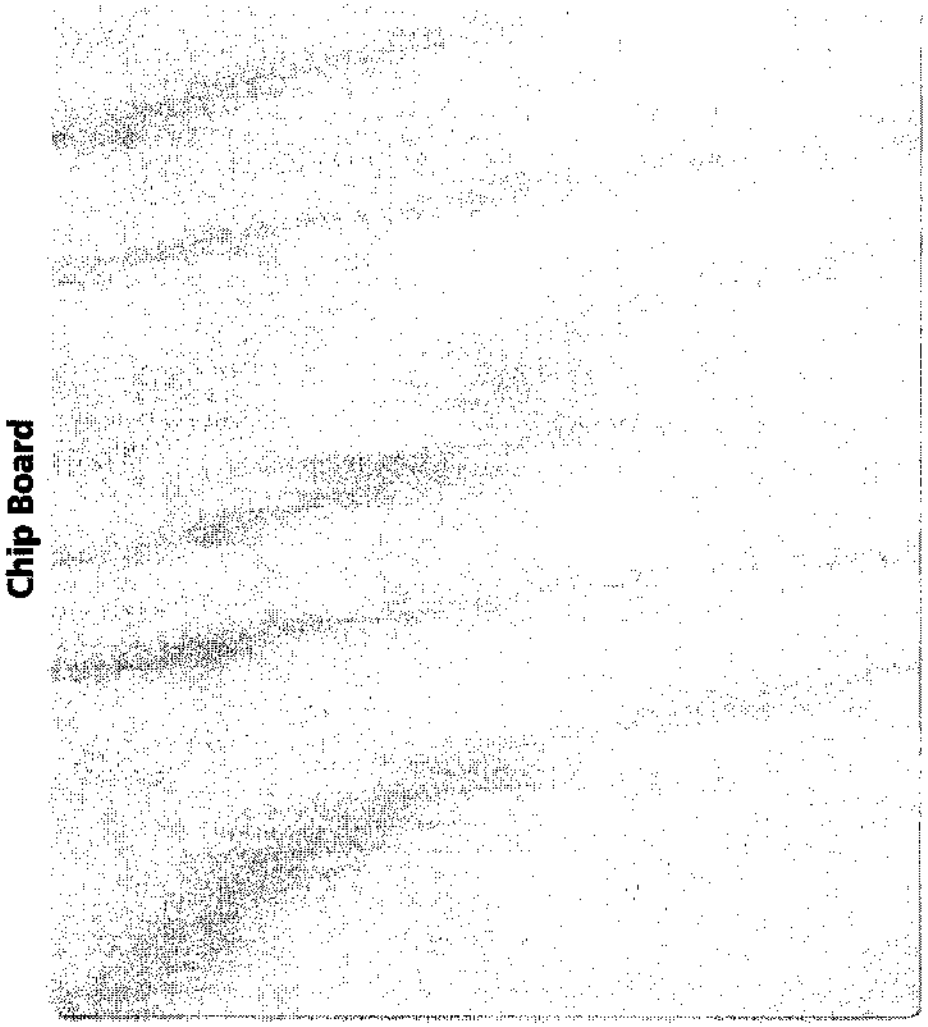
c. Then add 4 black chips and 4 red chips.

What will be the value of the chip board?

Write a number sentence to show your work.

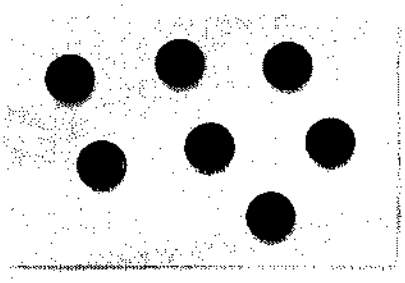
Name _____ Date _____ Class _____

Chip Board



Problem 2.3 Chip Model +/-

The chip board below shows a value of $^{+}7$.



There are two possible moves, one addition and one subtraction, that would change the value on the board to $^{+}1$. How would you complete the number sentences to represent each move?

$$^{+}7 + \blacksquare = {}^{+}1 \text{ and } {}^{+}7 - \blacksquare = {}^{+}1$$

There are two possible moves, one addition and one subtraction, that would change the value on the board to $^{+}13$. How would you complete the number sentences to represent each move?

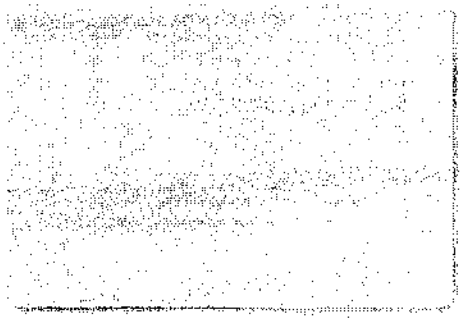
$$^{+}7 + \blacksquare = {}^{+}13 \text{ and } {}^{+}7 - \blacksquare = {}^{+}13$$

Can you describe a general relationship between addition and subtraction for integers?

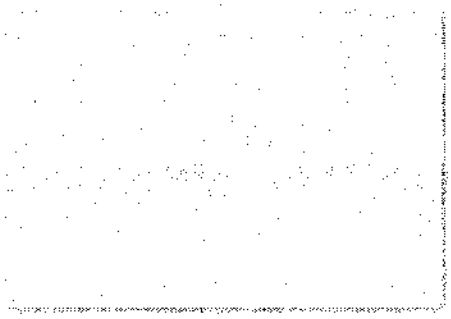
Name _____ Date _____ Class _____

Small Chip Boards

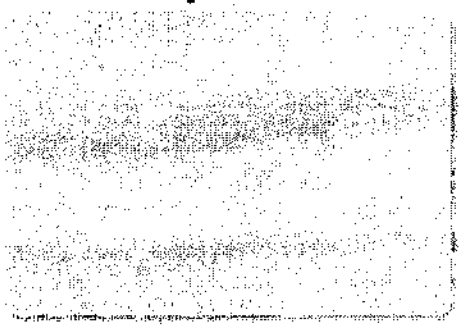
Chip Board



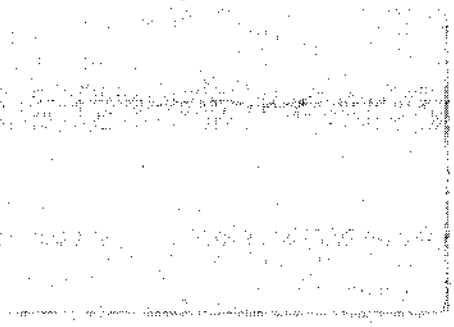
Chip Board



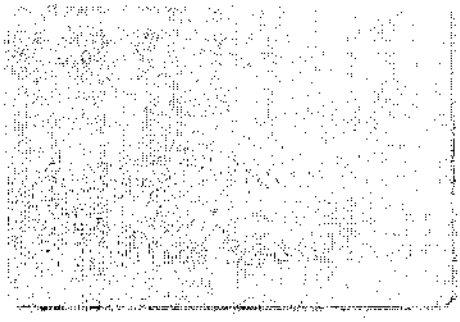
Chip Board



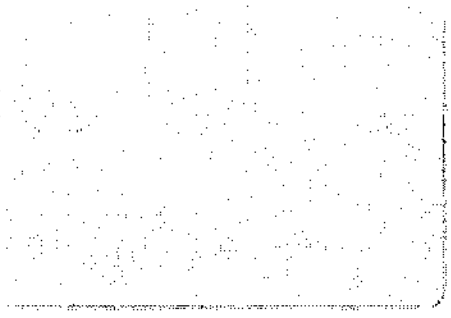
Chip Board



Chip Board



Chip Board



FACT FAMILY TABLE

a	b	a + b = c	c	b + a = c	a = c - b	b = c - a

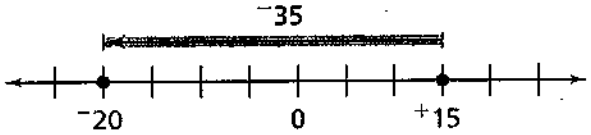
Applications

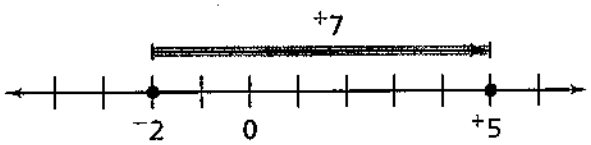
For Exercises 1–12, use your algorithms to find each sum without using a calculator.

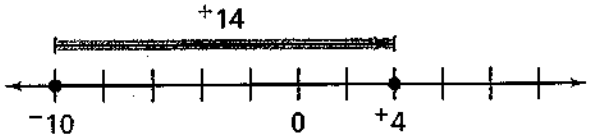
1. $^{+}12 + {}^{+}4$
2. $^{+}12 + {}^{-}4$
3. $^{-}12 + {}^{+}4$
4. $^{-}7 + {}^{-}8$
5. $^{+}4.5 + {}^{-}3.8$
6. $^{-}4.5 + {}^{+}3.8$
7. $^{-}250 + {}^{-}750$
8. $^{-}6{,}200 + {}^{+}1{,}200$
9. $^{+}0.75 + {}^{-}0.25$
10. $^{+}\frac{2}{3} + {}^{-}\frac{1}{6}$
11. $^{-}\frac{5}{12} + {}^{+}\frac{2}{3}$
12. $^{-}\frac{8}{5} + {}^{-}\frac{3}{5}$

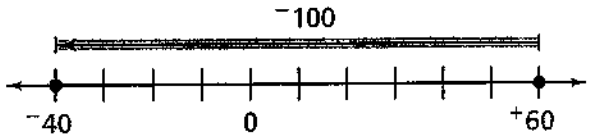
13. Find each sum.
    a. $^{+}3.8 + {}^{+}2.7$
    b. $^{-}3.8 + {}^{-}2.7$
    c. $^{-}3.8 + {}^{+}2.7$
    d. $^{+}3.8 + {}^{-}2.7$

14. Write an addition number sentence that matches each diagram.

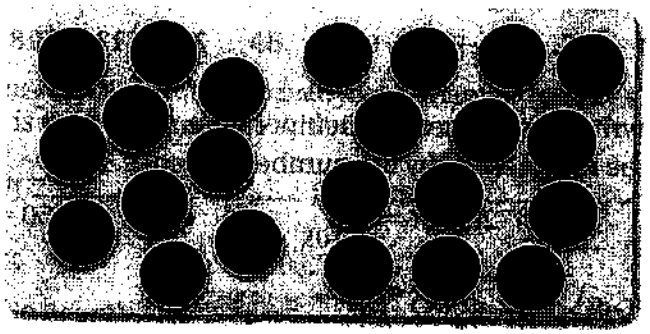
a.


b.


c.


d.


For Exercises 15 and 16, use the chip board below. The chip board has 10 black chips and 13 red chips.



15. What is the value shown on the board?

16. Write a number sentence to represent each situation. Then find the new value of the chip board.

a. Remove 5 red chips from the original board.

b. Then add 5 black chips.

c. Then add 4 black chips and 4 red chips.

17. Use properties of addition to find each value.

a. $^{+}43 + {}^{-}47 + {}^{-}43$ **b.** $^{+}5.2 + {}^{-}5.2 + {}^{-}\frac{4}{7}$ **c.** $^{+}5\frac{2}{5} + {}^{+}\frac{3}{7} + {}^{-}5\frac{2}{5}$

For Exercises 18–29, use your algorithms to find each difference without using a calculator. Show your work.

18. $^{+}12 - {}^{+}4$ **19.** $^{+}12 - {}^{+}12$ **20.** $^{-}12 - {}^{+}12$

21. $^{-}7 - {}^{+}8$ **22.** $^{+}45 - {}^{-}40$ **23.** $^{+}45 - {}^{-}50$

24. $^{-}25 - {}^{-}75$ **25.** $^{-}62 - {}^{-}12$ **26.** $^{+}0.8 - {}^{-}0.5$

27. $^{+}\frac{1}{2} - {}^{+}\frac{3}{4}$ **28.** $^{-}\frac{2}{5} - {}^{+}\frac{1}{5}$ **29.** $^{-}\frac{7}{10} - {}^{+}\frac{4}{5}$

30. Find each value without using a calculator.

a. $^{+}12 + {}^{-}12$ **b.** $^{+}4 - {}^{+}12$ **c.** $^{-}12 - {}^{+}4$

d. $^{-}12 - {}^{-}12$ **e.** $^{-}12 + {}^{-}12$ **f.** $^{-}12 + {}^{+}12$

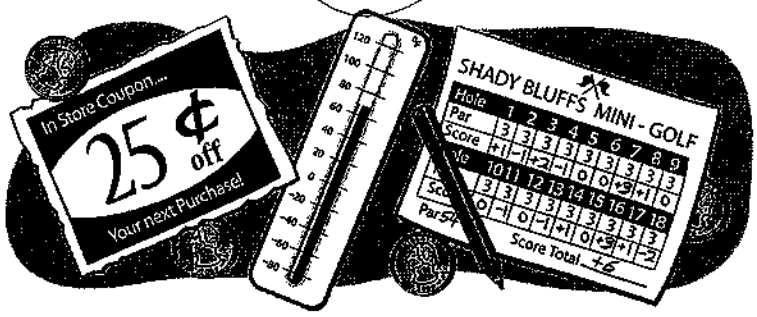
For Exercises 31–36, find each value.

