CCGPS Frameworks Student Edition

Mathematics

CCGPS Analytic Geometry
Unit 3: Circles and Volume

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"Making Education Work for All Georgians"

Unit 3:
Circles and Volume

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OVERVIEW

In this unit students will:

- Understand and Apply theorems about circles
- Find Arc Length and Area of Sectors of circles
- Explain Volume Formulas and Use them to solve problems

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight process standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Understand and apply theorems about circles

MCC9-12.G.C.1 Prove that all circles are similar.

MCC9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

MCC9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

MCC9-12.G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

MCC9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

<u>Explain volume formulas and use them to solve problems</u>

MCC9-12.G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

MCC9-12.G.GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

MCC9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

RELATED STANDARDS

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making

mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- Understand and Apply Theorems about Circles
- Find Arc Lengths and Areas of Sectors of Circles
- Explain Volume Formulas and Use them to solve problems

CONCEPTS/SKILLS TO MAINTAIN

The introduction to all of the parts of a circle and the relationships of all of those parts to each other will be new to students this year. The concepts of Area, Surface Area, and Volume of triangles, special quadrilaterals, and right rectangular prisms were introduced in the 6^{th} Grade Unit 5. This knowledge was built on in the 7^{th} Grade Unit 5 and expanded to include the slicing of right rectangular pyramids. The Volumes of Cones, Cylinders, and Spheres were previously covered in the 8^{th} Grade Unit 3. The purpose of re-visiting these formulas here in Analytic Geometry is to formalize the students understanding of the development of these formulas; to take them from a memorization and use of the formulas to an understanding and application level.

SELECTED TERMS AND SYMBOLS

- **Central Angle**-an angle whose vertex is at the center of a circle
- **Inscribed Angle**-an angle whose vertex is on the circle and whose sides contain chords of a circle
- **Arc**-an unbroken part of a circle; minor arcs have a measure less than 180^0; semi-circles are arcs that measure exactly 180^0; major arcs have a measure greater than 180^0
- **Chord**-a segment whose endpoints are on a circle
- **Tangent Line**-a line in the plane of a circle that intersects a circle at only one point, the point of tangency
- **Secant Line**-a line in the plane of a circle that intersects a circle at exactly two points
- **Secant Segment**-a segment that contains a chord of a circle and has exactly one endpoint outside of the circle
- **Inscribed Polygon**-a polygon whose vertices all lie on a circle
- **Inscribed Circle**-a circle enclosed in a polygon, where every side of the polygon is a tangent to the circle; specifically for this unit the polygon will be a triangle and so the center of the Inscribed Circle is the incenter of the triangle
- **Circumscribed Circle**-a circle containing an inscribed polygon; for this unit the polygon will be a triangle and so the center of the circle will be the circumcenter of the triangle.
- **Arc Length**-a portion of the circumference of the circle
- **Sector**-the region bounded by two radii of the circle and their intercepted arc

Circles and their Relationships among Central Angles, Arcs, and Chords

In this unit you will study properties of circles. As you progress through the unit, you will learn new definitions and theorems. In some cases you will discover, or develop and prove these theorems. It may be advantageous for you to keep a "**Circle Book**" or list that includes the definitions and theorems addressed in each task. With each definition and theorem you enter, you should also include an illustrative sketch.

We will begin by re-visiting the definition of a circle.

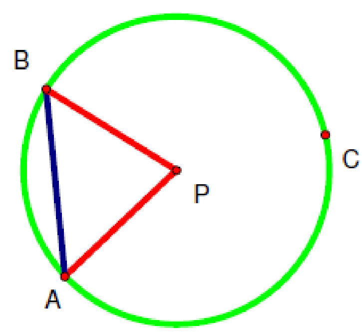
1. Use a compass to construct a circle on an unlined sheet of paper.
   Label the center of your circle with a Capital Letter.

a. What information do you need to determine a unique circle?

b. Prove that all circles are similar.

c. Use your answer to *item a* to help you write a definition of a circle.

