Calculus I
UNIT 3: Derivatives – REVIEW

Name: ___________________________
Date: ___________________________

UNIT 3: DERIVATIVES – STUDY GUIDE

Section 1: Limit Definition (Derivative as the Slope of the Tangent Line)

Section 2: Calculating Rates of Change (Average vs Instantaneous)

AVERAGE VELOCITY	INSTANTANEOUS VELOCITY
$v_{avg} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$	$s'(t) = v(t)$
AVERAGE ACCELERATION	**INSTANTANEOUS ACCELERATION**
$a_{avg} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$	$s''(t) = v'(t) = a(t)$

Section 3: Power Rule

POWER RULE
$y = x^n \rightarrow y' = n \cdot x^{n-1}$

Section 4: Product & Quotient Rules

PRODUCT RULE	QUOTIENT RULE
$y = f(x) \cdot g(x) \quad \rightarrow \quad y' = f(x) \cdot g'(x) + f'(x) \cdot g(x)$	$y = \dfrac{f(x)}{g(x)} \quad \rightarrow \quad y' = \dfrac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Section 5: Chain Rule

CHAIN RULE
$y = f\big(g(x)\big) \quad \rightarrow \quad y' = f'\big(g(x)\big) \cdot g'(x)$

Section 6: Complex Derivatives

Section 7: Trigonometric Derivative Rules

TRIGONOMETRIC DERIVATIVES					
$y = \sin x$	$y = \cos x$	$y = \tan x$	$y = \csc x$	$y = \sec x$	$y = \cot x$
$y' = \cos x$	$y' = -\sin x$	$y' = \sec^2 x$	$y' = -\csc x \cot x$	$y' = \sec x \tan x$	$y' = -\csc^2 x$

Section 8: Trigonometric Derivatives using Trig Identities

Section 9: Trigonometric Derivatives using Chain Rule

- ☐ Unusual Angle
- ☐ Raised to a Power
- ☐ Combination

Section 10: Higher Order Derivatives (Notation AND Projectile motion applications from Section 2)

f' NOTATION	y' NOTATION	D NOTATION	LEIBNIZ NOTATION
$f'(x)$	y'	$D_x y$	$\dfrac{dy}{dx}$

Section 11: Implicit Derivatives

Section 12: Derivative Rules without Functions

Section 13: Derivatives of Exponential & Logarithmic Functions

EXPONENTIAL BASE "e"	EXPONENTIAL BASE "a"	LOGARITHMIC BASE "e"	LOGARITHMIC BASE "a"
$y = e^{f(x)}$	$y = a^{f(x)}$	$y = \ln f(x)$	$y = \log_a f(x)$
$y' = e^{f(x)}$	$y' = a^{f(x)} \ln(a)$	$y' = \dfrac{f'(x)}{f(x)}$	$y' = \dfrac{f'(x)}{f(x) \ln(a)}$

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UNIT 3: DERIVATIVES – REVIEW

<u>Directions</u>: Find $f'(x)$.

1.) $f(x) = \frac{1}{2}x^4 - 2x^3 + x + 6$

2.) $f(x) = \frac{4}{x} + \frac{5}{x^2}$

3.) $f(x) = 4\sqrt{x} + 6\sqrt[3]{x^2}$

<u>Directions</u>: Find y'.

4.) $y = (x^2 + 3x)(x^3 - 1)$

5.) $y = 2x^5 \cot x$

6.) $y = \frac{3x}{5x^2-1}$

7.) $y = \frac{x^2-2x+5}{x^2+2x-3}$

<u>Directions</u>: Find $\frac{dy}{dx}$.

8.) $y = (3x^4 - 5)^6$

9.) $y = \frac{3}{(x^2+6x)^3}$

Directions: Use the appropriate rule when finding the derivative.

10.) If $f(2) = -2,\ f'(2) = 1$ and $g(2) = 13,\ g'(2) = 4$

a.) $(f+g)'(2) =$

b.) $(g-f)'(2) =$

c.) $(g \cdot f)'(2) =$

d.) $\left(\frac{f}{g}\right)'(2) =$

Directions: Find $D_x y$.

11.) $y = \csc^4 x$

12.) $y = \tan(5 - 4x^2)$

13.) $y = \cos^4(3x)$

Directions: Find $f'(x)$.

14.) $f(x) = \sec x \cos x + \sin^2 x \csc x$

15.) $f(x) = \tan x \cos x \cot x$

Directions: Find $f'''(x)$ and $f'''(-1)$ for the following function.

16.) $f(x) = 3x^6 - 5x^4 + 6x^3 - 7x$

Directions: Find the y'.

