CCGPS Frameworks Student Edition

Mathematics

CCGPS Coordinate Algebra

Unit 5: Transformations in the Coordinate Plane

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

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Unit 5
Transformations in the Coordinate Plane

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OVERVIEW

In this unit students will:

- use and understand definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined terms of point, line, distance along a line and length of an arc.
- describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, including translations and horizontal or vertical stretching
- represent and compare rigid and size transformations of figures in a coordinate plane using various tools such as transparencies, geometry software, interactive whiteboards, waxed paper, tracing paper, mirrors and digital visual presenters.
- compare transformations that preserve size and shape versus those that do not.
- describe rotations and reflections of parallelograms, trapezoids or regular polygons that map each figure onto itself.
- develop and understand the meanings of rotation, reflection and translation based on angles, circles, perpendicular lines, parallel lines and line segments.
- transform a figure given a rotation, reflection or translation using graph paper, tracing paper, geometric software or other tools.
- create sequences of transformations that map a figure onto itself or to another figure.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

Webinar Information

A two-hour course overview webinar may be accessed at
http://www.gpb.org/education/common-core/2012/02/28/mathematics-9th-grade

The unit-by-unit webinars may be accessed at
https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Experiment with transformations in the plane

MCC9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.
- Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes in general).
- Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- plotting points on a coordinate plane
- congruence of geometric figures and the correspondence of their vertices, sides, and angles
- recognizing line and rotational symmetry
- interpreting and sketching views from different perspectives
- calculate the perimeter and area of fundamental geometric plane figures
- use the concepts of ratio, proportion, and scale factor to demonstrate the relationships between similar plane figures

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school students. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Angle:** A figure created by two distinct rays that share a common endpoint (also known as a vertex). $\angle ABC$ or $\angle B$ or $\angle CBA$ indicate the same angle with vertex B.
- **Angle of Rotation:** The amount of rotation (in degrees) of a figure about a fixed point such as the origin.
- **Bisector:** A point, line or line segment that divides a segment or angle into two equal parts.
- **Circle:** The set of all points equidistant from a point in a plane.
- **Congruent:** Having the same size, shape and measure. $\angle A \cong \angle B$ indicates that angle A is congruent to angle B.
- **Corresponding angles:** Angles that have the same relative position in geometric figures.
- **Corresponding sides:** Sides that have the same relative position in geometric figures.
- **Endpoint:** The point at each end of a line segment or at the beginning of a ray.

- **Image:** The result of a transformation.
- **Intersection:** The point at which two or more lines intersect or cross.
- **Isometry:** a distance preserving map of a geometric figure to another location using a reflection, rotation or translation. $M \rightarrow M'$ indicates an isometry of the figure M to a new location M'. M and M' remain congruent.
- **Line:** One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely. $\overleftrightarrow{AB}$ indicates a line that passes through points A and B.
- **Line segment:** A part of a line between two points on the line. $\overline{AB}$ indicates the line segment between points A and B.
- **Parallel lines:** Two lines are parallel if they lie in the same plane and do not intersect. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ indicates that line AB is parallel to line CD.
- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ indicates that line AB is perpendicular to line CD.
- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.
- **Pre-image:** A figure before a transformation has taken place.
- **Ray:** A part of a line that begins at a point and continues forever in one direction. $\overrightarrow{AB}$ indicates a ray that begins at point A and continues in the direction of point B indefinitely.
- **Reflection:** A transformation of a figure that creates a mirror image, "flips," over a line.
- **Reflection Line (or line of reflection):** A line that acts as a mirror so that corresponding points are the same distance from the mirror.
- **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.
- **Segment:** See line segment.
- **Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.

- **Translation:** A transformation that slides each point of a figure the same distance in the same direction.

- **Vertex:** The location at which two lines, line segments or rays intersect.

Learning Task: Introduction to Reflections, Translations, and Rotations

Name________________________________ **Date**_________________

Mathematical Goals

- Explore transformations in the coordinate plane using appropriate tools.

Essential Questions

- What effects do transformations have on geometric figures in a coordinate plane?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measures to those that do not (e.g., translation versus horizontal stretch).