31. $^{+}50 + {}^{-}35$ **32.** $^{+}50 - {}^{-}20$ **33.** $^{-}19 - {}^{+}11$

34. $^{-}30 - {}^{+}50$ **35.** $^{-}35 + {}^{-}15$ **36.** $^{+}12 + {}^{-}18$

37. For each part below, write a problem about temperature, money, or game scores that can be represented by the number sentence.

a. $^{+}7 - {}^{-}4 = {}^{+}11$ **b.** $^{-}20 + n = {}^{+}30$ **c.** $^{-}n + {}^{-}150 = {}^{-}450$



38. Without doing any calculations, decide which expression is greater. Explain your reasoning.

a. $5{,}280 + {}^{-}768$ or $5{,}280 - {}^{-}768$

b. $1{,}760 - {}^{-}880$ or $1{,}760 - 880$

c. $1{,}500 + 3{,}141$ or $1{,}500 - {}^{-}3{,}141$

39. Without doing any calculations, determine which of the following results are positive and which are negative. Explain your reasoning.

a. $^{-}23 + 19$ **b.** $3.5 - {}^{-}2.7$

c. $^{-}3.5 - {}^{-}2.04$ **d.** $3.1 + {}^{-}6.2$

40. Find each missing part.

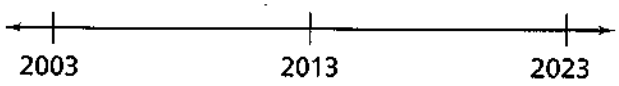
	Start With	Rule	End With
a.	●●	■	●●●●●●●
b.	●●●	■	●●●
c.	■	Add 5 ●	●●●
d.	■	Subtract 5 ●	●●

For Exercises 41–46, find each sum or difference. Show your work.

41. $15 + {}^{-}10$ **42.** ${}^{-}20 - 14$ **43.** $200 - {}^{-}125$

44. ${}^{-}20 - {}^{-}14$ **45.** ${}^{-}200 + 125$ **46.** $7 - 12$

47. Below is part of a time line with three years marked.



a. Write two sentences in words that refer to the year 2013. One should relate 2013 to 2003, and the other should relate 2013 to 2023.

b. Write two number sentences that refer to the year 2013. One should relate 2013 to 2003, and the other should relate 2013 to 2023.

c. Describe how these two number sentences are alike and different.

48. Compute each of the following.

a. $3 + {}^{-}3 + {}^{-}7$ **b.** $3 - 3 - 7$

c. ${}^{-}10 + {}^{-}7 + {}^{-}28$ **d.** ${}^{-}10 - 7 - 28$

e. $7 - 8 + {}^{-}5$ **f.** $7 + {}^{-}8 - 5$

g. ${}^{-}97 + {}^{-}35 - 10$ **h.** ${}^{-}97 - 35 + {}^{-}10$

i. What can you conclude about the relationship between subtracting a positive number and adding a negative number with the same absolute value? In other words, what is the relationship between a $(- {}^{+})$ situation and a $(+ {}^{-})$ situation?

49. Compute each of the following.

a. $3 - {}^{-}3 - {}^{-}7$ **b.** $3 + 3 + 7$

c. ${}^{-}10 - {}^{-}7 - {}^{-}28$ **d.** ${}^{-}10 + 7 + 28$

e. $7 + 8 + 5$ **f.** $7 - {}^{-}8 - {}^{-}5$

g. ${}^{-}97 - {}^{-}35 - 10$ **h.** ${}^{-}97 + 35 + {}^{-}10$

i. What can you conclude about the relationship between subtracting a negative number and adding a positive number with the same absolute value? In other words, what is the relationship between a $(- {}^{-})$ situation and a $(+ {}^{+})$ situation?

Multiple Choice In each set of calculations, one result is different from the others. Find the different result without doing any calculations.

50. **A.** $54 + {}^{-}25$ **B.** $54 - 25$

C. $25 - 54$ **D.** ${}^{-}25 + 54$

51. **F.** ${}^{-}6.28 - {}^{-}3.14$ **G.** ${}^{-}6.28 + 3.14$

H. $3.14 + {}^{-}6.28$ **J.** ${}^{-}3.14 - {}^{-}6.28$

52. **A.** $534 - 275$ **B.** $275 - 534$

C. ${}^{-}534 + 275$ **D.** $275 + {}^{-}534$

53. **F.** $175 + {}^{-}225$ **G.** $225 - 175$

H. $175 - 225$ **J.** ${}^{-}225 + 175$

54. Fill in the missing information for each problem.

a. $5 + \frac{3}{4} = \blacksquare$ **b.** $\frac{4}{8} + (-6) = \blacksquare$ **c.** $-3\frac{3}{4} - \left(-\frac{3}{4}\right) = \blacksquare$

d. $2\frac{2}{3} - \frac{1}{3} = \blacksquare$ **e.** $-2 + \blacksquare = -2\frac{1}{2}$ **f.** $-4.5 + \blacksquare = -5$

55. **Multiple Choice** Which is the correct addition and subtraction fact family for $-2 + 3 = 1$?

A. $-2 + 3 = 1$, $-2 + 1 = 3$, $3 - 1 = 2$

B. $-2 + 3 = 1$, $3 - 2 = 1$, $3 - 1 = 2$

C. $-2 + 3 = 1$, $1 - 3 = -2$, $1 - (-2) = 3$

D. $1 - 3 = -2$, $1 - (-2) = 3$, $3 - 1 = 2$

56. For each of the following, write a related equation. Then find the value of n.

a. $n - 7 = 10$ **b.** $-\frac{1}{2} + n = -\frac{5}{8}$ **c.** $\frac{2}{3} - n = -\frac{7}{9}$

57. Are ${}^{+}8 - {}^{+}8$ and $8 - 8$ equal? Explain.

58. Are ${}^{+}100 - {}^{+}99$ and $100 - 99$ equal? Explain.

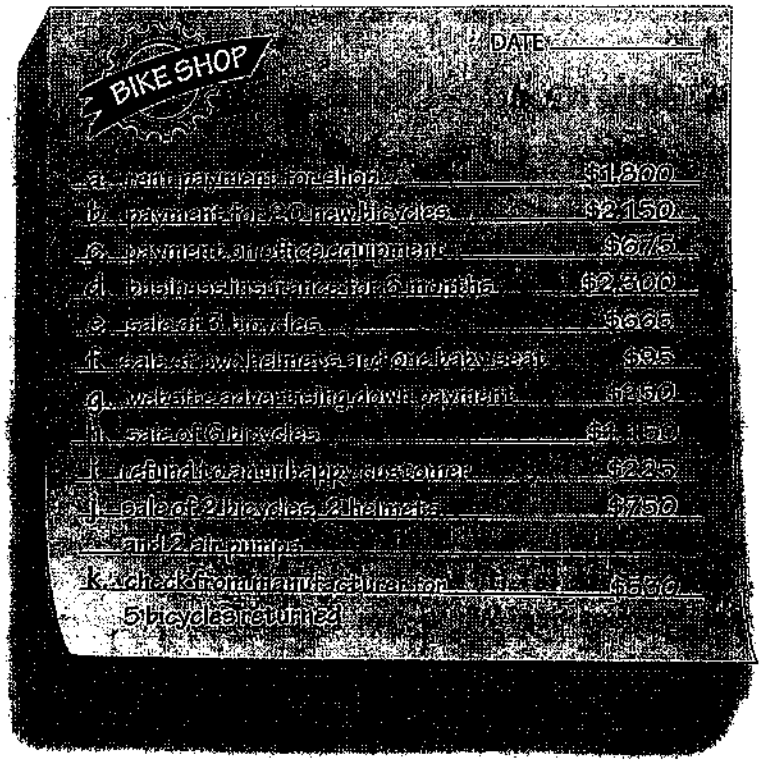
59. Are the expressions in each group below equivalent? If so, which form makes the computation easiest?

a. $8 + {}^{-}10$, $8 - {}^{+}10$, $8 - 10$

b. $3 + {}^{-}8$, $3 - {}^{+}8$, $3 - 8$

Connections

60. The Spartan Bike Shop keeps a record of their business transactions. They start their account at zero dollars. Write a number sentence to represent each transaction. Then find the new balance.



For Exercises 61 and 62, write a number sentence for each situation. Then answer the question.

61. The air temperature drops from 94°F to 72°F in 15 minutes. What is the change in temperature?

62. The Teacher's Pets team has 50 points in Math Fever. They miss a question worth 200 points. What is their new score?

63. Find four different numbers, in order from least to greatest, that lie between the two given numbers.

a. $^{-}4.5$ and $^{-}3.5$

b. $^{-}0.5$ and 0.5

Extensions

64. Which numbers, when added to $^{-}15$, give a sum

a. greater than 0?

b. less than 0?

c. equal to 0?

65. Use a number line to find the distance between each pair of numbers.

a. $^{+}8, ^{+}4$

b. $^{-}8, ^{+}4$

c. $^{+}8, ^{-}4$

d. $^{-}8, ^{-}4$

e. $^{-}3\frac{1}{2}, ^{+}\frac{3}{4}$

f. $^{+}5.4, ^{-}1.6$

66. Find each absolute value.

a. $|^{+}8 - ^{+}4|$

b. $|^{-}8 - ^{+}4|$

c. $|^{+}8 - ^{-}4|$

d. $|^{-}8 - ^{-}4|$

e. $|^{-}3\frac{1}{2} + ^{+}\frac{3}{4}|$

f. $|^{+}5.4 - ^{-}1.6|$

g. Compare the results of parts (a)–(f) with the distances found in Exercise 65. What do you notice? Why do you think this is so?

67. Replace n with a number to make each statement true.

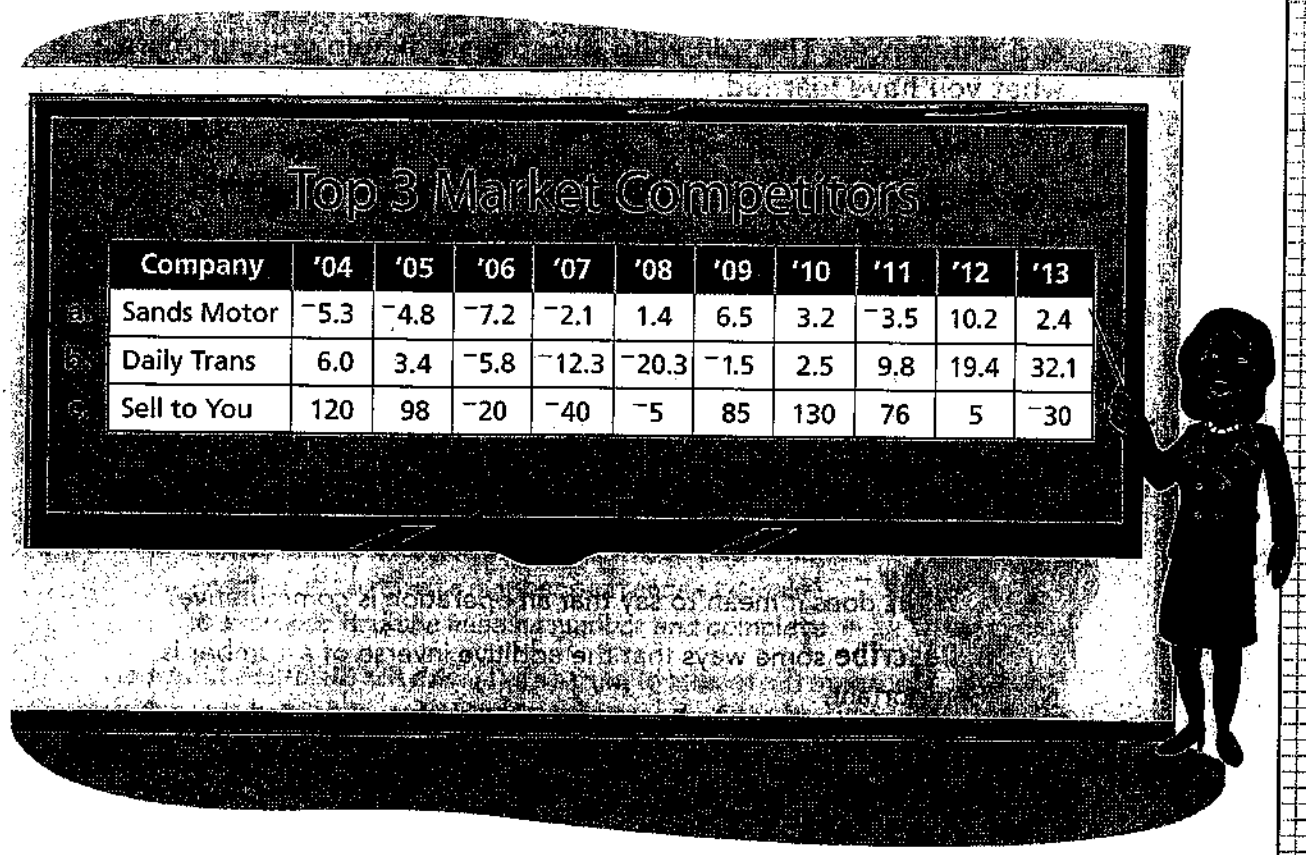
a. $n + ^{-}18 = 6$

b. $^{-}24 - n = 12$

c. $43 + n = ^{-}12$

d. $^{-}20 - n = ^{-}50$

68. The table shows the profits or losses (in millions of dollars) earned by three companies from 2004 to 2013. Find the range of the annual results and the overall profit (or loss) for each company over that time period.