17.) $y = e^{9x-4x^3}$

18.) $y = 5^{2x^3-7}$

19.) $\log(6x^2)$

20.) $\ln(4x^2 - 3x)$

Directions: Find the implicit derivative.

21.) $8x^2 - 2y^2 = 4$

22.) $\cos y + 6xy = 3x^2$

Directions: Find the slope and equation of the tangent line on the graph of the following.

23.) $f(x) = 2x^3 - 5x$ at the point $(2, 6)$

24.) $x^2y^2 = 9$ at the point $(1, 3)$

Directions: Answer the following questions.

25.) Find the point(s) on the graph of $f(x) = x^3 - 6x^2 - 10x$ where the slope is 5.

26.) The position of an object is given by $s(t) = t^3 + 4t$.
What is the ***average velocity*** of the object in the time interval $[1, 4]$?

27.) An object travels so that its position is given by $s(t) = 2t^4 + 5t^3 - t + 2$.
Find the ***instantaneous velocity*** at $t = 4$.

28.) A particle has a position function of $s(t) = 8t^5 - 5t^4$.
Find the ***instantaneous acceleration*** at $t = 3$.

29.) A particle has a position function of $s(t) = 3t^2 - 6t + 18$.
At what time will the velocity equal 6 units/sec?

30.) Phillies star, Shane Victorino "The Flyin' Hawaiian" jumps off a cliff into the water below and his position (feet) is defined after t seconds by the following: $s(t) = -16t^2 + 32t + 48$.

a.) What is his initial height? What is his initial velocity?

b.) When does he reach his maximum height? What is his maximum height?

c.) What other moment in time is his height the same as his initial height? What is his velocity?

d.) When does "The Flyin' Hawaiin" reach the water? What is his impact velocity?

31.) Show the setup for finding the derivative using the limit definition.

a.) $f(x) = x^2 - 2x + 5$ b.) $g(x) = \frac{2}{x^2+x}$ c.) $h(x) = \cos x$

SOLUTIONS

1.) $f'(x) = 2x^3 - 6x^2 + 1$

2.) $f'(x) = \frac{-4}{x^2} - \frac{10}{x^3}$

3.) $f'(x) = \frac{2}{\sqrt{x}} + \frac{4}{\sqrt[3]{x}}$

4.) $y' = 5x^4 + 12x^3 - 2x - 3$

5.) $y' = -2x^5 \csc^2 x + 10x^4 \cot x$

6.) $y' = \frac{-15x^2-3}{(5x^2-1)^2}$

7.) $y' = \frac{4x^2-16x-4}{(x^2+2x-3)^2}$

8.) $\frac{dy}{dx} = 72x^3(3x^4-5)^5$

9.) $\frac{dy}{dx} = \frac{-18x-54}{(x^2+6x)^4}$

10.) a.) 5 b.) 3 c.) 5 d.) $\frac{21}{169}$

11.) $D_x y = -4\csc^4 x \cot x$

12.) $D_x y = -8x\sec^2(5-4x^2)$

13.) $D_x y = -12\sin(3x)\cos^3(3x)$

14.) $f'(x) = \cos x$

15.) $f'(x) = -\sin x$

16.) $f'''(x) = 360x^3 - 120x + 36$

$f'''(-1) = -204$

17.) $y' = (9-12x^2)e^{9x-4x^3}$

18.) $y' = 6x^2 \cdot 5^{2x^3-7} \cdot \ln(5)$

19.) $y' = \frac{2}{x\ln(10)}$

20.) $y' = \frac{8x-3}{4x^2-3x}$

21.) $y' = \frac{4x}{y}$

22.) $y' = \frac{6x-6y}{-\sin y+6x}$

23.) $m_{tan} = 19$ $y = 19x - 32$

24.) $m_{tan} = -3$ $y = -3x + 6$

25.) $(-1,3)$ & $(5,-75)$

26.) $v_{avg} = 25\, units/sec$

27.) $v(4) = 751\, units/sec$

28.) $a(3) = 3780\, units/sec^2$

29.) $t = 2\, sec$

30.) a.) $s(0) = 48ft,\ v(0) = 32ft/sec$

b.) $t = 1\, sec,\ s(0) = 64ft$

c.) $t = 2\, sec,\ v(2) = -32ft/sec$

d.) $t = 3sec,\ v(3) = -64ft/sec$

31.) a.) $f'(x) = \lim_{h\to 0} \frac{(x+h)^2-2(x+h)+5-(x^2-2x+5)}{h}$

b.) $g'(x) = \lim_{h\to 0} \frac{\frac{2}{(x+h)^2+(x+h)} - \frac{2}{x^2+x}}{h}$

c.) $h'(x) = \lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h}$