CCGPS Frameworks Student Edition

Mathematics

Third Grade Unit Three
Operations and Algebraic Thinking:
Properties of Multiplication and Division

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"Making Education Work for All Georgians"

Unit 3
Operations and Algebraic Thinking: Properties of Multiplication and Division

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OVERVIEW

In this unit, students will:

- apply properties of operations (commutative, associative, and distributive) as strategies to multiply and divide
- understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
- fluently multiply and divide within 100, using strategies such as the patterns and relationships between multiplication and division
- understand multiplication and division as inverse operations
- solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.
- represent and interpret data

"Multiplication and division are commonly taught separately. However, it is very important to combine the two shortly after multiplication has been introduced. This will help the students to see the connection between the two." (Van de Walle and Lovin, Teaching Student-Centered Mathematics 3-5, p. 60)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, strategies, and properties-rules about how numbers work.

Common Multiplication and Division Situations

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

	Unknown Product	**Group Size Unknown** ("How many in each group? Division)	**Number of Groups Unknown** ("How many groups?" Division)
	3 × 6 = ?	**3 × ? – 18, and 18 ÷3 = ?**	**? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are6 inches long. How many pieces of string will you have?
Arrays[1], Area[2]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3times as long?	A red hat costs $18 and that is3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be18 cm long. How many times as long is the rubber band now as it was at first?
General	***a* ×*b* = ?**	***a* ×? = *p*, and *p*÷*a* = ?**	**?×*b* = *p*, and *p*÷*b* = ?**

Taken from: Common Core State Standards Glossary

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

STANDARDS FOR MATHEMATICAL CONTENT

Understand properties of multiplication and the relationship between multiplication and division.

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

Multiply and divide within 100

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Represent and interpret data.

MCC3.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

MCC3.3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

Students are expected to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

- Multiplication facts can be deduced from patterns.
- The associative property of multiplication can be used to simplify computation.
- The distributive property of multiplication allows us to find partial products and then find their sum.
- Patterns are evident when multiplying a number by ten or a multiple of ten.
- Multiplication and division are inverses; they undo each other.
- Multiplication and division can be modeled with arrays.
- Multiplication is commutative, but division is not.
- There are two common situations where division may be used.
    - Partition (or fair-sharing) - given the total amount and the number of equal groups, determine how many/much in each group
    - Measurement (or repeated subtraction) - given the total amount and the amount in a group, determine how many groups of the same size can be created.
- As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.
- There is a relationship between the divisor, the dividend, the quotient, and any remainder.

ESSENTIAL QUESTIONS

- How are multiplication and division related?
- How can multiplication and division be used to solve real world problems?
- How can multiplication help us repeatedly add larger numbers?
- How can multiplication products be displayed on a multiplication chart?
- How can the same array represent both multiplication and division?
- How can we connect multiplication facts with their array models?
- How can we determine numbers that are missing on a times table chart by knowing multiplication patterns?
- How can we model multiplication?
- How can we practice multiplication facts in a meaningful way that will help us remember them?
- How can we use patterns to solve problems?
- How can we write a mathematical sentence to represent a multiplication model we have made?

- How can you display data in a pictograph?
- How can you display data in a single bar graph?
- How can you use a graph to solve the answer to a question?
- How can you use multiplication facts to solve unknown factor problems
- How does drawing an array help us think about different ways to decompose a number?
- How does the order of the digits in a multiplication problem affect the product?
- How does understanding the commutative property help us create arrays?
- How does understanding the distributive property help us multiply large numbers?
- How does your graph communicate your data?
- How is division an unknown factor problem?
- How are multiplication and division used to solve a problem?
- How is the commutative property of multiplication evident in an array model?
- Is there more than one way of multiplying to get the same product?
- What are strategies for learning multiplication facts?
- What are the parts of a division problem?
- What are the steps involved in making and reading graphs?
- What patterns of multiplication can we discover by studying a times table chart?
- What strategies can be used to find factors or products?
- When can you use a line plot graph to organize data?
- When can you use multiplication or division in real life?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

In Grade 2, instructional time focused on four critical areas:

- Furthering their understanding for the base-ten system. Students worked with counting in fives, tens and multiples of hundreds, tens and ones. Students also recognize that the digits in each place of a number represent the amounts of thousands, hundreds, tens, or ones.
- Using their understanding of addition to develop fluency within 100. They solve problems within 1,000 by using models of addition and subtraction.
- Recognizing the need for units of measure (centimeter and inch) and understand how to use rulers and other measurement tools to get linear measurement.
- Developing an understanding of shapes by analyzing and describing them based on their sides and angles.

Specifically, it is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- odd and even numbers
- skip counting by twos, threes, fives, and tens
- determining reasonableness using estimation
- addition and subtraction as inverse operations
- multiplication of one-digit numbers
- commutative, associative, and identity properties of addition
- basic addition facts
- making tens in a variety of ways
- basic subtraction facts
- place value for ones, tens, hundreds, thousands, and tenths
- modeling numbers using base 10 blocks and on grid paper
- using addition to find the total number of objects in a rectangular array

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for **teacher reference only and are not to be memorized by the students**. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- array
- associative property of multiplication
- bar graph
- commutative property of multiplication
- distributive property
- dividend
- division
- divisor
- factor
- fourths
- halves
- identity property of multiplication
- inch
- line plot
- measurement division (or repeated subtraction)
- multiplicand
- multiplication
- multiplier

- partial products
- partition division
- picture graph
- product
- quotient
- remainder
- scale
- strategy
- unknown
- whole numbers

STRATEGIES FOR TEACHING AND LEARNING

(Information adapted from Grade 3 Mathematics Model Curriculum, Ohio Department of Education)

Understand properties of multiplication and the relationship between multiplication and division.

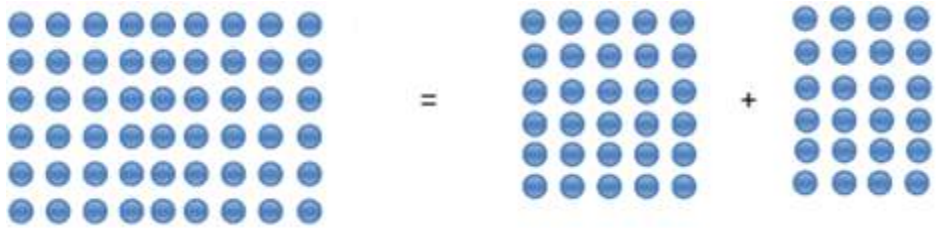
MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

Instructional Strategies
Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 (15) is the same as the result of multiplying 5 x 3 (15).

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply 5 x 7 x 2, students know that 5 x 2 is 10. Then, they can use mental math to find the product of 10 x 7 (70). Allow students to use their own strategies and share with the class when applying the associative property of multiplication.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.



The 6 groups of 5 is 30 and the 6 groups of 4 is 24. Students can write 6 x 9 as 6 x 5 + 6 x 4. Students' understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

Multiply and divide within 100
MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Strategies
Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.

To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division. Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

Represent and interpret data.

MCC3.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

MCC3.3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Instructional Strategies

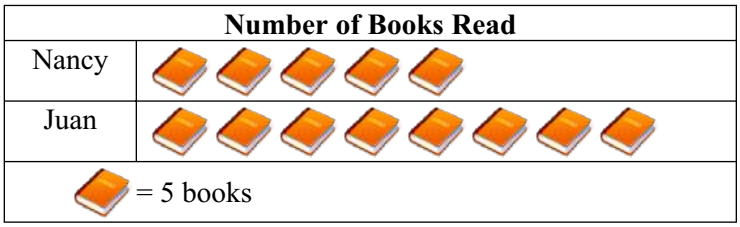
Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, ☺ represents 7 people. If there are three ☺ **,** students should use known facts to determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

Students are to draw picture graphs in which a symbol or picture represents more than one object. Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare quantities and use mathematical concepts and skills. Use symbols on picture graphs that student can easily represent half of, or know how many half of the symbol represents. Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole

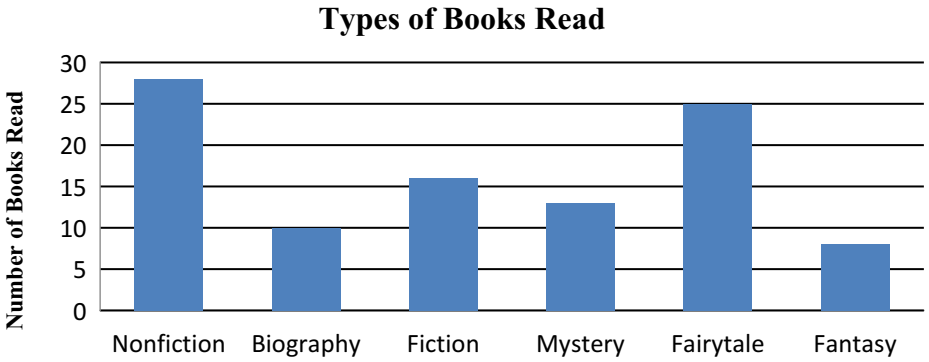
numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots.

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip- counting when determining the value of a bar since the scale is not in single units.

Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



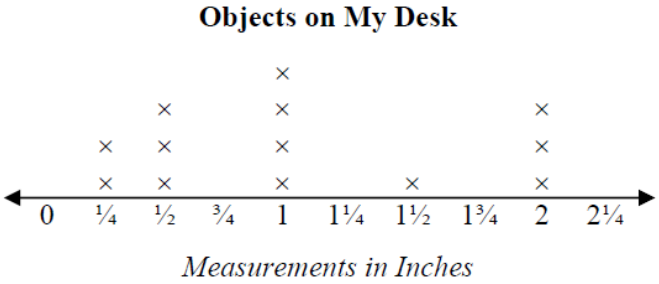
Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Line Plot Graphs: Students in second grade measured length in whole units using both metric and U.S. customary systems. It is important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.

This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.

Example:
Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc. …



Analyze and Interpret data:

- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre? Why?

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- use mental math to multiply and divide
- use estimation to determine reasonableness of products and quotients computed
- be able to read, interpret, solve, and compose simple word problems dealing with multiplication and division
- understand how to use inverse operations to verify accuracy of computation
- apply properties of operations (commutative, associative, and distributive) as strategies to multiply and divide
- understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
- fluently multiply and divide within 100, using strategies such as the patterns and relationships between multiplication and division
- represent and interpret data

TASKS

Scaffolding Task	Constructing Task	Practice Task	Performance Tasks
Tasks that build up to the constructing task.	Constructing understanding through deep/rich contextualized problem solving tasks	Games/activities	Summative assessment for the unit.

Task Name	Task Type *Grouping Strategy*	Skills
Arrays on the Farm	Scaffolding Task *Small Group/Partner*	Mental strategies and arrays
Seating Arrangements	Constructing Task *Individual/Partner Task*	Arrays and multiplication facts
Array-nging our Fact Families	Practice Task *Individual/Partner Task*	Models for multiplication and division
Family Reunion	Construction Task *Individual/Partner Task*	Multiplication and division patterns
Finding Factors	Constructing / Practice Task *Small Group/Partner*	Multiplication and division facts
Making Up Multiplication	Constructing Task *Individual/Partner Task*	Creating multiplication stories
Use What You Know	Practice Task *Individual/Partner Task*	Solving Unknown Factor Problems
Multiplication Chart Mastery	Practice Task *Individual/Small Group Task*	Identify patterns on the multiplication chart
Making the "Hard" Facts Easy	Constructing Task *Small Group/Partner Task*	Distributive Property of Multiplication
Find the Unknown Number	Practice Task *Individual/Partner Task*	Solving Unknown Factor Problems
Oh My Graphing	Constructing Task *Small Group/ Partner Task*	Single Bar Graphs and Pictographs
X Marks the Spot	Constructing Task *Small Group/ Partner Task*	Line Plot Graphs
My Special Day	Performance Task *Individual Task*	Multiplication and division

As this unit has no Culminating Task, you may pair/modify tasks to include all unit standards in combination.

SCAFFOLDING TASK: Arrays on the Farm
3-4 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) Use arrays, area models, and manipulatives to develop understanding of properties.