Now we will introduce some notation and terminology needed to study circles.
Consider the figure at right.
$m\angle APB = 75^\circ$



Circles are identified by the notation $\odot P$, where P represents the point that is the center of the circle.

A **central angle** of a circle is an angle whose vertex is at the center of the circle.
$\angle APB$ is a central angle of $\odot P$.

A portion of a circle's circumference is called an **arc**. An arc is defined by two endpoints and the points on the circle between those two endpoints. If a circle is divided into two unequal arcs, the shorter arc is called the **minor arc** and the longer arc is called the **major arc**. If a circle is divided into two equal arcs, each arc is called a **semicircle**.

In our figure, we call the portion of the circle between and including points A and B, arc AB notated by $\widehat{AB}$. We call the remaining portion of the circle arc ACB, or $\widehat{ACB}$. Note that major arcs are usually named using three letters.

We say that the central angle $\angle APB$ *intercepts* or has $\widehat{AB}$. We also say that $\widehat{AB}$*subtends* or has the central angle $\angle APB$. Note that when we refer to the arc of a central angle, we usually mean the minor arc unless otherwise stated.

Arcs are measured in two different ways - using degree measure and using linear measure. Usually when we refer to the **measure** of an arc, we are referring to the degree measure. The **measure** of a minor arc is defined to be the measure of the central angle that intercepts the arc. The measure of a major arc is 360° minus the measure of the minor arc with the same endpoints.

In the figure above, the measure of $\widehat{AB}$is 75° because that is the measure of its central angle. The measure of $\widehat{ACB}$ is 360° – 75° or 285°.

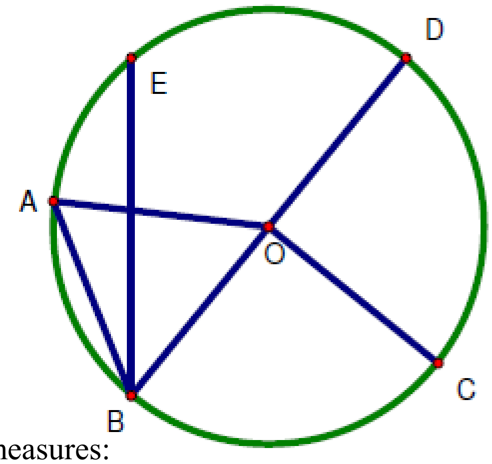
The **length** of an arc is different from its measure. The length is given in linear units and is determined as a portion of the length of the entire circumference of the circle. We will investigate the length of an arc in a later task. **Congruent arcs** have equal degree measures and equal lengths.

A **chord** is a *segment* whose endpoints lie on the circle. In the above figure, segment $\overline{AB}$ is a chord of ○P.

2. How many chords can be in a circle?

What is the longest chord in a circle?
Explain how you know?

3. Refer to the figure at the right.
Identify and name each of the following.
Be sure to use the correct notation.



a. Two different central angles
b. A minor arc
c. A major arc
d. A semicircle
e. Two different chords
f. The central angle subtended by $\widehat{AD}$

Use your protractor to help you find the following measures:
g. The measure of $\widehat{AC}$
h. The measure of $\widehat{DEC}$

4. Now it is time to use some of the terminology you have learned. Consider the following two theorems:

In the same circle or congruent circles, if two chords are congruent, their intercepted arcs are congruent.

In the same circle or congruent circles, if two arcs are congruent, then their chords are congruent.

a. Prove that each of the theorems is true.

b. Write the two theorems as one biconditional statement.

5. Use a compass to construct a circle on an unlined sheet of paper. Label the center of your circle.

a. Draw any chord, other than a diameter, on your circle.
Use your compass and a straightedge to construct a segment that represents the distance from the center of your circle to the chord. What is the relationship between the chord and the segment representing this distance?

b. Mary made the following conjecture: "If two chords of a circle are the same distance from the center of the circle, the chords are congruent." Mary is correct. Use what you learned in *Item 5a* to help convince Mary that her conjecture is correct.

c. State the converse of Mary's conjecture.

d. Write Mary's conjecture and its converse as a biconditional statement.

e. When a conjecture has been proven, it can be stated as a theorem. Write and illustrate this theorem in your *Circle Book*.

