4. Let x and y be functions of time t such that the sum of x and twice y is constant. Which of the following equations describes the relationship between the rate of change of x with respect to time and the rate of change of y with respect to time?

(A) $\frac{dx}{dt} = 2\frac{dy}{dt}$

(B) $\boxed{\frac{dx}{dt} = -2\frac{dy}{dt}}$

(C) $2\frac{dx}{dt} + \frac{dy}{dt} = 0$

(D) $\frac{dx}{dt} + 2\frac{dy}{dt} = K$, where K is a function of t

$$x + 2y = 0$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} = 0$$

$$-2\frac{dy}{dt} \qquad -2\frac{dy}{dt}$$

$$\frac{dx}{dt} = -2\frac{dy}{dt}$$

5. A right triangle has base x meters and height h meters, where h is constant and x changes with respect to time t, measured in seconds. The angle θ, measured in radians, is defined by $\tan\theta = \frac{h}{x}$. Which of the following best describes the relationship between $\frac{d\theta}{dt}$, the rate of change of θ with respect to time, and $\frac{dx}{dt}$, the rate of change of x with respect to time?

(A) $\frac{d\theta}{dt} = \left(\frac{-h}{x^2+h^2}\right)\frac{dx}{dt}$ radians per second

(B) $\frac{d\theta}{dt} = \left(\frac{h}{x^2+h^2}\right)\frac{dx}{dt}$ radians per second

(C) $\frac{d\theta}{dt} = \left(\frac{-h}{x\sqrt{x^2+h^2}}\right)\frac{dx}{dt}$ radians per second

(D) $\frac{d\theta}{dt} = \left(\frac{h}{x\sqrt{x^2+h^2}}\right)\frac{dx}{dt}$ radians per second

$$\frac{h}{x} = h \cdot x^{-1} = h \cdot -x^{-2}$$

$$\sec = \frac{\sqrt{x^2+h^2}}{x}$$

$$\frac{d}{dt}\tan\theta = \frac{h}{x}\frac{dx}{dt}$$

$$\frac{d\theta}{dt}\sec^2\theta = \frac{-h}{x^2}\frac{dx}{dt}$$

$$\left(\frac{\sqrt{x^2+h^2}}{x}\right)^2$$

$$\frac{d\theta}{dt}\frac{x^2+h^2}{x^2} = \frac{-h}{x}\frac{dx}{dy}$$

$$\frac{\frac{d\theta}{dt}\,\frac{x^2+h^2}{x^2}}{\frac{x^2+h^2}{x^2}} = \frac{\frac{-h}{x^2}\,\frac{dx}{dt}}{\frac{x^2+h^2}{x^2}}$$

$$\frac{-h}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{x^2+h^2} = \frac{-h}{x^2+h^2}$$

$$\frac{d\theta}{dt} = \frac{-h}{x^2+h^2}\,\frac{dx}{dt}$$ radians per second

6. In a certain factory, assume that the number of workers is constant. The number of minutes N that it takes to make a single unit of a product and the number of units U of the product that are made per day satisfy the relationship $U = \frac{k}{N}$, where k is a constant. Which of the following best describes the relationship between the rate of change, with respect to time t, of U and the rate of change, with respect to time t, of N ?

(A) $\frac{dU}{dt} = \frac{k}{\left(\frac{dN}{dt}\right)}$

(B) $\frac{dU}{dt} = \frac{-k}{\left(\frac{dN}{dt}\right)}$

(C) $\frac{dU}{dt} = \frac{k}{N^2}\left(\frac{dN}{dt}\right)$

(D) $\boxed{\frac{dU}{dt} = \frac{-k}{N^2}\left(\frac{dN}{dt}\right)}$

$$U = \frac{K}{N}$$

$$\frac{dU}{dt} = \frac{-K}{N^2} \frac{dN}{dt}$$

$$K \cdot N^{-1} = K \cdot -N^{-2}$$

$$= K \cdot \frac{-1}{N^2} = \frac{-K}{N^2}$$