Standards for Mathematical Practice

4. Model with mathematics.
5. Use appropriate tools strategically.

Learning Task: Introduction to Reflections, Translations, and Rotations

Name________________________________ **Date**_________________

Part I

1. On your graph paper draw and label a square. Describe its original position and size.

2. Rotate it 90º clockwise around any point.

3. Translate it so that it is in the 4^{th} quadrant.

4. Reflect it over a line *y* = "a number" so that the square is in the 1st quadrant.

5. Write **two** different ways that you can get the shape back in its original position.

Part II

6. On your graph paper draw and label a triangle. Describe its original position and size.

7. Rotate, translate, and/or reflect the triangle so that the two triangles create a parallelogram. List your steps here:

Part III

8. On your graph paper draw and label a parallelogram. Describe its original position and size.

9. Rotate, translate, and/or reflect the parallelogram several times, listing your steps here:

10. Now, challenge a friend to get the parallelogram back into its original position! (Don't reveal your steps.)

11. Record your friend's steps here:

12. Are the steps that your friend used the reverse of your steps, or are they different?

Learning Task: Exploring Reflections and Rotations

Name________________________________ **Date**_________________

Mathematical Goals

- Develop and demonstrate an understanding of reflections and rotations of figures in general and on a coordinate plane.

Essential Questions

- How are reflections and rotations similar and different?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

5. Use appropriate tools strategically.
7. Look for and make use of structure.

Learning Task: Exploring Reflections and Rotations

Name________________________________ **Date**__________________

Reflections:

1. Select the vertices of the **square** and go to Display → Label Points. Start with C so that your **square**'s vertices are labeled C, D, E, & F. Find the coordinates of each point:

   C (_____,_____); D (_____,_____); E (_____,_____); F (_____,_____)

2. Double click the line $x = 2$. You should see an animation on the line. This makes $x = 2$ the line of reflection (also called the line of symmetry).

3. Select the interior & the vertices of your **square**. Go to Transform → Reflect.

4. Select the vertices of the new **square** and go to Display → Show Labels.

5. How have the new points changed? ____

   ____

6. Double click on the *y*-axis to change it to the line of reflection. Select the interior & vertices of your **square**. Go to Transform → Reflect.

7. What has happened? ____

   Why is this reflection further away than the last one? ___________________________

   ____

   What effect did changing the reflection line have? _____________________________

   ____

8. Write out the coordinates of each square.

Original Square	Reflection over $x = 2$	Reflection over y-axis
C (___,___)	C' (___,___)	C' (___,___)
D (___,___)	D' (___,___)	D' (___,___)
E (___,___)	E' (___,___)	E' (___,___)
F (___,___)	F' (___,___)	F' (___,___)

How far apart are the original square and the first reflection? ______________________

The original square and the second reflection? __________________________________

How far is the original square from $x = 2$ and how far is the first reflection from $x = 2$?

____

How far is the original square from the y-axis and the second reflection from y-axis?

____

9. Delete the two reflections so that only your original square remains.

10. Double click the line $y = 0.5x - 5$ to make it the new line of reflection. If you were to reflect the **square** over $y = 0.5x - 5$, **predict** where would the new vertices be?

Original Square	**Prediction** of Reflection over $y = 0.5x - 5$
C (___,___)	C' (___,___)
D (___,___)	D' (___,___)
E (___,___)	E' (___,___)
F (___,___)	F' (___,___)

11. ***After*** you have made your prediction, select the vertices and interior of the **square**. Go to Transform → Reflect. Select the vertices of the new **square** and go to Display → Show Labels. How does your prediction compare with the actual reflection?

____

____

12. Make a general conclusion about what happens to coordinates of a point when they are reflected over a line.

____

____

Rotations:

13. Delete the reflection so only the original square remains.

14. Using the Selection Arrow Tool, double click the point B. A brief animation should show. Point B is now your center of rotation.

15. Select the interior and the vertices of your **square**. Go to Transform → Rotate. A box labeled "Rotate" should come up. (Move the box over to the side so you can still see your sketch.) You should see a lighter colored square that has appeared on your sketch. Change the number of degrees **BUT DON'T CLICK ANY BUTTONS**. The image should change where it goes. Try different angles (between 0º and 360º). Explain what is happening to the square and the points.

____

____

16. Try negative angles (once again **without clicking any buttons**). What do you notice about 90º and –270º? 180º and –180º? 45º and –315º?

____

Why do you think this is? ____

____

Are there any other pairs of measures that have the same phenomenon? How could we predict additional pairs?

____

17. Type in 270º (clockwise) for the angle and click the button "rotate". Write down the new coordinates and compare them to the old coordinates.

Original Square	Rotation around B
C (___,___)	C' (___,___)
D (___,___)	D' (___,___)
E (___,___)	E' (___,___)
F (___,___)	F' (___,___)

What relationship is there between the vertices of the two figures?