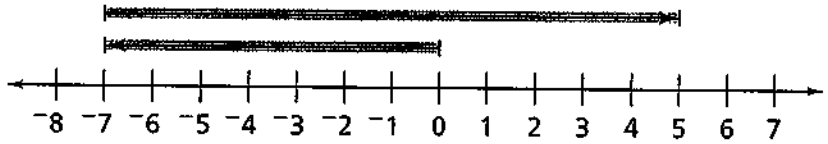


Top 3 Market Competitors

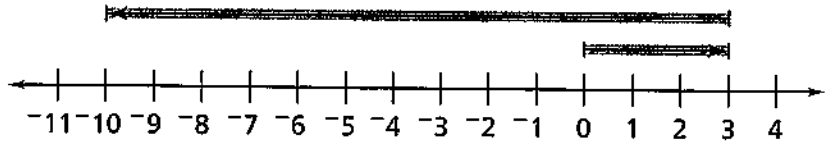
Company	'04	'05	'06	'07	'08	'09	'10	'11	'12	'13
Sands Motor	−5.3	−4.8	−7.2	−2.1	1.4	6.5	3.2	−3.5	10.2	2.4
Daily Trans	6.0	3.4	−5.8	−12.3	−20.3	−1.5	2.5	9.8	19.4	32.1
Sell to You	120	98	−20	−40	−5	85	130	76	5	−30

69. Starting from 0, write an addition sentence for the diagrams below.

a.



b.



Mathematical Reflections

In this Investigation you applied your informal ideas about rational numbers to develop algorithms for calculating any sums and differences. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. **What** algorithm(s) will produce the correct result for the sum "$a + b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

   b. **What** algorithm(s) will produce the correct result for the difference "$a - b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works.

2. **How** can any difference "$a - b$" be restated as an equivalent addition statement, where a and b each represent any rational number?

3. a. **What** does it mean to say that an operation is *commutative*?

   b. **Describe** some ways that the additive inverse of a number is important.

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Sophie described her thoughts in the following way:

We noticed there was a relationship between addition and subtraction in Problem 2.4. If you know that $7 + {}^{-}8 = {}^{-}1$, then you can write two equivalent subtraction problems: ${}^{-}1 - 7 = {}^{-}8$ and ${}^{-}1 - {}^{-}8 = 7$. You can also write subtraction number sentences as addition problems. These patterns enabled us to rewrite number sentences to make it easier to find the missing number and complete the sentence.

Common Core Standards for Mathematical Practice

MP7 Look for and make use of structure.

- What other Mathematical Practices can you identify in Sophie's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Accentuate the Negative Investigation 2 Quiz

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

Vocab:

Word Bank:		
A-Commutative property	B- Additive inverse	C- Additive identity
D- Absolute value	E- Algorithm	

_____ The distance of a number from 0, on a number line.
_____ A set of rules for performing a procedure
_____ Adding zero to a rational number is this
_____ Two numbers that are the same but have opposite signs.
_____The order of the addition or multiplication of two numbers does not change the result.

Solve the following problems (7.NS.A.1d)

1. $-3 + 9 =$
2. $-76 - -6 =$
3. $23 - 31 + - 31 =$
4. $2432 - 9878 - (- 10{,}789) =$
5. $-7 + 20 =$

Using number lines and/or chip board diagrams please show how you can add/subtract the following problems. (7.NS.A.1c)

6. $-5 + -1 =$
7. $0 - 3 - -7 =$
8. $4 + 8 - 5 =$
9. $-4 + -6 + 0 - 1 + 4$

Answer the following word problems. Show all work! (7.NS.3)

10. Bill Gates is a very rich man. His bank account on Monday had a total of $3,000,000. On Tuesday he decided to give $100,000 to a person he just met. On Wednesday he gave his two sons $10,000 each for the work they did around the house at his company. On Thursday he was given his paycheck from his company's makings, a total of $12,000,000. On Friday he decides to give $3,300,000 to a local Boys and Girls Club. How much money does he have on Saturday in his bank account?

11. Lil' Wayne signed his first record label for $100,000. The first thing he bought was a BMW which was $70,000. He wanted to put rims on the BMW and the ones he wanted were $1,500. After tricking out his ride how much money does he have left from his first label deal?

12. Ryan owns a local construction company. His men dig holes for posts. The men can dig a total of 8 feet a day by hand. If they have sunny weather for 5 days, how deep can they dig their hole?

13. Ryan finally bought a machine that digs holes faster than his men. His machine can dig up to 20 feet per day on a sunny day. When it rains the hole fills with dirt 6 feet per day. Over 5 days, there were 3 sunny days and 2 rainy days. What is the depth of the hole after 5 days?

Answer the following by solving for the unknown number. (7.NS.A.1b,7.EE.B.3)

14. $8+ n = -10$
15. $-99 - n = 130$
16. $n + 45 = 25$
17. $n - - 13 = 13$
18. $67- n = 0$
19. $-71 + n = -98$

Accentuate the Negative Investigation 2 Quiz - KEY

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

Vocab:

Word Bank:		
A-Commutative property	B- Additive inverse	C- Additive identity
D- Absolute value	E- Algorithm	

__D__ The distance of a number from 0, on a number line.
__E__ A set of rules for performing a procedure
__C__ Adding zero to a rational number is this
__B__ Two numbers that are the same but have opposite signs.
__A__ The order of the addition or multiplication of two numbers does not change the result.

Solve the following problems

1. $-3 + 9 =$ 6
2. $-76 - -6 =$ -70
3. $23 - 31 + -31 =$ -39
4. $2432 - 9878 - (-10{,}789) =$ 3,343
5. $-7 + 20 =$ -13

Using number lines and/or chip board diagrams please show how you can add/subtract the following problems.

6. $-5 + -1 =$ -6
7. $0 - 3 - -7 =$ 4
8. $4 + 8 - 5 =$ 7
9. $-4 + -6 + 0 - 1 + 4 =$ -7

Answer the following word problems. Show all work!

10. Bill Gates is a very rich man. His bank account on Monday had a total of $3,000,000. On Tuesday he decided to give $100,000 to a person he just met. On Wednesday he gave his two sons $10,000 each for the work they did around the house at his company. On Thursday he was given his paycheck from his company's makings, a total of $12,000,000. On Friday he decides to give $3,300,000 to a local Boys and Girls Club. How much money does he have on Saturday in his bank account? $11,580,000

11. Lil' Wayne signed his first record label for $100,000. The first thing he bought was a BMW which was $70,000. He wanted to put rims on the BMW and the ones he wanted were $1,500. After tricking out his ride how much money does he have left from his first label deal? $28,500

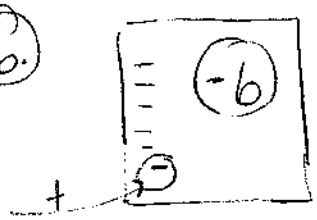
12. Ryan owns a local construction company. His men dig holes for posts. The men can dig a total of 8 feet a day by hand. If they have sunny weather for 5 days, how deep can they dig their hole? 40ft

13. Ryan finally bought a machine that digs holes faster than his men. His machine can dig up to 20 feet per day on a sunny day. When it rains the hole fills with dirt 6 feet per day. Over 5 days, there were 3 sunny days and 2 rainy days. What is the depth of the hole after 5 days? 48 ft.

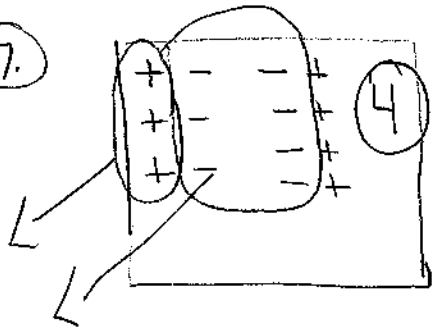
Answer the following questions for solving for the unknown number.

14. $8 + n = -10$ $n = -18$
15. $-99 - n = 130$ $n = 31$
16. $n + 45 = 25$ $n = -20$
17. $n - -13 = 13$ $n = 0$
18. $67 - n = 0$ $n = 67$
19. $-71 + n = -98$ $n = -27$

6.



7.



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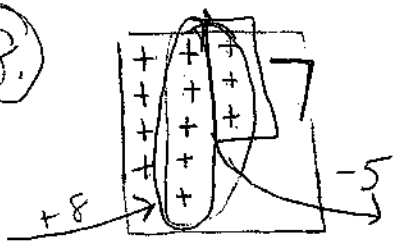

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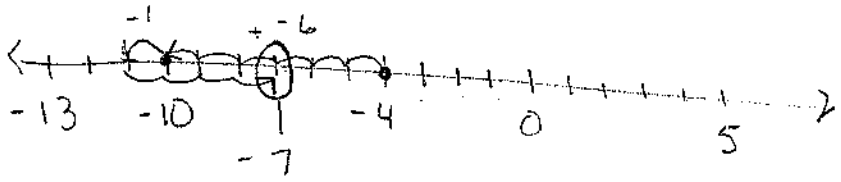


10. $3,000,000 - 100,000 = 2,900,000$

$2,900,000 - 20,000 = 2,880,000$

$2,880,000 + 12,000,000 = 14,880,000$

$14,880,000 - 3,300,000 = \$11,580,000$

11. $100,000 - 70,000 = 30,000$

$30,000 - 1,500 = \$28,500$

12. $8 \times 5 = 40$ ft.

13. $20 + 20 + 20 = 60$

$60 - 12 = 48$

Investigation 3: Multiplying and Dividing Rational Numbers

Lesson Focus Questions: How do we multiply and divide rational numbers?

Standards: 7.NS.A.2, 7.NS.A.2a, 7.NS.A.2b, 7.NS.A.2c, 7.NS.A.3, 7.EE.B.3, MP.2, MP.4

Learner Outcomes (KUD):
Students will:
Develop and use models to represent multiplication and division of integers and rational numbers.
Develop algorithms for multiplying and dividing integers and rational numbers.
Write and use fact families for multiplication and division to solve simple equations.
Demonstrate an understanding of and use the Commutative Property to multiply integers and rational numbers.
Interpret and write mathematical sentences to show relationships and solve problems.
Model with mathematics.
Reason abstractly and quantitatively.

Pre-Requisites: Understanding multiplication, division and previous models for integers

Key Concepts/Vocabulary: Multiplicative identity and multiplicative inverses

Activating Strategy: The teacher provides students with ten sets of numbers to multiply. The class discusses the correct answers and justifies their answers using a number line. Then the teacher replaces one of the numbers in the original set with a negative number and asks them to answer the same questions as before but using a number line to justify their answers to each. Extension: Have students create additional problems and quiz each other.

Learner Activities: (5 Days)
Problems 3.1 Multiplication Patterns with Integers (Day 1): The problem starts with a Number Relay Race in which runners are running on a number line that goes from –50 to +50. This race gives students practice in "reading" number lines that extend from positive to negative numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Teacher presents students with the introduction to Problem 3.1 on page 55 of the Student Edition. It might be helpful if a number line is created on the board or floor and have students act out the situation. Use the following questions to make sure students understand the context: How far does each racer run? *(Racers 1–4 run 100 meters. Racer 5 runs only 50 meters.)*Explain what the first leg of the relay would look like. *(Racer 1 takes off from the −50 point and runs as fast as possible to the 50 point and hands off the baton to Racer 2.)* Explain the last leg of the race. *(Racer 5 receives the baton from Racer 4 and runs from the −50 point to the 0 point.)*
3. The teacher has 1 student read the directions for Question A of Problem 3.1. The goal of Question A is to have students use their knowledge of repeated addition to make sense of multiplication situations that involve both positive and negative integers. Make sure they pay close attention to the use of negative and positive numbers in the problem.