STANDARDS FOR MATHEMATCIAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 85)

"It is not intuitively obvious that 3 x 8 is the same as 8 x 3 or that, in general, the order of the numbers makes no difference (the order or commutative property). A picture of 3 sets of 8 objects cannot immediately be seen as 8 piles of 3 objects. Eight hops of 3 land at 24, but it is not clear that 3 hops of 8 will land at the same point.

The array, by contrast, is quite powerful in illustrating the order property, as shown in Figure 3.9. Children should draw or build arrays and use them to demonstrate why each array represents two different multiplications with the same product."

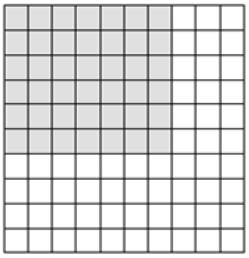
In this task the students use arrays to solve multiplication problems. Farmers grow their crops in arrays to make them easier to look after and to harvest.

Arrays provide a quick and efficient way to count things. For example, this can be done by adding the numbers in each row together. However, it is quickest to determine the number of objects in an array by multiplying the number in the width by the number in the height using area models.

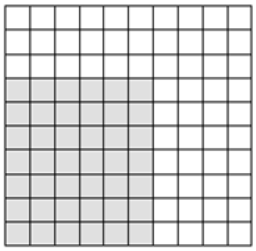
As an important side issue to this task, we ask the students to be involved in guessing and estimating. These are both useful skills that take time to develop. This task provides some practice in this area.

This task provides different contexts to explore multiplication concepts using arrays such as the one below. This array is an area model.

6 rows of 7
or
6 x 7 = 42



7 rows of 6
or
7 x 6 = 42



ESSENTIAL QUESTIONS

- What are strategies for learning multiplication facts?
- How can we practice multiplication facts in a meaningful way that will help us remember them?
- How can we connect multiplication facts with their array models?
- How is the commutative property of multiplication evident in an array model?
- How can we model multiplication?

MATERIALS

- Counter or small manipulatives
- Large pieces of paper for recording
- scissors
- Henry's Array Farm recording sheet
- Array Circles recording sheet
- Enlarged Small Station Task problems
- Station Task problems individual recording sheet

GROUPING

Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I Scaffolding

1. Begin the task with the "Farmer's Problem"
   Pose the problem:
   Henry's farm produces potatoes every fall. He has 36 potato plants. How could Henry arrange his plants into equal rows and columns?

   Provide a pile of counters on an overheard, use a smart board with a shape tool (or plant image), or ask volunteers to come and show what the first row of plants might look like.
   Is there only one way to arrange Henry's potato plants?

   It's important for the students to understand what a row is so they can make sense of the problem.

   What are some predictions? How many different strategies will Henry be able to choose from? Record predictions on the board or someplace the class can see.

   2. Arrange the class into small mixed ability groups with 3 or 4 students in each. Give each group a large sheet of paper. Ask them to fold their piece of paper so it makes 4 boxes.

Strategy 2	**Strategy 4**
Strategy 1	**Strategy 3**

   Allow some time for each group to see if they can come up with 4 different ways to solve the Farmer's Problem and record their method in the 4 boxes.
   The teacher should circulate to prompt groups and challenge their thinking.

   Ask: *Are there more efficient ways to determine how many plants there are without having to count each one?*

3. Ask the groups to cut their large sheet into the 4 boxes. Each student will take a different strategy from their group. They must be aware that they will explain the strategy and ready to do so. Gather the class in a circle or position them so each one can see the presenting student and strategy. Ask the groups to share what they think is their most interesting strategy. Place each group's strategy in the middle of the circle or area as they are being shared. Once each group has contributed, ask the students to offer strategies that no one has shared yet.

Likely strategies	Possible teacher responses
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ….	Can you think of a quicker way to work out how many plants there are? How many plants are there in one row?
6,12,18,24,30,36	Do you know what 6 + 6 =? Can that help you solve this problem quicker?
6 x 6 = 36	What if Henry had 9 rows of plants and there were 8 plants in each row?
6 + 6 = 12; 12 + 12 = 24; 24 + 12 = 36	You used adding to work that out. How could you have used multiplication?
2 x 6 = 12; 12 + 12 + 12 = 36	If 2 x 6 = 12, what does 3 x 6 =? How could you work out 6 x 6 from this?
3 x 6 = 18 and then doubled it	Awesome, clever you, could you work out 9 rows of 6 for me?
5 x 6 = 30; and 6 more = 36	

4. The shared strategies can be put into similar groups.
Ask: *Who used a strategy like this one?* Show Henry's Array Farm Format sheet. *How do you think we could use this to solve Henry's problem?* Send groups off to experiment with Henry's Array Farm Format sheet. Again circulate in the classroom and observe what the groups are doing.

5. As a class, share the different ways that students used the array to solve Henry's Farm problem.

6. Challenge the students to use the array in the same way as another group did and pose the following problems.

7. Review predictions made before breaking up into small groups? Which predications were most reasonable?

Henry plants 2 more rows of 6 potato plants. How many rows does he have all together now? How many plants is that all together?	*Henry's sister, Katie has a farm exactly the same size with the same amount of plants. How many plants do they have all together?*

Part II Constructing

Before the students work at small group tasks, you will need to prepare the material from Henry's Array Farm recording sheet, Array Circles sheet and Station Task sheets.

The next few days will involve working on similar problems. Allow students the opportunity to choose intervention, if they are finding the problems challenging and are unable to get started independently. This is an opportunity for the teacher to go over more examples and clarify any misunderstandings.

Begin by role playing the Airplane Rides Over the Farm problem from the Station Task sheet to get the students started. Ask 6 students to pretend to be the passengers on the plane. Ask them to bring their chair to the front and sit in a row. Leave a gap in the middle to show where the aisle is. Then choose a second group of 6 to make a second row behind the first row of 6 students.

Ask students to think about:
How many passengers would there would be if there were 4 rows of 6?
What about if there were 9 rows?
Encourage students to share and demonstrate their strategies.

Place the enlarged Station Task problem cards at each station with Henry's Array Farm, Circle Array sheets and counters or small manipulatives. (You may want to have multiple stations of each problem depending on your class size. Make sure the Station Task number is large, so when students are ready to move they can identify an open station easily)
Read the Station Tasks to the class one at a time to clarify any misunderstandings.

Explain the Station Task Recording Sheet and how it is used. (As students have finished a Station Task, they complete the tracking box and continue to the next problem. Encourage students to show their math language, working knowledge and solutions on this sheet.)

Set/remind student of classroom procedures for partner work.

Assign the partners and allow students to choose which Station Task to solve first. Spend the session circulating the stations and questioning students.

How many did you think there would be to start with?
Why did you predict that to start with?
What gave you a clue to make that prediction?
Can you think of another way to use the array to solve that problem?

Can you think of a way to solve the problem without using the array?
Which way do you think is faster, why?

Task Station 1 (have counters, Henry's Array Farm and Array Circles available)

Peanut Farm (Part 1)	Peanut Farm (Part 2)
Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all? Prediction Solution	Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant? How many peanut plants should be in each row? Show all the different ways Paul could plant his peanut plants in rows by making your own array on a blank piece of paper.

Task Station 2 (have counters or manipulatives available)

Onion Farm (Part 1)	Onion Farm (part 2)
Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm? Prediction Solution	Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son? Draw all the possibilities

Task Station 3

Watermelon Farm (Part 1)	Watermelon Farm (Part 2)
Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are 4 rows. How many watermelon plants does she have in all? Prediction Solution	Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after… 1 year 2 years 3 years Record your strategies on an array

Task Station 4

Airplane Rides Over the Farm Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold? Prediction Solution	Airplane Rides Over the Farm Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane? Prediction Solution

At the end of each session allow for a sharing time to discuss what students were finding interesting and challenging. Share some strategies used.

Part III
Conclusion/Reflection

On the final day of the task, ask the students to make up their own multiplication problems for their partner to solve.

Set the problem: Tell the students they are going to pretend to be tomato growers. They are to decide how many rows of tomato plants they want in each row and how many rows they will have altogether.

Now they challenge their partner to see if the partner can work out how many tomato plants they will have altogether. Ask for volunteers to share their problems and the partner share what strategy they used to solve the problem.

Challenge the students to make up 2 more problems for their partner. Suggest that they solve the problem using an array.

Conclude the lesson by talking about the types of problems we have explored and solved over the past few days. They were doing multiplication problems! Let them know there are many ways of solving these problems and that the array is just one of these ways.

FORMATIVE ASSESSMENT QUESTIONS

- What are two strategies you used to solve the problems?
- How can the same problem have two different arrays that are correct?
- How does an array model show repeated addition?
- Can you think of a quicker way to work out how many _____ there are?
- How many ____ are there in one row?
- Do you know what 6 + 6 =? Can that help you solve this problem quicker?
- What if _____ had 9 rows of _____ and there were 8 _____ in each row?
- You used adding to work that out. How could you have used mulitiplication?
- If 2 x 6 = 12, what does 3 x 6 =? How could you work out 6 x 6 from this?
- Awesome, clever you, could you work out 9 rows of 6 for me?

DIFFERENTIATION

Extension

- Replace Station Task amounts with greater numbers.
- Provide larger numbers and challenge the students to create as many possible arrays as they can for each farm

Intervention

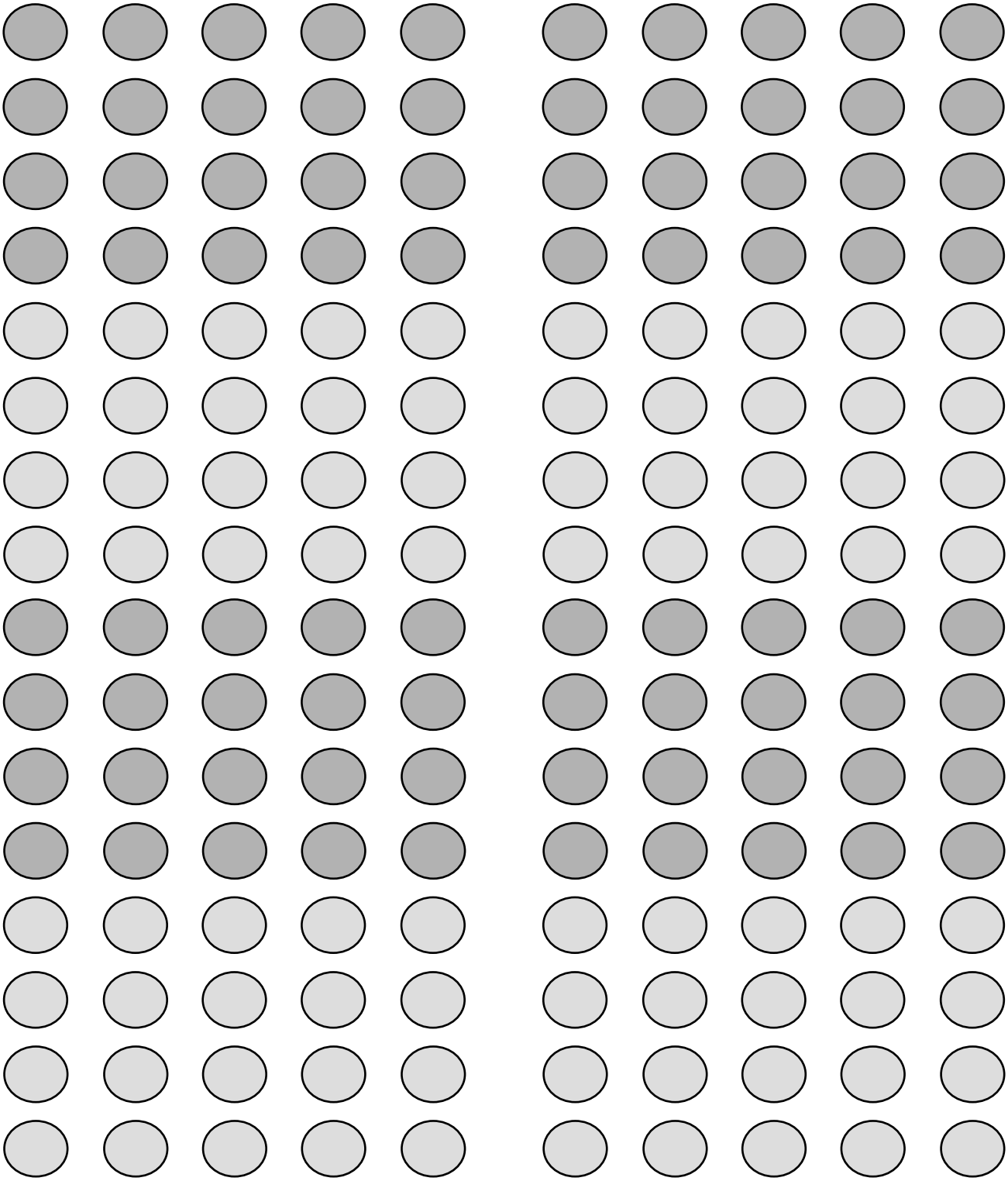
- Replace Station Tasks with lower amounts
- Provide Station Tasks with partial arrays that the student needs to complete

TECHNOLOGY CONNECTION

http://www.multiplication.com/
Practice games for multiplication facts as well as teacher resource pages with instructional ideas on how to introduce multiplication.
Note: This site contains advertising.