6. Ralph made the following conjecture: "A radius perpendicular to a chord bisects the chord."
a. Use your construction from *item 5a* to help convince yourself that Ralph's conjecture and the converse are true.

b. Write Ralph's conjecture and its converse as a biconditional statement and illustrate it in your *Circle Book*.

c. Ralph also believes that a radius perpendicular to a chord bisects the arc intercepted by the chord.

Is this true? How do you know?

7. Tevante examined his construction and his partner's construction. He believes that *any* line that is a perpendicular bisector of a chord of a circle must also contain the center of the circle. Is he right? How do you know?

8. An investigator working for the Georgia Bureau of Investigation's crime lab has uncovered a jagged piece of a circular glass plate believed to have been used as a murder weapon. She needs to know the diameter of the plate. How might you use the information you learned in problem 7 to help determine the diameter of the circular plate?
Use a compass, a straightedge, and a ruler to illustrate your answer.

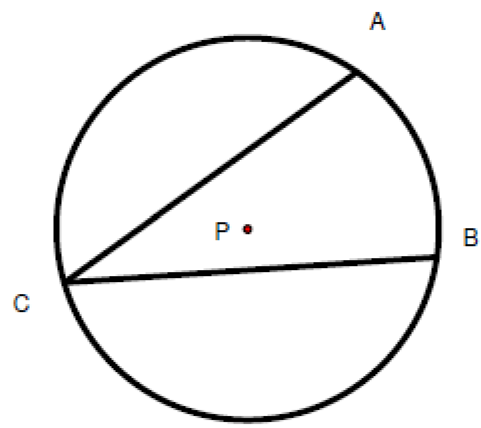
Investigating Angle Relationships in Circles

In this task, you will be investigating, discovering, and proving two theorems that involve circles and their inscribed angles. Afterwards, you will be expected to memorize and apply these theorems, and several others that you will be shown, to solve problems.

Part 1: Inscribed Angles

Definition: an **inscribed angle** is an angle whose vertex lies on the circle and whose sides are chords of the circle.

In ○P, ∠ACB is an inscribed angle.



1. Sketch another inscribed angle in ○P.

2. Now, you need to investigate the measure of an inscribed angle and its intercepted arc by following your teacher's instructions.

3. Write your conjecture here:

Remember that a conjecture is not a theorem until it has been proved.

Part 2: Quadrilaterals Inscribed in a Circle

4. Define quadrilateral.

A polygon is **inscribed** in a circle when every vertex of the polygon is on the circle.

5. Sketch a picture of a circle P with an inscribed quadrilateral ABCD.

6. Now, you will investigate the relationships among the angles of the quadrilateral inscribed in a circle.

7. Write your conjecture here:

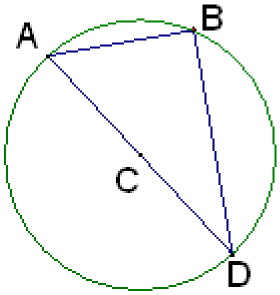
8. Write a proof of the theorem using your sketch from above.

Part 3: Graphic Organizer for Angel Theorems

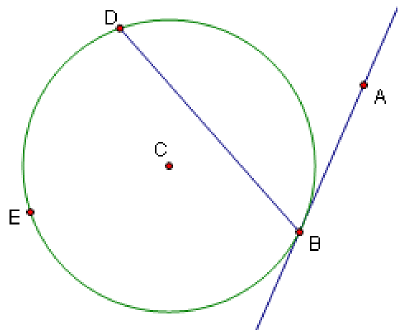
Location of the Vertex	Picture	Theorem
Inside the circle **At the Center**		
Not at the center		
Outside of the circle		
On the circle		

Part 4: Apply these theorems to solve these special cases of inscribed angles.

