CCGPS Frameworks Student Edition

Mathematics

CCGPS Analytic Geometry
Unit 1: Similarity, Congruence, and Proofs

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"Making Education Work for All Georgians"

Unit 1
Similarity, Congruence, and Proofs

Table of Contents

OVERVIEW

In this unit students will:

- verify experimentally with dilations in the coordinate plane.
- use the idea of dilation transformations to develop the definition of similarity.
- determine whether two figures are similar.
- use the properties of similarity transformations to develop the criteria for proving similar triangles.
- use AA, SAS, SSS similarity theorems to prove triangles are similar.
- use triangle similarity to prove other theorems about triangles.
- using similarity theorems to prove that two triangles are congruent.
- prove geometric figures, other than triangles, are similar and/or congruent.
- use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
- know that rigid transformations preserve size and shape or distance and angle; use this fact to connect the idea of congruency and develop the definition of congruent.
- use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS.
- prove theorems pertaining to lines and angles.
- prove theorems pertaining to triangles.
- prove theorems pertaining to parallelograms.
- make formal geometric constructions with a variety of tools and methods.
- construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. The first unit of Analytic Geometry involves similarity, congruence, and proofs. Students will understand similarity in terms of similarity transformations, prove theorems involving similarity, understand congruence in terms of rigid motions, prove geometric theorems, and make geometric constructions. During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** in the Comprehensive Course Overview and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Understand similarity in terms of similarity transformations

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand congruence in terms of rigid motions

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

MCC9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

SMP = Standards for Mathematical Practice

*****Although the language of mathematical argument and justification is not explicitly expressed in the standards, it is embedded in the Standards for Mathematical Practice (3. Construct viable arguments and critique the reasoning of others.). Using conjecture, inductive reasoning, deductive reasoning, counterexamples and multiple methods of proof as appropriate is relevant to this and future units. Also, understanding the relationship between a statement and its converse, inverse and contrapositive is important.

ENDURING UNDERSTANDINGS

- Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is performed. However, a segment that passes through the center remains unchanged.
- Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.
- Use the idea of dilation transformations to develop the definition of similarity.
- Given two figures determine whether they are similar and explain their similarity based on the equality of corresponding angles and the proportionality of corresponding sides.
- Use the properties of similarity transformations to develop the criteria for proving similar triangles: AA.
- Use AA, SAS, SSS similarity theorems to prove triangles are similar.
- Prove a line parallel to one side of a triangle divides the other two proportionally, and its converse.
- Prove the Pythagorean Theorem using triangle similarity.

- Use similarity theorems to prove that two triangles are congruent.
- Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
- Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
- Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria: ASA, SSS, and SAS.
- Prove vertical angles are congruent.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.
- Prove points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- Prove the measures of interior angles of a triangle have a sum of 180º.
- Prove base angles of isosceles triangles are congruent.
- Prove the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.
- Prove the medians of a triangle meet at a point.
- Prove properties of parallelograms including: opposite sides are congruent, opposite angles are congruent, diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
- Copy a segment and an angle.
- Bisect a segment and an angle.
- Construct perpendicular lines, including the perpendicular bisector of a line segment.
- Construct a line parallel to a given line through a point not on the line.
- Construct an equilateral triangle so that each vertex of the equilateral triangle is on the circle.
- Construct a square so that each vertex of the square is on the circle.
- Construct a regular hexagon so that each vertex of the regular hexagon is on the circle.

CONCEPTS/SKILLS TO MAINTAIN

Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency. Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified. Dilations and similarity, including the AA criterion, are investigated in Grade 8, and these experiences should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

The Pythagorean Theorem and its converse are proved and applied in Grade 8. In high school, another proof, based on similar triangles, is presented. The alternate interior angle theorem and its converse, as well as properties of parallelograms, are established informally in Grade 8 and proved formally in high school.

Properties of lines and angles, triangles and parallelograms are investigated in Grades 7 and 8. In high school, these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.

The theorem about the midline of a triangle can easily be connected to a unit on similarity. The proof of it is usually based on the similarity property that corresponding sides of similar triangles are proportional.

Students should be expected to have prior knowledge/experience related to the concepts and skills identified below. Pre-assessment may be necessary to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Understand and use reflections, translations, and rotations.
- Define the following terms: circle, bisector, perpendicular and parallel.
- Solve multi-step equations.
- Understand angle sum and exterior angle of triangles.
- Know angles created when parallel lines are cut by a transversal.
- Know facts about supplementary, complementary, vertical, and adjacent angles.
- Solve problems involving scale drawings of geometric figures.
- Draw geometric shapes with given conditions.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Draw polygons in the coordinate plane given coordinates for the vertices.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Adjacent Angles:** Angles in the same plane that have a common vertex and a common side, but no common interior points.
- **Alternate Exterior Angles**: Alternate exterior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. When the two other lines are parallel, the alternate exterior angles are equal.
- **Alternate Interior Angles**: Alternate interior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. When the two other lines are parallel, the alternate interior angles are equal.
- **Angle:** Angles are created by two distinct rays that share a common endpoint (also known as a vertex). $\angle ABC$ or $\angle B$ denote angles with vertex B.
- **Bisector:** A bisector divides a segment or angle into two equal parts.
- **Centroid:** The point of concurrency of the medians of a triangle.
- **Circumcenter:** The point of concurrency of the perpendicular bisectors of the sides of a triangle.
- **Coincidental:** Two equivalent linear equations overlap when graphed.
- **Complementary Angles:** Two angles whose sum is 90 degrees.
- **Congruent:** Having the same size, shape and measure. Two figures are congruent if all of their corresponding measures are equal.
- **Congruent Figures**: Figures that have the same size and shape.

- **Corresponding Angles:** Angles that have the same relative positions in geometric figures.
- **Corresponding Sides:** Sides that have the same relative positions in geometric figures
- **Dilation**: Transformation that changes the size of a figure, but not the shape.
- **Endpoints:** The points at an end of a line segment
- **Equiangular:** The property of a polygon whose angles are all congruent.
- **Equilateral:** The property of a polygon whose sides are all congruent.
- **Exterior Angle of a Polygon:** an angle that forms a linear pair with one of the angles of the polygon.
- **Incenter:** The point of concurrency of the bisectors of the angles of a triangle.
- **Intersecting Lines:** Two lines in a plane that cross each other. Unless two lines are coincidental, parallel, or skew, they will intersect at one point.
- **Intersection:** The point at which two or more lines intersect or cross.
- **Inscribed Polygon:** A polygon is inscribed in a circle if and only if each of its vertices lie on the circle.
- **Line:** One of the basic undefined terms of geometry. Traditionally thought of as a set of points that has no thickness but its length goes on forever in two opposite directions. $\overleftrightarrow{AB}$ denotes a line that passes through point A and B.
- **Line Segment or Segment:** The part of a line between two points on the line. $\overline{AB}$ denotes a line segment between the points A and B.
- **Linear Pair:** Adjacent, supplementary angles. Excluding their common side, a linear pair forms a straight line.
- **Measure of each Interior Angle of a Regular n-gon:** $\frac{180°(n-2)}{n}$
- **Median of a Triangle:** A segment is a median of a triangle if and only if its endpoints are a vertex of the triangle and the midpoint of the side opposite the vertex.
- **Midsegment:** A line segment whose endpoints are the endpoint of two sides of a triangle is called a midsegment of a triangle.

- **Orthocenter:** The point of concurrency of the altitudes of a triangle.
- **Parallel Lines:** Two lines are parallel if they lie in the same plane and they do not intersect.
- **Perpendicular Bisector:** A perpendicular line or segment that passes through the midpoint of a segment.
- **Perpendicular Lines:** Two lines are perpendicular if they intersect at a right angle.
- **Plane:** One of the basic undefined terms of geometry. Traditionally thought of as going on forever in all directions (in two-dimensions) and is flat (i.e., it has no thickness).
- **Point:** One of the basic undefined terms of geometry. Traditionally thought of as having no length, width, or thickness, and often a dot is used to represent it.
- **Proportion**: An equation which states that two ratios are equal.
- **Ratio**: Comparison of two quantities by division and may be written as r/s, r:s, or r to s.
- **Ray:** A ray begins at a point and goes on forever in one direction.
- **Reflection:** A transformation that "flips" a figure over a line of reflection
- **Reflection Line:** A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.
- **Regular Polygon:** A polygon that is both equilateral and equiangular.
- **Remote Interior Angles of a Triangle:** the two angles non-adjacent to the exterior angle.
- **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction.
- **Same-Side Interior Angles**: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are between the other two lines. When the two other lines are parallel, same-side interior angles are supplementary.
- **Same-Side Exterior Angles**: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are outside the other two lines. When the two other lines are parallel, same-side exterior angles are supplementary.

- **Scale Factor**: The ratio of any two corresponding lengths of the sides of two similar figures.
- **Similar Figures**: Figures that have the same shape but not necessarily the same size.
- **Skew Lines:** Two lines that do not lie in the same plane (therefore, they cannot be parallel or intersect).
- **Sum of the Measures of the Interior Angles of a Convex Polygon:** $180^\circ(n - 2)$.
- **Supplementary Angles:** Two angles whose sum is 180 degrees.
- **Transformation**: The mapping, or movement, of all the points of a figure in a plane according to a common operation.
- **Translation:** A transformation that "slides" each point of a figure the same distance in the same direction
- **Transversal:** A line that crosses two or more lines.
- **Vertical Angles:** Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

FORMATIVE ASSESSMENT LESSONS (FALs)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on types of Formative Assessment Lessons may be found in the Comprehensive Course Guide.

Dilations in the Coordinate Plane

Adapted from Stretching and Shrinking: Similarity, *Connected Mathematics*, Dale Seymour Publications

Mathematical Goals

- Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is performed.
- Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.
- Use the idea of dilation transformations to develop the definition of similarity.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.
8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order. Do not connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (do not connect the dots).

Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Set 1	**Set 1**	**Set 1**	**Set 1**	**Set 1**	**Set 1**
(6, 4)	(12, 8)	(18, 4)	(18, 12)	(6, 12)	(8, 6)
(6, -4)	(12, -8)	(18, -4)	(18, -12)	(6, -12)	(8, -2)
(-6, -4)	(-12, -8)	(-18, -4)	(-18, -12)	(-6, -12)	(-4, -2)
(-6, 4)	(-12, 8)	(-18, 4)	(-18, 12)	(-6, 12)	(-4, 6)
Set 2	**Set 2**	**Set 2**	**Set 2**	**Set 2**	**Set 2**
(1, 1)	(2, 2)	(3, 1)	(3, 3)	(1, 3)	(3, 3)
(1, -1)	(2, -2)	(3, -1)	(3, -3)	(1, -3)	(3, 1)
(-1, -1)	(-2, -2)	(-3, -1)	(-3, -3)	(-1, -3)	(1, 1)
(-1, 1)	(-2, 2)	(-3, 1)	(-3, 3)	(-1, 3)	(1, 3)
Set 3	**Set 3**	**Set 3**	**Set 3**	**Set 3**	**Set 3**
(4, -2)	(8, -4)	(12, -2)	(12, -6)	(4, -6)	(6, 0)
(3, -3)	(6, -6)	(9, -3)	(9, -9)	(3, -9)	(5, -1)
(-3, -3)	(-6, -6)	(-9, -3)	(-9, -9)	(-3, -9)	(-1, -1)
(-4, -2)	(-8, -4)	(-12, -2)	(-12, -6)	(-4, -6)	(-2, 0)
Set 4	**Set 4**	**Set 4**	**Set 4**	**Set 4**	**Set 4**
(4, 2)	(8, 4)	(12, 2)	(12, 6)	(4, 6)	(6, 4)
(-4, 2)	(-8, 4)	(-12, 2)	(-12, 6)	(-4, 6)	(-2, 4)

After drawing the six figures, compare Figure 1 to each of the other figures and answer the following questions.

