1. A particle moves on the hyperbola $xy = 15$ for time $t \geq 0$ seconds. At a certain instant, $x = 3$ and $\frac{dx}{dt} = 6$. Which of the following is true about y at this instant?

(A) y is decreasing by 10 units per second.

(B) y is increasing by 10 units per second.

(C) y is decreasing by 5 units per second.

(D) y is increasing by 5 units per second.

$$\frac{xy}{x} = \frac{15}{x}$$

$$y = \frac{15}{3}$$

$x = 3$

$\frac{dx}{dt} = 6$

$y = 5$

$$xy = 15$$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$3\frac{dy}{dt} + 5(6) = 0$$

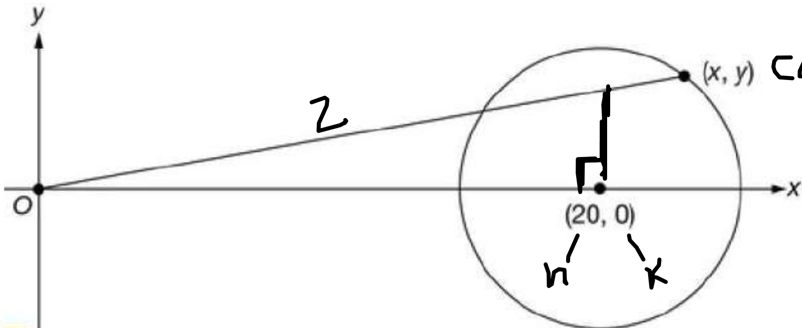
$$3\frac{dy}{dt} + 30 = 0$$

$$\frac{3\frac{dy}{dt}}{3} = \frac{-30}{3}$$

$$\frac{dy}{dt} = -10$$

*The neg. sign indicates decreasing

2.



$x=23$
$y=4$
$\frac{dx}{dt}=2$
$\frac{dy}{dt}=\frac{-3}{2}$

A model car travels around a circular track with radius 5 feet. Let Z denote the distance between the model car and a fixed point that is 20 feet to the left of the center of the circular track. The diagram above indicates the fixed point at the origin, the center of the circular track at the point $(20,0)$, and the position of the car at the point (x,y). Z is the length of the line segment from the origin to the point (x,y). If x and y are functions of time t, in seconds, what is the rate of change of Z when $x=23$, $y=4$, and $\frac{dx}{dt}=2$? (The equation of a circle with radius r and center (h,k) is $(x-h)^2+(y-k)^2=r^2$.)

$(x-h)^2+(y-k)^2=r^2$

$2(x-20)\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$2(23-20)\cdot 2+2(4)\frac{dy}{dt}=0$

$12+8\frac{dy}{dt}=0$ $\quad \frac{dy}{dt}=-\frac{3}{2}$

$\frac{8\frac{dy}{dt}}{8}=\frac{-12}{8}$

(A) ~~$\frac{dZ}{dt} = 0$, so the distance between the model car and the fixed point is constant.~~

(B) $\frac{dZ}{dt} = \frac{40}{\sqrt{545}}$, so the model car is moving away from the fixed point at a rate of approximately 1.7 feet per second.

(C) $\frac{dZ}{dt} = \frac{40}{\sqrt{545}}$, so the model car is moving toward the fixed point at a rate of approximately 1.7 feet per second.

(D) ~~$\frac{dZ}{dt} = 80$, so the model car is moving away from the fixed point at a rate of 80 feet per second.~~

$x = 23$

$y = 4$

$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = -\frac{3}{2}$

$Z = \sqrt{x^2 + y^2}$

$$\frac{dZ}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}}$$

$$\frac{dZ}{dt} = \frac{2(23)(2) + 2(4)\left(\frac{-3}{2}\right)}{2\sqrt{23^2 + 4^2}}$$

5. A tube is being stretched while maintaining its cylindrical shape. The height is increasing at the rate of 2 millimeters per second. At the instant that the radius of the tube is 6 millimeters, the volume is increasing at the rate of 96π cubic millimeters per second. Which of the following statements about the surface area of the tube is true at this instant? (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$. The surface area S of a cylinder, not including the top and bottom of the cylinder, is $S = 2\pi rh$.)

(A) The surface area is increasing by 28π square millimeters per second.

(B) The surface area is decreasing by 28π square millimeters per second.

(C) The surface area is increasing by 32π square millimeters per second.

(D) The surface area is decreasing by 32π square millimeters per second.

Find $\frac{dS}{dt}$ when $r = 6$ $\frac{dV}{dt} = 96\pi$ $\frac{dh}{dt} = 2$

Volume $\rightarrow$ $V = \pi r^2 h$

$96\pi = \frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

$= 2\pi(6) h \frac{dr}{dt} + \pi(36)(2) \rightarrow h\frac{dr}{dt} = 2$

surface area $S = 2\pi r h$

$\frac{dS}{dt} = 2\pi\left(r\frac{dh}{dt} + h\frac{dr}{dt}\right) = 2\pi(6\cdot 2 + 2)$

$= 2\pi(14) = 28\pi$