4. Students are asked to write number sentences for each problem in Question A. It is important that they do this in order to make sense of what happens when they multiply different combinations of integers. The teacher should do the first problem in Question A with the students to draw their attention to the relationship between repeated addition and multiplication and to make sure they are making sense of the labeling for positive and negative situations. Use the following questions to check for student understanding: What does 5 meters per second mean? *(that every 5 meters takes him 1 second, or that in 1 second, he runs 5 meters)* Which way is Hahn running? Left or right? Positive or negative? *(right and positive)* What number sentence could you write to show where Hahn will be in 6 seconds if he is going to the right and passes the 0 point running 5 meters per second? *(If he runs 5 meters each second, and he runs for 6 seconds, then 5+5+5+5+5+5=30 meter point or 6×5=30 meter point.)* What do the 5 and the 6 stand for? *(The 5 stands for the 5 meters that is covered each second and the 6 stands for the 6 seconds that Hahn is running after he passes the 0 point in the race.)* Suppose Hahn was running 5 meters per second to the left. How would this change the number sentence? *(The speed, or rate, would be –5 meters per second instead of 5 meters per second, so you would add –5 six times or multiply –5 times 6.)* What would the number sentence look like in this situation? *[(−5)+(−5)+(−5)+(−5)+(−5)+(−5)=−30 meter point or 6×(−5)=−30 meter point.]* What do the –5, the 6, and the –30 indicate in this situation? *(Hahn is running 5 meters per second, but going to the left. He runs for 6 seconds, or 6 times –5. At the end of the 6 seconds, he will be at the −30 meter point on the playing field.)* Remind students to write the mathematical sentences for Question A. This will help them see what they were thinking so they can make sense of the problems in Questions C and D.
5. The teacher circulates as pairs work, making sure they are writing complete mathematical sentences reflecting totals and not just expressions. The teacher reminds students that they are looking for patterns with multiplying positive and negative integers. In Question A, remind students that they need to be able to explain why each number in their mathematical sentences (factors and product) is positive or negative in the context of the relay race. When discussing Question A, part (2) students should not use the words *positive* and *negative*. Instead they should focus on the meter markers on the Number Relay and their placement with respect to the center 0. The goal is for students to be able to interpret the signs of the factors as the direction from 0 (left versus right) or points in time (earlier versus later). Students should interpret the sign of the resulting product as a location, left or right of zero. **Note: If students are struggling to make sense of the Number Relay Race problems in Question A, the teacher may want to summarize Questions A and B before going on to Questions C and D. If not, have students work through all parts of the problem.**
6. Discuss the answers to Questions B-D. As students are explaining their answers the teacher should make special note of the patterns that they are seeing to help them summarize their thinking in developing an algorithm for multiplying a positive by a negative and negative time a positive. Formalize these algorithms on the board so students can write them in their notes.

7. Students will do an assortment of problems from the ACE problems (page 66) for homework. Applications #1-9, Connections #37, Extensions # 49-53.

Problem 3.2 Multiplication of Rational Numbers (Day 2): This problem has the same goal as problem 3.1: to help students understand and use multiplication. However, the numbers used now are rational numbers expressed as decimals, fractions, whole numbers, and integers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. The teacher has 1 student read the introduction to Problem 3.2 Multiplication of Rational Numbers and relates this problem to the work they did the previous day.
4. Students will work in pairs to complete Problem 3.2, Question A-E. When they have finished they are to compare their answers with other groups as they finish making sure that they are justifying their answers as they go.
5. When all groups are finished, have one group report out for Question A, another for Question B, and so on through Question E.
6. Students will do an assortment of problems from the ACE problems (page 66) for homework. Applications #10-13, Connections #38-41, Extensions # 54.

Problem 3.3 Division of Rational Numbers (Day 3): Students have explored fact families in previous investigations. Here they explore fact families as a way of understanding how a multiplication problem can be rearranged into different forms.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Teacher writes $36 \div 4 = 9$ on the board. The teacher then asks students: What does this sentence mean? *(This equation means that there are 36 things that are put into 4 groups of equal size and that there will be 9 things in each group—sharing reasoning. Or, it can be thought of as putting 36 things into groups of 4 and finding that there are enough to make 9 groups of 4—grouping reasoning.)* How are the operations of multiplication and division related? *(Multiplication and division are opposite, or inverse, operations; they undo each other. Multiplication puts equal-sized groups together to find the total, and division partitions the total to find either the size of the groups or the number of groups of a given size.)* If multiplication and division are opposite operations that undo each other, what number sentence would undo the sentence $3 \cdot 12 = 36$? *(This multiplication sentence says that the total of 3 groups of 12 each is 36. If you start with the total and divide by 3, you are partitioning the total to find the size of each equal-sized group. $36 \div 3 = 12$.)*
4. The teacher reminds students that they used fact families to write related addition and subtraction sentences in order to make sense of subtracting integers and asks students to examine the tables in the introduction to Problem 3.3 (Page 60). The teacher then explains that they will use fact families and their understanding of the relationship between multiplication and division to do Problem 3.3 and that they will also find patterns that will help them predict the quotient of any two rational numbers, including integers.
5. Have students work on Problem 3.3 either individually or in pairs. As students

are working through the problem the teacher circulates to address any misunderstandings or struggles.

6. The teacher will have students display their solutions to Question B. If there are disagreements about any of the solutions, have students discuss their solutions and explain why they make sense.
7. The teacher will facilitate a conversation for the rest of Problem 3.3 to help students summarize and generalize some rules for dividing positive and negative integers. Use questions like these as students discuss their solutions: How can you use your algorithm for multiplication to decide whether the quotient is positive or negative? *(We know $3(-4)=-12$, so $-12\div(-4)=3$ and $-12\div3=-4$.)* How did you decide if the quotient is positive or negative? *(When the divisor and dividend are opposite signs, the quotient is negative. When the divisor and dividend are the same sign, the quotient is positive.)* Why is the rule "a negative divided by a negative is positive" reasonable? *(For example, $-96\div(-4)=24$, you can interpret the problem as having a total of –96 and wanting to find out how many groups of –4 you can make by partitioning the total into equal groups. You can make 24 groups of –4 from –96. Also, the quotient must be positive because –4 must be multiplied by a positive 24 to have a product of –96.)* How can you find the quotient when you divide a negative integer by a positive integer? For example, why does it make sense that $-99\div11=-9$? *(You can think of $-99\div11=-9$ as sharing a total of –99 into 11 groups of equal size. The groups must be negative amounts, since you are sharing a negative amount, so the quotient must be negative.)* How can you find the quotient when you divide a positive number by a negative number? It might be helpful to use fact families to answer this question. *For example, if you want to show $99\div(-11)=-9$, you know that $-11\cdot(-9)=99$. Then you know that $99\div(-11)$ must be –9 and $99\div(-9)$ must equal -11.)* Compare the algorithms for division of integers and rational numbers. *(T he algorithms are the same for both integers and rational numbers. When dividing two numbers with the same sign, the quotient is positive. When dividing two numbers with opposite signs, the quotient is negative.)* Which operations on integers are commutative? Give examples to support your conclusions. Are these conclusions the same for the operations on rational numbers? Explain. (*Addition and multiplication of integers are commutative. $5+(-7)=-7+5$ and $5\cdot(-7)=-7\cdot5$. Addition and multiplication are both commutative for rational numbers.)*
8. Students will do an assortment of problems from the ACE problems (page 66) for homework which will be due the end of **Day 4**. Applications #14-35, Connections #42-45, Extensions # 55-59.

Problem 3.4 Playing the Integer Product Game (Day 4): In this investigation students will apply the algorithms they created for multiplication and division of rational numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. The teacher introduces the students to the Integer Product Game and Problem 3.4 (page 64 – 65) of the their student edition. Play one game as a class to help explain the rules
3. Pairs of students are to play the game against each other several times, then pairs of students play against other pairs several times.

4. Students complete Problem 3.4. As they finish they should discuss their answers with other.
5. Students complete ACE problems (page 66) that were assigned **Day 3**. Applications #14-35, Connections #42-45, Extensions # 55-59.
6. Teacher assigns Mathematical Reflection Sheet (page 75-76) from Accentuate the Negative for homework

Review (Day 5)

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems and Mathematical Reflection Sheet (page 75-76) from Accentuate the Negative to guide review for quiz.

Quiz (Day 5)

2. Students will take Accentuate the Negative Investigation 3 Quiz.
3. As students finish their quiz, have them read and take notes on the Introduction to Investigation 4 (pg77-78) and do Problem 4.1 (This should be a review of what they learned in 5th and 6th grades.)

Summarizing Strategy:
Mathematical Reflections 3 (Pg. 75-76) from Accentuate The Negative Student Edition and discuss.

Formative Assessment: Problems 3.1, 3.2, 3.3, 3.4; ACE problems; Mathematical Reflection	**Summative Assessment:** Investigation 3 Quiz

Teaching Strategies/Tips Use CMP lesson plans to make sure all necessary materials are available. Have students explain how they are determining their solutions.
To make efficient use of the time for discussing homework, (ACE problems) provide students with answers and then discuss only those questions that students had difficulty with.

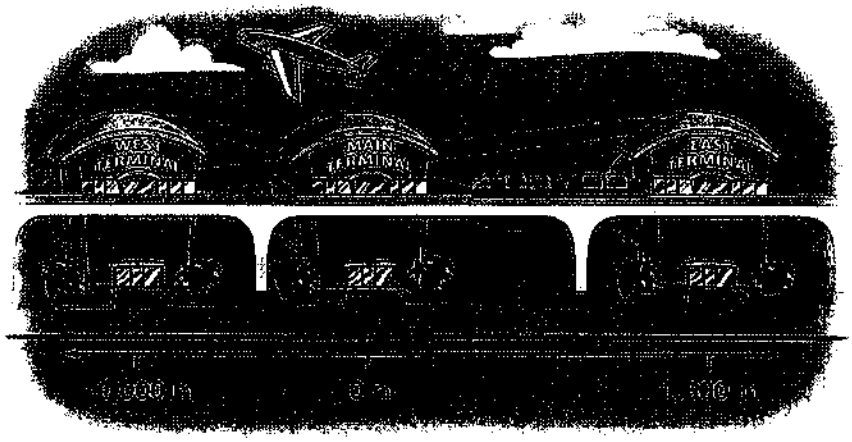
Differentiation (content/process/product):
Shorten assignments and give notes to students that need help with note taking and organization skills. Look for ACE problems that are related to the day's problem and give the students about 3-6 extra problems per day.

Resources: Pearson Connected Mathematics Project (CMP3), Math-Drills.com (fluency), http://www.mathdashboard.com/cmp3/

Attachments (assessments, rubrics, graphic organizers, projects, etc.):
ACE Problems, Teaching Aid 3.4: Integer Product Game, Mathematical Reflection Sheet (page 75-76) from Accentuate the Negative, Investigation 3 Quiz, Investigation 3 Quiz Key

Applications

1. At some international airports, trains carry passengers between the separate terminal buildings. Suppose that one such train system moves along a track like the one below.



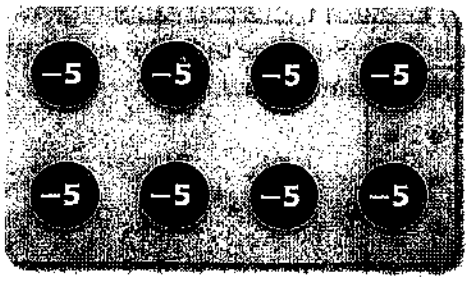
   a. A train leaves the main terminal going east at 10 meters per second. Where will it be in 10 seconds? When will it reach the east terminal?

   b. A train passes the main terminal going east at 10 meters per second. Where was that train 15 seconds ago? When was it at the west terminal?

   c. A train leaves the main terminal going west at 10 meters per second. Where will it be in 20 seconds? When will it reach the west terminal?

   d. A train passes the main terminal going west at 10 meters per second. When was it at the east terminal? Where was it 20 seconds ago?

2. Julia thinks a bit more about how to use red and black chips to model operations with integers. She draws the following chip board. She decides it represents $8 \times (-5) = -40$ and $-40 \div 8 = -5$. Explain why Julia's reasoning makes sense.