1. Which figures are similar? Use the definition of similar figures to justify your response.

2. Describe any similarities and/or differences between Figure 1 and each of the similar figures.
   - Describe how corresponding sides compare.
   - Describe how corresponding angles compare.

3. How do the coordinates of each similar figure compare to the coordinates of Figure 1? Write general rules for making the similar figures.

4. Is having the same angle measurement enough to make two figures similar? Why or why not?

5. What would be the effect of multiplying each of the coordinates in Figure 1 by ½?

6. Create a similar Figure 7 to Figure 1 where the center of dilation is not the origin but (-6, -4) instead. Also Figure 7 is twice as big as Figure 1. What are the sets of points used to create Figure 7?

Similar Triangles

Mathematical Goals

- Discover the relationships that exist between similar figures using the scale factors, length ratios, and area ratios

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- **c.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **d.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.
4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

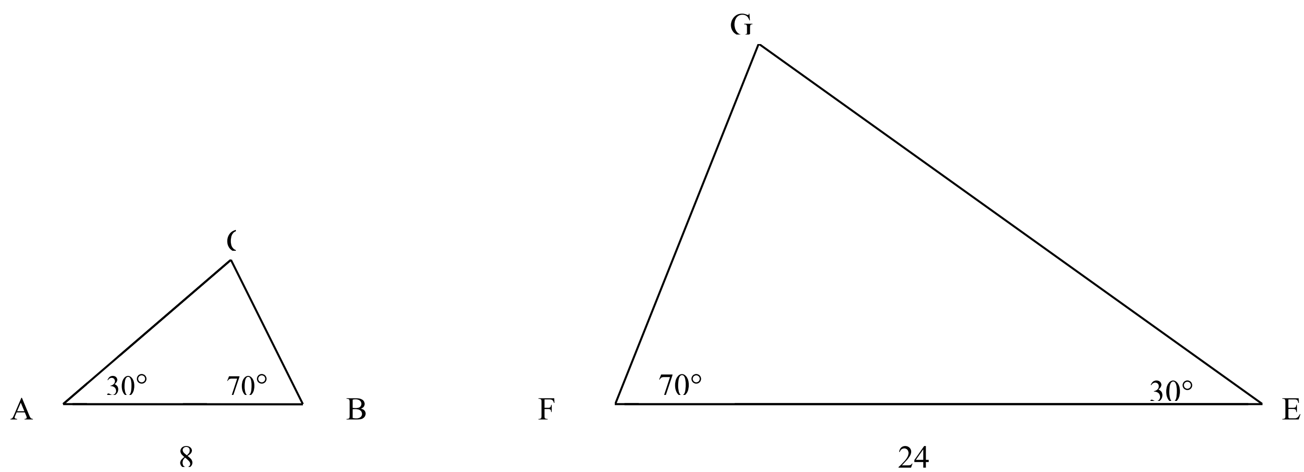
The sketch below shows two triangles, ΔABC and ΔEFG. ΔABC has an area of 12 square units, and its base (AB) is equal to 8 units. The base of ΔEFG is equal to 24 units.

a. How do you know that the triangles are similar?

b. Name the pairs of corresponding sides and the pairs of corresponding angles. How are the corresponding sides related and how are the corresponding angles related? Why is this true?

c. What is the area of Δ EFG? Explain your reasoning.

d. What is the relationship between the area of ΔABC and the area of Δ EFG? What is the relationship between the scale factor and the ratio of the areas of the two triangles? Use an area formula to justify your answer algebraically.



Shadow Math

Mathematical Goals

- Determine missing side lengths and areas of similar figures.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

Jeannie is practicing on the basketball goal outside her house. She thinks that the goal seems lower than the 10 ft. goal she plays on in the gym. She wonders how far the goal is from the ground. Jeannie can not reach the goal to measure the distance to the ground, but she remembers something from math class that may help. First, she needs to estimate the distance from the bottom of the goal post to the top of the backboard. To do this, Jeannie measures the length of the shadow cast by the goal post and backboard. She then stands a yardstick on the ground so that it is perpendicular to the ground, and measures the length of the shadow cast by the yardstick. Here are Jeannie's measurements:

Length of shadow cast by goal post and backboard: 5 ft. 9 in.

Length of yardstick's shadow: 1 ft. 6 in.

Draw and label a picture to illustrate Jeannie's experiment. Using her measurements, determine the height from the bottom of the goal post to the top of the backboard.

If the goal is approximately 24 inches from the top of the backboard, how does the height of the basketball goal outside Jeannie's house compare to the one in the gym? Justify your answer.

Proving Similar Triangles

Mathematical Goals

- Identify Similar Triangles.
- Use similarity theorems to prove that two triangles are similar.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

Introduction

This task identifies the three ways to prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons. Examples and practice problems are provided.

You can always prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons.

1.) Show that corresponding angles are congruent AND
2.) Show that corresponding sides are proportional.

However, there are 3 simpler methods.

Angle-Angle Similarity Postulate (AA~) If two angles of one triangle are congruent to two angles of another triangle then the triangles are similar.

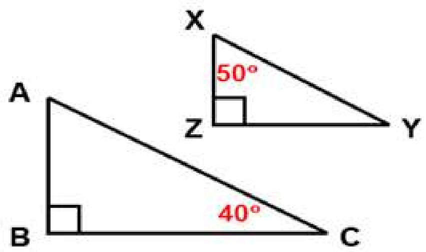
Examples of AA~

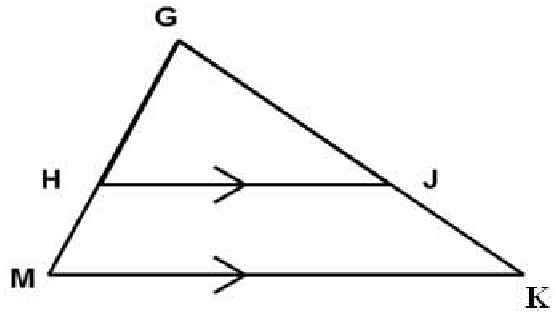
The ΔABC ~ ΔXZY are similar by AA~ because

1) They are both right triangles; therefore they both have a 90 degree angle.
2) All triangles add up to 180 degrees, since angle C is 40 degrees in ΔABC angle A will be 50 degrees. Therefore, ∠ A and ∠ X are congruent.

The ΔGHJ ~ ΔGMK are similar by AA~ because

1) ∠ H and ∠ M are congruent by Corresponding Angles Postulate.
2) ∠ HGJ and ∠ MGK are congruent since they are the same angle.





Side-Side-Side Similarity (SSS~): If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.

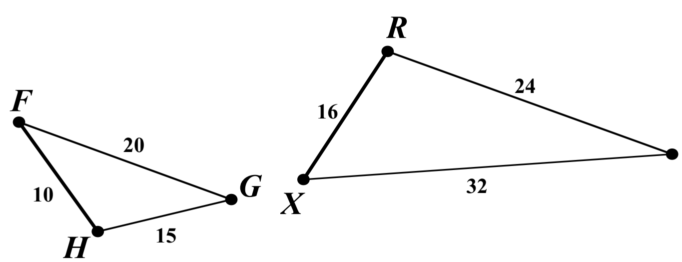
Example of SSS~

ΔFHG ~ ΔXRS because three sides of one triangle are proportional to three sides of another triangle.

$\frac{FH}{XR} = \frac{10}{16} \equiv \frac{5}{8}$

$\frac{HG}{RS} = \frac{15}{24} = \frac{5}{8}$

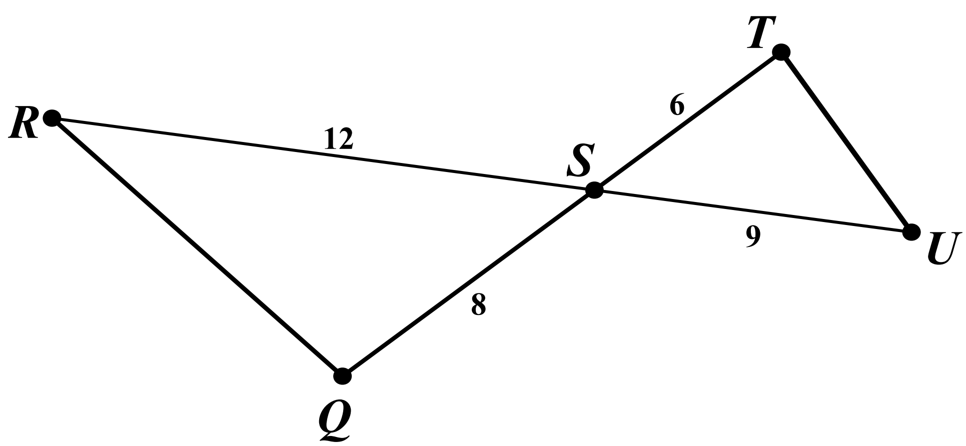
$\frac{FG}{XS} = \frac{20}{32} = \frac{5}{8}$



Side-Angle-Side Similarity (SAS~): If two sides of one triangle are proportional to two sides of another triangle and the included angles of these sides are congruent, then the two triangles are similar.

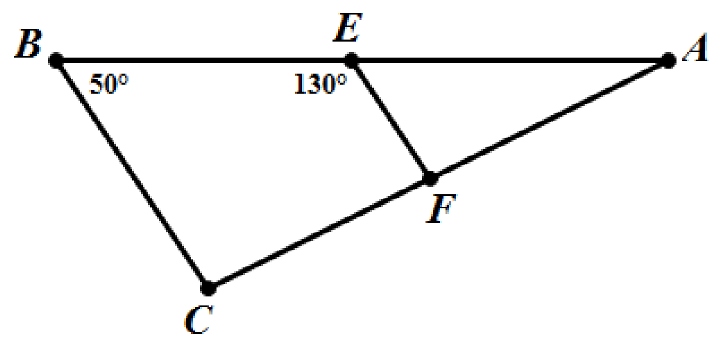
Example of SAS~
ΔRSQ ~ ΔUST because
1) ∠ RSQ ≅ ∠UST since Vertical Angles are Congruent

2) $\frac{RS}{US} = \frac{12}{9} = \frac{4}{3}$, Two sides of one triangle are proportional to two sides of another triangle.
$\frac{SQ}{ST} = \frac{8}{6} = \frac{4}{3}$

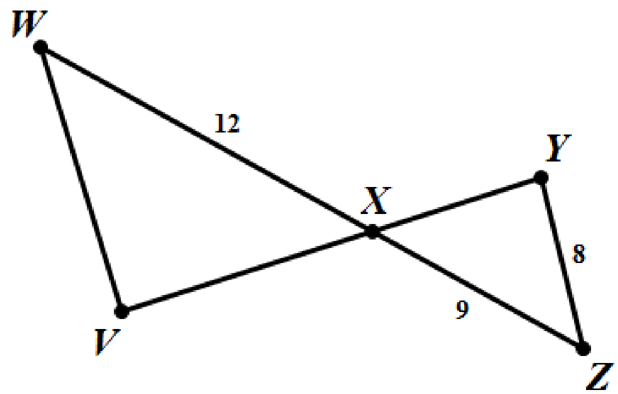


Can the two triangles shown be proved similar? If so, state the similarity and tell which method you used.