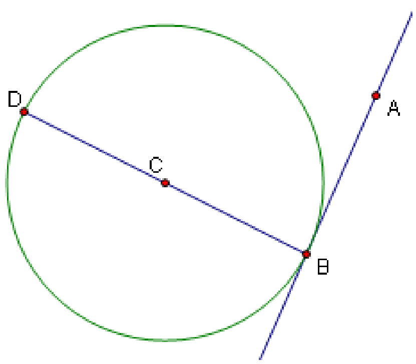
1. Find the m∠ABD, the inscribed angle of ○C.



2. Find the m∠ABD, the inscribed angle of ○C, if $m\widehat{BED} = 300°$.



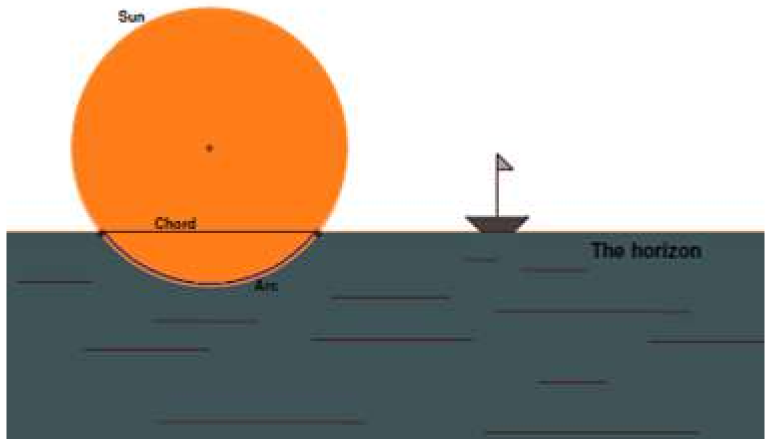
3. Find the m∠ABD, the inscribed angle of ○C.



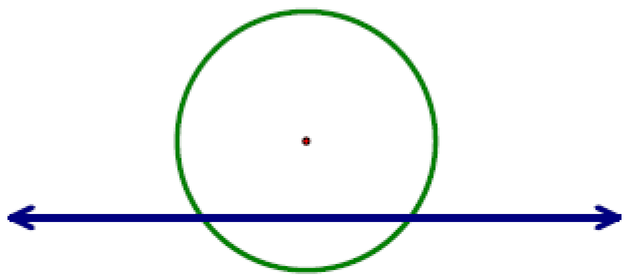
Chords, Secants, and Tangents

Part 1: Sunrise on the First Day of the New Year

It is customary for people in Asia to visit the seashores on the eastern sides of their countries on the first day of the year. While watching the sun rise over the ocean, visitors wish for good luck in the New Year.



As the sun rises, the horizon cuts the sun at different positions. Although a circle is not a perfect representation of the sun, we can simplify this scene by using a circle to represent the sun and a line to represent the horizon.



1. Using the simplified diagram above, sketch and describe the different types of intersections the sun and the horizon may have.

2. Definitions:
A **tangent line** is a line that intersects a circle in exactly one point.
A **secant line** intersects a circle in two points.

Do any of your sketches contain tangent or secant lines?
If so, label them.

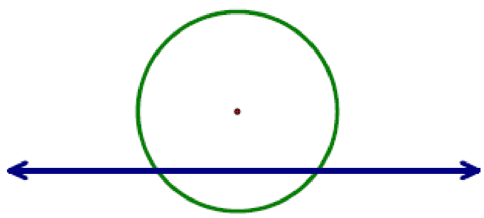
Is it possible for a line to intersect a circle in 3 points? 4 points?
Explain why or why not.

3. When a secant line intersects a circle in two points, it creates a chord. As you have already learned, a **chord** is a segment whose endpoints lie on the circle. How does a chord differ from a secant line?

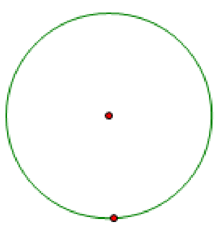
4. Look again at our representation of the sun and the horizon.

Let *d* represent the distance between the center of a circle and a line *l.* Let *r* represent the length of a radius of the circle.