Use Julia's reasoning from Exercise 2 to find each value.

3. $10 \times (-5)$ **4.** $4 \times (-15)$ **5.** $3 \times (-5)$

6. $-14 \div 2$ **7.** $-14 \div 7$ **8.** $-35 \div 7$

9. Find each product.

a. $7 \cdot 2$ **b.** $-7 \cdot (-2)$ **c.** $7 \cdot (-2)$

d. $-7 \cdot 2$ **e.** $8 \cdot 2.5$ **f.** $-9 \cdot (-4)$

g. $12 \cdot (-3)$ **h.** $-1.5 \cdot 4$ **i.** $3.5 \cdot 7$

j. $-8.1 \cdot (-1)$ **k.** $1 \cdot (-6)$ **l.** $-2\frac{1}{2} \cdot 1$

10. Tell whether each product is greater than or less than zero.

a. $5 \cdot (-7)$ **b.** $-3.2 \cdot 1.5$

c. $10.5 \cdot (-4)$ **d.** $-2 \cdot (-3) \cdot (-1)$

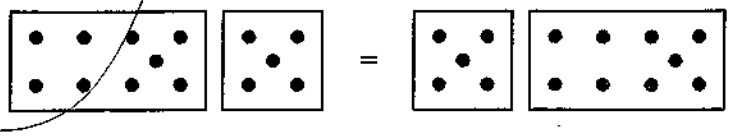
e. $-\frac{2}{3} \cdot 2\frac{3}{4}$ **f.** $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \left(-\frac{7}{4}\right)$

g. $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \frac{7}{4}$ **h.** $-\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \left(-\frac{7}{4}\right) \cdot \left(-2\frac{3}{8}\right)$

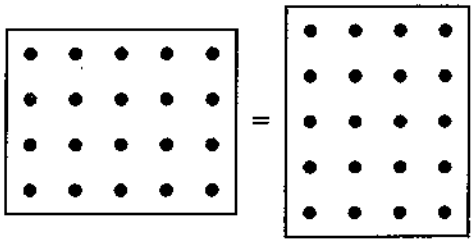
i. $\frac{3}{4} \cdot \left(-1\frac{5}{6}\right) \cdot \frac{7}{4} \cdot \left(-2\frac{3}{8}\right)$ **j.** $\frac{3}{4} \cdot 1\frac{5}{6} \cdot \frac{7}{4} \cdot \left(-2\frac{3}{8}\right)$

The dot patterns illustrate commutative properties for operations on whole numbers. Write a number sentence for each case.

11.



12.



13. Find the values for each pair.

a. $4 \cdot (-3)$ and $-3 \cdot 4$

b. $2 \cdot (-4)$ and $-4 \cdot 2$

c. $-2 \cdot (-3)$ and $-3 \cdot (-2)$

d. $\frac{1}{5} \cdot \left(-\frac{4}{9}\right)$ and $-\frac{4}{9} \cdot \frac{1}{5}$

e. What can you conclude about multiplication with negative numbers?

14. You have located fractions such as $-\frac{5}{7}$ on a number line. You have also used fractions to show division: $\frac{-5}{7} = -5 \div 7$ and $\frac{5}{-7} = 5 \div (-7)$. Tell whether each statement is *true* or *false*. Explain.

a. $\frac{-1}{2} = \frac{1}{-2}$

b. $-\frac{1}{2} = \frac{-1}{-2}$

15. For each number sentence, find a value for n that makes the sentence true.

a. $24 \div 2 = n$

b. $-24 \div (-2) = n$

c. $24 \div n = -12$

d. $n \div 2 = -12$

e. $5 \div 2.5 = n$

f. $-12 \div n = 3$

g. $n \div (-3) = -4$

h. $(-16) \div \frac{1}{4} = n$

For Exercises 16–18, write four related multiplication and division facts for each set of integers.

Sample 27, 9, 3

$9 \cdot 3 = 27$

$3 \cdot 9 = 27$

$27 \div 9 = 3$

$27 \div 3 = 9$

16. 7, −3, −21 **17.** −4, −5, 20 **18.** 1.5, −3, −4.5

For Exercises 19–24, determine whether the product of or quotient of each expression is greater than, less than, or equal to 0 without doing any calculations. Explain your reasoning.

19. $-1{,}105.62 \div 24.3$ **20.** $0 \cdot (-67)$

21. $-27.5 \cdot (-63)$ **22.** $0 \div 89$

23. $-54.9 \div (-3)$ **24.** $-2{,}943 \cdot 1.06$

25. Use the multiplication and division algorithms you developed to find each value.

a. $12 \cdot 9$ **b.** $5 \cdot (-25)$ **c.** $-220 \div (-50)$

d. $48 \div (-6)$ **e.** $-63 \div 9$ **f.** $\frac{2}{-3} \cdot \left(\frac{-4}{5}\right)$

g. $\frac{-99}{33}$ **h.** $-2.7 \div (-0.3)$ **i.** $-36 \cdot 5$

j. $52.5 \div (-7)$ **k.** $-2\frac{1}{2} \cdot \left(-\frac{2}{3}\right)$ **l.** $9 \div 5$

m. $-9 \cdot (-50)$ **n.** $\frac{-96}{24}$ **o.** $6 \cdot 1\frac{1}{2}$

p. $-\frac{5}{8} \cdot \frac{8}{5}$ **q.** $4 \cdot \left(-1\frac{1}{4}\right)$ **r.** $-2.5 \cdot 2\frac{1}{5}$

Multiple Choice For Exercises 26 and 27, find each value.

26. $-24 \div 4$

A. -96 **B.** -6 **C.** 6 **D.** 96

27. $-10 \cdot (-5)$

F. -50 **G.** -2 **H.** 2 **J.** 50

Use properties of multiplication and division to find each value. State which properties you use.

28. $\dfrac{\frac{2}{7} \cdot 9}{\frac{2}{7}}$

29. $\frac{3}{8} \cdot (-4) \cdot 3.5 \cdot \frac{8}{3}$

30. $-1.3 \cdot 5 \cdot (-6) \cdot 0.2$

31. $\dfrac{4.9 \cdot 5.8}{4.9}$

For Exercises 32–35, state whether each fraction will *terminate* or *repeat*. Write each fraction as a decimal.

32. $\frac{-5}{9}$ **33.** $\frac{7}{-8}$ **34.** $\frac{-4}{-11}$ **35.** $\frac{5}{16}$

36. Chris and Elizabeth are making a version of the Integer Product Game in which players need three products in a row to win.

Chris and Elizabeth's Product Game

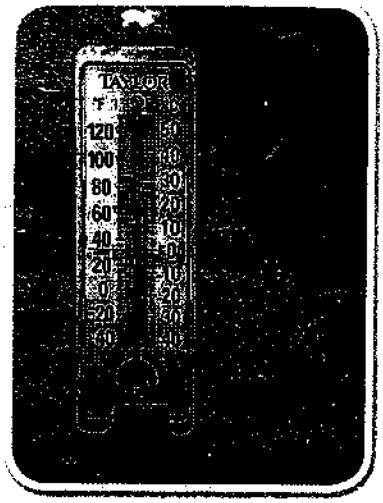
4	−4	6	−6
9	−9	10	−10
15	−15	25	−25

Factors:

What six factors do they need for their game? Explain your reasoning.

Connections

37. The temperature changed $-2°F$ per hour from noon on Tuesday until 10:00 A.M. the next morning. The temperature at noon on Tuesday is shown.



a. What was the temperature at 4:00 P.M. on Tuesday?

b. What was the temperature at 9:00 A.M. on Wednesday?

Write a number sentence to represent each situation. Then answer the question.

38. The Extraterrestrials have a score of -300. They answer four 50-point questions incorrectly. What is their new score?

39. The Super Computers answered three 100-point questions incorrectly. They now have 200 points. What was their score before answering the three questions incorrectly?

40. A football team is at its own 25-yard line. In the next three plays, it loses an average of 4 yards per play. Where is the team after the three plays?

41. A new convenience store wishes to attract customers. For a one-day special, the store sells gasoline for \$.25 per gallon below its regular cost per gallon. Suppose the store sells 5,750 gallons of gas that day. What is the store's profit or loss in comparison to the amount the store would have made without the special?

42. Multiply or divide. Show your work.

a. $52 \cdot 75$ **b.** $52 \cdot (-75)$ **c.** $-2{,}262 \div (-58)$

d. $\frac{2}{3} \cdot \frac{4}{5}$ **e.** $-9{,}908 \div 89$ **f.** $-7.77 \div (-0.37)$

g. $-34 \cdot 15$ **h.** $53.2 \div (-7)$ **i.** $\frac{-2}{3} \cdot \frac{6}{8}$

j. $90 \div 50$ **k.** $-90 \cdot (-50)$ **l.** $-108 \div 24$

m. $19.5 \div (-3)$ **n.** $-8.4 \cdot 6$ **o.** $6 \cdot 2\frac{1}{2}$

p. $-3\frac{2}{3} \cdot (-9)$ **q.** $4 \cdot \left(-1\frac{1}{4}\right)$ **r.** $-2.5 \cdot 2\frac{1}{5}$

43. The list below gives average temperatures (in °C) for a city for each month of the year, from January through December.

$$-25, -20, -13, -2, 9, 15, 17, 14, 7, -4, -16, -23$$

a. What is the median?

b. What is the range?

c. What is the mean?

d. Number the months from 1 (for January) through 12 (for December). Plot a graph of the (month, temperature) data.

44. Find the sum, difference, product, or quotient.

a. $-5 - 18$ **b.** $-23 + 48$ **c.** $\frac{3}{4} \cdot \left(-\frac{5}{9}\right)$

d. $119 + (-19.3)$ **e.** $-1.5 - (-32.8)$ **f.** $12 \div 15$

g. $-169 \div (-1.3)$ **h.** $0.47 - 1.56$ **i.** $6 \cdot (-3.5)$

j. $\frac{2}{-3} \div \frac{5}{6}$ **k.** $\frac{7}{12} - \left(-\frac{2}{3}\right)$ **l.** $-\frac{4}{5} \div \left(-\frac{1}{4}\right)$

45. Estimate the sum, difference, product, or quotient.

a. $-52 - 5$ **b.** $-43 + (-108)$ **c.** $2\frac{3}{4} \cdot \left(-\frac{5}{9}\right)$

d. $79 + (-25.3)$ **e.** $-12.5 - (-37.3)$ **f.** $89 \div 15$

g. $-169 \div (-13)$ **h.** $6.3 - 1.86$ **i.** $61 \cdot (-3.9)$

j. $-\frac{2}{3} \div \left(1\frac{5}{6}\right)$ **k.** $5\frac{7}{12} - \left(-\frac{2}{3}\right)$ **l.** $-\frac{4}{5} \div \left(-\frac{1}{4}\right)$

Find integers to make each sentence true.

46. ■ • ■ = 30

47. ■ • ■ = −30

48. −24 ÷ ■ = ■

Extensions

Determine whether each statement is *always*, *sometimes*, or *never true*. Explain.

49. If m and n are positive rational numbers, then $m + n$ is positive.

50. If m and n are negative rational numbers, then $m + n$ is negative.

51. If m is a positive rational number and n is a negative rational number, then $m + n$ is negative.

52. If m and n are positive rational numbers, then $m \times n$ is positive.

53. If m and n are negative rational numbers, then $m \times n$ is negative.

54. To add $5 + 3 + 2$, you might think that it is easier to add the $3 + 2$ and then add the answer to the 5. The mathematical property that allows you to change the grouping of addends (or factors) is called the *Associative Property*.

Test the Associative Property for addition and multiplication of integers by simplifying the expressions below. Find the values within the parentheses first. When you need a grouping symbol like parentheses inside another set of parentheses, you can use brackets to make it easier to read. For example, $(4 - (-6))$ can be written as $[4 - (-6)]$.

a. $[3 \cdot (-3)] \cdot 4$ and $3 \cdot (-3 \cdot 4)$

b. $(-5 \cdot 4) \cdot (-3)$ and $-5 \cdot [4 \cdot (-3)]$

c. $[-2 \cdot (-3)] \cdot (-5)$ and $-2 \cdot [-3 \cdot (-5)]$

d. $(3 \cdot 4) \cdot (-5)$ and $3 \cdot [4 \cdot (-5)]$

e. $[3 + (-3)] + 4$ and $3 + (-3 + 4)$

f. $(-5 + 4) + (-3)$ and $-5 + [4 + (-3)]$

g. $[-2 + (-3)] + (-5)$ and $-2 + [-3 + (-5)]$

h. $(3 + 4) + (-5)$ and $3 + [4 + (-5)]$

i. Does the Associative Property work for addition and multiplication of integers?