Make arrays and see the associated fact
http://www.haelmedia.com/OnlineActivities_txh/mc_txh3_002.html

Array of Circles



Arrays on the Farm: Task Station Cards (enlarge)

Peanut Farm (Part 1)

Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all?

Prediction

Solution

Peanut Farm (part 2)

Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant?

How many peanut plants should be in each row?

Show all the different ways Paul could plant his peanut plants in rows by making your own array on a blank piece of paper.

Onion Farm (Part 1)

Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm?

Prediction

Solution

Onion Farm (part 2)

Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son?

Draw all the possibilities

Watermelon Farm (Part 1) 

Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are 4 rows. How many watermelon plants does she have in all?

Prediction

Solution

Watermelon Farm (Part 2)

Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after…

1 year

2 years

3 years

Record your strategies on an array

Airplane Rides Over the Farm (Part 1)

Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold?

Prediction

Solution

Airplane Rides Over the Farm (Part 2)

Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane?

Prediction

Solution

Name _____

Arrays on the Farm
Student Recording Sheet

I. Station Tasks

Stations	Initial when Completed
Peanut Farm	
Onion Farm	
Watermelon Farm	
Delta's New Plane	

Peanut Farm (part 1)
Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all?

Prediction:

Show your math language and work to find the answer here

Peanut Farm (part 2)
Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant?

How many peanut plants should be in each row?

Show all the different ways Paul could plant his peanut plants in rows by making your own array on a blank piece of paper.

Draw all the ways you can think of here

Onion Farm (Part 1)

Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm?

Prediction

Show your math language and work to find the answer here

Onion Farm (part 2)

Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son?

Draw all the ways you can think of here

Watermelon Farm (Part 1)

Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are
4 rows. How many watermelon plants does she have in all?

Prediction

Show your math language and work to find the answer here

Watermelon Farm (Part 2)

Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after…

Record your strategies on an array

1 year

2 years

3 years

Airplane Rides Over the Farm (Part 1)

Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold?

Prediction

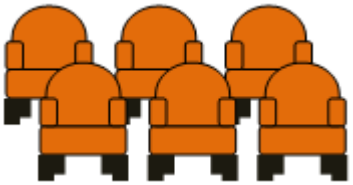
Show your math language and work to find the answer here

Airplane Rides Over the Farm (Part 2)

Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane?

Prediction

Show your math language and work to find the answer here



CONSTRUCTING TASK: Seating Arrangements

1 Day

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 62)

"In the beginning, students will be able to use the same models – sets and number lines – for all four operations. A model not generally used for addition but extremely important and widely used for multiplication and division is the array. An array is any arrangement of things in rows and columns, such as a rectangle of square tiles or blocks.

To make clear the connection to addition, early multiplication activities should also include writing and addition sentence for the same model."

ESSENTIAL QUESTIONS

- How does understanding the commutative property help us create arrays?
- How does drawing an array help us think about different ways to decompose a number?

MATERIALS

- "Seating Arrangements" recording sheet
- Grid paper, if needed
- Manipulatives, if needed

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will solve a word problem requiring them to make arrays using the number 24.

You may want to provide grid paper or have students draw the arrays on plain copy paper. Students should develop the following arrays: 1 x 24, 2 x 12, 3 x 8, 4 x 6, 24 x 1, 12 x 2, 8 x 3, and 6 x 4. As students examine both the 4 x 6 array and the 6 x 4 array, for instance, help them understand that while both arrays have the same area, their orientation can make a difference. For example, when arranging chairs in a room, the shape of the room could dictate whether there are 6 rows of 4 chairs or 4 rows of 6 chairs.

Task Directions

Students will follow the directions below from the "Seating Arrangements" recording sheet.

Your class is going to have a special presentation and your teacher has asked you to figure out a good way to place 24 chairs in your room for seating. There is only one requirement. All the chairs must be placed in an array.

1. Draw pictures to show all the ways you can arrange the chairs in an array.
2. Label and write matching number sentences for each array.
3. Choose your favorite arrangement and explain why you think it would be the best arrangement so that every student could see the presentation.

FORMATIVE ASSESSMENT QUESTIONS

- Explain how you built each array.
- With 24 blocks, can you have an array with 7 in each row? Why or why not?
- Is there a way to determine the measurements of an array for 24 without building it with blocks or drawing a diagram?
- How many different solutions do you think there are to this problem? Is there a way to check to see if you have found all possible solutions?

DIFFERENTIATION

Extension

- Using 24, or another appropriate number, have students multiply to find the number of chairs needed for 2, 3, 4, 5, and 6 third grade classrooms that use twenty-four chairs each. Ask students to develop a strategy to solve the problem. Then allow students to share their strategies.
- Replace 24 chairs with 30, 36 or 72 for students who wish to work with larger numbers.

Intervention

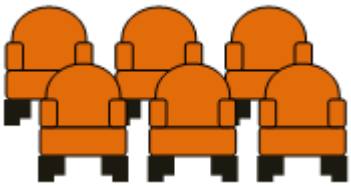
- Replace 24 with a smaller number such as 12, 18 or 20.
- Model this task or a similar one in a small group setting.

TECHNOLOGY CONNECTION

http://illuminations.nctm.org/LessonDetail.aspx?id=U109 Numerous ideas for introducing multiplication, including the array model.

Name _____ Date _____

Seating Arrangement



Your class is going to have a special presentation and your teacher has asked you to figure out a good way to place 24 chairs in your room for seating. There is only one requirement. All the chairs must be placed in an array.

1. Draw pictures to show all the ways you can arrange the chairs in an array.
2. Label and write matching number sentences for each array.
3. Choose your favorite arrangement and explain why you think it would be the best arrangement so that every student can see the presentation.

PRACTICE TASK: Array-nging Our Fact Families
1 Day

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, pages 62-63)

"Students can benefit from a few activities with models and no context. The purpose of such activities is to focus the meaning of the operation and the associated symbolism.

Make sure to draw students' attention to the dimension of the rectangles (length and width). You want students to make the connection that the factors in the multiplication expression they have written indicate the number of rows and columns (the dimensions) in a rectangle that consist of the given number of squares."

ESSENTIAL QUESTIONS

- How are multiplication and division related?
- How can the same array represent both multiplication and division?

MATERIALS

- Grid paper
- Colored pencils or markers
- "Array-nging Our Fact Families" recording sheet

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will make models on grid paper of arrays that show both multiplication and division number sentences. This task makes important connections between multiplication and division. Students will become familiar with division as the inverse operation of multiplication as they learn that the numbers in a multiplication sentence can also be used in a related division sentence.

PART 1
Give students 12 blocks that represent the total area of an array. Have them arrange the blocks in an array and identify the dimensions of their array, noting different arrays are possible for 12. Then ask if there is a way they can make a division sentence with the dividend represented by the total area of the array. For example, a student may make a 4 x 3 array. The dividend (area of 12) can be divided by 4 or 3, both factors of 12. Both dimensions are utilized, one as the divisor and the other as the quotient.

PART 2
Task Directions
Students will follow the directions below from the "Array-nging Our Fact Families" recording sheet.

1. Draw the following arrays:
   6 by 3
   4 by 8
   2 by 7
2. Use the example to complete the following for each array:
   - Label the dimensions and total area.
   - Write a multiplication sentence and tell the factors and the product.
   - Write a division sentence and indicate the divisor, dividend, and quotient.
3. Select one of your arrays and write two story problems that can be modeled with the array, one for multiplication and one for division.

FORMATIVE ASSESSMENT QUESTIONS

- How can you describe your array?
- How does the array show both multiplication and division?
- What does the word "by" mean in the directions (i.e. 6 by 3)?

- What is the difference between a factor and a product? With what operation would you use these words?
- Explain the meaning of the divisor, dividend, and quotient in a division sentence?

DIFFERENTIATION

Extension

- Have students build an array of their choice and have a partner describe the dimensions and area of the array and all related vocabulary relating to both multiplication and division.
- Have students build arrays for multiplication and division that involve larger numbers. Limit the dimensions to a three-digit number times a one-digit number.

Intervention

- If students are not ready to transition to grid paper without the use of the base-ten blocks, allow the use of these manipulatives to guide student work.

TECHNOLOGY CONNECTION

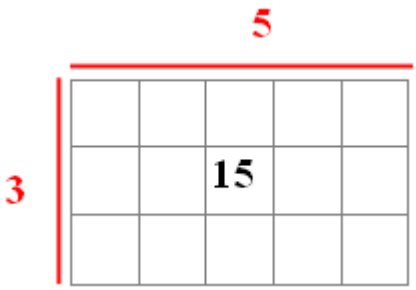
http://www.eduplace.com/math/mw/background/3/08/te_3_08_overview.html Provides background information on the relationship between multiplication and division.

Name _____ Date _____

Array-nging Our Fact Families

This 3 by 5 array has a total of 15 square units.
3 x 5 = 15.

3 and 5 are **factors**.
15 is the **product**.



Fifteen divided by three equals five.
15 ÷ 3 = 5
15 is the **dividend.**
3 is the **divisor.**
5 is the **quotient.**

1. Draw the following arrays listed in the table below.
2. Following the example above, complete the following for each array:
   - ☑ Label the dimensions and total area.
   - ☑ Write a multiplication sentence and label the factors and the product.
   - ☑ Write a division sentence and label the divisor, dividend, and quotient.

6 by 3	4 by 8	2 by 7

3. Select one of your arrays. On the back of this paper, write two story problems that can be modeled with the array, one for multiplication and one for division.

CONSTRUCTION TASK: Family Reunion

1 Day

STANDARDS FOR MATHEMATCIAL CONTENT

Understand properties of multiplication and the relationship between multiplication and division.

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 61)

Students need a good understanding of how to manipulate pattern blocks in order to solve tasks. Students should have had prior experiences with the manipulatives; they should be aware of how to use the blocks as a tool for problem solving. Emphasize the connection between multiplication and division in these tasks. Students should also have an understanding of what to do with remainders. Sometimes remainders can be discarded, could force the number to the next highest whole, or rounded to the next whole for an approximate answer. In this activity they may experience someone being left out. Students need to understand how the remainder will be handled. In the case of seating, the students will have to force the number to the next whole.

ESSENTIAL QUESTIONS

- How is the associative property of multiplication used in solving a problem?
- How can multiplication and division be used to solve real world problems?
- How can we use patterns to solve problems?

MATERIALS

- Item cards for Part I (printed on three different color paper or write them on three different colors of index cards)
- "Family Reunion" recording sheet
- Pattern Blocks

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task will be in two parts: Part I Introducing and scaffolding the Associative Property and then in Part II will be constructing the associative property.

Part I task, students will be giving the three integers and asked to find the product. Students will be able to determine that the arrangement of the integers do not affect the product.

Task Directions
Students will be given an item card containing one type of food they will need to pack for the family reunion.
1. They have to determine how much of each item is needed and how to pack the coolers.
2. Distribute the item cards (one per student)
3. Challenge the students to find out how much of each type of food to pack. There will be 36 people at the family reunion.
4. Create a three columned chart where students can record their findings, label each column: Over the Amount, Close to the Amount, Not Enough.