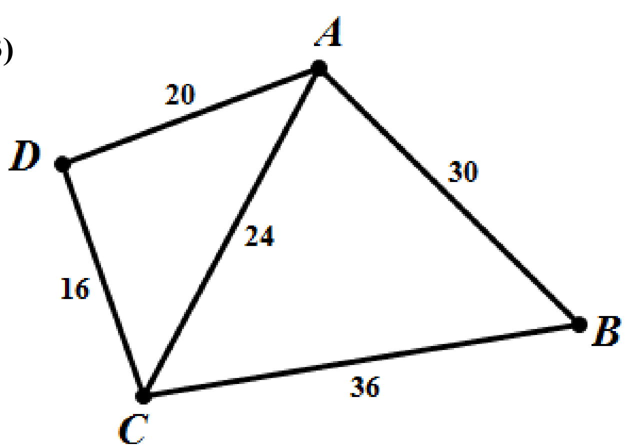
1)



2)



3)



Formative Assessment Lesson: Hopewell Geometry

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=499

ESSENTIAL QUESTION:

- How do you prove triangles are similar using the Pythagorean Theorem?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Hopewell Geometry*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=127&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=499

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=500

STANDARDS ADDRESSED IN THIS TASK:

Prove theorems involving similarity

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice
This task uses all of the practices with emphasis on:

3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

Pythagorean Theorem using Triangle Similarity

Mathematical Goals

- Prove the Pythagorean Theorem using triangle similarity.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.
8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Introduction

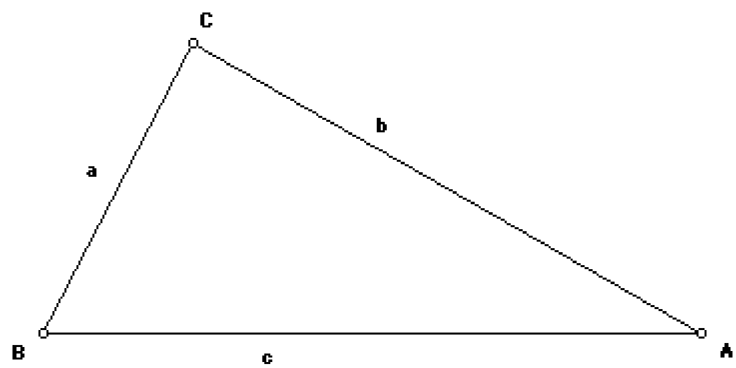
This task has students use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a^2 + b^2 = c^2$) and thus obtain an algebraic proof of the Pythagorean Theorem.

Materials

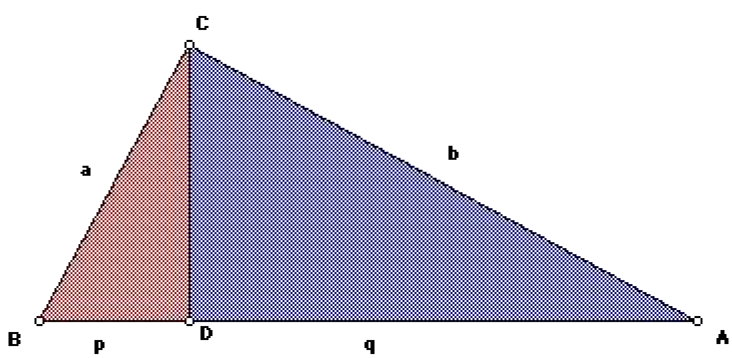
- Cardboard
- Straightedge

Use cardboard cutouts to illustrate the following investigation:

In the next figure, draw triangle ABC as a right triangle. Its right angle is angle C.



Next, draw CD perpendicular to AB as shown in the next figure.

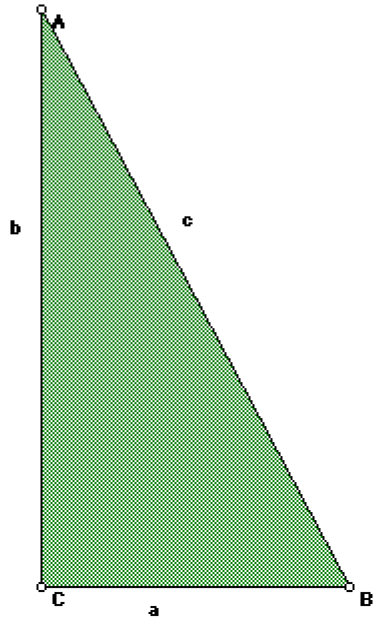


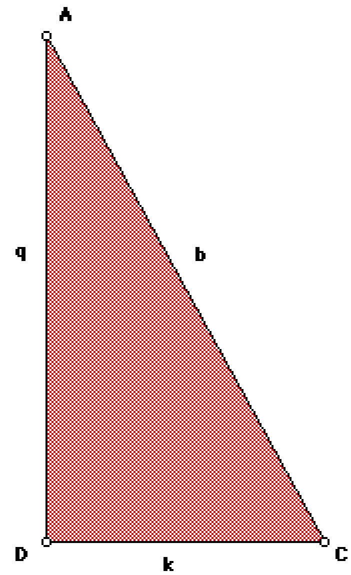
How many triangles do you see in the figure?

Why are the three triangles similar to each other?

<u>Compare triangles 1 and 3</u>:

Triangle 1 (green) is the right triangle that we began with prior to constructing CD. Triangle 3 (red) is one of the two triangles formed by the construction of CD.

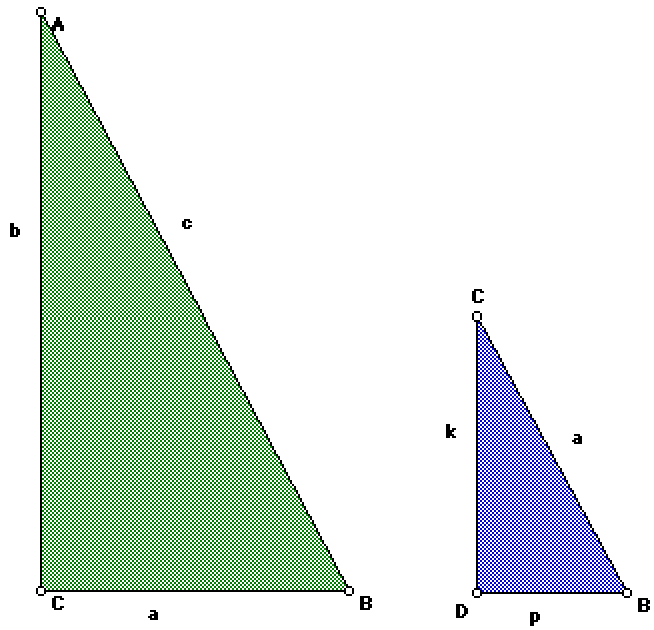




By comparing these two triangles, we can see that $\frac{c}{b} = \frac{b}{q}$ and $b^2 = cq$

Compare triangles 1 and 2:

Triangle 1 (green) is the same as above. Triangle 2 (blue) is the other triangle formed by constructing CD. Its right angle is angle D.



By comparing these two triangles, we see that $\frac{c}{a} = \frac{a}{p}$ and $a^2 = cp$

By adding the two equations:

$$a^2 + b^2 = cp + cq$$
$$a^2 + b^2 = c(p + q)$$

CD, we have that (p + q) = c. By substitution, we get

$$a^2 + b^2 = c^2$$

Lunch Lines

Mathematical Goals

- Prove vertical angles are congruent.
- Understand when a transversal is drawn through parallel lines, special angles relationships occur.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent.

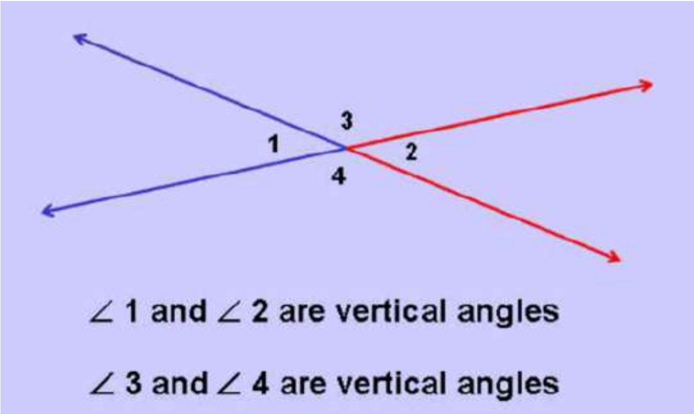
STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

Two angles are vertical angles if their sides form two pairs of opposite rays.



How do you know that vertical angles are congruent?

$m\angle 1 + m\angle 3 = 180°$ because the Linear Pair postulate
$m\angle 2 + m\angle 3 = 180°$ because the Linear Pair postulate

Set the two equations equal to each other since they both equal 180 degrees.

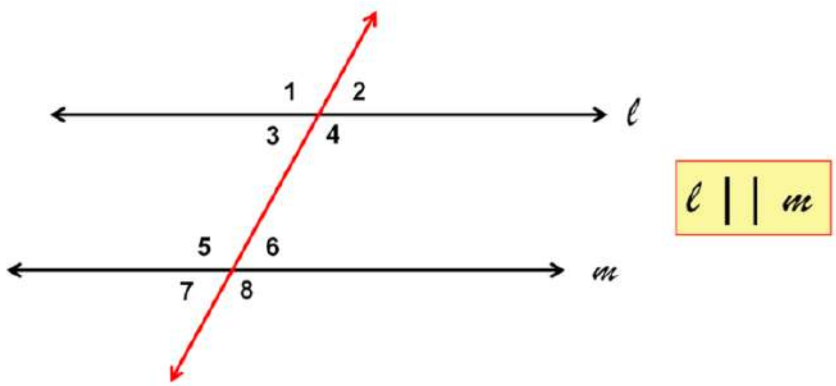
$m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$
$\quad - m\angle 3 \qquad\qquad - m\angle 3$
$m\angle 2 = m\angle 1$

Therefore: $\angle 2 \cong \angle 1$

Prove that $\angle 3 \cong \angle 4$ using a similar method.

When a transversal crosses parallel lines, there are several pairs of special angles. Let's look at a few together.

Corresponding Angle Postulate: If two parallel lines are cut by a transversal, then corresponding angles are congruent.



Using this postulate, name a pair of congruent angles.

How do we know that $\angle 3 \cong \angle 6$?

Alternate Interior Angle Theorem: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Prove this theorem using the figure above.

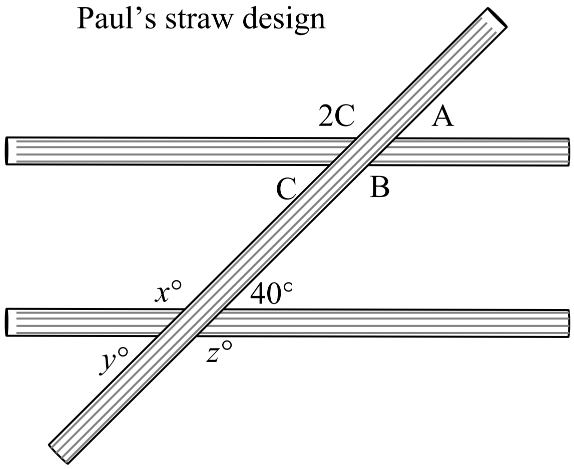
How do we know that $\angle 3 \cong \angle 5$ are supplementary?