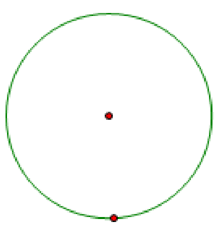
a. Draw and describe the relationship between *d* and *r* when *l* is a secant line



b. Draw and describe the relationship between *d* and *r* when *l* is a tangent line



c. Draw and describe the relationship between *d* and *r* when *l* does not intersect the circle



5. You just compared the length of a radius of a circle to the distance from the center of the circle to a tangent line. What does this comparison tell you about the relationship of a tangent line to a radius at the point of tangency. Explain your thinking. Relate this to the problem you solved in the task *Investigating Angle Relationships in Circles* (Part 4, question #3).

Part 2: The Segment Theorems Graphic Organizer
In the remaining items of this task, we will work with the relationships between the lengths of the segments created when these lines intersect.

Picture	Type	Theorem
	2 tangents **vertex outside**	
	2 secants **vertex outside**	
	Secant and tangent **vertex outside**	
	2 secants **VERTEX INSIDE**	

Part 3: Apply the Theorems to Solve Problems
Sketch a picture for each problem, choose a theorem, set up an equation, and then solve.

1. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. $AE = 2$, $CE = 4$, and $ED = 3$. Find *EB*.

2. A diameter of a circle is perpendicular to a chord whose length is 12 inches. If the length of the shorter segment of the diameter is 4 inches, what is the length of the longer segment of the diameter?

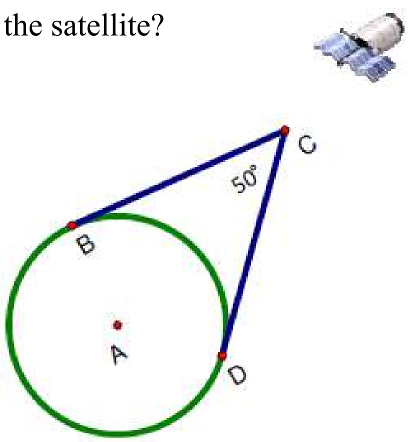
3. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. $AE = 5$, CE $= 10$, $EB = x$, and $ED = x-4$. Find *EB* and *ED*.

4. Two secant segments are drawn to a circle from a point outside the circle. The external segment of the first secant segment is 8 centimeters and its internal segment is 6 centimeters. If the entire length of the second secant segment is 28 centimeters, what is the length of its external segment?

5. A tangent segment and a secant segment are drawn to a circle from a point outside the circle. The length of the tangent segment is 15 inches. The external segment of the secant segment measures 5 inches. What is the measure of the internal secant segment?

6. The diameter of a circle is 19 inches. If the diameter is extended 5 inches beyond the circle to point *C*, how long is the tangent segment from point *C* to the circle?

7. A satellite orbits the earth so that it remains at the same point above the Earth's surface as the Earth turns. If the satellite has a 50° view of the equator, what percent of the equator can be seen from the satellite?

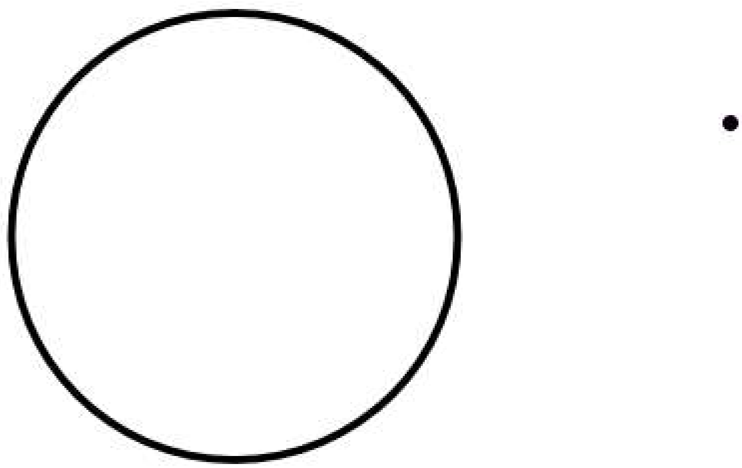


8. The average radius of the Earth is approximately 3959 miles.
a. How far above the Earth's surface is the satellite described in *Problem 7?*