Over the Amount Needed	Close to the Amount Needed	Not Enough
Ex. (5 packages x 8 hot dogs) x 2 coolers = (2 coolers x 5 packages) x 8 hot dogs = 80 hot dogs	Ex. (3 packages x 8 hot dogs) x 2 coolers = (2 coolers x 3 packages) x 8 hotdogs = 48 hotdogs	Ex. (2 packages x 8 hot dogs) x 2 coolers = (2 coolers x 2 packages) x 8 hotdogs = 32 hot dogs

If needed, just distribute the card for hot dogs and work on solving this problem independently, then distribute the other food item cards and separate into groups. (This may be used for remedial groups.)

5. Ask students to get in groups of three. Each student must have a different type of card (one food, one package and one cooler card in each group.)
6. If they multiply their items, will they have enough, not enough or just the right amount of everything for everyone to have one of each item.
7. Challenge the groups of three to see if they can get a different product by switching places. Ask them if it matters what position they are in?
8. Students can record the number sentences and products either on chart paper, smart board or math notebook.
9. Allow time for the students to determine what the best equation will be for each food group.
10. Discuss the findings and how they figured out the different amounts.
11. Discuss the associative property and how it will affect the product when multiplying three integers.
12. Challenge: find how the equations would change if everyone at the family reunion would take two of each item, three… etc.

Part II task, students will use models of tables to decide how many tables must be used to seat a given number of guests.

Task Directions
Students will follow the directions below from the "Family Reunion" recording sheet.

1. Help set up tables for your upcoming family reunion. Thirty-six relatives need a place at a table to sit and enjoy their food and drinks. You may use the following table styles:
    - Square tables that seat one person to a side for a total of four people at a square table.
    - Circular bistro tables that seat exactly three people.
    - Hexagonal tables that seat one person to a side for a total of 6 people.
    - Rectangular tables that seat twelve people.
    - Pentagonal tables that seat one person to a side for a total of five people.
2. Which table would you need the most? Show how you figured out how many of those tables you would need.
3. Which table would you need the least? Show how you know.
4. Choose two types of tables and draw your method for seating all 36 relatives for the family reunion. Write a number sentence to describe what you've drawn.
5. Suppose the only tables you had were pentagonal. Explain how you would seat all of your relatives.

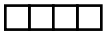
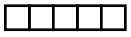
FORMATIVE ASSESSMENT QUESTIONS

- What combinations of tables have you tried so far?
- How will you know when you find the right combination?
- Do you think there is more than one right solution for this task? Why do you think so? Do you have a way of finding out?
- How many _____ (square, circular, hexagonal, rectangular, or pentagonal) tables do you need? How do you know?

DIFFERENTIATION

Extension

- Use square tables that seat one person to a side, but this time push the tables together end to end and find out how many relatives can be seated. Continue adding tables this same way until you have enough tables to seat everyone. Enter the information in a table and describe any patterns you see. How many square tables pushed end to end would it take?

# Tables	# People Seated	Sketch	Number Pattern
1	4		(1 x 4) - 0 = 4 (1 x 2) + 2 = 4
2	6		(2 x 4) – 2 = 6 (2 x 2) + 2 = 6
3	8		(3 x 4) – 4 = 8 (3 x 2) + 2 = 8
4	10		(4 x 4) – 6 = 10 (4 x 2) + 2 = 10
5	12		(4 x 5) – 8 = 12 (5 x 2) + 2 = 12
. . .			
17	36		(17 x 2) + 2 = 36

Two possible number patterns are shown.
The first is the number of seats for the tables, minus the sides lost when tables are pushed together.
The second pattern is the number of seats along the top and bottom plus the seat at each end.

- Choose another pattern block shape and see if the same pattern holds as you push the tables together.
- Experiment to see if it will take more or less tables if a hole is left in the center or if all tables touch another table on all sides except the side where the guests will sit.
- Use a different number of relatives or allow students to make up additional types of tables (octagonal, rhomboidal, triangular, or trapezoidal).
- Rather than two types of tables, let students use three types that still yield seating for 36 people.

Intervention

- Use a smaller number of relatives, such as 12 or 20.
- Guided practice that simulates the task, done ahead of time, will enable students to develop problem solving strategies, particularly if the teacher models the strategies students are developing.

TECHNOLOGY CONNECTION

- http://www.arcytech.org/java/patterns/patterns_d.shtml Allows students to work with pattern blocks in an interactive applet and easily print their work.

Item cards for Part I

1 package	2 packages
3 packages	4 packages
5 packages	6 packages
7 packages	8 packages
9 packages	10 packages

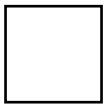
1 cooler	2 coolers
3 coolers	4 coolers
5 coolers	6 coolers
7 coolers	8 coolers
9 coolers	10 coolers

Hamburgers (8 in a package)	Hamburger Buns (4 in a package)
Hot Dogs (7 in a package)	Hog Dog Buns (3 in a package)
Bag of Individual Chips (5 in a package)	Juice Drinks (10 in a package)
Sodas (6 in a package)	Ice Pops (9 in a package)

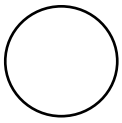
Name _____ Date _____

Family Reunion

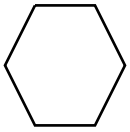
1. Help set up tables for your upcoming family reunion. Thirty-six relatives need a place at a table to sit and enjoy their food and drinks. You may use the following table styles:



Square tables that seat one person to a side for a total of four people at a square table

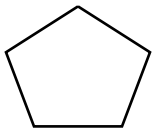


Circular bistro tables that seat exactly three people



Hexagonal tables that seat one person to a side for a total of 6 people

Rectangular tables that seat twelve people



Pentagonal tables that seat one person to a side for a total of five people

2. Of which table would you need the most? Show how you figured out how many of those tables you would need.

3. Of which table would you need the least? Show how you know.

4. Choose two types of tables and draw your method for seating all 36 relatives for the family reunion. Write a number sentence to describe what you've drawn.

5. Suppose the only tables you had were pentagonal ones that only seat five people per table. Explain how you would seat all of your relatives.

SCAFFOLDING TASK: Finding Factors

Approximately 3-4 Days

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 64)

"Students look for patterns in the factors they find for numbers, such as the number of factors, the type of factors, the shape of the resulting array, and so on. Rather than always assigning numbers that have several factors, this activity suggests including numbers that have only a few factors so that differences between numbers become more distinct."

Make sure to draw student's attention to the dimensions of the rectangles (length and width). You want students to make the connections that the factors in the multiplication expression they have written indicate the number of rows and columns (the dimensions) in a rectangle that consist of the given number of squares. Your class will undoubtedly want to decide if a rectangle that is 3 by 8 should be counted differently from one that is 8 by 3. Leave the decision to the class but take advantage of the opportunity to discuss how 3 rows of 8 are the same as 8 rows of 3. Note that if sets rather than arrays are made, 3 sets of 8 look very different from 8 sets of 3.

ESSENTIAL QUESTIONS

- How can multiplication products be displayed on a multiplication chart?
- What strategies can be used to find factors or products?
- How does the order of the digits in a multiplication problem affect the product?

MATERIALS

Part I:
- Square tiles
- Finding Factors with Arrays Student recording Sheet

Part II:
- Counters
- Blank Multiplication chart
- Right angle guide (on colored card stock)
- Colored card stock cut out or Paper dessert plates or coffee filters or index cards
- Finding Factors Using Equal Sets Student Recording Sheet (will also use for Multiplication Chart Mastery Task)

GROUPING

Small Group/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I:

Begin the lesson by writing 6 x 4 = 24 on the board/overhead/smart board. Ask three volunteers to label the two factors and the one product. Discuss and review the terms from the task before to ensure retention. You may do a few together then ask the students to write their favorite multiplication sentence (expression) on their desk/wipe off board/ paper and label. Circulate and check for comprehension.

Next, write a large number 24 on the board. Ask the class to think if there is one factor, or more than one factor for the number 24. Is there more than 1? 2?. Ask for three predictions for how many factors there are for the number 24. After taking the predictions, use square tiles (square magnets, square image on the smart board) to have students model their thinking. Have three sets of 24 prepared and ask three volunteers (or have a set for each student/pair of students) and ask them to create an array using the 24 tiles. Show the student where and how to label the length and width. Share the results. Discuss commutative property. Decide, as a class, if 4 x 6 and 6 x 4 are the same. If they did not find all the factors, prompt to see if they can find more ways. There are four arrays for 24: 1 x 24 or 24 x 1, 2 x 12 or 12 x 2, 3 x 8 or 8 x 3 and 4 x 6 or 6 x 4. Students can draw and label the different arrays on the board. Identify the factors and the product. Students can then record the different arrays for the product 24 on their Finding Factors with Arrays Student Response Sheet.

Part II:

Divide the students into pairs or small groups of mixed abilities. Prepare stations (you may need multiple) with bags of 12, 18, 30 and 36 square tiles. Allow the students to create arrays for each product. They may draw and label the arrays on their Finding Factors Student Recording Sheet. Circulate and encourage with questions such as: Is there another way to create an array with your product? How many factors do you think your product will have? Is there a difference between these two arrays? Can you explain why these arrays are/aren't different?

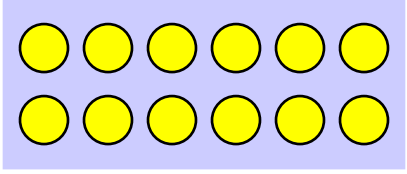
Conclude with sharing the arrays and discuss the student's findings. End the lesson with reviewing factors and products.

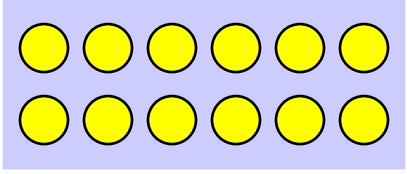
Part III:

Find Factors using Equal Sets Student Recording Sheet to build the multiplication chart.

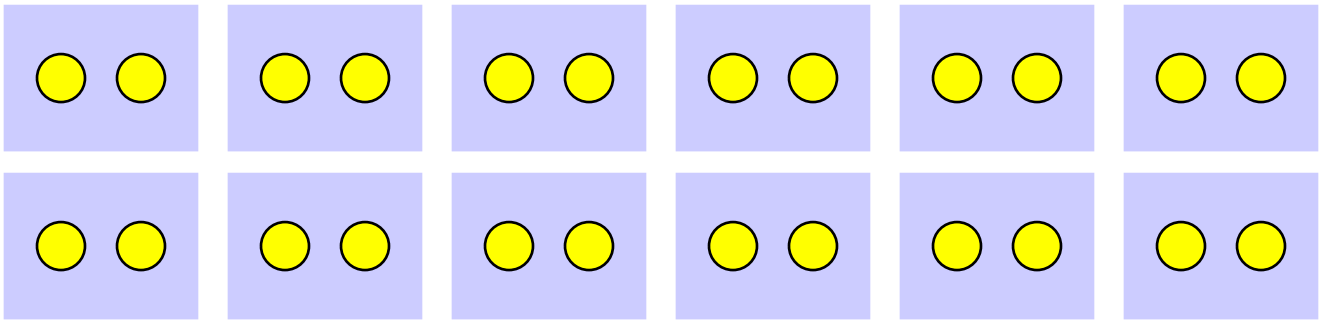
Students will continue to build on the concept of factors and products using counters and index cards/coffee filters/ paper dessert plates or the cut out from the right angle guide. The teacher will model with the number 24. Using two sets of 24 counters (one for each student or you can choose to use the smart board) the teacher will ask two volunteers to bring their Student Response Sheet from yesterday. These students will decide on one multiplication fact whose product is 24. For example, the students choose 2 x 12 = 24 or 12 x 2 = 24. Each student selects a factor and receives that amount of cards (index, cut out from right angle guide, coffee filter, paper dessert plate) and 24 counters. Ask each student to evenly divide the counters onto each card. Ask each student to use math language to create a division sentence and multiplication sentence for their model.