Same-Side Interior Angle Theorem: If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

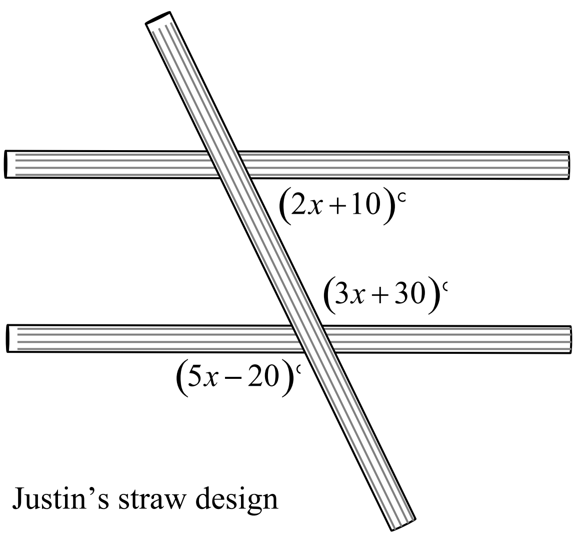
Prove this theorem using the figure above.

Paul, Jane, Justin, and Opal were finished with lunch and began playing with drink straws. Each one was making a line design using either 3 or 4 straws.
They had just come from math class where they had been studying special angles.
Paul pulled his pencil out of his book bag and labeled some of the angles and lines. He then challenged himself and the others to find all the labeled angle measurements in Paul and Justin's straw designs and to determine whether the lines that appear to be parallel really are parallel.

Paul's straw design



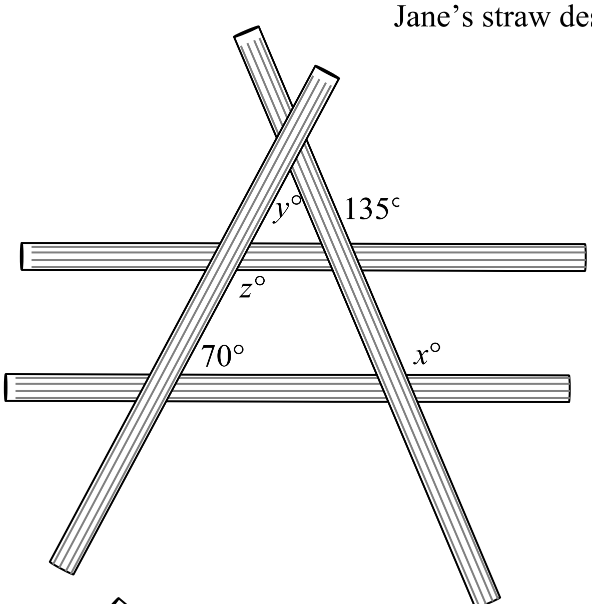
- Find all of the labeled angle measurements.
- Determine whether the lines that appear to be parallel really are parallel.
- Explain the reasoning for your results.



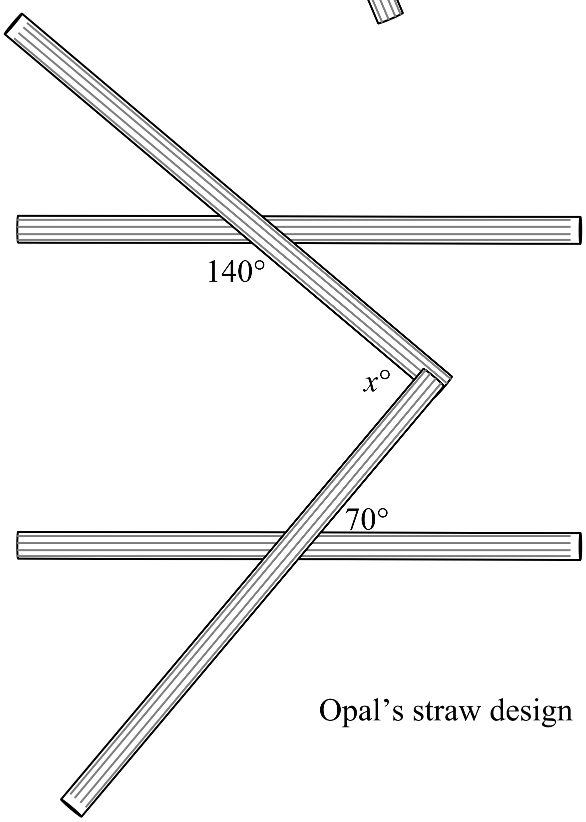
Justin's straw design

Paul then challenged himself and the others to find all the labeled angle measurements in Jane and Opal's straw designs knowing that the lines created by the straws in their designs were parallel.

Jane's straw design



- Find all of the labeled angle measurements (knowing that the lines created by the straws are parallel).
- Explain the reasoning for your results



Opal's straw design

Triangle Proportionality Theorem

Mathematical Goals

- Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
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8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Draw two parallel lines that contain points A and B. (Refer to Figure 1.)

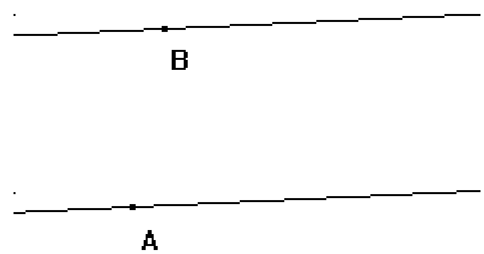


Figure 1

Draw line $\overleftrightarrow{AB}$. Create a new point C on $\overleftrightarrow{AB}$ and draw another transversal. Label the intersections of this line with the parallel lines as points D and E as shown in Figure 2.

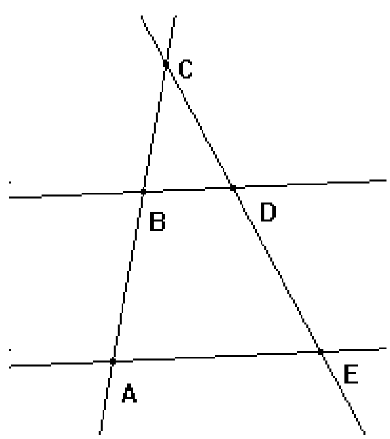


Figure 2

	Side 1	Side 2	Side 3	Perimeter
ΔCBD	$CB =$	$CD =$	$BD =$	$Perim_{\Delta CBD} =$
ΔCAE	$CA =$	$CE =$	$AE =$	$Perim_{\Delta CAE} =$
Ratio	$\frac{CB}{CA} =$	$\frac{CD}{CE} =$	$\frac{BD}{AE} =$	$\frac{Perim_{\Delta CBD}}{Perim_{\Delta CAE}} =$

Answer the following questions:

1. Explain why $\overleftrightarrow{AB}$ (illustrated in figure 2) is a transversal.

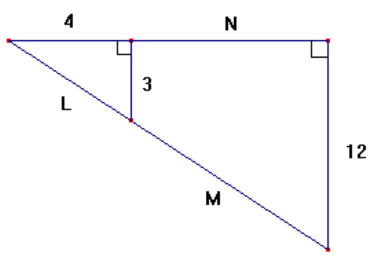
2. Explain why segments $\overline{CB}$ and $\overline{CA}$ are called corresponding segments.

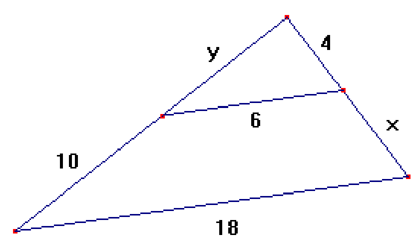
3. In view of the last row of results in the table, what appears to be true about the ratio of lengths defined by two transversals intersecting parallel lines?

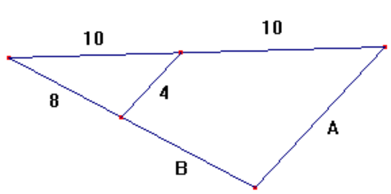
4. Grab different points and lines in the construction and move them around, if possible. While all of the measurements will change, one relationship will continue to hold no matter how the construction is changed. What is that relationship?

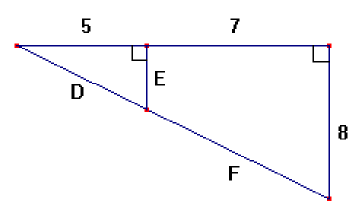
5. Compare your findings from question 4 to those of a classmate. Did everyone discover the same relationship?

6. Use the relationship that you have observed to solve for the unknown quantities in each of the following figures. You may assume that lines which look parallel in each figure are parallel.

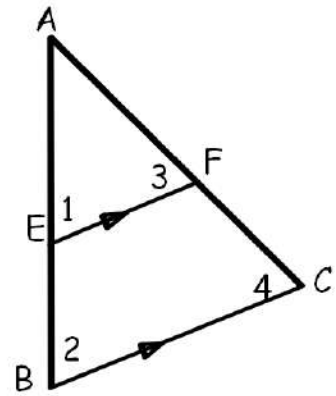








Triangle Proportionality Theorem
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



Proof #1:

Given: $\overleftrightarrow{EF} \parallel \overline{BC}$

Prove: $\frac{AE}{EB} = \frac{AF}{FC}$

Complete the proof:

Show that ΔAEF ~ ΔABC
Since $\overleftrightarrow{EF} \parallel \overline{BC}$, you can conclude that $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by __________

So ΔAEF ~ ΔABC by ____________

Use the fact that corresponding sides of similar triangles are proportional to complete the proof

$\frac{AB}{AE} =$ ____________ Corresponding sides are proportional

$\frac{AE + EB}{AE} =$ ____________ Segment Addition Postulate

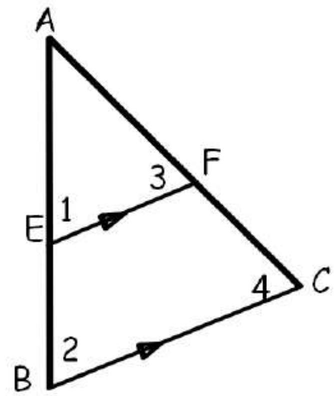
$1 + \frac{EB}{AE} =$ _______________ Use the property that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{EB}{AE} =$ _______________ Subtract 1 from both sides.

$\frac{AE}{EB} =$ ________________ Take the reciprocal of both sides.

Converse of the Triangle Proportionality Theorem
If a line divides two sides of a triangle proportionally, then it is parallel to the third side

Proof #2



Given: $\frac{AE}{EB} = \frac{AF}{FC}$

Prove: $\overleftrightarrow{EF} \parallel \overline{BC}$

Complete the proof. Show that ΔAEF ~ ΔABC
It is given that $\frac{AE}{EB} = \frac{AF}{FC}$ and taking the reciprocal of both sides show that __________________.

Now add 1 to both sides by adding $\frac{AE}{AE}$ to the left side and $\frac{AF}{AF}$ to the right side.

____

Adding and using the Segment Addition Postulate gives _______________________.

