CCGPS Frameworks Student Edition

Common Core Georgia Performance Standards

Mathematics

Accelerated CCGPS Coordinate Algebra / Analytic Geometry A
Unit 5: Transformations in the Coordinate Plane

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Unit 5
Transformations in the Coordinate Plane

Table of Contents

OVERVIEW

In this unit students will:

- use and understand definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined terms of point, line, distance along a line and length of an arc.
- describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, including translations and horizontal or vertical stretching
- represent and compare rigid and size transformations of figures in a coordinate plane using various tools such as transparencies, geometry software, interactive whiteboards, waxed paper, tracing paper, mirrors and digital visual presenters.
- compare transformations that preserve size and shape versus those that do not.
- describe rotations and reflections of parallelograms, trapezoids or regular polygons that map each figure onto itself.
- develop and understand the meanings of rotation, reflection and translation based on angles, circles, perpendicular lines, parallel lines and line segments.
- transform a figure given a rotation, reflection or translation using graph paper, tracing paper, geometric software or other tools.
- create sequences of transformations that map a figure onto itself or to another figure.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Experiment with transformations in the plane

MCC9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between

equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper,

concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.
- Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes in general).
- Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- plotting points on a coordinate plane
- congruence of geometric figures and the correspondence of their vertices, sides, and angles
- recognizing line and rotational symmetry

- interpreting and sketching views from different perspectives
- calculate the perimeter and area of fundamental geometric plane figures
- use the concepts of ratio, proportion, and scale factor to demonstrate the relationships between similar plane figures

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school students. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Angle:** A figure created by two distinct rays that share a common endpoint (also known as a vertex). $\angle ABC$ or $\angle B$ or $\angle CBA$ indicate the same angle with vertex B.

- **Angle of Rotation:** The amount of rotation (in degrees) of a figure about a fixed point such as the origin.

- **Bisector:** A point, line or line segment that divides a segment or angle into two equal parts.

- **Circle:** The set of all points equidistant from a point in a plane.

- **Congruent:** Having the same size, shape and measure. $\angle A \cong \angle B$ indicates that angle A is congruent to angle B.

- **Corresponding angles:** Angles that have the same relative position in geometric figures.
- **Corresponding sides:** Sides that have the same relative position in geometric figures.
- **Endpoint:** The point at each end of a line segment or at the beginning of a ray.
- **Image:** The result of a transformation.
- **Intersection:** The point at which two or more lines intersect or cross.
- **Isometry:** a distance preserving map of a geometric figure to another location using a reflection, rotation or translation. $M \rightarrow M'$ indicates an isometry of the figure M to a new location M'. M and M' remain congruent.
- **Line:** One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely. $\overleftrightarrow{AB}$ indicates a line that passes through points A and B.
- **Line segment:** A part of a line between two points on the line. $\overline{AB}$ indicates the line segment between points A and B.
- **Parallel lines:** Two lines are parallel if they lie in the same plane and do not intersect. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ indicates that line AB is parallel to line CD.
- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ indicates that line AB is perpendicular to line CD.
- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.
- **Pre-image:** A figure before a transformation has taken place.
- **Ray:** A part of a line that begins at a point and continues forever in one direction. $\overrightarrow{AB}$ indicates a ray that begins at point A and continues in the direction of point B indefinitely.
- **Reflection:** A transformation of a figure that creates a mirror image, "flips," over a line.
- **Reflection Line (or line of reflection):** A line that acts as a mirror so that corresponding points are the same distance from the mirror.

- **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.

- **Segment:** See line segment.

- **Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.

- **Translation:** A transformation that slides each point of a figure the same distance in the same direction.

- **Vertex:** The location at which two lines, line segments or rays intersect.

Introduction to Reflections, Translations, and Rotations

1. On your graph paper draw and label a square. Describe its original position and size.

   Rotate it 90 degrees. Translate it so that it is in the 4th quadrant.
   Reflect it over a line y="a number" so that the square is in the 1st quadrant.

   Write 2 distinctly different ways that you can get the shape back in its original position.

2. On your graph paper draw and label a triangle. Describe its original position and size.

   Rotate, Translate, and Reflect the triangle so that the one side is touching an original side in such a way that it forms a parallelogram. List your steps here:

3. On your graph paper draw and label a parallelogram. Describe its original position and size.

Rotate, Translate, and Reflect the parallelogram several times, listing your steps here:

Now, challenge a friend to get the parallelogram back into its original position! Are the steps that your friend used the reverse of your steps, or are they different?