Student 1: Choose Factor 2 (receives 2 cards, divides 24 counters evenly)





Student 2: Choose Factor 12 (receives 12 cards, divides 24 counters evenly)



Discuss with the class who has 2 groups of 12 and who has 12 groups of 2. Provide time to discuss if this is the same or different and why. Discuss the commutative property and the relationship that multiplication has with division. Repeat with new volunteers and the remaining factors for 24. Create and record the multiplication expressions and corresponding division sentences for the factors of 24. Discuss what the division sentences have in common. (The product from the multiplication expression is first in the division sentence.)

Divide the students into pairs or small groups of mixed abilities. Prepare stations (you may need multiple) with bags of 12, 18, 30 and 36 counters and up to 36 index cards (coffee filters, cut outs, paper dessert plates). Allow the small groups or partners to go to each station to model the different factors using the counters and index cards (or what you have chosen to be the designated grouping). They may draw and label grouping on their Finding Factors Student Recording Sheet from the previous task (if the students are working the same groups as Part 1 or new sheets for today's grouping). Circulate and encourage with questions like:

- *Is there another way to create a multiplication sentence with your product?*
- *How many factors do you think your product will have?*
- *Is there a difference between these two facts?*
- *Can you explain why or why not these facts are different?*
- *What does the division sentence look like?*
- *What do you notice about how you arranged the division sentence?*

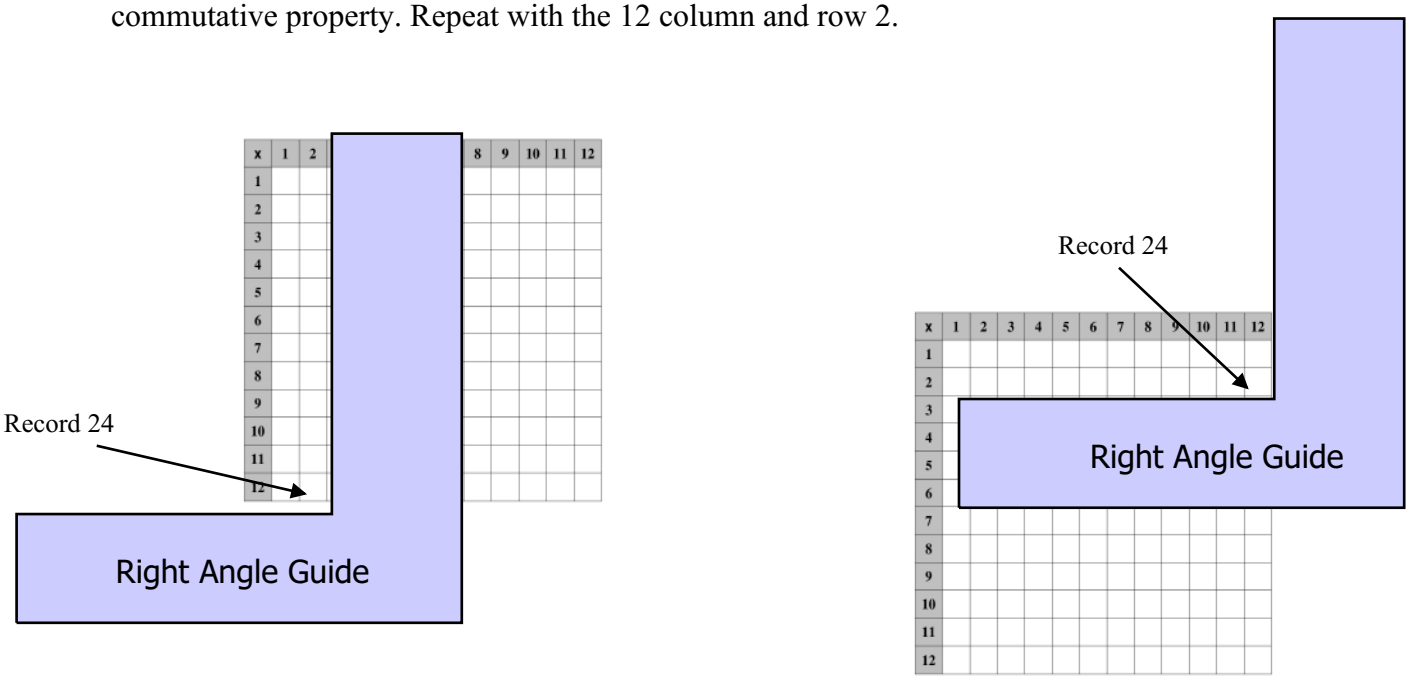
Part IV

Building the Multiplication Chart using the factors from the numbers 12, 18, 24, 30 and 36.

Today the students will begin recording the products on the multiplication chart. The teacher will demonstrate using a whole class setting or may choose to use a small group setting while students are finishing work from Part I or Part II. Students will use the student response sheets from both days or they may choose to use one strategy.

Starting with the 2 the teacher will line up the vertical Right Angle Guide with the two column. Ask the class what was the other factor that multiplies with 2 to get the product 24. Find the row with 12 and line up the horizontal Right Angle Guide. This will show the student where

to write the product 24 in the multiplication chart. Discuss with the class or small group the commutative property. Repeat with the 12 column and row 2.



Continue to complete the factors of 24 as a class or with the small group. Circulate and check for comprehension. The students may work in pairs or individually to complete the facts for 12, 18, 30 and 36.

FORMATIVE ASSESSMENT QUESTIONS

- How can multiplication products be displayed on a multiplication chart?
- What strategies can be used to find factors or products?
- How does the order of the digits in a multiplication problem affect the product?
- How does a division sentence use the factors and product from a multiplication expression?

DIFFERENTIATION

Extension

- Provide the student with larger numbers to determine factors
- Provide the student with odd numbers, lead to discussion of remainders

Intervention

- Provide the student with one of the two factors
- Provide the student with two factors to determine the product using one strategy
- Focus on one strategy in both lessons
- Some students will need to fill every box within the multiplication chart.
- For a visual, see the Square of Pythagoras- http://moosehuntress.blogspot.com/2011/03/square-of-pythagoras-first-presentation.html

NAME _____ DATE _____

Multiplication Chart

x	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

NAME _____ DATE _____

Finding Factors with Arrays
Student Recording Sheet

Directions:
Record as many arrays you can for each number given.
Label each array with factor, factor and product.

Product _12__

Product 18

Product 24

Product 30

Product 36

Checklist: List the factors for each product									
12									
18									
24									
30									
36									

NAME _____ DATE _____

Finding Factors with Equal Sets
Student Recording Sheet

Directions:
Record as many equal sets you can for each number given.
Label each set with factor, factor and product.
Write the multiplication and division sentences for each number.

Product _12__

Product 18

Product 24

Product 30

Product 36

Checklist: List the factors for each product									
12									
18									
24									
30									
36									

CONSTRUCTING TASK: Making up Multiplication
3-4 Days

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson. *****

BACKGROUND KNOWLEDGE

The basic concept of multiplication is an important one because of it practicality (how much do 5 shirts cost at $10 each) and efficiency (it is quicker to determine 5 x 10 than 10 + 10 + 10 + 10 + 10). Multiplication is used in a number of situations. Here we think about multiplication as repeated addition and in rate problems, comparison problems and array problems. This latter type of problems helps students to see the commutative property of numbers, the order of the numbers in not important in the multiplication of two numbers.

ESSENTIAL QUESTIONS

- How can we write a mathematical sentence to represent a multiplication model we have made?
- How can multiplication help us repeatedly add larger numbers?

- How does the order of the digits in a multiplication problem affect the product?
- Is there more than one way of multiplying to get the same product?
- How can we model multiplication?

MATERIALS

- Interlocking cubes
- Number strip
- Graph paper
- Two ten-sided dice
- Index cards
- Making up Multiplication, Student recording sheet
- Wipe off boards (optional)

GROUPING

Partners/Independent

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

Students should begin by working with several equivalent group/set problems before asking them to pose their own problems. For example: There are 3 cars. Each has 2 people in it. How many people altogether? OR There are 6 fish bowls. Each contains 4 goldfish. How many goldfish are there altogether? OR There are 7 tables. Each table has 4 legs. How many legs altogether?

The students can model these and similar types of problems with:

Interlocking cubes
Jumps on a number strip
Drawing a picture
Creating an array

It is important to link environmental examples of equivalent sets with the idea of multiplication as repeated addition. As well as modeling with manipulatives, students should write the same equation using repeated addition and using multiplication. For example: 2 + 2 + 2 =6 or 3 x 2 = 6.

Identify the factors and product in each multiplication problem you solve with the students. Identify the addend and sum in each repeated addition problem.

Ask the students to make up story problems using the number sentence structure above with different factors and products. The students can write their problems on index cards and swap with other students. The student can solve the student created problem on a wipe off board or paper showing the repeated addition and multiplication sentences. Ask the students to label

the two factors and one product in the problem. Ask for volunteers to share the problem and how they got the answers.

Did all of the student-created problems work? If some did not work due to wording or values too high, collectively decide how to adjust the problem so it is able to be solved. Return the cards to the owners. Have the owners solve their problem on the back of the index card. The students should also label the factors, product, addends and sum. Collect and review for accuracy and content.

Part II: Exploring different types of story problems

First begin with comparison problems *("times as many"),* where possible, using a context familiar to students. Brainstorm with the class and decide on three separate topics to write their comparison problems. (Possible topic: lunch, pets, toys, types of cards, pencils)

For example: Anna has 3 carrots in her lunchbox and Jessie has 3 *times as many* carrots as Anna. How many carrots does Jessie have in her lunchbox?

Have student identify the factors and product in each multiplication problem you solve with the students.

Pose a few questions to the whole group and have students solve collaboratively with their partner and then share their thinking. Ask them to pose their own problems using *"times as many"* in their problem. They can use index cards, wipe off boards or paper to present their own problems to partners to solve. Ask the students to label the two factors and one product in the problem. Share problems, solutions and strategies used.

Next, work with several array problems (equivalent groups). Pose problems such as: The students are lined up in 3 teams. Each team has 6 members. How many students are there altogether? Identify and label the two factors and product.

Students can use graph paper, peg boards, bead strings, or interlocking cubes to create arrays to solve the problems. Students can then create their own array problem for a partner to solve. Ask the students to label the two factors and one product in the problem. Share problems, solutions and strategies used.

Next, work with unknown-factor problems. Allow students to solve a few problems working with their partners, and then ask them to pose their own problems.

If you need 1 car for 5 people, how many cars with you need for 15 people? OR If you need 1 fish bowl for 2 goldfish, how many bowls will you need for 18 goldfish? OR If each table seats 4 people, how many tables will you need for 28 people?

Now ask the students to make up word problems using the unknown-factor problem structure. They can use index cards, wipe off boards or paper to present their own problems to

partners to solve. Ask the students to label the two factors and one product in the problem. They can use repeated subtraction as well as unknown-factor multiplication.

Finally, ask students to share their problems and solutions. Ask if they could write their answers using a division sign.

Part III

Use the Making up Multiplication, Student recording sheet as evaluation.
Teachers may decide to use this each day to record the student's progress by having the student create and solve the type of multiplication story problem after each session. Then use another copy to assess at the end of the task as record of performance. Or the teacher may use as mid-unit assessment.

FORMATIVE ASSESSMENT QUESTIONS

- Explain one strategy you can use to solve a multiplication word problem?
- How can you find the unknown-factor in a multiplication problem?
- What is a factor?
- What is a product?

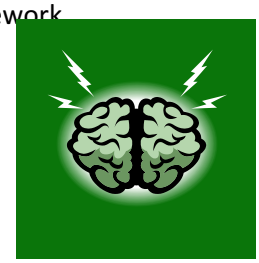
DIFFERENTIATION

Extension

- Create problem using greater values such a two digit numbers multiplied by one digit numbers. (Keeping the product under 100)
- Give the student the product and ask them to find out how many different factors could be used to solve the problem. Which factors would be the most likely? Explain the reasoning and strategy used.

Intervention

- Guided practice that simulates the task, done ahead of time, will enable students to develop problem solving strategies, particularly if the teacher models the strategies students are developing.
- Using manipulatives instead of graph paper
- Use Making Up Multiplication, Student Response sheet with problem already created. The student will solve the problem and label the parts of the number sentence only.