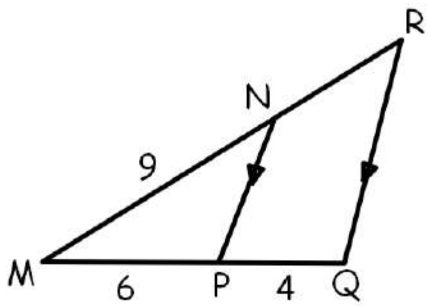
Since $\angle A \cong \angle A, \Delta AEF \sim \Delta ABC$ by ____________________.

As corresponding angles of similar triangles, ∠AEF ≅ ________________.

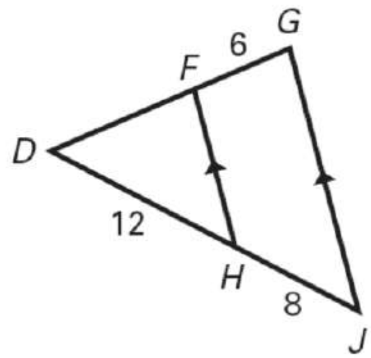
So, $\overleftrightarrow{EF} \parallel \overline{BC}$ by _________________.

Let's practice finding the length of a segment since you know how to prove the Triangle Proportionality Theorem and its converse.

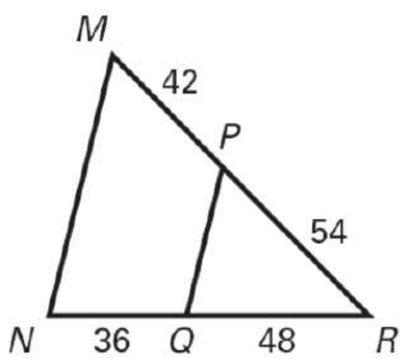
1. What is the length of NR?



2. What is the length of DF?



3. Given the diagram, determine whether *MN* is parallel to *PQ*.



Challenges from Ancient Greece

Mathematical Goals

- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Standards for Mathematical Practice

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Introduction

In this task, students will investigate and perform geometric constructions using Euclidean tools. The focus of this task is to learn how to copy line segments, copy an angle, bisect a segment, and bisect an angle.

It is highly recommended that students have access to tools such as a Mira™ or reflective mirror and Patty Paper™ to assist developing conceptual understandings of the geometry. During construction activities, students should also use technology, such as Geometer's Sketchpad to reinforce straight edge and compass work and to assist with dexterity challenges.

Materials

- compass and straightedge
- Mira™ or reflective mirror

- graph paper
- patty paper or tracing paper (optional)

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

- A straight edge without any markings
- A compass

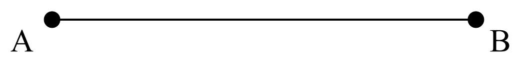
The straight edge could be used to construct lines, the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge. What constructions can you create?

Your First Challenge: Can you copy a line segment?

Step 1	Construct a circle with a compass on a sheet of paper.
Step 2	Mark the center of the circle and label it point A.
Step 3	Mark a point on the circle and label it point B.
Step 4	Draw $\overline{AB}$.

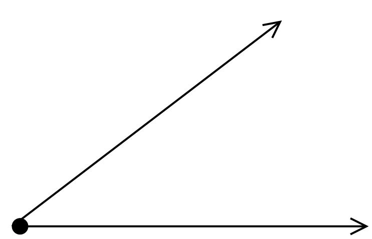
Your Second Challenge: Can you copy any line segment?

Below is a line segment $\overline{AB}$. Using only an unmarked straight edge and compass, can you construct another line segment the same length beginning at point C? Write instructions that explain the steps you used to complete the construction. (*Hint: An ancient geometer would require you to "cut off from the greater of two lines" a line segment equal to a given segment.*)

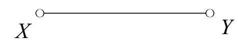
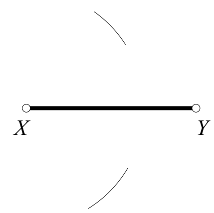
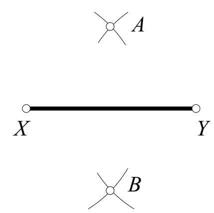
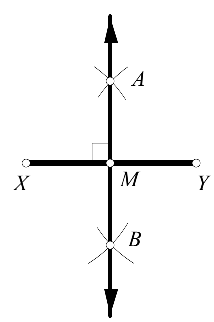


Your Third Challenge: Can you copy an angle?

Now that you know how to copy a segment, copying an angle is easy. How would you construct a copy of an angle at a new point? Discuss this with a partner and come up with a strategy. Think about what congruent triangles are imbedded in your construction and use them to justify why your construction works. Be prepared to share your ideas with the class.



Your Fourth Challenge: Can you bisect a segment?

1. Begin with line segment *XY*.	
2. Place the compass at point *X*. Adjust the compass radius so that it is more than (½)*XY*. Draw two arcs as shown here.	
3. Without changing the compass radius, place the compass on point *Y*. Draw two arcs intersecting the previously drawn arcs. Label the intersection points *A* and *B*.	
4. Using the straightedge, draw line *AB*. Label the intersection point *M*. Point *M* is the midpoint of line segment *XY*, and line *AB* is perpendicular to line segment *XY*.	

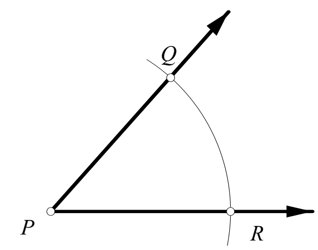
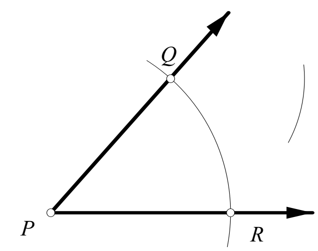
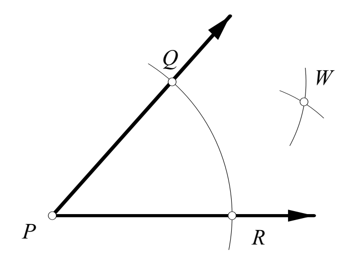
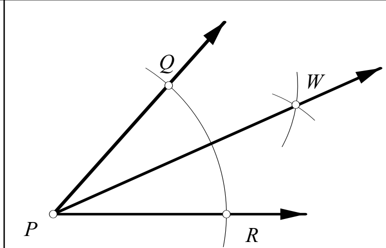
Construct the perpendicular bisector of the segments. Mark congruent segments and right angles. Check your work with a protractor.

1.

2.

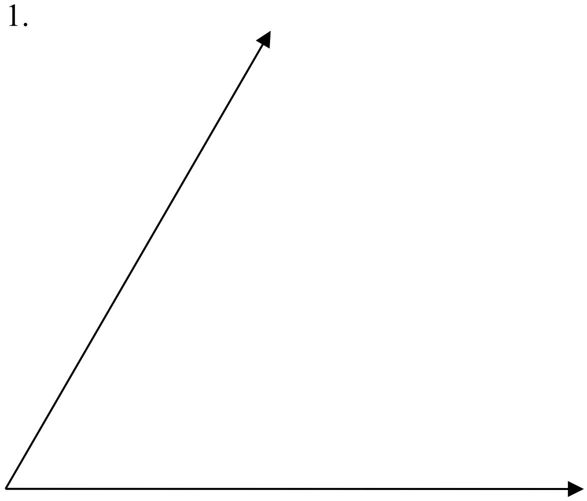
3.

Your Fifth Challenge: Can you bisect an angle?

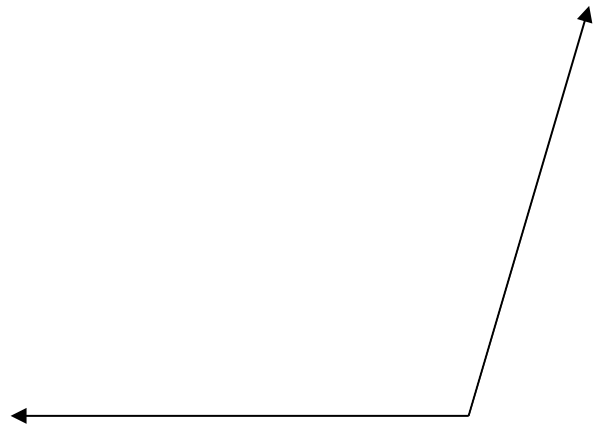
1. Let point *P* be the vertex of the angle. Place the compass on point *P* and draw an arc across both sides of the angle. Label the intersection points *Q* and *R*.	
2. Place the compass on point *Q* and draw an arc across the interior of the angle.	
3. Without changing the radius of the compass, place it on point *R* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *W*.	
4. Using the straightedge, draw ray *PW*. This is the bisector of $\angle QPR$.	

Construct the angle bisector. Mark congruent angles. Check your construction by measuring with a protractor.

1.



2.



Constructing Parallel and Perpendicular Lines

Mathematical Goals

- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Standards for Mathematical Practice

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

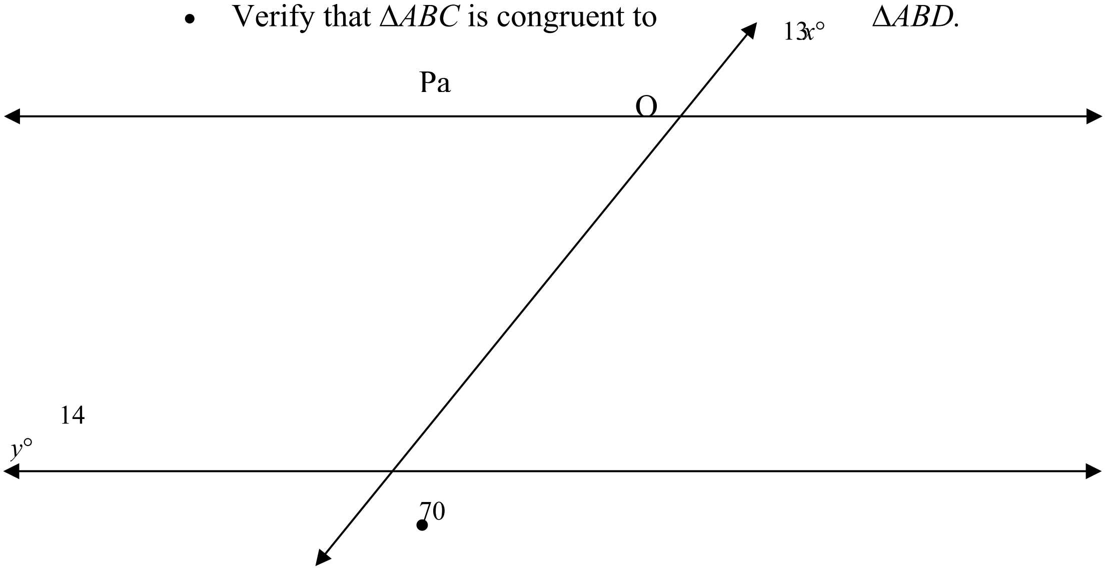
8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Let's start by exploring features of parallel lines.

In the figure below, lines *m* and *n* are parallel and the line *t* intersects both.

- Label a new point *C* anywhere you choose on the line *m*. Connect *B* and *C* to form Δ*ABC*.
- Construct a point *D* on line *n* so that points D and C are on opposite sides of line *t* and *AC* = *BD*.