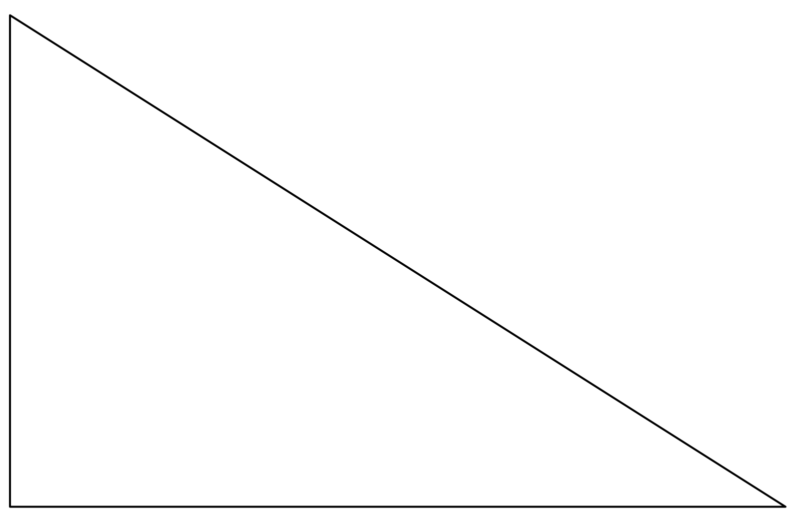
b. What is the length of the longest line of sight from the satellite to the Earth's surface? Identify this line of sight using the diagram.

Part 4: Constructions in Euclidean Geometry

1. Using a Compass and Straight Edge, Construct a Tangent Line to a circle from a given exterior point.



2. Using a compass and a straight edge, **Construct** the Inscribed Circle for the given triangle.



Arc Length and Area of a Sector

Define Arc Length.

Define Sector of a Circle.

Part 1: Hands on Activity - COOKIE LAB

Materials: Large Soft Cookie, String, Protractor, Ruler, Knife, Paper Towel

1. Find the circumference of the cookie in cm using the string and the ruler.

   Circumference = _______cm

2. Find the measure of the diameter in cm.

   Diameter = ________cm

3. What is the ratio of the Circumference to the Diameter?

   $\frac{C}{d} =$ ______

4. The formula for Area of a circle is πr^2; where r = radius of circle

5. Find the Area of the cookie. ____________cm^2

*Cut the cookie in half on the diameter. Then cut each half of the cookie into two **unequal** sectors. You will have 4 different pieces of cookie.*

6. Using the protractor, find the Angle Measure of each sector's central angle.

   Angle 1 = _______° Angle 2 = _______°

   Angle 3 = _______° Angle 4 = _______°

You may now eat your cookie!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

7. The formula for the **length of an arc** in a circle is $Arc\ Length = \frac{2\pi r\theta}{360}$

where r = radius of circle and θ= central angle

8. Using the Arc Length formula, find the measure of each sector's arc length.

Arc Length 1 = ________cm

Arc Length 2 = ________cm

Arc Length 3 = ________cm

Arc Length 4 = ________cm

9. What is the total length of the 4 arcs? _________cm

10. How does it compare to the circumference of the cookie?

11. The formula for the **Area of a Sector** is $Area\ of\ Sector = \frac{\pi r^2\theta}{360}$

where r = radius of circle and θ = central angle

12. Find the Area of each sector.

Area of sector 1 = _________cm^2

Area of sector 2 = _________ cm^2

Area of sector 3 = _________ cm^2

Area of sector 4 = _________ cm^2

13. What is the total area of the four sectors? _________ cm^2

14. How does it compare to the area of the original cookie?

15. Explain why the 4 arc lengths should add to the circumference of your circle. If they did not add to the circumference of your cookie, explain why they did not.

