1. Let C be the curve defined by $x^2y = 4$. Which of the following statements is true of curve C at the point $(2, 1)$?

(A) It has a relative minimum because $y' = 0$ and $y'' > 0$.

(B) It has a relative maximum because $y' = 0$ and $y'' < 0$.

(C) It is decreasing and concave up because $y' < 0$ and $y'' > 0$.

(D) It is decreasing and concave down because $y' < 0$ and $y'' < 0$.

$\frac{d}{dx}$ $2xy + x^2y' = 0$

$-2xy \qquad -2xy$

$\frac{x^2y'}{x^2} = \frac{-2xy}{x^2} = \frac{-2y}{x}$

$\frac{d}{dx} = -\frac{2y}{x}$

$y' = -\frac{2(1)}{2}$

$y' = -1 < 0$

$y'' = \frac{x(-2y') + (-2y)(1)}{x^2}$

$y'' = \frac{-2x\left(\frac{-2y}{x}\right) + 2y}{x^2}$

$y'' = \frac{4y}{x^2} = \frac{4}{2^2}$ $\frac{4}{4} > 0$

2. Consider the curve in the xy-plane defined by $x^2 - \frac{y^2}{5} = 1$. It is known that $\frac{dy}{dx} = \frac{5x}{y}$ and $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$. Which of the following statements is true about the curve in Quadrant IV ?

(A) The curve is concave up because $\frac{dy}{dx} > 0$.

(B) The curve is concave down because $\frac{dy}{dx} < 0$.

(C) The curve is concave up because $\frac{d^2y}{dx^2} > 0$.

(D) The curve is concave down because $\frac{d^2y}{dx^2} < 0$.

In QIV $y<0$

2	1
3	4

Therefore $\frac{d^2y}{dx^2} = \frac{-25}{y^3} > 0$

Concave up

5. If $x^2y - \frac{x}{y} = -2$, then $\frac{dy}{dx} =$

xy^{-1}

(A) $\frac{y}{x+x^2y^2}$

(B) $\frac{y-2xy^3}{x+x^2y^2}$

(C) $\frac{y+2xy^3}{x-x^2y^2}$

(D) $\frac{y-2xy^3}{x^2y^2-x}$

$$2xy + x^2\left(\frac{dy}{dx}\right) - \frac{y - x\left(\frac{dy}{dx}\right)}{y^2} = 0$$

$$2xy^3 + x^2y^2\left(\frac{dy}{dx}\right) - y + x\left(\frac{dy}{dx}\right) = 0$$

$$\frac{\frac{dy}{dx}\left(x+x^2y^2\right)}{x+x^2y^2} = \frac{y-2xy^3}{x+x^2y^2}$$

7. Consider the curve in the xy-plane defined by $y\sqrt{y^3+1} = x$ for $y > -1$. For what value of y, $y > -1$, does the derivative of y with respect to x not exist?

$$y'\sqrt{y^3+1} + y\left(\frac{3y^2}{2\sqrt{y^3+1}}\right) = y' = \frac{2\sqrt{y^3+1}}{5y^3+2}$$

(A) $y = \left(-\frac{2}{5}\right)^{\frac{1}{3}}$ ✓

y' will not exist when the denominator equals 0

(B) $y = \left(-\frac{1}{4}\right)^{\frac{1}{3}}$

(C) $y = 0$

$5y^3 + 2 = 0$ when $y = \left(-\frac{2}{5}\right)^{\frac{1}{3}}$

(D) The derivative of y with respect to x exists for all $y > -1$.

9. At how many points on the curve $x^{\frac{2}{5}} + y^{\frac{2}{5}} = 1$ in the xy-plane does the curve have a tangent line that is horizontal?

(A) None

(B) One

(C) Two

(D) Three

Answer C

Correct. To find the number of points at which the tangent line is horizontal, find where the derivative $\frac{dy}{dx}$ is equal to 0. Using implicit differentiation yields $\frac{2}{5}x^{-\frac{3}{5}} + \left(\frac{2}{5}y^{-\frac{3}{5}}\right)\left(\frac{dy}{dx}\right) = 0$, which can be solved for $\frac{dy}{dx}$ to get $\frac{dy}{dx} = \frac{-y^{\frac{3}{5}}}{x^{\frac{3}{5}}}$. The derivative is equal to 0 at points (x, y) on the curve in which $y = 0$ and $x \neq 0$. Letting $y = 0$ in the equation of the curve yields $x^{\frac{2}{5}} = 1$, which has two solutions, $x = -1$ and $x = 1$. The curve has a horizontal tangent line at the two points $(-1, 0)$ and $(1, 0)$ and no others.

10. Which of the following describes the y-coordinates of the points on the curve $e^x = \sin y$ in the xy-plane where the curve has a vertical tangent line?

(A) The values $\frac{\pi}{2} + n\pi$ for all nonnegative integers n

(B) The values $\frac{\pi}{2} + 2n\pi$ for all nonnegative integers n

(C) The values $\frac{\pi}{2} + n\pi$ for all integers n

(D) The values $\frac{\pi}{2} + 2n\pi$ for all integers n

Answer D

Correct. To find the points at which the tangent line is vertical, find where the derivative y' can be written in the form of a quotient in which the denominator is equal to 0 and the numerator is nonzero. Using implicit differentiation yields $e^x = (\cos y)\, y'$, so that $y' = \frac{e^x}{\cos y}$. The numerator e^x is nonzero for all values of x. The points (x, y) on the curve at which the denominator of y' is equal to 0 satisfy $\cos y = 0$, which is true for $y = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$, or $y = \frac{\pi}{2} + n\pi$ for all integers n. However, since the equation $e^x = \sin y$ must also be satisfied, and since $e^x > 0$ for all values of x, the values of y for which $\sin y < 0$ are not coordinates of points on the curve; they must be excluded. That is, only the values $y = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ and $y = -\frac{3\pi}{2}, -\frac{7\pi}{2}, -\frac{11\pi}{2}, \ldots$ satisfy all of the conditions, and these values are represented by $\frac{\pi}{2} + 2n\pi$ for all integers n.