Tell whether each statement is *true* or *false*. Explain.

55. $-1 = -1 + 0$

56. $-3\frac{3}{8} = -\frac{21}{8}$

57. $-6.75 = -6 + \left(-\frac{3}{4}\right)$

For Exercises 58 and 59, write a story for a problem that is answered by finding the value of *n*.

58. $-4n = -24$

59. $\frac{n}{2} = 16$

60. Find a set of addends to make a Sum Game. Each sum on the board below should be the sum of two numbers (possibly a single number added to itself). Each pair of numbers should add to a sum on the board.

Hint: You need 11 numbers, all with different absolute values.

Sum Game

–24	–22	–20	–18	–16	–14
–12	–11	–10	–9	–8	–7
–6	–5	–4	–3	–2	–1
0	1	2	3	4	5
6	7	8	9	10	11
12	14	16	18	20	22

Factors:

Problem 3.4 Integer Product Game

Integer Product Game Board

–36	–30	–25	–24	–20	–18
–16	–15	–12	–10	–9	–8
–6	–5	–4	–3	–2	–1
1	2	3	4	5	6
8	9	10	12	15	16
18	20	24	25	30	36

Factors:

–6 –5 –4 –3 –2 –1 1 2 3 4 5 6

Mathematical Reflections 3

In this Investigation, you studied ways to use multiplication and division of rational numbers to answer questions about speed, time, distance, and direction of motion. You used the results of those calculations to develop algorithms for multiplying and dividing any two rational numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Give** an example of a multiplication problem, involving two integers, in which the product is

   a. less than 0. **b.** greater than 0. **c.** equal to 0.

   d. In general, describe the signs of the factors for each product in parts (a)–(c).

2. **Give** an example of a division problem, involving two integers, in which the quotient is

   a. less than 0. **b.** greater than 0. **c.** equal to 0.

   d. In general, describe the signs of the dividend and divisor for each quotient in parts (a)–(c).

3. **a.** Suppose three numbers are related by an equation of the form $a \cdot b = c$, where a, b, and c are not equal to 0. Write two related number sentences using division.

   b. Suppose three numbers are related by an equation of the form, $a \div b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication.

4. **Which** operations on integers are commutative? Give numerical examples to support your answer.

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Hector described his thoughts in the following way:

Jake and I knew that a negative number times a negative number is positive. This helped us predict the sign of the product of a set of numbers in Problem 3.2. If there is an even number of numbers that are negative, we can group these numbers in pairs. The product of each pair is positive. Then we are left with positive numbers, so the final product is positive. If there are an uneven number of negative numbers, the product is negative because there will always be a negative number that is not paired with another negative number. And a negative times a positive is negative.

Common Core Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

- What other Mathematical Practices can you identify in Hector's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Accentuate the Negative Investigation 3 Quiz

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

Part A (7.NS.A.2)

1. Describe an algorithm for multiplying/dividing two positive rational numbers. Use specific numbers in your description.

2. Describe an algorithm for multiplying/dividing a positive rational numbers and a negative rational number. Use specific numbers in your description.

3. Describe an algorithm for multiplying/dividing two negative rational numbers. Use specific numbers in your description.

Part B(7.NS.A.2a)

Find each value

4. $6 \cdot 10$

5. $-7 \cdot 15$

6. $-1.2 \cdot (-10)$

7. $21 \cdot (-8)$

8. $-24 \div -6$

9. $12 \div (-10)$

10. $-27 \div 9$

Part C (7.NS.A.3, 7.EE.B.3)

11. A football team loses an average of 3 yards per play. How many yards have they lost after 4 plays? Write a number sentence to represent your answer.

12. Together, siblings Jack, Joe and Mary owe their parents $100. Write a number sentence to represent how much each owes their parents if they split the debt equally.

13. A new convenience store wishes to attract customers. For a one-day special, the store sells gasoline for $.25 per gallon below its regular cost per gallon. Suppose the store sells 5,750 gallons of gas that day. What is the store's profit or loss in comparison to the amount the store would have made without the special?

Accentuate the Negative Investigation 3 Quiz - KEY

Name: _____ Date: _____ Period:____

ANSWER ALL QUESTIONS ON A SEPARATE PIECE OF PAPER.

Part A (7.NS.A.2)

1. Describe an algorithm for multiplying/dividing two positive rational numbers. Use specific numbers in your description. 2 x 3 is 2 sets of positive 3 so that would be positive 6 [+/+ = +] [+ • + = +]

2. Describe an algorithm for multiplying/dividing a positive rational numbers and a negative rational number. Use specific numbers in your description. 2 x -3 is 2 sets of -3 so that would (-3)+(-3) is -6 [− • × = −]

3. Describe an algorithm for multiplying/dividing two negative rational numbers. Use specific numbers in your description. -2 x -3 is the opposite of 2 sets of -3 which is -6 so the opposite of that is 6 [(−)•(−) = +]

Part B(7.NS.A.2a)

Find each value

4. 6 •10 60
5. -7 •15 −105
6. -1.2 • (-10) 12
7. 21 • (-8) −168
8. -24 ÷ -6 4
9. 12 ÷ (-10) −1.2
10. -27 ÷ 9 −3

Part C (7.NS.A.3, 7.EE.B.3)

11. A football team loses an average of 3 yards per play. How many yards have they lost after 4 plays? Write a number sentence to represent your answer. 4 • -3 = -12
    They have lost 12 yards
12. Together, siblings Jack, Joe and Mary owe their parents $100. Write a number sentence to represent how much each owes their parents if they split the debt equally. -$100 ÷ 3 = -$33.33
13. A new convenience store wishes to attract customers. For a one-day special, the store sells gasoline for $.25 per gallon below its regular cost per gallon. Suppose the store sells 5,750 gallons of gas that day. What is the store's profit or loss in comparison to the amount the store would have made without the special?

    5,750 • -.25 = -$1437.50

    They loose $1,437.50

Investigation 4: Properties of Operations

Lesson Focus Questions: What properties of can be used to simplify expressions and solve equations and contextual problems?

Standards: 7.NS.A.1D, 7.NS.A.2A, 7.NS.A.2C, 7.NS.A.3, 7.EE.B.3, MP.2, MP.6

Learner Outcomes (KUD):
Students will:
Recognize situations in which one or more operations of rational numbers are needed.
Interpret and write mathematical sentences to show relationships and solve problems.
Use parentheses and the Order of Operations in computations.
Apply the Distributive Property to simplify expressions and solve problems.
Reason abstractly and quantitatively.
Attend to precision.

Pre-Requisites: Understanding and use of exponents, operations with rational numbers, Order of Operations, and properties of operations.

Key Concepts/Vocabulary: Distributive Property, expanded form, factored form, number sentence, and Order of Operations

Activating Strategy:
Teacher displays Teaching Aid 4.1B: Soccer Jersey Example and asks students to look at the problem and predict whether Pedro or David is correct. Make sure that students are justifying their predictions using Order of Operations. If students are having difficulty agreeing on an answer, the teacher should display Teaching Aid 4.1B: Order of Operations and review the Order of Operations. To check for student understanding the teacher presents the following problems: Use parentheses and the + and – signs and the four numbers in the given order to write an equality 5 11 9 7 = 27 (5+11(9−7)=27) What other numbers can you obtain by using ( ), +, and – with these four numbers? *((5+11)−(9+7)=0, 5−11(9−7)=−17, (5−11)+(9+7)=10)*

Learner Activities: (6 Days)
Problem 4.1 Order of Operations (Day 1): Examine the Order of Operations with positive and negative numbers. Students deal with complicated addition and subtraction computation strings and see why rules for computational order are necessary.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Teacher writes the following problem on the board and tells students to take a couple of minutes to do what the problem suggests.
   What is the greatest result you can make from two of the following numbers? Three? Four? -25 +2 -3 +3
3. Students will share their answers to see who has the greatest result, If exponents were not used in their expressions, the teacher may want to call attention to this and how the results may change. Since students will be working with exponents, the teacher reviews this notation with a few simple examples such as $3^2=3\cdot3$, $2^3=2\cdot2\cdot2$, and $2^4=2\cdot2\cdot2\cdot2$ and to get students to think deeper about the use of exponents ask: Do 3^2, -3^2, and $(-3)^2$ all have the same value? *(No; $3^2=9$, $-3^2=-9$, $(-3)^2=9$).* What number is being squared in each case? *(3^2 means $3\cdot3$. -3^2 means the opposite of 3^2, or $-(3\cdot3)$. $(-3)^2$ means $(-3)\cdot(-3)$.*

4. Students will work in pairs on Problem 4.1. As groups finish, have them compare their answers with other groups that are done.
5. As a class discuss Problem 4.1. Go over Question A and use the discussion to summarize the strategies students have used so far to help them write and interpret mathematical sentences. Throughout the discussion, have students say in words how they should compute the expressions they have written or have seen. Each of the parts in Question C asks students to find the greatest and least values. Ask students to share strategies that helped them use parentheses to make values less and strategies that helped make values greater. Question E can be a challenge for students because of its length and complexity. If all students have not started this question, give them a few minutes now to work on it before discussing it. Be sure to discuss this question in steps so that students can reason through it and apply the Order of Operations when the string of symbols is long.
   Students are to display their thinking in the discussion of this problem.
6. To check for understanding students will complete Problem 4.1 Exit Ticket individually.
7. Students will do an assortment of problems from the ACE problems and finish whatever is not done for homework: Applications #1-7, Connections #20-45, Extensions #53-63

Problem 4.2 The Distribute Property (Day 2-3): Students encounter more complicated strings of computations. They have to use their knowledge of the Order of Operations to carry out the needed computations. Students work in both directions: expanding and factoring expressions that involve positive and negative numbers.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.
3. Have several students take turns reading the Problem 4.2 introduction on page 81 in the student edition. The teacher should call attention to the vocabulary and further explain the examples if necessary. Use Teacher Aid 4.2: Distributive Property to review and summarize the distributive property for multiplication and division.
4. Students work on Problem 4.2 individually to begin with. After they have completed Question A through C, they pair up with another student who has also completed those questions and discuss their answers. Students then complete Questions D through F with their new partner.
5. As a class, students report the Questions D through F.
6. To check for understanding, students will complete **Problem 4.2 Exit Ticket** individually.
7. Students will do an assortment of problems from the ACE problems and finish whatever is not done for homework: Applications #8-18, Connections #46-52, Extensions #66-69

Problem 4.3 What Operations Are Needed? (Day 4): This problem builds on what students learned in Problem 4.1 and 4.2 through contextual situations.

1. Fluency Drill - Math-Drills.com (see Fluency Binder)
2. Discuss previous day's ACE problems.

3. Students work in pair to complete Problem 4.3.
4. Discuss Problem 4.3 as a class. Questions A and B are very important. Call on more than one student to present their solutions and to describe their solutions strategies. For Questions B, have students display their number sentences on the board.
5. Students will complete Mathematical Reflection Sheet page 93 from Accentuate the Negative and at least 10 questions about the problems in this investigation and all previous investigations for homework.

Review (Day 5) Review consists of students' questions from their homework, Mathematical Reflections from all investigations, all previous quiz material and selected ACE problems and reworking problems to clarify their understanding.

Unit Test **(Day 6):**

1. Students will complete Unit Test.

Summarizing Strategy: Mathematical Reflection Sheet

Formative Assessment: Problems 4.1 and 4.2, 4.3; Problem 4.1 Exit Ticket; Problem 4.2 Exit Ticket; ACE problems; Mathematical Reflections 4	**Summative Assessment:** Unit Assessment

Teaching Strategies/Tips: Use CMP lesson plans to make sure all necessary materials are available. Have students explain how they are determining their solutions.

Differentiation (content/process/product): Shorten assignments and give notes to students who need help with note taking and organization skills. For extensions, provide ACE problems that are related to the days problem and give the students about 3-6 extra problems per day.

Resources: Pearson Connected Mathematics Project (CMP3), Math-Aids.com (homework/worksheets), http://www.mathdashboard.com/cmp3/

Attachments (assessments, rubrics, graphic organizers, projects, etc.): Teaching Aid 4.1B: Soccer Jersey, Teaching Aid 4.1B: Order of Operations, Problem 4.1 Exit Ticket and Key, Teacher Aid 4.2: Distributive Property, Problem 4.2 Exit Ticket Mathematical ACE Problems, Mathematical Reflections 4, Unit Assessment, Key and Rubric.

Problem 4.1 Soccer Jersey Example

The soccer team orders 20 new jerseys. The manufacturer charges a $100 setup fee and $15 per shirt. The total cost is represented by the equation $C = 100 + 15n$, where C is the cost in dollars and n is the number of jerseys ordered.