____

18. What conclusions can you make about what happens to coordinates when rotated?

____

____

EXTENSION

19. Is there ever a time when a rotation is the same as a reflection? Explain.

____

____

20. Create an animation in a new Geometer's Sketchpad window to support your answer to number 19. Email your animation to your teacher.

Learning Task: Exploring Reflections and Rotations (Alternative)

Name________________________________ **Date**_________________

Reflections:

1. On a piece of graph paper, graph the following points to create square CDEF
   C (3, 0); D (4, 1); E (5, 0); F (4, -1)

2. Draw the line: $x = 2$.

3. Using either Mira, patty paper or a transparency reflect the square over the $x = 2$ line.

4. How have the new points changed? ____

   ____

5. Using the original square, now reflect it over the y-axis.

6. What has happened? ____

   Why is this reflection further away than the last one? ___________________________

   ____

   What effect did changing the reflection line have? ______________________________

   ____

7. Write out the coordinates of each square.

Original Square	Reflection over $x = 2$	Reflection over y-axis
C (___,___)	C' (___,___)	C' (___,___)
D (___,___)	D' (___,___)	D' (___,___)
E (___,___)	E' (___,___)	E' (___,___)
F (___,___)	F' (___,___)	F' (___,___)

How far apart are the original square and the first reflection? ______________________

The original square and the second reflection?__________________________________

How far is the original square from $x = 2$ and how far is the first reflection from $x = 2$?

____

How far is the original square from the y-axis and the second reflection and the y-axis?

____

8. Draw the line $y = 0.5x - 5$. If you were to reflect the **original square** over $y = 0.5x - 5$, **predict** where the new vertices would be.

Original Square	**Prediction** of Reflection over $y = 0.5x - 5$
C (___,___)	C' (___,___)
D (___,___)	D' (___,___)
E (___,___)	E' (___,___)
F (___,___)	F' (___,___)

9. ***After*** you have made your prediction, use the Mira, patty paper, or transparency to reflect the original square over the $y = 0.5x - 5$ line. How does your prediction compare with the actual reflection?

____

____

10. Make a general conclusion about what happens to coordinates of a point when it is reflected over a line.

____

____

Rotations:

11. Return to the original square.

12. Plot the point B(2,0). Point B is now your center of rotation.

13. Experiment with rotating the square about point B using either patty paper or a transparency. Try different numbers of degrees (less than 360) counter clockwise. Explain what is happening to the square and the points.

____

____

14. Try rotating negative degrees (clockwise). What do you notice about 90º and –270º? 180º and –180º? 45º and –315º?

____

Why do you think this is?____

____

Are there any other pairs of measures that have the same phenomenon? How could we predict additional pairs?

____

15. Rotate the square about point B, 270º clockwise. Write down the new coordinates and compare them to the old coordinates.

Original Square	Rotation around B
C (___,___)	C' (___,___)
D (___,___)	D' (___,___)
E (___,___)	E' (___,___)
F (___,___)	F' (___,___)

What relationship is there between the points of the two figures?

____

16. What conclusions can you make about what happens to coordinates when rotated?

____

____

EXTENSION

17. Is there ever a time when a rotation is the same as a reflection? Explain.

____

____

18. Create a sequence of transformations to support your answer to number 17.

Performance Task: Mirrored Mappings

Name_________________________________ **Date**__________________

Mathematical Goals

- Develop and demonstrate an understanding of reflections of figures on a coordinate plane.

Essential Questions

- How do we reflect points in a coordinate plane?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

5. Use appropriate tools strategically.
7. Look for and make use of structure.

Performance Task: Mirrored Mappings

Name________________________________ **Date**_________________

Antonio and his friend Brittany were at summer math camp where the counselors had drawn a large coordinate plane on the gym floor. Antonio challenged Brittany to mirror him as he walked in the first quadrant. Map both of their travels on the same coordinate plane.

Antonio began at (2, 1) and walked to (3, 5); Brittany decided to begin at (–2, 1), then tried to mirror Antonio by walking to (–3, 5). Antonio jumped to (5, 5) and side-stepped to (4, 3); Brittany jumped to (–5, 5) then side-stepped to (–4, 3). Antonio returned to (2, 1) and Brittany returned to (–2, 1).