16. Explain why the 4 sector areas should add to the area of your cookie. If they did not sum to equal the area, explain why.

Part 2: Understanding the Formulas

Investigating the Area of a Circle

1. Cut out a Circle and Fold it then Cut it into at least 8 Congruent Sectors (or pizza slices).

2. Lay the slices next to each other to create a rectangle like shape.

3. Think about the dimensions of your "rectangle" in terms of the original circle's Circumference and radius. Sketch and label it here.

4. Since the Area of a Rectangle is found by multiplying the length of its base by its height, Find the Area of your "rectangle" by doing this calculation as well.

   My Rectangle created from a circle has an approximate area of:

5. How does your formula compare with the formula you know to be the Area of a Circle?

Investigating Arc Length and the Area of a Sector

In this portion of the investigation you will look at the relationship between a central angle and its intercepted arc.

1. Decide how long you would like your radius to be. r = __________ (don't forget units!)

2. Using your compass, draw a circle with the above radius on a separate piece of paper. What is the circumference of your circle? Don't forget units!

3. Divide your circle, using a protractor, into four equal parts (hint: use two diameters). What is the measure of each central angle in the circle you constructed?

4. Write a fraction that compares the measure of the central angle to the total number of degrees in a circle. Then simplify this fraction.

5. Keeping in mind your answer above, what would be the length of the arc formed by one of the central angles?

6. Write a fraction that compares the arc length computed above to the total circumference of the circle. Then simplify this fraction.

7. Make a conjecture about the proportion of the measure of a central angle and the proportion of its intercepted arc? (Hint: What do you notice about the fraction you found in question 4 and the fraction you found in question 6?) **Come up with a general formula for this conjecture.**

8. Compare your findings and formula with your peers around you. Does your formula work with different sized circles?

Write your discovery as:

Arc Length Proportion **Arc Length Formula**

In the next portion of this investigation we will look at the relationship between the central angle and the sector area it creates.

9. What is the area of your circle? Don't forget units!

10. Using your divided circle from the beginning of the investigation, what is the area of one of the sectors?

11. Write a fraction that compares the sector area computed above to the total area of the circle. Then simplify this answer.

12. What do you notice about the simplified fraction of the sector area and the fraction you found in problem 4 of the central angle?

13. Make a conjecture about the proportion of the measure of a central angle and the proportion of its sector area? **Come up with a general formula for this conjecture.**

14. Compare your findings and formula with your peers around you. Does your formula work with different sized circles?

Write your discovery as:

Area of Sector Proportion **Area of Sector Formula**

Volumes of Cylinders, Cones, Pyramids, and Spheres

Bonaventura Francesco Cavalieri (1598 – November 30, 1647) was an Italian mathematician. We will use his works as the basis for this task. Cavalieri's Principle states:

> The volumes of two solids are equal if the areas of corresponding sections drawn parallel to some given plane are equal.

This can best be understood by looking at a stack of pennies. The volume of two stacks of the same number of pennies is the same, even if one stack is not vertically aligned.

Let's apply this to derive the formula for the volume of a right cylinder. If you cut a super thin slice, or cross section, out of a cylinder what shape would result?

What is the Area Formula for that shape? $A =$

How many slices would you need to stack up to create your cylinder?

Complete the Volume of a Cylinder Formula:

$V = Area\ of\ the\ Base \bullet height$

$V = B \bullet h$

$V =$

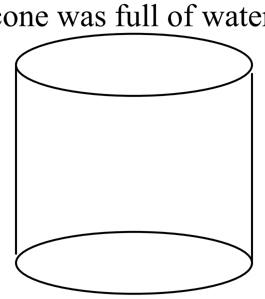
Cavalieri principle. *Encyclopedia of Mathematics*. URL: http://www.encyclopediaofmath.org/index.php?title=Cavalieri_principle&oldid=17072

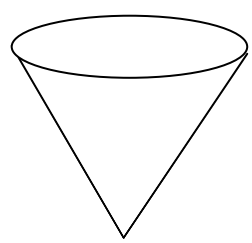
Part 1: Understanding the Formulas

Let's investigate the relationship between a Cone and its corresponding Cylinder with the same height and radius. LABEL the height, h and radius, r on each diagram below.