Name _____

Making up Multiplication
Student Recording Sheet

Directions:

A) Write your own a word problem for each type.
B) Solve the problem and show your fabulous math work.
C) Label the parts of the number sentence.

Problem Type 1:
How many times as many: (Comparison Problem)

_____

_____

Problem Type 2:
Array problem (Equivalent groups problem)

_____

_____

Problem Type 3:
If I have…, then I need…problems (Unknown-factor problem)

_____

_____

Name _____

Making up Multiplication
Student Recording Sheet

Directions:

A) Write your own a word problem for each type.
B) Solve the problem and show your fabulous math work.
C) Label the parts of the number sentence.

Problem Type 1:
How many times as many: (Comparison Problem)

Amber has 3 stickers on her notebook, and Jill has 4 times as many stickers on her notebook. How many stickers does Jill have on her notebook?

Problem Type 2:
Array problem (Equivalent groups problem)

Rob has 5 rows of flowers in his garden. Each row has 9 flowers. How many flowers are there in all?

Problem Type 3:
If I have…, then I need…problems (Unknown-factor problem)

If you need 1 box for 6 markers, how many boxes will you need for 24 markers?

SCAFFOLDING TASK: Use What You Know

1 Day to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 123)

This missing-factor approach is likely to be invented by some students if they are solving measurement problems such as the following: "Grace can put 6 pictures on one page of her photo album. If she has 82 pictures, how many pages will she need?" Alternatively, you can simply pose a task such as 82 ÷ 6 and ask students, "What number times 6 would be close to 82?" and continue from there.

ESSENTIAL QUESTIONS

- How are multiplication and division related?
- How is division an unknown factor problem?
- What are the parts of a division problem?

MATERIALS

- Unknown Factor Record sheet
- Counters
- Index cards

GROUPING

Partner/Independent

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Using multiplication facts students either know or can solve using a practiced strategy, students will identify the unknown factor in a division problem, record and label parts of a division problem in both formats. Students will then answer the provided problems as well as create two of their own.

To begin, distribute one index card to each student. Ask them to write one multiplication problem, including the product on the card. (You may want to assign tables, rows, groups etc different numbers. For example one group writes different 4s facts, another writes different 6s facts.) Collect the cards. Choose one card from the stack. Reserve the rest of the stack for partner/independent practice.

Part I:

Write the known fact on the board/overhead/smart board. Discuss the fact and how it can be solved. (Check retention of strategies). Create a story problem from the fact. For example if the fact is 4 x 7 = 28, the story problem could be "28 people are riding the roller coaster. There are four people in each row. How many rows are in the roller coaster?" Provide students (individually or at work groups) counters and blank index cards. Ask them to count out 28 counters (Great opportunity to discuss different strategies to count out 28 counters using counting by twos, threes, fours). Next, ask the students to identify the other given number in the problem. Discuss what has to happen next. Ask for suggestions on how to divide 28 into groups of four. Will they need four index cards to be the roller coaster rows? Will they need to count out four roller coaster riders and put them on an index card to represent each row? Discuss with the class which model would correctly show the problem? Ask if there are any other strategies the students could have used to solve this problem. Ask, what multiplication problem that we already know or are working on would have helped us with the story problem? Discuss, "Well, if we know 4 x 7 = 28, how can we use this fact to find the answer to the problem? Write the division equation and discuss the reasoning.

After the division equation has been written, introduce or reintroduce the parts of a division equation; divisor, dividend and quotient. Label the correct parts.

$$28 \div 4 = 7$$

Dividend is 28
Divisor is 4
Quotient is 7

$$4\overline{)28}\quad 7$$

Repeat, using the student-made multiplication problems from the beginning of the lesson. You, the teacher, can make up the story problems for the first few and then challenge the class to

create a few. Always give the product and one factor to create the story problem. Work it through with volunteers using the counters and index cards to group. Make sure to discuss what known multiplication problem would have helped to solve the story problem. Write the division sentence and label the parts. Allow time and encourage students to share how they solved the story problem.

Part II:

Same day or on the second day, distribute the Unknown Factor Record Sheet. The students can be in partners, working groups or individual. Complete the first problem as a class. Let the students use the counters and index cards to model their math thinking. Allow time for them to explain. Discuss methods used to solve the unknown factor. Label the parts of the problem together. Complete the second as a class or allow students to complete on their own or with partners. After enough time for the students to come up with an answer, call the class back and discuss strategies used to find the unknown factor. Check student work and labeling. Continue as partners to solve the final problems.

Part III:

On the second or third day, hang or place the student written multiplication facts around the room. On the back of the Unknown Factor Recording sheet, have the students divide it into four sections. Each pair (or individual) will tip toe to a multiplication problem somewhere in the room. They must write their own unknown factor problem using the format the class practiced. Instruct them to write the multiplication problem. Write the unknown factor story problem. Write the division equation that solves the problem and label the parts of the division equation. Repeat until the partners or individual has four different unknown factor problems written. Have counters and index cards available if students would like to use them.
Finally, ask for students to share the unknown factor problem they have created and how they solved it. Discuss strategies used. Students can also challenge a class mate to solve their unknown factor problem and model it using the counters and index cards.
Extension: collect student record sheets and create a follow up task or station using the student created unknown factor problems.

FORMATIVE ASSESSMENT QUESTIONS

- What strategy can you use to solve an unknown factor problem?
- Describe how multiplication and division are alike.
- What are the parts of a division problem?

DIFFERENTIATION

Extension:

- Students can create story problems that have a two step process.

Intervention:

- Students may use manipulatives or multiplication chart

Name _____ Date _____

Unknown Factor Stories

Student Record Sheet

Directions:
1. Use the known multiplication facts to solve the unknown factor problem.
2. Write a division equation that solves the problem.
3. Label the parts of the division equation (Dividend, Divisor, Quotient)

Bonus: Write and label both ways to write a division equation

I know 10 x 7 = 70. How can I use that fact to find the answer to this problem? Mr. Adams bought 70 new notebooks for the class. There were 10 notebooks in each package. How many packages did Mr. Adams buy?
I know 4 x 6 = 24. How can I use that fact to find the answer to this problem? Janey would like to put 24 pictures of her friends in a photo album. She can put 4 pictures on each page. How many pages in the photo album will have pictures of her friends on them?
I know 2 x 4 = 8. How can I use that fact to find the answer to this problem? PetSmart has 8 hamsters. How many hamsters will fit in 4 cages?
I know 5 x 7 = 35. How can I use that fact to find the answer to this problem? It took Sam 35 minutes to run each lap. If Sam ran 7 laps, how long did each lap take?
I know 6 x 8 = 48. How can I use that fact to find the answer to this problem? Tanya spent $48 at Old Navy. She bought eight new shirts at the same price. What was the price of each shirt?
I know 3 x 9 = 27. How can I use that fact to find the answer to this problem? Kim spent $27 on candy for her nine friends. How much did Kim spend on each friend?

PRACTICE TASK: Multiplication Chart Mastery

Approximately 2 Days to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

When learning about multiplication, students need a wide variety of experiences and opportunities to explore and discover patterns on their own. Students need a good understanding of how to read rows and columns on a multiplication chart and how to find products using the chart as a tool. Students should also have an understanding of the commutative property.

ESSENTIAL QUESTIONS

- What patterns of multiplication can we discover by studying a times table chart?
- How can we determine numbers that are missing on a times table chart by knowing multiplication patterns?

MATERIALS

- "Multiplication Chart Mastery" recording sheet
- Manipulatives
- Blank Multiplication Chart (partially filled in from Finding Factors Task)

GROUPING

Individual/Small group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will explain and describe the patterns they find in the multiplication chart.

As students discover and verbalize patterns in the multiplication chart, they find more strategies with which to remember multiplication and division facts. The more familiar students become with patterns and predicting successive numbers in patterns, the better prepared they will be for further understanding.

This task would work well as a math conference interview. Consider using it as an assessment during the year, adding, deleting or changing questions as well as parts of the chart to uncover students' thinking and learning. Be sure to make manipulaives available to students who may need them.

Part I:
Students may begin to fill in their own multiplication chart. Challenge them to fill in the facts they know. Discuss what patterns they discover. How will they find the products they are missing?

Part II:
Students will answer the questions on the "Multiplication Chart Mastery" recording sheet. Be sure to give students an opportunity to discuss their answers with peers and the teacher.

FORMATIVE ASSESSMENT QUESTIONS

- What patterns do you notice in the ___ column?
- If you think of 8 x 4 as 8 x 2 doubled, what is the product of 8 x 4? Will this strategy always work? How do you know?

- What strategy could you use to find the products for the eight facts?
- Where are examples of the commutative property on the multiplication chart?

DIFFERENTIATION

Extension:

- Have a student fill in a multiplication chart and purposely put six wrong items. Trade with a partner and try to be the first to identify the incorrect numbers on the chart and make corrections.

Intervention:

- Students compare the multiplication chart in this task with a completed chart. Discuss ideas from the students about ways the charts are similar and different.

Name _____ Date _____

Multiplication Chart Mastery

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18			27	30
4	8	12	16	20	24			36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36			54	60
7	14		28	35		49		63	70
8	16			40			64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

1. Mike filled in this chart to practice his multiplication facts. Which fact does he seem to know best?

_____

How do you know?

_____

_____

2. Mike has all his nines facts correct, even though he has not memorized them. Explain one strategy he might have used to fill in his nines on the chart.

_____

_____

_____

3. Mike is missing some of threes and fours facts. Fill them in for him and explain how you would teach him to find these answers.

4. How could Mike use the fours facts to help him find the eights facts? Fill those in for him and explain your strategy.

5. Mike has done a great job filling in all the numbers on the diagonal. What do you notice about these numbers?

6. Do you see any other patterns on the multiplication chart? Describe at least one.

7. Explain how the commutative property helps you fill in facts on the multiplication chart. Give an example.

CONSTRUCTION TASK: Making the "Hard" Facts Easy

Approximately 2 Days to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 113)

For multiplication, the ability to break numbers apart in flexible ways is even more important than in addition or subtraction. The distributive property is another concept that is important in multiplication computation. For example, to multiply 43 x 5, one might think about breaking 43 into 40 and 3, multiplying each by 5, and then adding the results. Children require ample opportunities to develop these concepts by making sense of their own ideas and those of their classmates.

Third grade students will not be required to use the distributive property for products greater than one hundred.

ESSENTIAL QUESTIONS

- How does understanding the distributive property help us multiply large numbers?
- What are strategies for leaning multiplication facts?

MATERIALS

- Connecting cubes (bags of 100)
- Graph paper
- Making the Hard Facts Easy Recording Sheet
- Crayons or colored pencils
- Scissors
- Glue

GROUPING

Small Group (Part 1)
Partners or Independent (Part 2)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part 1
(Students should be divided into small mixed ability groups)

Today we are going to make those "hard" multiplication facts easy. Before we tackle the hard facts. Who can tell me what the easy multiplication facts are? (Allow groups to discuss) Record or let students come up and record the "easy" multiplication facts on the board/overhead or smart board. Discuss why these are easy? ("Easy" multiplication facts consist of but are not limited to 1s, 2s, 5s, 10s). Understanding that this is an opinion question, allow this to be an open discussion for the students in a respectful environment.

Next, ask the class what are the "hard" facts or facts they haven't yet mastered? (Allow groups to discuss). List the numbers on the board. If there are too many, decide on the two "hardest" facts (generally 7s and 9s). With a quick show of hands, let the class decide which are the "hardest" facts. Use these facts to model for the class. (For this example, the 7s facts will be used, however you will use the facts the class decides upon)

Distribute bags of connecting cubes to each group. Instruct the groups to build seven rows of nine using the connecting cubes OR which ever "hardest fact" your class has chosen. Follow the example using 7 x 9. (But insert your factors) Arrange the seven rows of nine into an array and ask "From our knowledge of arrays, who can say the multiplication expression is represented in this array?" Correct response would be 7 x 9. Acknowledge that this is one of the "hard" facts. Can we break the number 7 into 5 + 2, agree or disagree? Separate two rows of nine from the array you have created. What are the two arrays you have now? (Answer: 5 rows of nine and 2 rows of nine) Using our knowledge of arrays, can you find the product of each array? (5 rows of 9 is 45 and 2 rows of 9 is 18). Now, push the two arrays back together and ask "What

is the total when we push together the two arrays? 45 + 18? Answer 63 so what would be the product of 7 x 9? This is called the Distributive Property. We can break down or distribute one factor in a multiplication problem to create two expressions, then add the products. You may repeat modeling this a second time with a different fact.