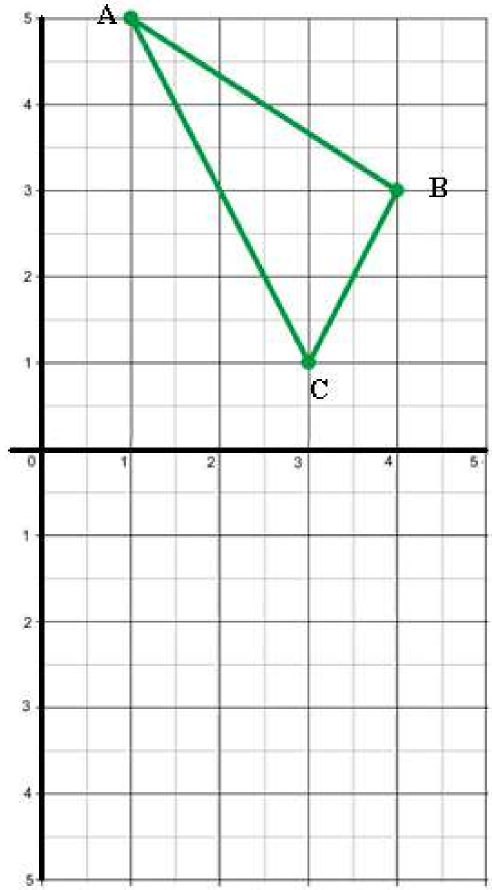
1. Did Brittany mirror Antonio?

a. If you answered no, identify the incorrect coordinates Brittany used and find the correct coordinates. Explain your decision and identify the line of symmetry she should have used as a mirror. How did you know that this should have been the line of symmetry?

b. If you answered yes, identify the line of symmetry Brittany used as a mirror. How did you know it was the line of symmetry?

2. If Brittany had instead begun at (–2,1), walked to (–4,3), side-stepped to (–5,5), jumped to (–3,5) and then returned to (–2,1), could she claim that she created a mirror image of Antonio's path? Justify your answer.

Antonio and Brittany decided to change the game and use some lettered blocks to mark points they visited on the grid. Antonio placed blocks A, B and C as indicated by the points below, then drew a chalk line between them.



3. Label the coordinates Antonio used, and then construct the graph of where Brittany would place her blocks if she correctly reflected Antonio's figure across the *x*-axis.

4. Describe how you determined where to place Brittany's blocks.

5. Each block Brittany placed corresponds to one that Antonio placed. List each pair of coordinates that correspond.

6. What can you observe about the distances between each of Antonio's blocks and the corresponding block Brittany placed?

7. If Antonio walked 2 feet from his block A toward his block C, and Brittany mirrored his movement by walking 2 feet from the blocks corresponding to A and C, would Brittany and Antonio be the same distance from the reflection line? How can you be certain?

8. How would you define a reflection now that you have analyzed some of the properties of reflected images using the coordinate plane?

Homework Task: Coordinating Translations

Name________________________________ **Date**_________________

Mathematical Goals

- Translate a geometric figure based on given directions and then explain the general rule.

Essential Questions

- How do we translate a geometric figure in the coordinate plane?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
8. Look for and express regularity in repeated reasoning.

Homework Task: Coordinating Translations

Name________________________________ **Date**__________________

Create any polygon you want on the coordinate plane, and then create polygons congruent to the one you designed using the three directions described below. Use the same coordinate plane for all transformations.

1. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x + 4, y)$. Record your answers in the table below.

2. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x, y – 3)$. Record your answers in the table below.

3. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x – 4, y + 1)$. Record your answers in the table below.

Original polygon's vertices (x, y)	**#1** $(x + 4, y)$	**#2** $(x, y – 3)$	**#3** $(x – 4, y + 1)$

4. What kind of transformations are these?

5. Can you create a translation $(x + 2, y + 2)$? Is it necessary that the same number is added or subtracted to the x and y coordinates of the polygon? Why or why not?

EXTENSION:

Provide a description of each of the following translations, where c represents a positive number.

6. $(x + c, y)$

7. $(x, y - c)$

8. $(x - c, y)$

9. $(x , y + c)$

Learning Task: Transformations in the Coordinate Plane

Name________________________________ **Date**_________________

Mathematical Goals

- Find rules to describe transformations in the coordinate plane.

Essential Questions

- How do we describe a transformation or series of transformations that take place in the coordinate plane?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Make sense of problems and persevere in solving them.

Learning Task: Transformations in the Coordinate Plane

Name________________________________ **Date**_________________

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order.
  Do **not** connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (don't connect the dots).