Pedro and David calculate the total cost.

Pedro's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 100 + 300 \\ &= \$400 \end{aligned}$$

David's calculation:

$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 115 \times 20 \\ &= \$2{,}300 \end{aligned}$$

Who did the calculations correctly?

Problem 4.1 Order of Operations

1. Compute any expressions within parentheses.

Example 1

$(-7 - 2) + 1 = -9 + 1 = -8$

Example 2

$(1 + 2) \times (-4) = 3 \times (-4) = -12$

2. Compute any exponents.

Example 1

$-2 + 3^2 = -2 + 9 = 7$

Example 2

$6 - (-1 + 4)^2 = 6 - 3^2 = 6 - 9 = -3$

3. Multiply and divide in order, from left to right.

Example 1

$1 + 2.8 \times 4.01 = 1 + 11.228$ Multiplication first

$= 12.228$

Example 2

$200 \div 10 \times \frac{1}{2} = 20 \times \frac{1}{2}$ Division first

$= 10$ Multiplication second

4. Add and subtract in order, from left to right.

$1 - 2 + 3 \times 4 = -1 + 12$ Subtraction first

$= -11$ Addition second

Name: _____ Date: _____
Class: _____

Exit Ticket Problem 4.1

Use the Order of Operations to specify the sequence of computations and find the value. Show each step separately

1. $2^2+7\cdot(-3)-5$

2. $(2^2+7)\cdot(-3)-5$

3. $(2^2+7)\cdot(-3-5)$

4. What different values can you get from this expression by inserting parentheses? $-5+3\cdot7-2+6\cdot4$

Name: _____ Date: _____
Class: _____

Exit Ticket Problem 4.1 - Key

Use the Order of Operations to specify the sequence of computations and find the value. Show each step separately

1. $2^2+7\cdot(-3)-5$ *(The order of exponents followed by multiplication, then addition, and then subtraction yields $4-21-5=-22$.)*

2. $(2^2+7)\cdot(-3)-5$ *(The order of parentheses followed by multiplication and then subtraction yields $(11)\cdot(-3)-5=-33-5=-38$.)*

3. $(2^2+7)\cdot(-3-5)$ *(The order of parentheses followed by multiplication yields $(4+7)\cdot(-8)=(11)(-8)=-88$.)*

4. What different values can you get from this expression by inserting parentheses? $-5+3\cdot7-2+6\cdot4$ *Answers will vary.*
   Sample: $(-5+3)\cdot(7-2)+(6\cdot4)=(-2)(5)+24=14$

Problem 4.2 Distributive Property

You can use the Distributive Property to rewrite an expression as one that is easier to calculate or gives new information.

You can do this in two ways.

1. The expression given is written as the product of two factors, one of which is a sum. You can use the Distributive Property to multiply the first factor by each number in the second factor and add the two resulting products. This is called *expanding* the expression.

$$-3 \cdot (4 + 8) = -3 \cdot 4 + (-3) \cdot 8$$

With a variable: $-2 \cdot (x + 6) = -2 \cdot x + (-2) \cdot 6$

2. The expression given is written as a sum and the numbers have a common factor. You can use the Distributive Property to rewrite the expression as the common factor multiplied by the sum. This is called *factoring* the expression.

$$20 + 35 = 55$$

$$5 \cdot 4 + 5 \cdot 7 = 5 \cdot (4 + 7)$$

With a variable: $40 + 8x = 8 \cdot 5 + 8x$

$$8 \cdot 5 + 8x = 8 \cdot (5 + x)$$

Name: _____ Date: _____
Class: _____

Exit Ticket Problem 4.2

Calculate the following:

1. $4+3(6+8)-10$

2. $4-3(6+8)-10$

3. $4-3(6-8)-10$

Name: _____ Date: _____
Class: _____

Exit Ticket Problem 4.2 - Key

Calculate the following:

1. $4+3(6+8)-10$ *(4+3(14)−10=4+42−10=46−10=36)*

2. $4-3(6+8)-10$ *(4−3(14)−10=4−42−10=−38−10=−48)*

3. $4-3(6-8)-10$ *(4−3(−2)−10=4+6−10=10−10=0)*

Applications

1. Find the values of each pair of expressions.

a. $-12 + (-4 + 9)$ $\quad$ $[-12 + (-4)] + 9$

b. $(14 - 20) - 8$ $\quad$ $14 - (20 - 8)$

c. $[14 + (-20)] + (-8)$ $\quad$ $14 + [-20 + (-8)]$

d. $-1 - [-1 + (-1)]$ $\quad$ $[-1 - (-1)] + (-1)$

e. Which cases lead to expressions with different results? Explain.

For Exercises 2–7, find the value of each expression.

2. $(5 - 3) \div (-2) \cdot (-1)$

3. $2 + (-3) \cdot 4 - (-5)$

4. $4 \cdot 2 \cdot (-3) + (-10) \div 5$

5. $-3 \cdot [2 + (-10)] - 2^2$

6. $(4 - 20) \div 2^2 - 5 \cdot (-2)$

7. $10 - [50 \div (-2 \cdot 25) - 7] \cdot 2^2$

For Exercises 8–11, rewrite each expression in an equivalent form to show a simpler way to do the arithmetic. Explain how you know the two results are equal without doing any calculations.

8. $(-150 + 270) + 30$

9. $(43 \cdot 120) + [43 \cdot (-20)]$

10. $23 + (-75) + 14 + (-23) - (-75)$

11. $[0.8 \cdot (-23)] + [0.8 \cdot (-7)]$

12. Without doing any calculations, determine whether each number sentence is true. Explain. Then check your answer.

a. $50 \cdot 432 = (50 \cdot 400) + (50 \cdot 32)$

b. $50 \cdot 368 = (50 \cdot 400) - (50 \cdot 32)$

c. $-50 \cdot 998 = [-50 \cdot (-1{,}000)] + [-50 \cdot 2]$

d. $-50 + (400 \cdot 32) = (-50 + 400) \cdot (-50 + 32)$

e. $(-70 \cdot 20) + (-50 \cdot 20) = (-120 \cdot 20)$

f. $6 \cdot 17 = 6 \cdot 20 - 6 \cdot 3$

For each part, use the Distributive Property to write an equivalent expression.

13. $-2 \cdot [5 + (-8)]$

14. $(-3 \cdot 2) - [-3 \cdot (-12)]$

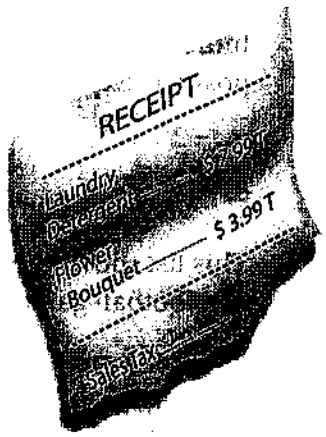
15. $x \cdot (-3 + 5)$

16. $-7x + 4x$

17. $2x \cdot [2 - (-4)]$

18. $x - 3x$

19. A grocery store receipt shows 5% state tax due on laundry detergent and a flower bouquet. Does it matter whether the tax is calculated on each separate item or the total cost? Explain.



Connections

For Exercises 20–37, find the sum, difference, product, or quotient.

20. $3 \cdot 12$

21. $3 \cdot (-12)$

22. $-3 \div (-12)$

23. $-10 \cdot (-11)$

24. $-10 + 11$

25. $10 - 11$

26. $-24 - (-12)$

27. $\frac{-24}{-12}$

28. $-18 \div 6$

29. $50 \cdot 70$

30. $50 \cdot (-70)$

31. $2{,}200 \div (-22)$

32. $-50 \cdot (-120)$

33. $-139 + 899$

34. $5{,}600 - 7{,}800$

35. $-4{,}400 - (-1{,}200)$

36. $\frac{-9{,}900}{-99}$

37. $-580 + (-320)$

38. When using negative numbers and exponents, you sometimes need parentheses to make it clear what you are multiplying.

You can think of -5^4 as "the opposite of 5^4" or
$-(5^4) = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$

You can think of $(-5)^4$ as "negative five to the fourth power" or
$(-5)^4 = -5 \cdot (-5) \cdot (-5) \cdot (-5) = 625$

Indicate whether the following expressions will be negative or positive. Explain your answers.

a. -3^2 **b.** $(-6)^3$ **c.** $(-4)^4$

d. -1^6 **e.** $(-3)^4$ **f.** -2^3

39. This list shows the yards gained and lost during the first several plays of a football game:

$$8, 4, 3, 7, -15, 20, 5, -12, 32, 1$$

Write an expression that shows how to compute the team's average gain or loss per play. Then compute the average.

40. Complete each number sentence.

a. $-34 + (-15) = \blacksquare$

b. $-12 \cdot (-23) = \blacksquare$

c. $-532 \div \blacksquare = -7$

d. $-777 - \blacksquare = -740$

e. Write a fact family for part (a).

f. Write a fact family for part (b).

For Exercises 41–44, write a related fact. Use it to find the value of n that makes the sentence true.

41. $n - (-5) = 35$ **42.** $4 + n = -43$

43. $-2n = -16$ **44.** $\frac{n}{4} = -32$

45. Insert parentheses (or brackets) in each expression if needed to make the equation true.

a. $1 + (-3) \cdot (-4) = 8$

b. $1 + (-3) \cdot (-4) = 13$

c. $-6 \div (-2) + (-4) = 1$

d. $-6 \div (-2) + (-4) = -1$

e. $-4 \cdot 2 - 10 = -18$

f. $-4 \cdot 2 - 10 = 32$

46. Multiple Choice Which set of numbers is in order from least to greatest?

A. 31.4, −14.2, −55, 75, −0.05, 0.5, 3.140

B. $\frac{2}{5}, \frac{-3}{5}, \frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, \frac{5}{3}$

C. −0.2, −0.5, 0.75, 0.6, −1, 1.5

D. None of these

47. Find the absolute values of the numbers for each set in Exercise 46. Write them in order from least to greatest.

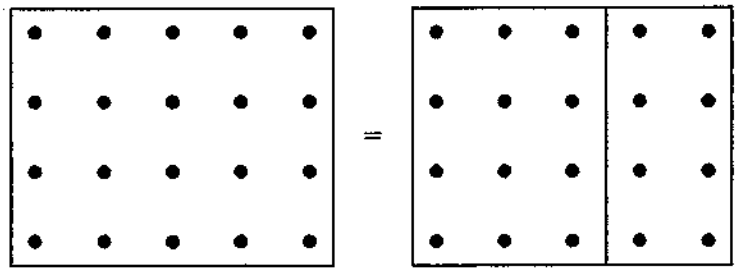
For Exercises 48–50, decide whether each statement is correct, and explain your answer.

48. $|-2 + 3| = |-2| + |3|$

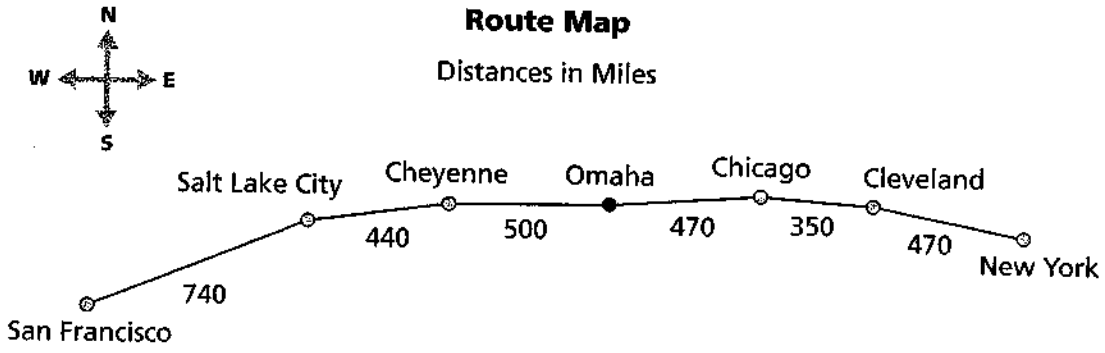
49. $5 - |-2 + 3| = 5 - |-2| + |3|$

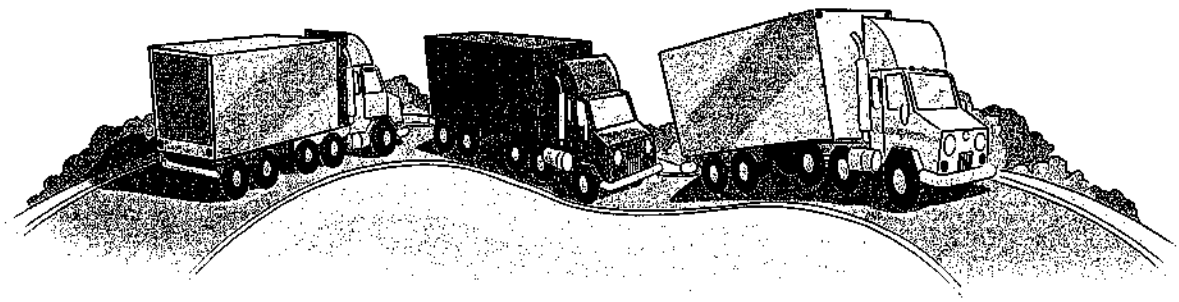
50. $|-2 - 3| = |-2| + |-3|$

51. You can use dot patterns to illustrate the distributive properties for operations on whole numbers. Write a number sentence to represent the pair of dot patterns.