- Verify that Δ*ABC* is congruent to Δ*ABD*.

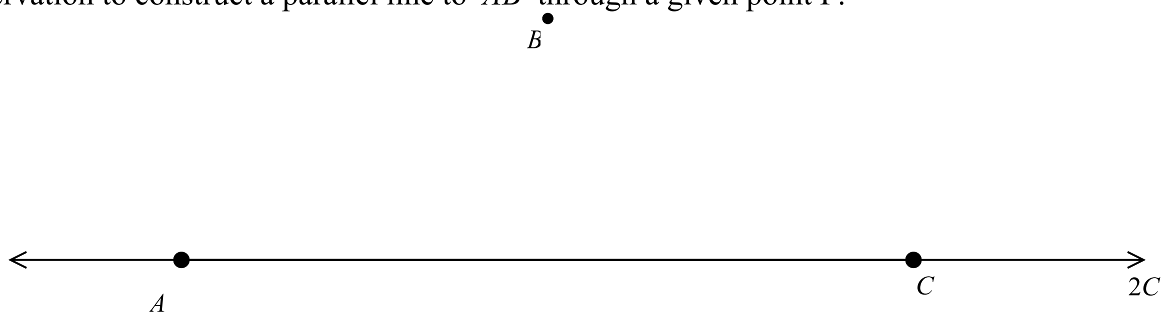


1. Name all corresponding and congruent parts of this construction.

2. What can you conclude about ∠CAB and ∠DBA? Will this always be true, regardless of where you choose *C* to be? Does it matter how line *t* is drawn? (*In other words could line t be perpendicular to both lines? Or slanted the other way*?)

3. What type of quadrilateral is *CADB*? Why do you think this is true?

Drawing a line that intersects two parallel lines creates two sets of four congruent angles. Use this observation to construct a parallel line to $\overline{AB}$ through a given point P.



4. Construct a perpendicular line to $\overline{AB}$ that passes through *P*. Label the intersection with line *m* as Q.

Constructions Inscribed in a Circle

Adapted from http://www.mathopenref.com/ 2009 Copyright Math Open Reference.

Mathematical Goals

- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Standards for Mathematical Practice

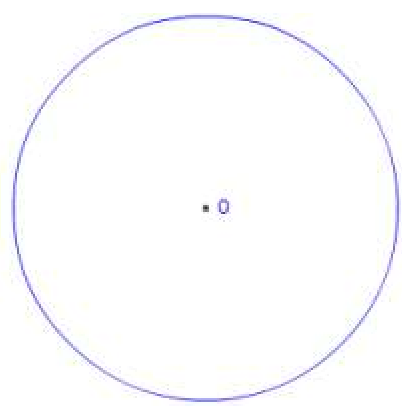
4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

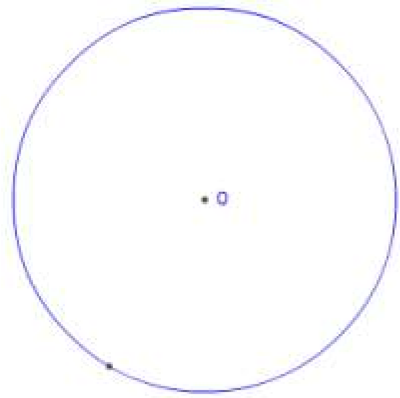
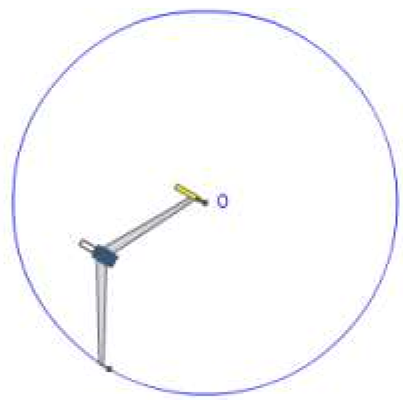
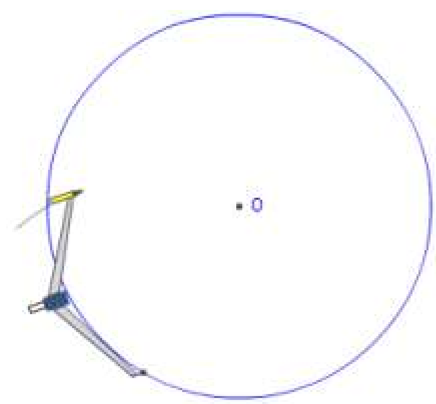
5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

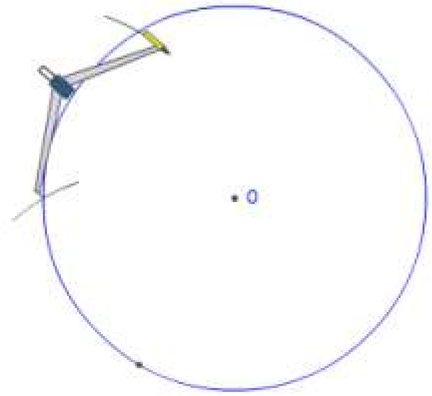
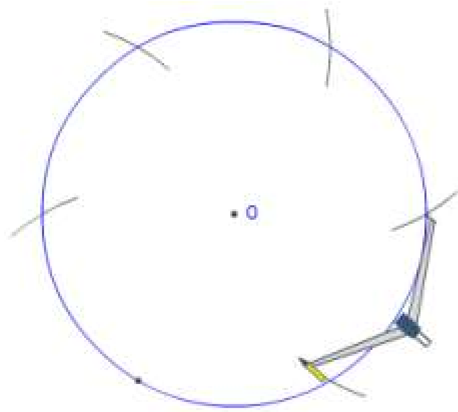
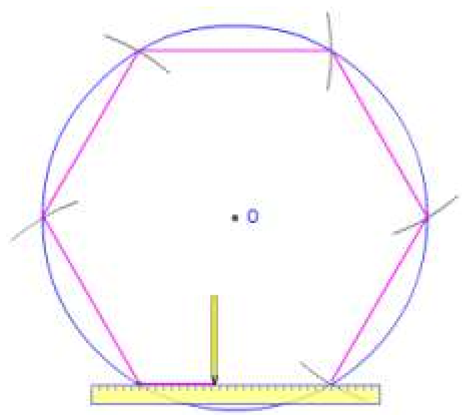
6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

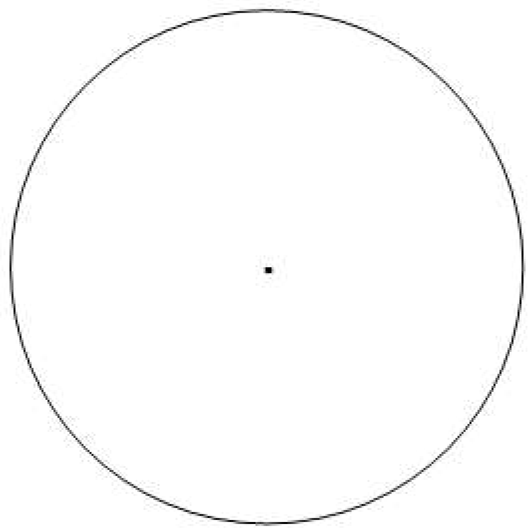
We start with the given circle, center O.	

1. Mark a point anywhere on the circle. Label this point P. This will be the first vertex of the hexagon.	
2. Set the compass on point P and set the width of the compass to the center of the circle O. The compass is now set to the radius of the circle $\overline{OP}$.	
3. Make an arc across the circle. This will be the next vertex of the hexagon. Call this point Q. (It turns out that the side length of a hexagon is equal to its circumradius - the distance from the center to a vertex).	

4. Move the compass on to the next vertex Q and draw another arc. This is the third vertex of the hexagon. Call this point R.	
5. Continue in this way until you have all six vertices. PQRSTU	
6. Draw a line between each successive pairs of vertices, for a total of six lines.	
7. Done. These lines form a regular hexagon inscribed in the given circle. Hexagon PQRSTU	

Try the example below using the steps to construct a hexagon inscribed in a circle using a compass and straightedge. Then brainstorm with a partner on how to construct an equilateral triangle inscribed in a circle.

1. Construct the largest regular hexagon that will fit in the circle below.



2. How would you construct an equilateral triangle inscribed in a given circle?

3. How would you construct a square inscribed in a given circle?

Proving Two Triangles are Congruent

Mathematical Goals

- Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
- Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS, AAS.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Standards for Mathematical Practice

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Triangle Investigations: Station 1

1. Draw a triangle of any size and shape and label it ABC.
2. Using the ruler and protractor, measure and record the angles and sides of the triangle. Remember the total degrees in any triangle should be _______?
3. Draw ΔDOG where AB=DO, BC=OG, and AC=DG.
4. Measure and record the angles of Δ DOG, how are they related to the angles of ABC?
5. Do you have enough information to conclude that ΔABC and ΔDOG are congruent? Why or why not?

Triangle Investigations: Station 2

1. Draw Δ KIT where KI=5 cm.
2. Using a protractor and point K as a vertex draw a 60^o angle with side length 11cm (label point T).
3. Connect point T to point I.
4. Measure the sides and angles of Δ KIT.
5. Now draw a line segment of 11cm. Name it FG.
6. Using point F as your vertex draw a 60^o angle with side length 5 cm. label point H.
7. Connect point G to point H.
8. Measure the sides and angles of Δ FGH.
9. Are triangles KIT and FGH congruent? Why or why not?

Triangle Investigations: Station 3

1. Draw a line segment that is 7cm long. Label it ML.
2. Using point L as a vertex draw a 38^o angle.
3. Draw a line segment beginning at point M that is 5 cm long and hits the side of the angle?
4. Repeat steps 1-2. This time connect the 5 cm segment at a different point on the side of the angle.
5. Are the 2 triangles congruent? Why or why not?

Triangle Investigations: Station 4

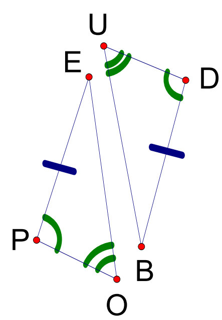
1. Draw a line segment LM that is 7 inches long.
2. Using point L as a vertex draw a 35^o angle.
3. Using point M as a vertex draw a 57^o angle.
4. Label the point of intersection of the two angles as N. This is triangle LMN.
5. Draw line segment ST that is 7 inches long.

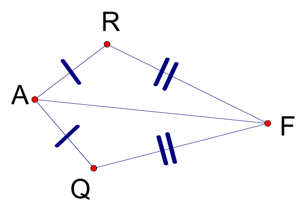
6. Using point S as a vertex draw a 35^o angle.
7. Using point T as a vertex draw a 57^o angle.
8. Label the point of intersection of the two angles as U. This is triangle STU
9. Hold up the two triangles - are they congruent? Why or why not?

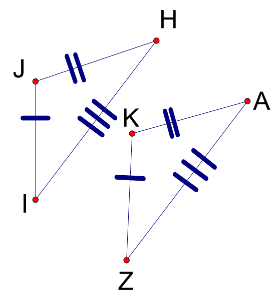
Triangle Investigations: Station 5

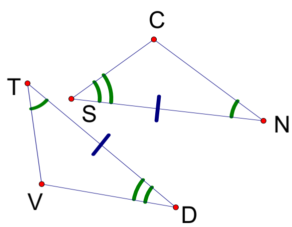
Practice with Triangle Congruence

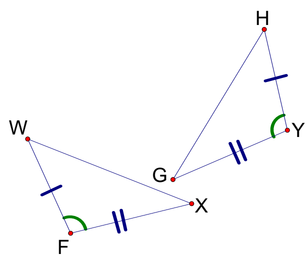
State whether each pair of triangles is congruent by SSS, SAS, ASA, AAS, or HL; if none of these methods work, write N. If congruent, make a congruence statement for the triangles.