FIGURE 1	**FIGURE 2**	**FIGURE 3**	**FIGURE 4**	**FIGURE 5**	**FIGURE 6**
Set 1					
For Set 1, connect the points in order, and connect the last point to the first point in the set.					
(6, 4)	(12, 8)	(18, 4)	(18, 12)	(6, 12)	(8, 6)
(6, -4)	(12, -8)	(18, -4)	(18, -12)	(6, -12)	(8, -2)
(-6, -4)	(-12, -8)	(-18, -4)	(-18, -12)	(-6, -12)	(-4, -2)
(-6, 4)	(-12, 8)	(-18, 4)	(-18, 12)	(-6, 12)	(-4, 6)
Set 2					
For Set 2, connect the points in order, and connect the last point to the first point in the set.					
(1, 1)	(2, 2)	(3, 1)	(3, 3)	(1, 3)	(3, 3)
(1, -1)	(2, -2)	(3, -1)	(3, -3)	(1, -3)	(3, 1)
(-1, -1)	(-2, -2)	(-3, -1)	(-3, -3)	(-1, -3)	(1, 1)
(-1, 1)	(-2, 2)	(-3, 1)	(-3, 3)	(-1, 3)	(1, 3)
Set 3					
For Set 3, connect the points in order, but DO NOT connect the last point to the first point.					
(4, -2)	(8, -4)	(12, -2)	(12, -6)	(4, -6)	(6, 0)
(3, -3)	(6, -6)	(9, -3)	(9, -9)	(3, -9)	(5, -1)
(-3, -3)	(-6, -6)	(-9, -3)	(-9, -9)	(-3, -9)	(-1, -1)
(-4, -2)	(-8, -4)	(-12, -2)	(-12, -6)	(-4, -6)	(-2, 0)
Set 4					
For Set 4, don't connect the dots at all.					
(4, 2)	(8, 4)	(12, 2)	(12, 6)	(4, 6)	(6, 4)
(-4, 2)	(-8, 4)	(-12, 2)	(-12, 6)	(-4, 6)	(-2, 4)

After drawing the six figures, **compare Figure 1 to each of the other figures** and answer the following questions.

1. Describe any similarities and/or differences between Figure 1 and each of the other figures.

2. Consider how the coordinates of each figure compare to the coordinates of Figure 1. Write general rules for making Figures 2-6.

3. Translate, reflect, and rotate (between 0° and 180°) Figure 1 so that it lies entirely in Quadrant III on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

4. Translate, reflect, and rotate (between 0º and 180°) Figure 1 so that it lies entirely in Quadrant IV on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

