A particle moves on the hyperbola $xy = 18$ for time $t \geq 0$ seconds. At a certain instant, $y = 6$ and $\frac{dy}{dt} = 8$. Which of the following is true about x at this instant?

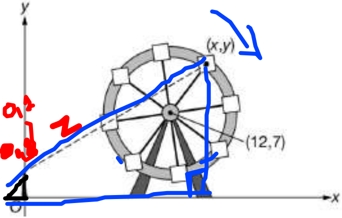
(A) x is decreasing by 4 units per second.

(B) x is increasing by 4 units per second.

(C) x is decreasing by 1 unit per second.

(D) x is increasing by 1 unit per second.

$$xy = 18$$
$$x(6) = 18$$
$$x = 3$$
$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$
$$3(8) + 6\frac{dx}{dt} = 0$$
$$\frac{6\frac{dx}{dt}}{6} = \frac{-24}{6}$$
$$\frac{dx}{dt} = -4$$



$Z = \sqrt{x^2 + (y-2)^2}$

$\frac{dz}{dt}$

General Form Derivative of Pythag. Theorm

$$\boxed{\frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}}}$$

The figure above shows a Ferris wheel with radius 5 meters as Jalen, whose eye level is at point $(0, 2)$, watches his friend, Ashanti, ride in one of the cars as the wheel turns. Let Z denote the distance from Jalen to Ashanti's car. The diagram indicates the center of the Ferris wheel at the point $(12, 7)$ and the position of Ashanti's car at the point (x, y). If x and y are functions of time t, in seconds, what is the rate of change of Z when $x = 15$, $y = 11$, and $\frac{dx}{dt} = 1$? (The equation of a circle with radius r and center (h, k) is $(x-h)^2 + (y-k)^2 = r^2$.)

$(x-h)^2 + (y-k)^2 = r^2$

$(x-12)^2 + (y-7)^2 = 25$

$2(x-12)\frac{dx}{dt} + 2(y-7)\frac{dx}{dt} = 0$

$2(y-7)\frac{dy}{dt} = -2(x-12)\frac{dx}{dt}$

$2(11-7)\frac{dy}{dt} = -2(15-12)1$

$\frac{8\frac{dy}{dt}}{8} = \frac{-6}{8}$

$\frac{dy}{dt} = -\frac{3}{4}$

$$\frac{dz}{dt} = \frac{2x\frac{dx}{dt} + 2(y-2)\frac{dy}{dt}}{2\sqrt{x^2+(y-2)^2}}$$

$$\frac{dz}{dt} = \frac{2(15)(1) + 2(11-2)\frac{-3}{4}}{2\sqrt{15^2+(11-2)^2}}$$

$$= \frac{30 + \frac{-27}{2}}{2\sqrt{225+81}}$$

$\frac{18}{1}\cdot\frac{-3}{4}$

A piece of rubber tubing maintains a cylindrical shape as it is stretched. At the instant that the inner radius of the tube is 2 millimeters and the height is 20 millimeters, the inner radius is decreasing at the rate of 0.1 millimeter per second and the height is increasing at the rate of 3 millimeters per second. Which of the following statements about the volume of the tube is true at this instant? (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

- The volume is increasing by 4π cubic millimeters per second.
- The volume is decreasing by 4π cubic millimeters per second.
- The volume is increasing by 20π cubic millimeters per second.
- The volume is decreasing by 20π cubic millimeters per second.

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 3 \frac{mm}{sec}$$

$$r = 2mm$$

$$\frac{dr}{dt} = -0.1 \frac{mm}{sec}$$

$$h = 20mm$$

$$\frac{dV}{dt} = 2\pi(2)(20)(0.1) + \pi(4)(3)$$

$$-8\pi \quad + 12\pi$$

$$\frac{dV}{dt} = 4\pi$$

4.

x	2.8	3.0	3.2	3.4
$g'(x)$	1.05	−1.2	−0.8	1.3

Selected values of the derivative of the function g are given in the table above. It is known that $g(3) = 17$. What is the approximation for $g(3.2)$ found using the line tangent to the graph of g at $x = 3$?

(A) 16.76

(B) 16.80

(C) 16.84

(D) 17.40

$(3, 17)$

$y - y_1 = m(x - x_1)$

$y - 17 = -1.2(x - 3)$

$y = 17 - 1.2(x - 3)$

$y = 17 - 1.2(3.2 - 3)$

$y = 17 - 1.2(.2)$

$y = 17 - .24$

$y = 16.76$

Let f be a differentiable function such that $f(4) = 7$ and $f'(4) = \frac{1}{5}$. The graph of f is concave up on the interval $(3, 5)$. Which of the following is true about the approximation for $f(3.5)$ found using the line tangent to the graph of f at $x = 4$?

$(4, 7)$

(A) $f(3.5) \approx 6.9$ and this approximation is an overestimate of the value of $f(3.5)$.

(B) $f(3.5) \approx 6.9$ and this approximation is an underestimate of the value of $f(3.5)$.

(C) $f(3.5) \approx 7.1$ and this approximation is an overestimate of the value of $f(3.5)$.

(D) $f(3.5) \approx 7.1$ and this approximation is an underestimate of the value of $f(3.5)$.

$y - y_1 = m(x - x_1)$

$y = 7 + \frac{1}{5}(3.5 - 4)$

$y = 6.9$

6. Let f be a function such that at each point (x, y) on the graph of f, the slope is given by slope $\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{4}y^2$. The graph of f passes through the point $(1, -2)$ and is concave up on the interval $1 < x < 1.5$. Let k be the approximation for $f(1.3)$ found by using the locally linear approximation of f at $x = 1$. Which of the following statements about k is true?

(A) $k = -2.65$ and is an underestimate for $f(1.3)$.

(B) $k = -2.65$ and is an overestimate for $f(1.3)$.

(C) $k = -2.15$ and is an underestimate for $f(1.3)$.

(D) $k = -2.15$ and is an overestimate for $f(1.3)$.

$(1, -2)$

$\frac{dy}{dx} = \frac{1}{2}(1) - \frac{1}{4}(-2^2)$

$\frac{1}{2} - \frac{1}{4}(4)$

$\frac{dy}{dx} = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y = -2 - \frac{1}{2}(1.3 - 1)$

$y = -2.15$