52. A trucking company carries freight along a highway from New York City to San Francisco. Its home base is in Omaha, Nebraska, which is about halfway between the two cities. Truckers average about 50 miles per hour on this route.





Make a number line to represent this truck route. Put Omaha at 0. Use positive numbers for cities east of Omaha and negative numbers for cities west of Omaha. Then write number sentences to answer each question.

a. A truck leaves Omaha heading east and travels for 7 hours. About how far does the truck go? Where on the number line does it stop?

b. A truck leaves Omaha heading west and travels for 4.5 hours. About how far does the truck go? Where on the number line does it stop?

c. A truck heading east arrives in Omaha. About where on the number line was the truck 12 hours earlier?

d. A truck heading west arrives in Omaha. About where on the number line was the truck 11 hours earlier?

Extensions

Copy each pair of expressions in Exercises 53–57. Insert $<$ or $>$ to make a true statement.

53. $-23 \square -45$

54. $-23 + 10 \square -45 + 10$

55. $-23 - 10 \square -45 - 10$

56. $-23 \cdot 10 \square -45 \cdot 10$

57. $-23 \cdot (-10) \square -45 \cdot (-10)$

For Exercises 58–60, refer to your results in Exercises 53–57. Complete each statement. Test your ideas with other numerical cases, or develop another kind of explanation, perhaps using chip board or number line ideas.

58. If $a > b$, then $a + c \square b + c$.

59. If $a > b$, then $a - c \square b - c$.

60. If $a > b$, then $a \cdot c \square b \cdot c$.

For Exercises 61–63, find the value for n that makes the sentence true.

61. $n - (-24) = 12$

62. $2.5n = -10$

63. $2.5n + (-3) = -13$

64. Complete each pair of calculations.

a. $12 \div (-8 + 4) = \square$ $\quad [12 \div (-8)] + (12 \div 4) = \square$

b. $-12 \div [-5 - (-3)] = \square$ $\quad [-12 \div (-5)] - [-12 \div (-3)] = \square$

c. $(-2 - 6) \div 4 = \square$ $\quad (-2 \div 4) - (6 \div 4) = \square$

d. $(5 + 6) \div 3 = \square$ $\quad (5 \div 3) + (6 \div 3) = \square$

e. What can you conclude from parts (a)–(d) about the Distributive Property?

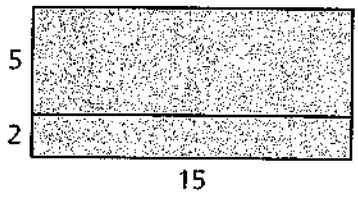
65. When you find the mean (average) of two numbers, you add them together and divide by 2.

a. Does the order in which you do the operations matter? Give examples.

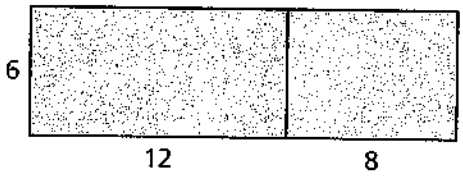
b. Does multiplication distribute over the averaging operation? That is, will a number a times the average of two numbers, x and y, give the same result as the average of ax and ay? Give examples.

For Exercises 66–69, write equivalent expressions to show two different ways to find the area of each rectangle. Use the ideas of the Distributive Property.

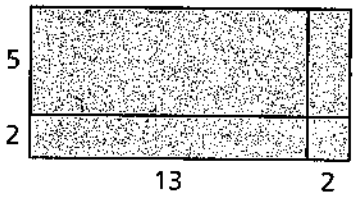
66.



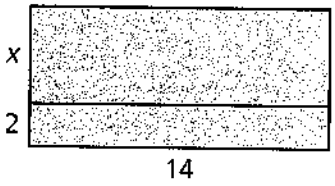
67.



68.



69.



For Exercises 70–73, draw and label the edges and areas of a rectangle to illustrate each pair of equivalent expressions.

70. $(3 + 2) \cdot 12 = 3 \cdot 12 + 2 \cdot 12$

71. $9 \cdot 3 + 9 \cdot 5 = 9 \cdot (3 + 5)$

72. $x \cdot (5 + 9) = 5x + 9x$

73. $2 \cdot (x + 8) = 2x + 16$

Mathematical Reflections

In this Investigation, you compared important properties of arithmetic with positive numbers to properties of arithmetic with negative numbers. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. **What** is the Order of Operations? Why is the Order of Operations important?

   b. **Give** an example of a numerical expression in which the use of parentheses changes the result of the computation.

2. **Describe** how the Distributive Property relates addition and multiplication. Give numerical examples.

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Elena described her thoughts in the following way:

While working on Problem 4.1, we noticed the importance of where the parentheses are in an expression. We were given an expression and had to put parentheses in to make the greatest and least possible values. To help us with this, we needed a good understanding of the Order of Operations, so we would know where to put the parentheses.

Common Core Standards for Mathematical Practice

MP6 Attend to precision.

- What other Mathematical Practices can you identify in Elena's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Name: _____ Date: _____
Class: _____

Accentuate the Negative Unit Assessment

ANSWER ALL QESTIONS ON A SEPARATE SHEET OF PAPER.

1. Find each sum or difference. Use either a number line or chip diagram to model your answer. (7.NS.A.1)

a.) $30 - (-17)$ b.) $-17 - (-30)$
c.) $-150 + 75$ d.) $1.5 - 2.7$
e.) $-14 + -15$ f.) $\frac{3}{4} + (-\frac{1}{2})$

2. Find the value of the expressions below. Write out each step to show your work. (7.NS.A.1, 7.NS.A2)

a.) $10 + 9 \cdot (-6) - (-1 + 8)$
b.) $(5 - 3) \div (-2) \cdot (-1)$
c.) $(4 - 20) \div 2^2 - 5 \cdot (-2)$

3. Without doing any calculations, determine whether each number sentence is true. EXPLAIN. Then check your answer. (7.NS.A2)

a.) $50 \cdot 432 = (50 \cdot 400) + (50 \cdot 32)$
b.) $-50 \cdot 998 = [-50 \cdot (-1000)] + [-50 \cdot 2]$
c.) $(-70 \cdot 20) + (-50 \cdot 20) = (-120 \cdot 20)$

4. Find each product or quotient. (7.NS.A2)

a.) $13 \cdot (-7)$
b.) $-8 \cdot (-20)$
c.) $99 \div (-3)$
d.) $-3.6 \div 1.8$
e.) $1/3 \cdot (-5/7)$

5. a.) Write two subtraction sentences to complete the fact family for $-8 + n = 62$. (7.EE.B.3, 7.EE.B.4)

b.) Use one of the fact family sentences to find the value of n.

6. Malique wants to take four of her friends to a movie. She knows it cost $5.50 for a ticket and $3.25 for popcorn. (7.NS.A.3, 7.EE.B.3)

a.) How much will it cost if she pays for the movie and popcorn for all five people?
b.) Write a number sentence to show how you found the total cost.
c.) Write a new number sentence that shows a different way to find the total cost. Explain.

7. Find the values of x that satisfy the inequality $x + 5 < 7$. Then graph the solutions for x on a number line. (7.EE.B.4)

8. The table below gives monthly average low temperatures (in degrees Fahrenheit) from November through March for International Falls, Minnesota. (7.EE.B.3)

Average Low Temperatures for International Falls

November	December	January	February	March
17°F	0°F	-9°F	-3°F	10°F

a. What is the mean of the monthly low temperatures? Show your work.
b. What is the difference of the highest and lowest temperatures?

Name: _____ Date: _____
Class: _____

Accentuate the Negative Unit Assessment - KEY

ANSWER ALL QESTIONS ON A SEPARATE SHEET OF PAPER.

8. Find each sum or difference. Use either a number line or chip diagram to model your answer. (7.NS.A.1)

a.) 30 + (+17) 47
b.) -17 + (+30) 13
c.) -150 + 75 - 75
d.) 1.5 – 2.7 -1.2
e.) -14 + -15 -29
f.) ¾ + (- ½) $\frac{1}{4}$.

9. Find the value of the expressions below. Write out each step to show your work. (7.NS.A.1, 7.NS.A2)

d.) 10 + 9 • (-6) – (-1 + 8)
e.) (5 – 3) ÷ (-2) • (-1)
f.) (4 – 20) ÷ 2^2 – 5 • (-2)

d.) 10 + 9 • (-6) – (-1 + 8)
10 + -54 - 7
-44 - 7
-51

e.) (5 - 3) ÷ (-2) • (-1)
2 ÷ (-2) • (-1)
(-1) • (-1)
1

10. Without doing any calculations, determine whether each number sentence is true. EXPLAIN. Then check your answer. (7.NS.A2)

d.) 50 • 432 = (50 • 400) + (50 • 32) T
e.) -50 • 998 = [-50 • (-1000)] + [-50 • 2] F
f.) (-70 • 20) + (-50 • 20) = (-120 • 20) T

11. Find each product or quotient. (7.NS.A2)

a.) 13 • (-7) -91
b.) -8 • (-20) 160
c.) 99 ÷ (-3) -33
d.) -3.6 ÷ 1.8 -2
e.) 1/3 • (-5/7) $-\frac{5}{21}$

12. a.) Write two subtraction sentences to complete the fact family for -8 + n = 62. (7.EE.B.3, 7.EE.B.4)

n + -8 = 62
62 - (-8) = n
62 - n = -8

b.) Use one of the fact family sentences to find the value of n. 62 - (-8) = n
70 = n

13. Malique wants to take four of her friends to a movie. She knows it cost $5.50 for a ticket and $3.25 for popcorn. (7.NS.A.3, 7.EE.B.3)

a.) How much will it cost if she pays for the movie and popcorn for all five people?
b.) Write a number sentence to show how you found the total cost.
c.) Write a new number sentence that shows a different way to find the total cost. Explain.

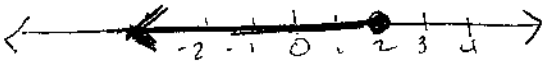
a.) 5($5.50) + 5($3.25)
b.) 27.50 + 16.25
$43.75
It will cost $43.75

c.) 5($5.50 + 3.25)
5($8.75)
$43.75
Distributive Property

14. Find the values of x that satisfy the inequality $x + 5 < 7$. Then graph the solutions for x on a number line. (7.EE.B.4)

$-5 \quad -5$

$x < 2$



8. The table below gives monthly average low temperatures (in degrees Fahrenheit) from November through March for International Falls, Minnesota. (7.EE.B.3)

Average Low Temperatures for International Falls

November	December	January	February	March
17°F	0°F	-9°F	-3°F	10°F

a. What is the mean of the monthly low temperatures? Show your work.
b. What is the difference of the highest and lowest temperatures?

$17 + 0 + -9 + -3 + 10 = \frac{15}{3} = 5°F$ is the mean of the monthly temperatures

$17°F - (-9°F) = 26°F$ is difference between the highest and lowest temperatures

Accentuate the Negative Unit Assessment – Scoring Rubric

Mark appropriate rating for each item

	0 **No Attempt** Irrelevant response, does not attempt a solution or does not address conditions of the problems	**1** **Inadequate Response** Incomplete; explanation is insufficient or not understandable, shows little understanding of mathematical concepts, or fails to address essential conditions of the problem	**2** **Partial Response** Response may be unclear or lacks detail, shows some understanding of mathematical concepts and satisfies some essential conditions of the problem	**3** **Reasonably Complete Response** Response may lack detail in explanation, shows understanding of most of the mathematical concepts and satisfies most of the essential conditions of the problem	**4** **Complete Response** Response has clear, coherent explanations, shows understanding of mathematical concepts and satisfies all essential conditions of the problem
#1 7.NS.A.1					
#2 7.NS.A.1 7.NS.A2					
#3 7.NS.A2					
#4 7.NS.A2					
#5 7.EE.B.3 7.EE.B.4					
#6 7.NS.A.3 7.EE.B.3					
#7 7.EE.B.4					
#8 7.EE.B.3					