Finally, challenge the groups to use the distributive property on their own. Practice with these facts 7 x 8, 7 x 4 and 7 x 3. Circulate and check, asking groups how they distributed the one of the facts. Conclude with a class discussion about the distributive property. How does it work? Did you distribute one or both factors? What do you add together?

Part 2

Making the Hard Facts Easy recording sheet (Choose one or use the blank one to create a sheet specific to each student, or groups depending on level)

Today we are going to continue using the distributive property with the multiplication "hard" facts. Review one or two multiplication facts using the connecting cubes from Part 1. Discuss how the distributive property works. (Distribute one factor into two numbers from the easy facts, create the two arrays and add the products.) We are going to use this distributive strategy using graph paper and the Making the Hard Facts Easy recording sheet. Distribute these sheets to either partners or individuals. Model the first question. Circulate and check for comprehension of the directions. Continue to model the second question or students may complete with partners or individually.

FORMATIVE ASSESSMENT QUESTIONS

- How does the distributive property help us find the product of "hard" facts?
- When can you use the distributive property?

DIFFERENTIATION

Extension:

- Students can create their own multiplication problems to solve using the distributive strategy. (Product does not need to be greater than 100)

Intervention:

- Students can continue to use manipulatives or repeated addition

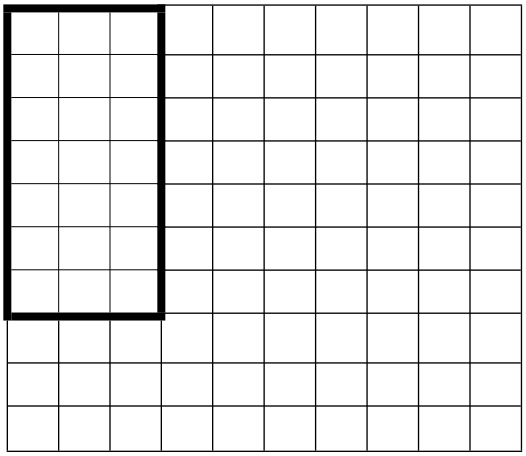
Name __**Sample**_____ Date _____

Making the "Hard" Facts Easy

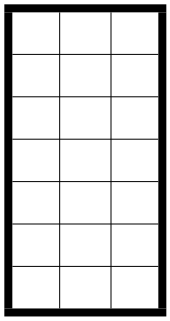
Student Recording Sheet

Example: 7 x 3

1. Outline 7 rows of 3 on graph paper

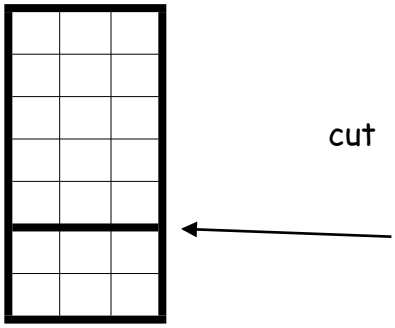


2. Cut out 7 rows of 3 from graph paper



3. Record what two numbers 7 can distribute to _**5**__+ _**2**___= 7

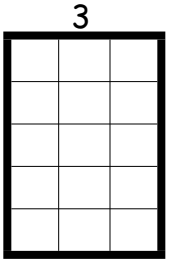
4. Cut 7 rows of 3 to be two arrays, 5 rows of 3 and 2 rows of 3



5. Paste both arrays and label.

3

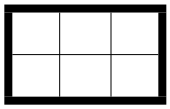
5



= 15

3

2



= 6

6. Record multiplication sentences

5 x 3 = 15
2 x 3 = 6

15 + 6 = 21 so 7 x 3 = 21

Name _____ Date _____

Making the "Hard" Facts Easy

Student Recording Sheet

"Hard" 7s facts

Hard fact _____

1. Outline the array on graph paper.
2. Cut out the array from graph paper.
3. Record the two numbers 7 can distribute to **_5__**+ **_2___**= 7.
4. Cut ___ rows of ___ to be two arrays, __ rows of __ and __ rows of __
5. Paste both arrays and label.
6. Record multiplication sentences.

Name _____ Date _____

Making the "Hard" Facts Easy

Student Recording Sheet

"Hard" 9s facts

Hard fact _____

1. Outline the array on graph paper.
2. Cut out the array from graph paper.
3. Record the two numbers 9 can distribute to ___+ ____= 9.
4. Cut ___ rows of ___ to be two arrays, __ rows of __ and __ rows of __
5. Paste both arrays and label.
6. Record multiplication sentences.

Name _____ Date _____

Making the "Hard" Facts Easy

Student Recording Sheet

My "Hard" facts

Hard fact _____

1. Outline the array on graph paper.
2. Cut out the array from graph paper.
3. Record the two numbers __ can distribute to _____+ _____= __.
4. Cut ___ rows of ___ to be two arrays, __ rows of __ and __ rows of __.
5. Paste both arrays and label.
6. Record multiplication sentences.

PRACTICE TASK: Find the Unknown Number

1 Day to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 123)

This missing-factor approach is likely to be invented by some students if they are solving measurement problems such as the following: “Grace can put 6 pictures on one page of her photo album. If she has 82 pictures, how many pages will she need?” Alternatively, you can simply pose a task such as $82 \div 6$ and ask students, “What number times 6 would be close to 82?” and continue from there.

ESSENTIAL QUESTIONS

- How can you use multiplication facts to solve unknown factor problems
- How are multiplication and division related?

MATERIALS

- Ten sided dice or number cards (1-10) or playing cards Ace through 10
- Find the Unknown Number partner cards (You may want to put this in a sheet protector or laminate for reuse)

GROUPING

Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Partners will take turns rolling the ten sided dice or choosing a card from a face down pile. If Partner 1 can use the number he/she rolled/chose to correctly complete any equation on the "Find the Unknown" partner card, they can record the number in the correct spot. Partner 2 then takes a turn. The partner to fill in the division equations correctly first wins that round. Partners can play again or switch cards with another pair of partners.

FORMATIVE ASSESSMENT QUESTIONS

- How do multiplication facts help in solving division equations?
- What strategy can you use to solve division equations?

DIFFERENTIATION

Extension:

- Create a game board that has two missing components. Use number cards to 100, or use a spinner (top) on a 100s chart. (must place books on each side to keep spinner (top) on the 100s chart.

Intervention:

- Allow student to use multiplication chart or reduce the amount of rounds.

Find the Unknown Number (Partner Card A)

Partner 2

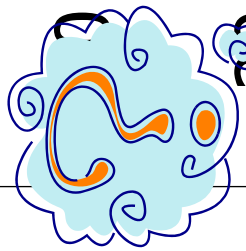
18 ÷ 2 = ____

____ ÷ 1 = 7

8 ÷ 4 = ____

____ ÷ ____ = 6

____ ÷ ____ = 2



Partner 1

12 ÷ ____ = 2

16 ÷ 4 = ____

5 ÷ 5 = ____

18 ÷ ____ = 6

____ ÷ 2 = 4

Find the Unknown Number (Partner Card B)

Partner 2

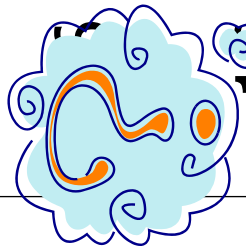
$20 \div 2 = \_\_\_\_$

$\_\_\_\_ \div 1 = 3$

$36 \div 4 = \_\_\_\_$

$\_\_\_\_ \div \_\_\_\_ = 6$

$\_\_\_\_ \div \_\_\_\_ = 2$



Partner 1

$24 \div \_\_\_\_ = 6$

$12 \div 4 = \_\_\_\_$

$6 \div 6 = \_\_\_\_$

$60 \div \_\_\_\_ = 6$

$\_\_\_\_ \div 2 = 5$

Find the Unknown Number (Partner Card C)

Partner 2

32 ÷ 8 = ____

____ ÷ 1 = 7

16 ÷ 4 = ____

____ ÷ 3 = 7

____ ÷ 13 = 2

Partner 1

80 ÷ ____ = 8

32 ÷ 4 = ____

7 ÷ 7 = ____

18 ÷ ____ = 6

____ ÷ 2 = 3

CONSTRUCTING TASK: Oh My Graphing! (OMG)
4 Days to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

MCC3.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson. *****

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 331)

"Bar graphs remain important tools throughout the grades and are frequently seen in newspapers and on the TV news. In grades 3-5, two qualitative differences in their bar graphs can be developed. First, students can easily use centimeter grid paper to construct their own graphs with little guidance. Students can decide on their own scales for their graphs. Labeling of graphs should be less a matter of teacher direction and more a matter of making a graph that communicates and answers the question that is being asked."

"A second element of progression may involve using a single tally or picture element in the graph to represent more than one count. For example, if a graph were being made to show how many seconds different students required to run around the gym floor, using one square of the grid to represent 10 seconds may be useful."

ESSENTIAL QUESTIONS

- How can you display data in a single bar graph?
- How can you display data in a pictograph?
- How does your graph communicate your data?
- What are the steps involved in making and reading graphs?
- How can you use a graph to solve the answer to a question?

MATERIALS

- Our Favorite Sports student record sheet
- Favorite Presents student record sheet
- Favorite Present Family survey sheet
- Skittles student record sheet
- Centimeter grid paper
- 4-5 other teacher's permission to survey their class

GROUPING

Small group, independent

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I: Scaffolding

Today the class will construct single bar graphs using data that is collected and answer questions about the data. They can survey each other to collect data to make a single bar graph representing the class' opinions or the small groups can survey another class. Finally, they will create and answer questions about our class' single bar graph.

Group the students into working groups, no more than four. Introduce the task by asking the students "What do you think are the five most popular sports?" Write student responses on the board or chart paper. Distribute the Our Favorite Sports student record sheet and have the class record what they have determined as the five most popular sports. (At this point you can collect data from your class and use it with an intervention group that needs a more guided lesson. Distribute index cards or post- its and have each student choose their favorite sport from the five determined earlier by the class.) Other small groups can go and survey the pre-arranged classes. Once the working groups have returned, provide materials to each group to begin constructing their single bar graph.

Circulate and check the progress of each group while they are constructing their single bar graph. Each working group does not have to have the same scale as long as the scale works. Prompt groups that choose 1 as their scale to try a different number. After all the steps are completed, let each group share their single bar graph. Let the class ask each group questions. Did every group try a different scale? Which scales worked better?

Finally, ask the groups to look at their single bar graphs. You are going to ask the groups some questions that they can answer with the data from the single bar graphs. If the group knows the answer they must stand up, but they must explain how they got the answer using math language and evidence from the single bar graph.

Questions:

1. Which sport had the most votes?
2. Which sport had the least votes?
3. Which sport had the same amount of votes?
4. How many votes total?
5. How many students voted for _____ and _____?
6. How many students did not vote for _____ and _____?
7. Which sport got half as many votes as _____?
8. Which sport got twice as many votes as _____?

When the working groups have completed their single bar graphs, allow time for each working group to present their single bar graph. When each group has completed their presentation, ask each group to create three questions that could be answered using their single bar graph. Ask each working group to write their questions on a sheet of paper. Next, each working group will ask the class or another working group to answer their questions using the single bar graph. Conclude by sharing some of the questions and how the answers were found. Finally, compare the two sets of data. Which class had more votes for _____ (pick a sport that is on both single bar graphs). How does a single bar graph help you to answer question about a set of data. Display the graphs with questions in the classroom for future reference.

Part II: Constructing

The next two days we are going to continue to construct a single bar graph using data collected from our class as well as construct a picture graph using the same data.