Exploring Reflections and Rotations

Reflections:

1. Select the vertices of the **square** and go to Display → Label Points. Start with C so that your **square**'s vertices are labeled C, D, E, & F. Find the coordinates of each point:
C = (___,___); D = (___,___); E = (___,___); F = (___,___)
2. Double click the line x=2. You should see an animation on the line. This makes x=2 the line of reflection (also called the line of symmetry).
3. Select the interior & the vertices of your **square**. Go to Transform → Reflect.
4. Select the vertices of the new **square** and go to Display → Show Labels.
5. How have the new points changed? ____
____
6. Double click on the y-axis to change it to the line of reflection. Select the interior & vertices of your **square**. Go to Transform → Reflect.
7. What has happened? ____
Why is this reflection further away than the last one? ____________________________
____
What effect did changing the reflection line have? ______________________________
____
8. Write out the coordinates of each square.

Original Square		Reflection over x=2		Reflection over y-axis	
C	(___,___)	C'	(___,___)	C'	(___,___)
D	(___,___)	D'	(___,___)	D'	(___,___)
E	(___,___)	E'	(___,___)	E'	(___,___)
F	(___,___)	F'	(___,___)	F'	(___,___)

How far apart are the original square and the first reflection? ______________________
The original square and the second reflection?___________________________________
How far is the original square from x=2 and how far is the first reflection from x=2?
____
How far is the original square from the y-axis and the second reflection and the y-axis?
____

9. Delete the two reflections and their vertices.
10. Double click the line $y = 0.5x - 5$ to make it the new line of reflection. If you were to reflect the **square** over $y = 0.5x - 5$, **predict** where would the new vertices be?

Original Square		**Prediction** of Reflection over $y = 0.5x - 5$	
C	(___,___)	C'	(___,___)
D	(___,___)	D'	(___,___)
E	(___,___)	E'	(___,___)
F	(___,___)	F'	(___,___)

11. ***After*** you have made your prediction, select the vertices and interior of the **square**. Go to Transform → Reflect. Select the vertices of the new **square** and go to Display → Show Labels. How does your prediction compare with the actual reflection?

____

____

12. Make a general conclusion about what happens to coordinates of a point when they are reflected over a line.

____

____

Rotations:

13. Delete the reflection so only the original square remains.
14. Using the Selection Arrow Tool, double click the point B. A brief animation should show. Point B is now your point of rotation.
15. Select the interior & the vertices of your **square**. Go to Transform → Rotate. A box labeled "Rotate" should come up. Move the box over to the side so you can see your sketch. You should see a lighter colored square that has appeared on your sketch. Change the number of degrees **BUT DON'T CLICK ANY BUTTONS**. The image should change where it goes. Try different numbers of degrees (less than 360). Explain what is happening to the square and the points?

____

____

16. Try negative degrees (once again **without clicking any buttons**). What do you notice about 90 and –270? 180 and –180? 45 and –315?

____

Why do you think this is?____

____

Are there any other pairs of measures that have the same phenomena? How could we predict additional pairs?

____

17. Type in 270 for the degrees and click the button "rotate". Write down the new coordinates and compare them to the old coordinates.

Original Square		Rotation around B	
C	(___,___)	C'	(___,___)
D	(___,___)	D'	(___,___)
E	(___,___)	E'	(___,___)
F	(___,___)	F'	(___,___)

What relationship is there between the points of the two figures?

____

18. What conclusions can you make about what happens to coordinates when rotated?

____

____

EXTENSION

19. Is there ever a time when a rotation is the same as a reflection? Explain.

____

____

20. Create an animation in a new Geometer's Sketchpad window to support your answer to number 19. Email your animation to your teacher.

Exploring Reflections and Rotations (Alternative)

Reflections:

1. On a piece of graph paper, graph the following points to create Square CDEF
   C = (3, 0); D = (4, 1); E = (5, 0); F = (4, -1)
2. Draw the line: x = 2.
3. Using either Mira, patty paper or a transparency reflect the square over the x = 2 line.
4. How have the new points changed? ____
   ____
5. Using the original square, now reflect it over the y-axis.
6. What has happened? ____
   Why is this reflection further away than the last one? ___________________________
   ____
   What effect did changing the reflection line have? ______________________________
   ____
7. Write out the coordinates of each square.