1. If the cylinder was full of water and you poured it into the cone, how many times would it fill up the cone completely?

   If the cone was full of water, how much of the cylinder would it fill up?





2. Complete the Formulas below:
   Volume of a Cylinder Formula vs. Volume of a Cone Formula

$V = Area\ of\ the\ Base \bullet height$

$V = B \bullet h$ $\qquad\qquad$ $V =$

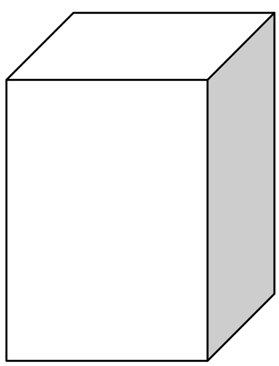
$V =$

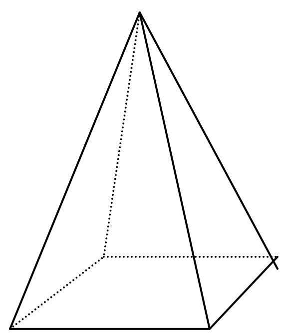
3. Describe the relationship between the formulas and the amount the cone filled the cylinder.

Let's investigate the relationship between a Pyramid and its corresponding Rectangular Prism with the same height, length, and width. LABEL the height, h, length, l, and width, w on each diagram below.

4. If the rectangular prism was full of water and you poured it into the pyramid, how many times would it fill up the pyramid completely?

   If the pyramid was full of water, how much of the rectangular prism would it fill up?





5. Complete the Formulas below:

Right Rectangular Prism Volume Formula vs. Right Pyramid Volume Formula

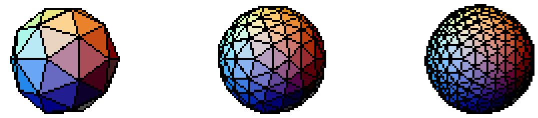
$V = Area\ of\ the\ Base \bullet height$

$V = B \bullet h$ $\qquad\qquad$ $V =$

$V =$

6. Describe the relationship between the formulas and the amount the pyramid filled the prism.

Let's investigate the Volume of a Sphere Formula**.**



An image from
http://mathworld.wolfram.com/GeodesicDome.html

What is the formula for the Surface Area of a Sphere?

$SA =$

If that surface is divided up into triangles that are actually the bases of triangular pyramids that fill up the entire space inside of the sphere, then the Volume of the Sphere would equal the sum of the Volumes of all those pyramids.

Let's see if we can use this concept to derive the formula for the volume of a sphere:

$V_{pyramid} =$

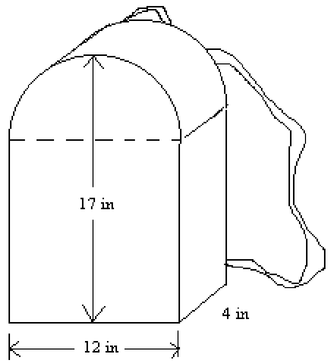
$V_{all\ the\ pyramids} =$

$=$

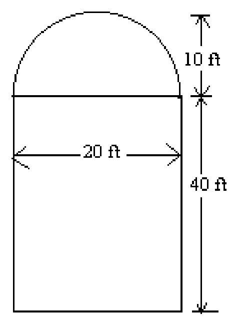
$V_{sphere} = \frac{4}{3}\pi r^3$

Part 2: Applications

1. Approximate the Volume of the Backpack that is 17in x 12in x 4in.



2. Find the Volume of the Grain Silo shown below that has a diameter of 20ft and a height of 50ft.



3. The diameter of a baseball is about 1.4 in.
   How much leather is needed to cover the baseball?
   How much rubber is needed to fill it?

4. The volume of a cylindrical watering can is $100cm^3$. If the radius is doubled, then how much water can the new can hold?