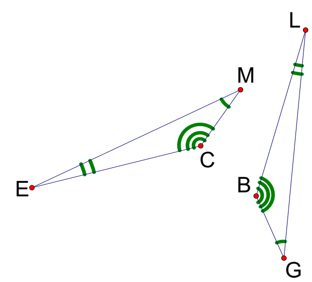
1. 

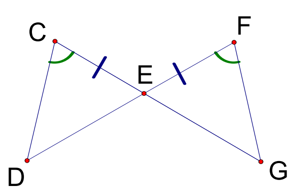
2. 

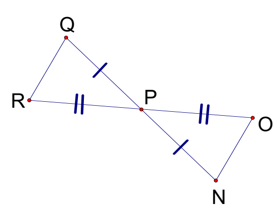
3. 

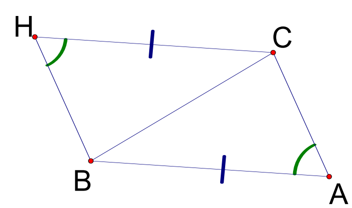
4. 

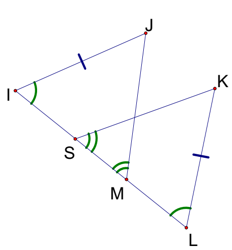
5. 

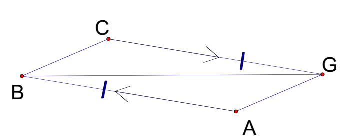
6. 

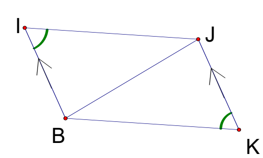
7. 

8. 

9. 

10. 

11. 

12. 

Triangle Proofs

Mathematical Goals

- Prove theorems pertaining to triangles.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.
8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Introduction

This task provides students an opportunity to prove several triangles theorems including the measure of interior angles of a triangle sum to 180 degrees, base angles of isosceles triangles are congruent, and the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length. Students will review CPCTC (Corresponding Parts of Congruent Triangles are Congruent) before beginning the proofs. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

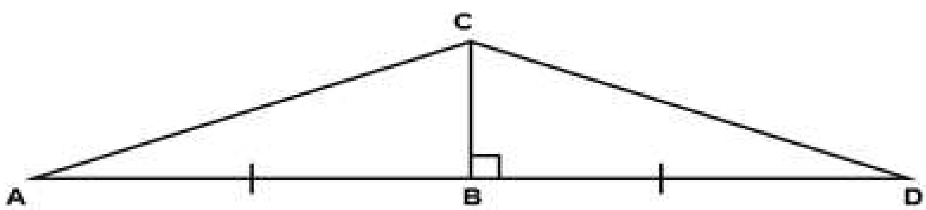
Materials

- Patty paper

Corresponding
Parts of
Congruent
Triangles are
Congruent

Remember the definition of congruent figures?
If two geometric figures are congruent, then their corresponding parts are congruent.

Example: In the figure below, how do we know that $\Delta ABC \cong \Delta DBC$?



Statements	Reason
1. $\overline{AB}$ $\overline{BD}$	1. Given
2. $\angle ABC \cong \angle DBC$	2. By Euclid's Perpendicular Postulate
3. $\overline{CB}$ $\overline{CB}$	3. Reflexive Property of Congruence
4. $\Delta ABC \cong \Delta DBC$	4. SAS Postulate

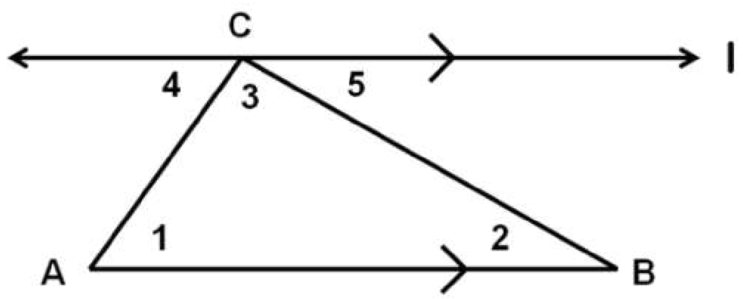
. . . And now that we know that the two triangles are congruent then by CPCTC all the other corresponding parts are congruent as well.

$\angle A \cong \angle D$
$\angle ACB \cong \angle DCB$
$\overline{AC}$ $\overline{DC}$

Let's start proving theorems about triangles using two column proofs. Fill in the missing statements and reasons in the proof below.

Theorem: The sum of the measure of the angles of any triangle is 180°.

Proof:
Given: The top line (that touches the top of the triangle) is running parallel to the base of the triangle.
Prove: $m\angle 1 + m\angle 3 + m\angle 2 = 180°$

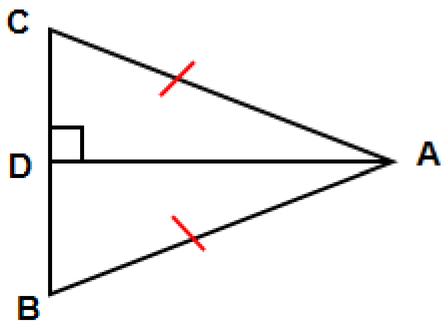


Statements	Reason
1. $m\angle 4 = m\angle 1$	1.
2. $m\angle 5 = m\angle 2$	2.
3.	3. Three angles form one side of the straight line
4. $m\angle 1 + m\angle 3 + m\angle 2 = 180°$	4.

Isosceles Triangles
Theorem: If two sides of a triangle are congruent, than the angles opposite those sides are congruent.

Fill in the following proof using postulates, theorems, and properties that you have learned.

Proof:
Given: $\overline{AC}$ $\overline{AB}$
Prove: $\angle C$ $\angle B$

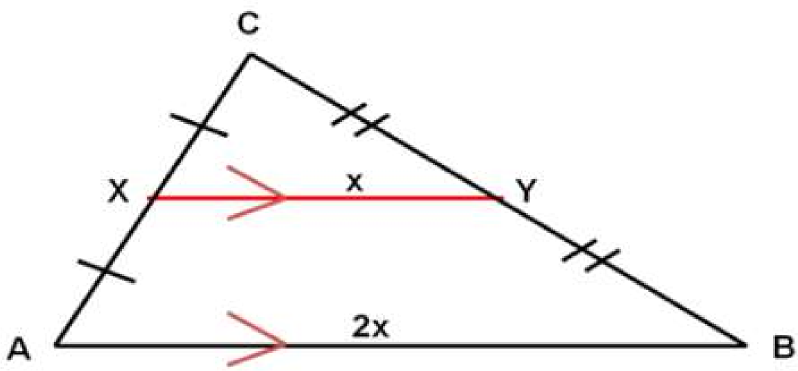


Statements	Reason
1.	1. Given
2. Draw $\overline{AD}$ $\overline{CB}$	2.
3.	3. Reflexive Property of Congruence
4. $\Delta\, CDA \cong \Delta\, BDA$	4.
5.	5.

Definition: A line segment whose endpoints are the midpoint of two sides of a triangle is called a midsegment of the triangle.

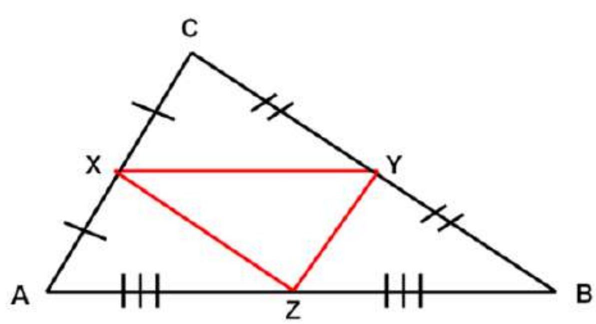
Theorem: The segment connecting the midpoints of two sides of the triangle is parallel to the third side and half the length of the third side.

Using the figure below. $\overline{XY} \parallel \overline{AB}$ and XY = ½ (AB) or AB = 2(XY)



Let's prove this theorem using a sheet of patty paper.

1) Draw Δ ABC on a sheet of patty paper.
2) Fold and pinch to locate the three midpoints of the triangle.
3) Draw and label the three midpoints X, Y, Z.
4) Draw segments $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.



Using your construction, verify:

$\overline{XY} \parallel \overline{AB}$, $\overline{YZ} \parallel \overline{XY}$, $\overline{CA}$, and $\overline{XZ} \parallel \overline{CB}$

Formative Assessment Lesson: Analyzing Congruency Proofs

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1302

ESSENTIAL QUESTION:

- How are triangles proven to be congruent?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Analyzing Congruency Proofs*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=452&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1302

STANDARDS ADDRESSED IN THIS TASK:

Understand similarity in terms of similarity transformations
MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Understand congruence in terms of rigid motions
MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Standards for Mathematical Practice
This lesson uses all of the practices with emphasis on:

3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

Centers of Triangles

Mathematical Goals

- Prove the medians of a triangle meet at a point.
- Bisect a segment and an angle.
- Construct perpendicular lines, including the perpendicular bisector of a line segment.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

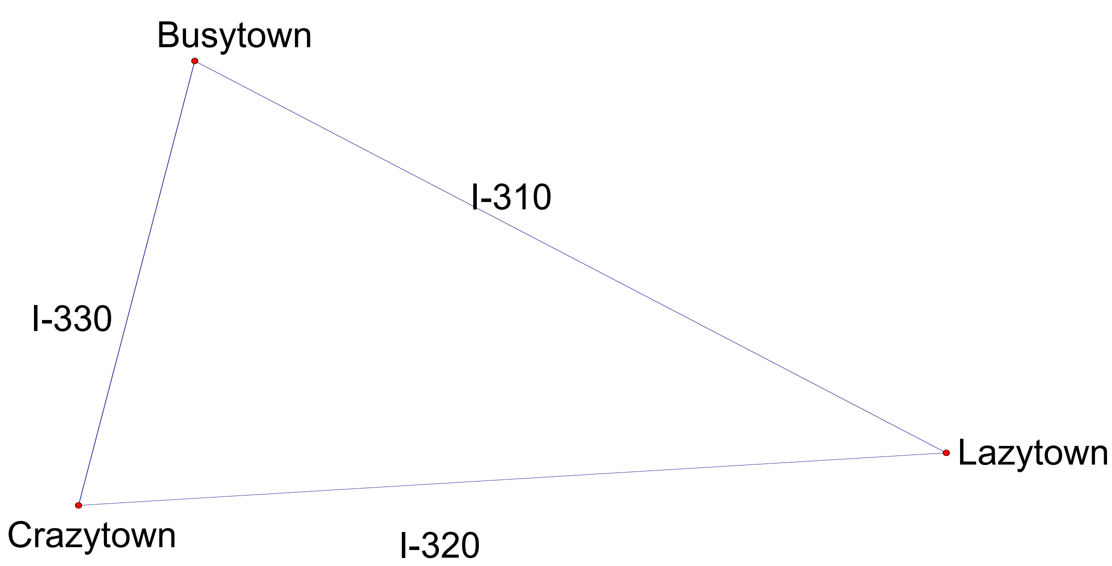
Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.
8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Triangle Center:	Point of Concurrency of:	Significance of:

Incenter	Angle bisectors	Center of inscribed circle Equidistant from the sides of the triangle
Circumcenter	Perpendicular bisectors	Center of the circumscribing circle Equidistant from the vertices of the
Orthocenter	Altitudes	
Centroid	Medians	Center of balance or gravity The distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

2. Now you will use some mathematical concepts to help you choose a location for the tower. Investigate the problem above by constructing the following:

a) all 3 medians of the triangle
b) all 3 altitudes of the triangle
c) all 3 angle bisectors of the triangle
d) all 3 perpendicular bisectors of the triangle

You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

3. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.