Original Square		Reflection over *x*=2		Reflection over y-axis	
C	(___,___)	C’	(___,___)	C’	(___,___)
D	(___,___)	D’	(___,___)	D’	(___,___)
E	(___,___)	E’	(___,___)	E’	(___,___)
F	(___,___)	F’	(___,___)	F’	(___,___)

How far apart are the original square and the first reflection? ______________________
The original square and the second reflection?________________________________
How far is the original square from *x*=2 and how far is the first reflection from *x*=2?
____
How far is the original square from the y-axis and the second reflection and the *y*-axis?
____

8. Draw the line $y = 0.5x - 5$. If you were to reflect the **original square** over $y = 0.5x - 5$, **predict** where would the new vertices be?

Original Square		**Prediction** of Reflection over $y = 0.5x - 5$	
C	(___,___)	C'	(___,___)
D	(___,___)	D'	(___,___)
E	(___,___)	E'	(___,___)
F	(___,___)	F'	(___,___)

9. ***After*** you have made your prediction, using the Mira, patty paper, or transparency reflect the original square over the $y = 0.5x - 5$ line. How does your prediction compare with the actual reflection?

____

____

10. Make a general conclusion about what happens to coordinates of a point when they are reflected over a line.

____

____

Rotations:

11. Return to the original square.
12. Plot the point B(2,0). Point B is now your point of rotation.
13. Experiment with rotating the square about point B using either patty paper or a transparency. Try different numbers of degrees (less than 360) counter clockwise. Explain what is happening to the square and the points.

____

____

14. Try rotating negative degrees (clockwise). What do you notice about 90 and –270? 180 and –180? 45 and –315?

____

Why do you think this is?____

____

Are there any other pairs of measures that have the same phenomena? How could we predict additional pairs?

____

15. Rotate the square, about point B, 270 degrees. Write down the new coordinates and compare them to the old coordinates.

Original Square		Rotation around B	
C	(___,___)	C’	(___,___)
D	(___,___)	D’	(___,___)
E	(___,___)	E’	(___,___)
F	(___,___)	F’	(___,___)

What relationship is there between the points of the two figures?

____

16. What conclusions can you make about what happens to coordinates when rotated?

____

____

EXTENSION

17. Is there ever a time when a rotation is the same as a reflection? Explain.

____

____

18. Create a sequence to support your answer to number 17.

Mirrored Mappings

Antonio and his friend Brittany were at summer math camp where the counselors had drawn a large coordinate plane on the gym floor. Antonio challenged Brittany to mirror him as he walked in the first quadrant. Map both of their travels on the same coordinate plane.

Antonio began at (2, 1) and walked to (3, 5); Brittany decided to begin at (–2, 1), then tried to mirror Antonio by walking to (–3, 5). Antonio jumped to (5, 5) and side-stepped to (4, 3); Brittany jumped to (–5, 5) then side-stepped to (–4, 3). Antonio returned to (2, 1) and Brittany returned to (–2, 1).

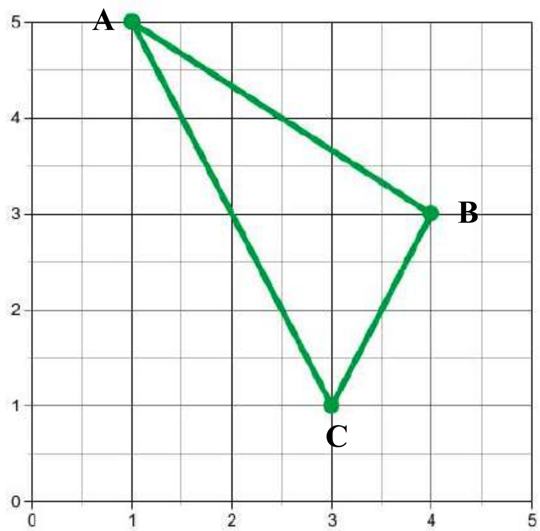
1. Did Brittany mirror Antonio?

    a. If you answered no, identify the incorrect coordinates Brittany used and find the correct coordinates. Explain your decision and identify the line of symmetry she should have used as a mirror. How did you know that this should have been the line of symmetry?

    b. If you answered yes, identify the line of symmetry Brittany used as a mirror. How did you know it was the line of symmetry?

2. If Brittany had instead begun at (–2,1), walked to (–4,3), side-stepped to (–5,5), jumped to (–3,5) and then returned to (–2,1), could she claim that she created a mirror image of Antonio's path? Justify your answer.

