

1.

$x$	-2	-1	0	1	2
$g(x)$	-4	-1	4	11	20

 $(\frac{1}{2}, 2)$  ✓

The table above gives values of a function  $g$  at selected values of  $x$ . Which of the following statements, if true, would be individually sufficient to conclude that there exists a number  $c$  in the interval  $[-2, 2]$  such that  $g(c) = 6$  ?

- $g$  is defined for all  $x$  in the interval  $[-2, 2]$ . X
- $g$  is increasing on the interval  $[-2, 2]$ .
- $g$  is continuous on the interval  $[-2, 2]$ . }

- (A) II only
- (B) III only
- (C) I and III only
- (D) I, II, and III


 $(-2, -4)$ 
 $(2, 20)$ 


2.

$x$	0	1	2	3	4	5
$f(x)$	1	-5	-4	2	-10	-15

Selected values of a continuous function  $f$  are given in the table above. What is the fewest possible number of zeros of  $f$  in the interval  $[0, 5]$ ?

- (A) Zero, because  $f(x)$  is not equal to 0 for any of the values in the table.
- (B) One, because  $f$  is continuous on the interval  $[0, 5]$  and  $f(0) > 0 > f(5)$ .
- (C) Two, because the values for  $f(x)$  in the table change from positive to negative twice.
- (D) Three, because  $f$  is continuous on the interval  $[0, 5]$  and  $f(0) > 0 > f(1)$ ,  $f(1) < 0 < f(3)$ , and  $f(3) > 0 > f(5)$ .

3   $f(x) = \frac{x^4 + x^3 + x^2 + x + 1}{880 - \ln\left(\frac{x^2 + 1}{2}\right)}$

Let  $f$  be the function defined above. The Intermediate Value Theorem applied to  $f$  on the closed interval  $[10, 12]$  guarantees a solution in  $[10, 12]$  to which of the following equations?

(A)  $f(x) = 0$

(B)  $f(x) = 27.372$

(C)  $f(x) = 42.421$

(D)  $f(x) = 67.205$

$f(x)$  is continuous

$f(10) = 29.6$   
 $f(12) = 60.3$

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

$$R(t) = \begin{cases} 41\sqrt{\frac{t}{20}} & \text{for } 0 \leq t < 20 \\ \frac{100t^2 + 1000}{t^2 + 600} & \text{for } t \geq 20 \end{cases}$$

Piecewise

The total accumulated revenue a company has received up to time  $t$  is modeled by the function  $R$  defined above, where  $R(t)$  is measured in millions of United States dollars and  $t$  is the time in years since 1980.

- (a) Find  $\lim_{t \rightarrow \infty} R(t)$ . Explain the meaning of  $\lim_{t \rightarrow \infty} R(t)$  in the context of the problem.

$$\lim_{t \rightarrow \infty} \frac{100t^2 + 1000}{t^2 + 600}$$

$$\frac{100t^2}{t^2} = 100$$

As time increases, the revenue approaches \$100 million

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

$$R(t) = \begin{cases} 41\sqrt{\frac{t}{20}} & \text{for } 0 \leq t < 20 \\ \frac{100t^2 + 1000}{t^2 + 600} & \text{for } t \geq 20 \end{cases}$$

$$\lim_{x \rightarrow 20} 41\sqrt{\frac{20}{20}} = 41\sqrt{1} = 41$$

$$\lim_{x \rightarrow 20} 41$$

The total accumulated revenue a company has received up to time  $t$  is modeled by the function  $R$  defined above, where  $R(t)$  is measured in millions of United States dollars and  $t$  is the time in years since 1980.

Yes, it is also defined at 20

(b) Is the function  $R$  continuous at  $t = 20$ ? Justify your answer.

$$\lim_{x \rightarrow 20^+} \frac{100t^2 + 1000}{t^2 + 600} = \frac{100(t^2 + 10)}{t^2 + 600} = \frac{100(20^2 + 10)}{20^2 + 600}$$

$$\frac{100(400 + 10)}{400 + 600} = \frac{41000}{1000} = 41$$

$$\lim_{x \rightarrow 20^+} 41$$

$$R(t) = \begin{cases} 41\sqrt{\frac{t}{20}} & \text{for } 0 \leq t < 20 \\ \frac{100t^2 + 1000}{t^2 + 800} & \text{for } t \geq 20 \end{cases}$$

[0.5] INT

$$-1 < 0 < 2.5$$

The total accumulated revenue a company has received up to time  $t$  is modeled by the function  $R$  defined above, where  $R(t)$  is measured in millions of United States dollars and  $t$  is the time in years since 1980.

(c) The company's total accumulated expenses up to time  $t$  is modeled by the function  $E$  defined by  $E(t) = 3\log_2(12t + 4)$ , where  $E(t)$  is measured in millions of United States dollars and  $t$  is the time in years since 1980. According to these models, is there a time  $t$ , for  $0 \leq t \leq 5$ , at which the total accumulated profit,  $R(t) - E(t)$ , is equal to 0? Justify your answer.

$$R(0) - E(0)$$

$$41\sqrt{\frac{0}{20}} - 3\log_2(12(0) + 4)$$

$$0 - 3\log_2(4)$$

$$R(0) - E(0) - 6 < 0$$

$$R(5) - E(5)$$

$$41\sqrt{\frac{5}{20}} - 3\log_2(12(5) + 4)$$

$$\frac{41}{2} - 3\log_2(64)$$

$$\frac{41}{2} - \frac{18}{1} = 2.5$$