5. Describe the transformations necessary to take the figure you created in question #4 on to Figure 6.

Coordinating Translations: Graph Paper

Name_________________________________ **Date**__________________

Figure 1

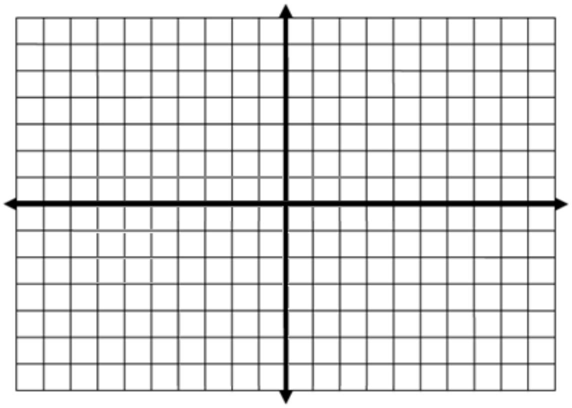


Figure 2

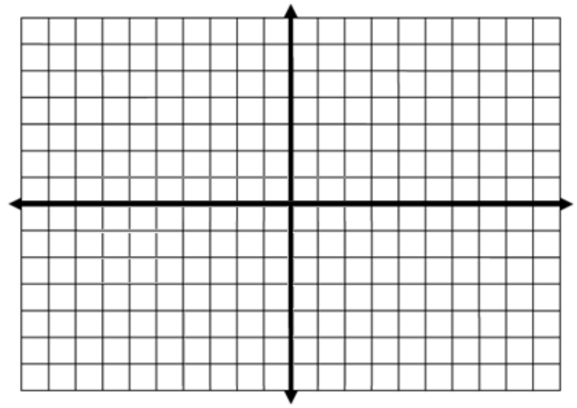


Figure 3

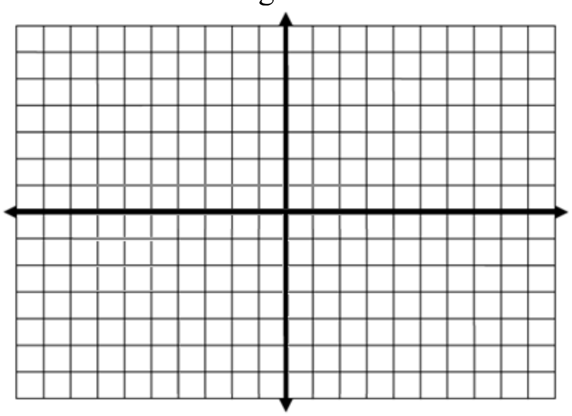


Figure 4

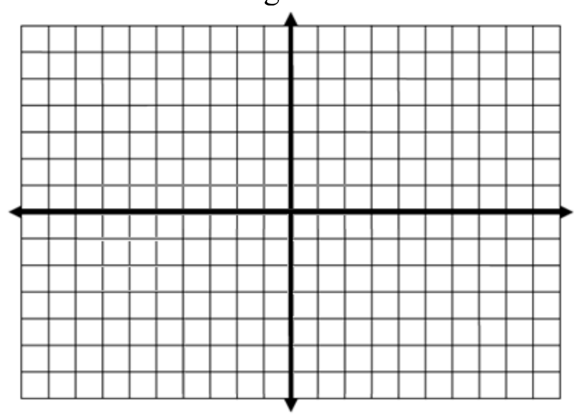


Figure 5

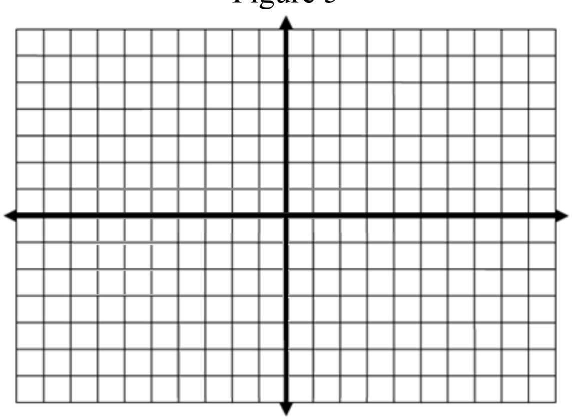
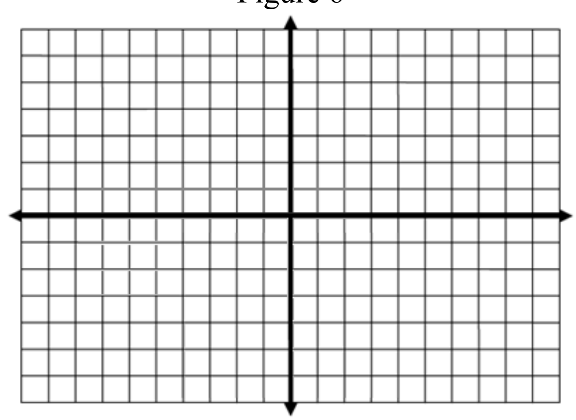


Figure 6



Representing and Combining Transformations (Formative Assessment Lesson (FAL))

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/mapbeta/download.php?fileid=1368

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Representing and Combining Transformations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=223&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/mapbeta/download.php?fileid=1368

Mathematical Goals

- Recognize and visualize transformations of 2-D shapes.
- Translate, reflect, and rotate shapes, and combine these transformations.

Essential Questions

- What effects do transformations have on geometric figures?
- What is the relationship between reflections, translations and rotations?
- How do you determine the type of transformation that has occurred?

Common Core Georgia Performance Standards

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
Students must understand and apply the various types of transformations.

3. Construct viable arguments and critique the reasoning of others.
Students must justify their choices in the card sort, matching transformations' visual representations to their verbal descriptions.

5. Use appropriate tools strategically.
Students use transparencies, pins, etc., to help them physically model transformations.

Background Knowledge

- Students should know how to apply translations, reflections, and rotations in the coordinate plane.
- Students should know how to describe transformations that map one figure onto another.

Common Misconceptions

- Students may confuse the various types of transformations.
- Students may struggle to use the correct center of rotation, especially if they have simply memorized a rule (e.g., $(x, y) \rightarrow (y, -x)$ for a 90º clockwise rotation works only if the center of rotation is the origin.)

Materials

- See FAL page.

Grouping

- Individual / partner / small group