Ask the class to close their eyes and picture a beautifully wrapped present. In their mind they are going to think about what their favorite gift they have received was (or what realistic favorite gift they would like to receive). Distribute index cards or small sticky notes and ask the students to write what favorite gift is in that beautifully wrapped box. Collect the cards/notes. Divide the favorite gifts into categories such as video games, DVD, books, clothes, games, money, technology (iPad, iPod, cell phones) etc. Divide the students into small working groups or partners. Using this data construct a single bar graph. Students may choose to use centimeter grid paper, chart paper or lined paper. Review the steps in constructing a single bar graph from the previous lesson. Circulate and check the progress, asking questions where needed. When the

groups or partners have completed their single bar graph, post the following question for the groups or partners to answer using their single bar graph (or have the question on index cards and give one question to each group, then have the groups explain the question and how they used the single bar graph to find the answer).

1. What is the title?
2. How many different gifts are shown?
3. What is the most popular gift?
4. What is the least popular gift?
5. Do any gifts show a tie?
6. How many people took this survey?
7. List the gifts from most popular to least popular.
8. What other ways could display this data?

A second way to display this data is with a picture graph. Using the data the class has collected about their favorite presents, model the procedure to create a picture graph. You may do this on the board, chart paper or smart board, while the students create one as well on the Favorite Present student record sheet. Discuss the selection of the picture that will represent the chart. Looking at the class' data, what number would be the best by which to count up? Could different numbers be used? Create a key at the bottom of your pictograph. Label each row and fill in the values using the pictures. Decide on a title for your picture graph. Discuss what the class had to do if the value of the picture did not equal the value of the data? Did they draw half of a picture? Repeat the questions above. Do the answers change even though the data did not, while the style of graph did?

For homework, have students take the Favorite Present student record sheet home and interview at least four family members. The next day, the class will present the data they have collected and combine to create a larger set of data. Then, students will work in small working groups or pairs to construct a new pictograph representing the new data. You can choose if the students will construct their pictographs on the Favorite Present student record sheet, chart paper or lined paper. Circulate and check student progress on their pictographs. When the class has completed, allow time for each group to present their pictographs and describe how they constructed them. You can ask the following questions for the students to answer using their pictographs (they can respond verbally or write the answers near their pictograph). After questioning, the students can create their own questions about the pictograph and partner with other groups to answer the questions.

1. How many people participated in this survey? How do you know?
2. How many people chose _____ as their favorite gift?
3. How many more people chose _____ than _____ as their favorite gift?
4. If _____ (insert a double digit number) more people chose _____ as their favorite gift, how would you show that in the pictograph?

Part III: Performance

Today we are going to practice constructing a single bar graph and a pictograph using data collected about Skittles. (Skittles were chosen because it is a peanut free candy). You can choose what works for your class as long as the amount of data is enough to show a variety of one type of item (M&Ms, Runts, Gobstoppers, fruit snacks) According to Google, fun size Skittles contain 25-30 pieces, regular bags contain about 60 pieces and one pound bags contain about 400 pieces. You can decide which amount your class can use to gather data. You can let the class use the Skittles student record sheet or they may use their own methods of collecting and organizing the data. The goal here is to see if they can take data and construct a single bar graph and pictograph that accurately displays the information and can be used to answer questions about the data.

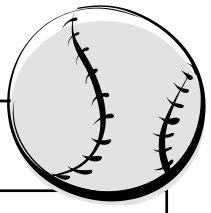
Allow time for students to present and explain their work.

FORMATIVE ASSESSMENT QUESTIONS

- How can you display data in a single bar graph?
- How can you display data in a pictograph?
- How does your graph communicate your data?
- What are the steps involved in making and reading graphs?
- How can you use a graph to solve the answer to a question?

DIFFERENTIATION

Extension: Students can create their own survey with questions about the school or community. (Best cafeteria food, types of books checked out, type of homework). They can present their data in both types of graphs. Share what they have found out about their community.
Intervention: Students can be given graphic organizers to help keep data organized as well as graph paper to allow the data to be filled in.



Name _____ Date _____

Favorite Sports

Favorite Sports	
Sport	Votes
Football	12
Basketball	6
Soccer	4
Baseball	8
Volleyball	6

Using the data in the frequency table, build your own single bar graph.

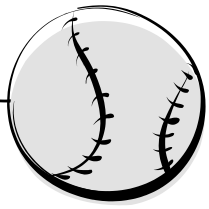
1. Write your title at the top of the single bar graph
2. Decide a number that would be easiest to count up by
3. Create your scale at the bottom of your single bar graph
4. Label each row
5. Fill in the value using a bar

Write three things you know about this class' favorite sports using the single bar graph.

1. _____

2. _____

3. _____



Name _____ Date _____

Our Favorite Sports

Student Record Sheet

Favorite Sports	
Sport	Votes

Using the data in the frequency table, build your own single bar graph.

1. Write your title at the top of the single bar graph
2. Decide a number that would be easiest to count up by
3. Create your scale at the bottom of your single bar graph
4. Label each row
5. Fill in the value using a bar

Write three things you know about this our class' favorite sports using the single bar graph.

1. _____

2. _____

3. _____

Name _____ Date _____

Favorite Presents

Favorite Presents	
Types of Presents	Votes

Using the data in the frequency table, build your own pictograph.

1. Write your title at the top of the pictograph
2. Decide on a picture that represents your pictograph
3. Decide on a number that would be easiest to count up by
4. Create your key at the bottom of your pictograph
5. Label each row
6. Fill in your values using pictures

Write 3 things you know about this class' favorite presents using the pictograph.

1. _____

2. _____

3. _____

Name _____ Date _____

Favorite Presents
Student Record Sheet- Homework

Favorite Presents	
Family Member	Favorite Present

Favorite Presents: Class Family Members Data	
Types of Presents	Votes

To be completed at school: Combine the data from the family member survey. Then using the data in from the entire class, build your own pictograph.

1. Write your title at the top of the pictograph
2. Decide on a picture that represents your pictograph
3. Decide on a number that would be easiest to count up by
4. Create your key at the bottom of your pictograph
5. Label each row
6. Fill in your values using pictures

Write 3 things you know about this class' favorite presents using the pictograph.

1. _____

2. _____

3. _____

Name _____ Date _____

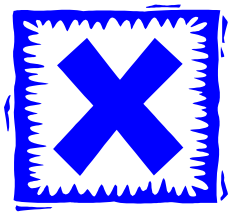
Skittles Graphing
Student Record Sheet

Sort your Skittles by color. Complete the frequency chart with the correct totals

Skittles Color Frequency Chart		
Colors	(Use tallies, addition, multiplication)	Total
Red		
Orange		
Yellow		
Green		
Purple		

Use the following area to create your rough draft of the single bar graph and the pictograph

Graph Checklist	
Appropriate scale	
Colors labeled	
Values labeled	
Title for graphs	
Picture represents graph	



CONSTRUCTING TASK: X Marks the Spot
3 Days to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

MCC3.3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson. *****

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 333)

" Line plots are useful counts of things along a numeric scale. To make a line plot, a number line is drawn and an X is made about the corresponding value on the line for every corresponding data element. One advantage of a line plot is that every piece of data is shown on the graph. It is also a very easy type of graph for students to make. It is essentially a bar graph with a potential bar for every variable."

ESSENTIAL QUESTIONS

- How does your graph communicate your data?
- What are the steps involved in making and reading graphs?
- How can you use a graph to solve the answer to a question?
- When can you use a line plot graph to organize data?

MATERIALS

Line Plot Graph student record sheet
Rulers or measuring tape
Index cards or post-its
Scale
Stopwatch
Possible items to graph (ice, pencils, shoes, backpacks)

GROUPING

Small group or partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Line plot graphs use a number line to display data. They are great for displaying measurement tasks using length, time and weight. This task will give you a few ideas to get started using line plots graphs. Students really enjoy exploring measurement and recording data using a line plot graph.

Measuring Length:

Introduce line plot graphs with the idea of measuring each student's pencil. Review how to use a ruler or measuring tape. Students, working in small groups or pairs will measure their favorite pencil. Record the measurement to the nearest whole number. Distribute one index card or post-it to each student. Next, have the students write the measurement on the card. Then, with the whole number written on the card, ask students to get in numerical order (challenge them to do it without talking). Students will have to decide what to do with the numbers that have more than one. Discuss the challenges or ease it took for the class to get organized. How did they know where to go? How could we display this data using a number line? Continue with creating a number line on the board, smart board or chart paper. Have the students decide what number the number line should start and end with and why it makes sense using the collected data. Create a title for the line plot. Display the line plot graph and create questions to answer using the graph.

Other measurement tasks could be to measure length student's pinkie fingers, shoes, or height. Choose at least one or create your own measurement line plot graph to repeat constructing a line plot graph for student retention or evaluation.

In conclusion, students can choose their own topic, collect data and present their own line plot graph using measurement. They can create a question and ask the class or the class can create questions and ask the student to find the answer using their line plot graph.

FORMATIVE ASSESSMENT QUESTIONS

- How does your graph communicate your data?

- What are the steps involved in making and reading graphs?
- How can you use a graph to solve the answer to a question?
- When can you use a line plot graph to organize data?

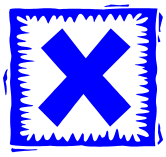
DIFFERENTIATION

Extension: Students can gather data from the entire grade level. Compare and contrast a single bar graph and a line plot graph.

Intervention: Students may use centimeter graph paper to organize data.

Name _____ Date _____

Line Plot Graph
Student Record Sheet



Topic or Item: _____

Data:

Item or Topic	Lengths

Line Plot Graph

FINAL PERFORMANCE TASK: My Special Day!
2 Days to complete

STANDARDS FOR MATHEMATCIAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
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5. Use appropriate tools strategically.
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8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 30)

"Assessment need not look different from instruction. The typical approach of an end-of-chapter test of skills may have some value but it is not appropriate as the main method of assessment. Assessment can and should happen every day as an integral part of instruction. If you restrict your view of assessment to tests and quizzes, you will miss seeing how assessment can help

students grow and inform instruction. "Assessment should focus on what students do know instead of what they do not know" (NCTM, 1989)."

ESSENTIAL QUESTIONS

- How is multiplication and division used to solve a problem?
- When can you use multiplication or division in real life?

MATERIALS

Planning Information sheet

GROUPING

Partner or Independent

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will plan a special day, just for them! They have to keep the cost of their day under $100. Each student will use the Planning recording sheet to decide how many guests will be invited to enjoy their special day with them. Students will figure out the cost of how much food, drinks and items they will need per guest. They have the choice of planning a small group with a lot of items or a large group with not as many items. The goal is to see which students use multiplication and division to determine the cost of the day and which students are using repeated addition or subtraction.

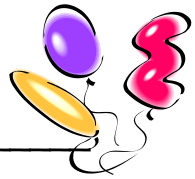
FORMATIVE ASSESSMENT QUESTIONS

How did you use multiplication or division to determine your guest list?

DIFFERENTIATION

Extension: Students could be challenged to create their own list of supplies. Increase the amount of the budget for the day.

Intervention: Provide the amount of guests that are coming for the day and then challenge the students to figure out what they could use on their day.



Name _____ Date _____

Planning Sheet

Student Recording Sheet

You get to plan a special day just for you! You parents are letting you pick what you want to do, eat, drink and who you will invite to share your special day with! Your task is to keep a budget under $100. You can invite whomever you want. You get to make all the decisions!

Use the price lists to find out how much you will spend. You must use at least one item in each category. Happy Planning!

FOOD	Cost per guest
Pizza	$2
Hamburgers	$2
Chicken Fingers	$3
Ribs	$5
Steak	$7
Crab Legs	$8
Japanese Steakhouse	$10

DRINK	Cost per guest
Bottled water	$1
Kool Aide	$1
Lemonade	$2
Fruit punch	$2
Soda	$2
Milkshakes	$3
Rootbeer Floats	$4

ACTIVITY	Cost per guest
Swimming	$3
Roller skating	$3
Bowling	$4
Lazer tag	$6
Limo ride	$8
Waterpark	$10
Amusement Park	$10

FAVORS for GUESTS	Cost per guest
Stickers	$1
Balloons	$1
Yo yo	$2
Frisbee	$2
Silly string	$2
Disposable camera	$6
Gift card	$10