Antonio and Brittany decided to change the game and use some lettered blocks to mark points they visited on the grid. Antonio placed blocks A, B and C as indicated by the points below, then drew a chalk line between them.



3. Label the coordinates Antonio used, and then construct the graph of where Brittany would place her blocks if she correctly reflected Antonio's figure across the x-axis.
4. Describe how you determined where to place Brittany's blocks.

5. Each block Brittany placed corresponds to one that Antonio placed. List each pair of coordinates that correspond.

6. What can you observe about the distances between each of Antonio's blocks and the corresponding block Brittany placed?

7. If Antonio walked 2 feet from his block A toward his block C, and Brittany mirrored his movement by walking 2 feet from the blocks corresponding to A and C, would Brittany and Antonio be the same distance from the reflection line? How can you be certain?

8. How would you define a reflection now that you have analyzed some of the properties of reflected images using the coordinate plane?

Coordinating Translations

Create any polygon you want on the coordinate plane, and then create polygons congruent to the one you designed using the three directions described below. Use the same coordinate plane for all transformations.

1. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x + 4, y)$.
2. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x, y - 3)$.
3. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x - 4, y + 1)$.
4. What kind of transformations are these?
5. Can you create a translation $(x + 2, y + 2)$? Is it necessary that the same number is added or subtracted to the x and y coordinates of the polygon? Why or Why not?

The vertices of your original polygon combined with their images must be mapped to points in all four quadrants of the coordinate plane to receive full credit.

***Option for differentiation*:**
Provide a description of each of the following translations, where c can represent any number.

1. $(x + c, y)$
2. $(x, y - c)$
3. $(x - c, y)$
4. $(x , y + c)$

Transformations in the Coordinate Plane

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order. Do **not** connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (don't connect the dots).

Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Set 1	**Set 1**	**Set 1**	**Set 1**	**Set 1**	**Set 1**
(6, 4)	(12, 8)	(18, 4)	(18, 12)	(6, 12)	(8, 6)
(6, -4)	(12, -8)	(18, -4)	(18, -12)	(6, -12)	(8, -2)
(-6, -4)	(-12, -8)	(-18, -4)	(-18, -12)	(-6, -12)	(-4, -2)
(-6, 4)	(-12, 8)	(-18, 4)	(-18, 12)	(-6, 12)	(-4, 6)
Set 2	**Set 2**	**Set 2**	**Set 2**	**Set 2**	**Set 2**
(1, 1)	(2, 2)	(3, 1)	(3, 3)	(1, 3)	(3, 3)
(1, -1)	(2, -2)	(3, -1)	(3, -3)	(1, -3)	(3, 1)
(-1, -1)	(-2, -2)	(-3, -1)	(-3, -3)	(-1, -3)	(1, 1)
(-1, 1)	(-2, 2)	(-3, 1)	(-3, 3)	(-1, 3)	(1, 3)
Set 3	**Set 3**	**Set 3**	**Set 3**	**Set 3**	**Set 3**
(4, -2)	(8, -4)	(12, -2)	(12, -6)	(4, -6)	(6, 0)
(3, -3)	(6, -6)	(9, -3)	(9, -9)	(3, -9)	(5, -1)
(-3, -3)	(-6, -6)	(-9, -3)	(-9, -9)	(-3, -9)	(-1, -1)
(-4, -2)	(-8, -4)	(-12, -2)	(-12, -6)	(-4, -6)	(-2, 0)
Set 4	**Set 4**	**Set 4**	**Set 4**	**Set 4**	**Set 4**
(4, 2)	(8, 4)	(12, 2)	(12, 6)	(4, 6)	(6, 4)
(-4, 2)	(-8, 4)	(-12, 2)	(-12, 6)	(-4, 6)	(-2, 4)

After drawing the six figures, **compare Figure 1 to each of the other figures** and answer the following questions.

1. Describe any similarities and/or differences between Figure 1 and each of the other figures.

2. How do the coordinates of each figure compare to the coordinates of Figure 1? If possible, write general rules for making Figures 2-6.

3. Translate, reflect, and rotate (between 0 and 90°) Figure 1 so that it lies entirely in Quadrant III on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

4. Translate, reflect, and rotate (between 0 and 90°) Figure 1 so that it lies entirely in Quadrant IV on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

5. Describe the transformations necessary to take the figure you created in question #4 on to Figure 6.