The intersection point of the medians is called the ***centroid*** of the triangle.
The intersection point of the angle bisectors is called the ***incenter*** of the triangle.
The intersection point of the perpendicular bisectors is called the ***circumcenter*** of the triangle.
The intersection point of the altitudes is called the ***orthocenter*** of the triangle.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

6. Which triangle center did you recommend for the location of the amusement park?

7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.

Constructing with Diagonals

Mathematical Goals

- Prove theorems pertaining to lines and angles.
- Prove theorems pertaining to triangles.
- Prove theorems pertaining to parallelograms.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.
8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Introduction

This task provides a guided discovery and investigation of the properties of quadrilaterals. Students will determine which quadrilateral(s) can be constructed based on specific information about the diagonals of the quadrilateral(s).
Sample proofs are given for each problem. The samples provided are not the only correct way these proofs can be written. Students should realize that proofs can be logically organized with differing orders of steps. They should also be given the opportunity to decide which type of proof they prefer writing.

Materials

There are many ways students can approach this task and the supplies will depend upon the method you choose for your students.

- Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can be used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.
- Students can use compasses, straightedges and protractors to construct the triangles.
- Geometer's Sketchpad, or similar software, is a good tool to use in these investigations.

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1. Construct two segments of different length that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

2. Repeat #1 with two congruent segments. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

3. Construct two segments that bisect each other but are not perpendicular. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

4. What if the two segments in #3 above are congruent in length? What type of quadrilateral is formed? What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

5. Draw a segment and mark the midpoint. Now construct a segment that is perpendicular to the first segment at the midpoint but is not bisected by the original segment. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

6. In the above constructions you have been discovering the properties of the diagonals of each member of the quadrilateral family. Stop and look at each construction. Summarize any observations you can make about the special quadrilaterals you constructed. If there are any

quadrilaterals that have not been constructed yet, investigate any special properties of their diagonals.

7. Complete the chart below by identifying the quadrilateral(s) for which the given condition is necessary.

Conditions	Quadrilateral(s)	Explain your reasoning
Diagonals are perpendicular.		
Diagonals are perpendicular and only one diagonal is bisected.		
Diagonals are congruent and intersect but are not perpendicular.		
Diagonals bisect each other.		
Diagonals are perpendicular and bisect each other.		
Diagonals are congruent and bisect each other.		
Diagonals are congruent, perpendicular and bisect each other.		

8. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible? Why does this make sense?

9. Name each of the figures below using as many names as possible and state as many properties as you can about each figure.

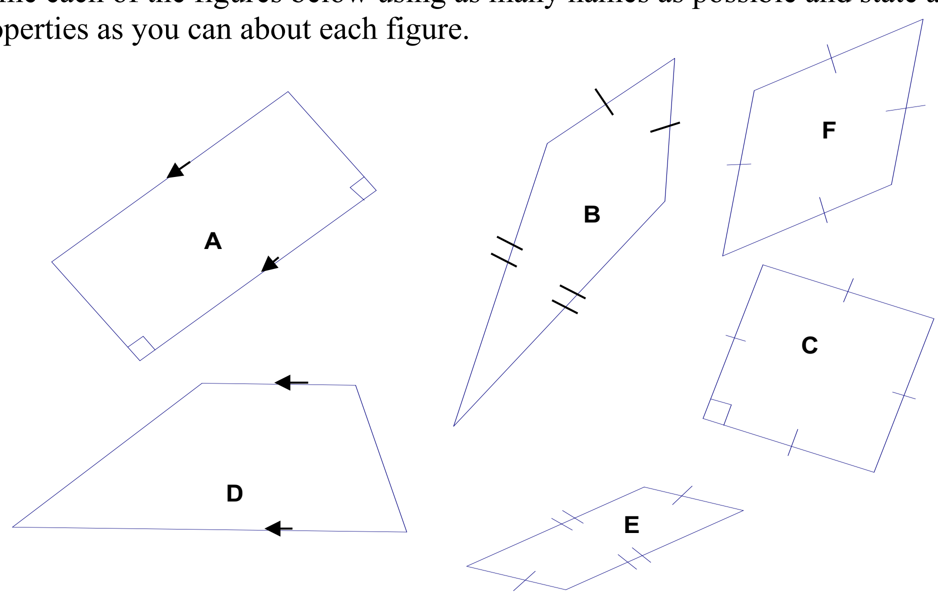

Figure	Names	Properties
A		
B		
C		
D		
E		
F		

10. Identify the properties that are always true for the given quadrilateral by placing an X in the appropriate box.

Property	Parallelogra m	Rectangle	Rhombus	Square	Isosceles Trapezoid	Kite
Opposite sides are parallel.						
Only one pair of opposite sides is parallel.						
Opposite sides are congruent.						
Only one pair of opposite sides is congruent.						
Opposite angles are congruent.						
Only one pair of opposite angles is congruent.						
Each diagonal forms 2 $\cong$ triangles.						
Diagonals bisect each other.						
Diagonals are perpendicular.						
Diagonals are congruent.						
Diagonals bisect vertex angles.						
All $\angle$s are right $\angle$s.						
All sides are congruent.						
Two pairs of consecutive sides are congruent.						

11. Using the properties in the table above, list the **minimum** conditions necessary to prove that a quadrilateral is:

a. a parallelogram

b. a rectangle

c. a rhombus

d. a square

e. a kite

f. an isosceles trapezoid

Proving Quadrilaterals in the Coordinate Plane

Mathematical Goals

- Prove theorems pertaining to lines and angles.
- Prove theorems pertaining to triangles.
- Prove theorems pertaining to parallelograms.

STANDARDS ADDRESSED IN THIS TASK

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
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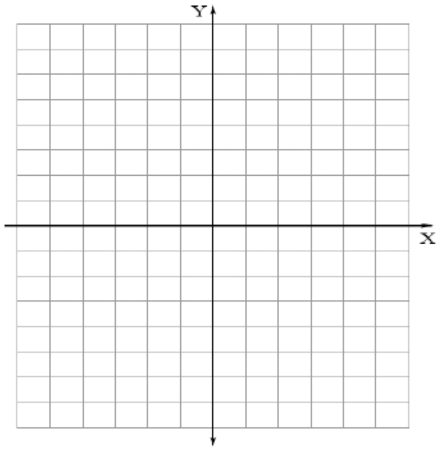
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.
8. **Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

Plot points A = (-3, -1), B = (-1, 2), C = (4, 2), and D = (2, -1).



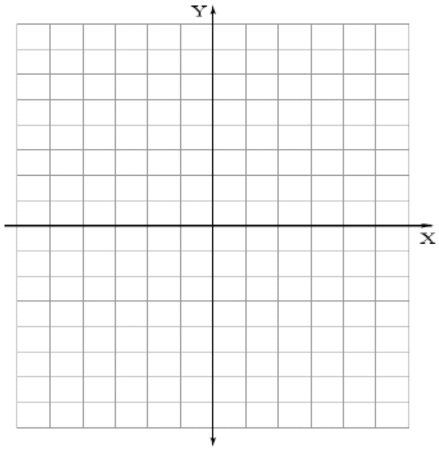
1. What specialized geometric figure is quadrilateral ABCD? Support your answer mathematically.

2. Draw the diagonals of ABCD. Find the coordinates of the midpoint of each diagonal. What do you notice?

3. Find the slopes of the diagonals of ABCD. What do you notice?

4. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points E = (1, 2), F = (2, 5), G = (4, 3) and H = (5, 6).



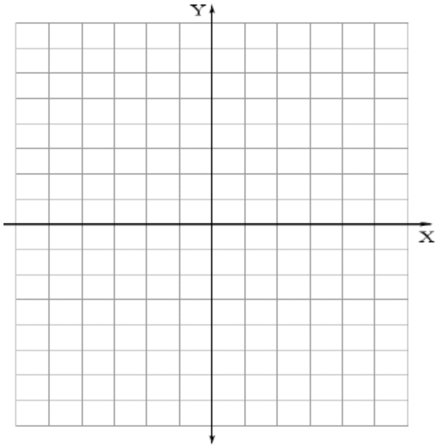
5. What specialized geometric figure is quadrilateral EFHG? Support your answer mathematically using two different methods.

6. Draw the diagonals of EFHG. Find the coordinates of the midpoint of each diagonal. What do you notice?

7. Find the slopes of the diagonals of EFHG. What do you notice?

8. The diagonals of EFHG create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points P = (4, 1), W = (-2, 3), M = (2,-5), and K = (-6, -4).



9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

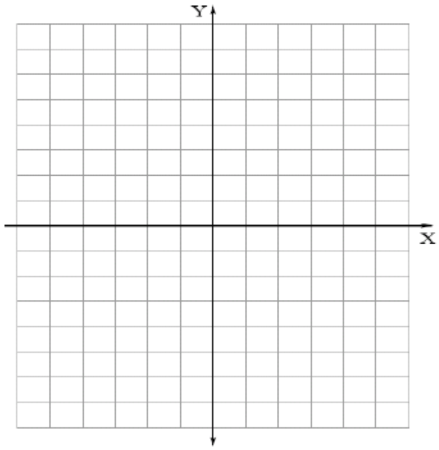
10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

11. Find the lengths of the diagonals of PWKM. What do you notice?

12. Find the slopes of the diagonals of PWKM. What do you notice?

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points A = (1, 0), B = (-1, 2), and C = (2, 5).



14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.

Formative Assessment Lesson: Evaluating Statements about Length & Area

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=675

ESSENTIAL QUESTION:

- Why are two areas equal or not equal?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Evaluating Statements about Length & Area*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=212&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=675

STANDARDS ADDRESSED IN THIS TASK:

Prove geometric theorems

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice
This lesson uses all of the practices with emphasis on:

2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Formative Assessment Lesson: Floor Pattern

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=768

ESSENTIAL QUESTION:

- How do you prove that a quadrilateral is a parallelogram?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Floor Pattern*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=258&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=768

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=769

STANDARDS ADDRESSED IN THIS TASK:

Prove geometric theorems

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice
This task uses all of the practices with emphasis on:

3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

Formative Assessment Lesson: Solving Geometry Problems: Floodlights

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1257

ESSENTIAL QUESTION:

- How do you make a mathematical model for similar triangles to solve problems?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Solving Geometry Problems: Floodlights*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=429&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1257

STANDARDS ADDRESSED IN THIS TASK:

Prove theorems involving similarity
MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Prove geometric theorems
MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the

segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice
This lesson uses all of the practices with emphasis on:

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
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