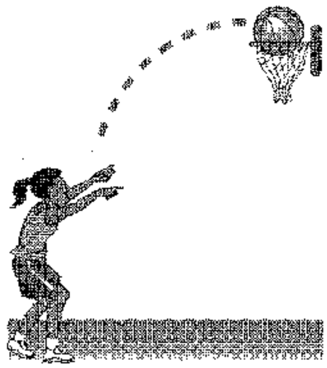
Projectile Motion and the Range Equation

Question: How can you predict the range of a launched marble?

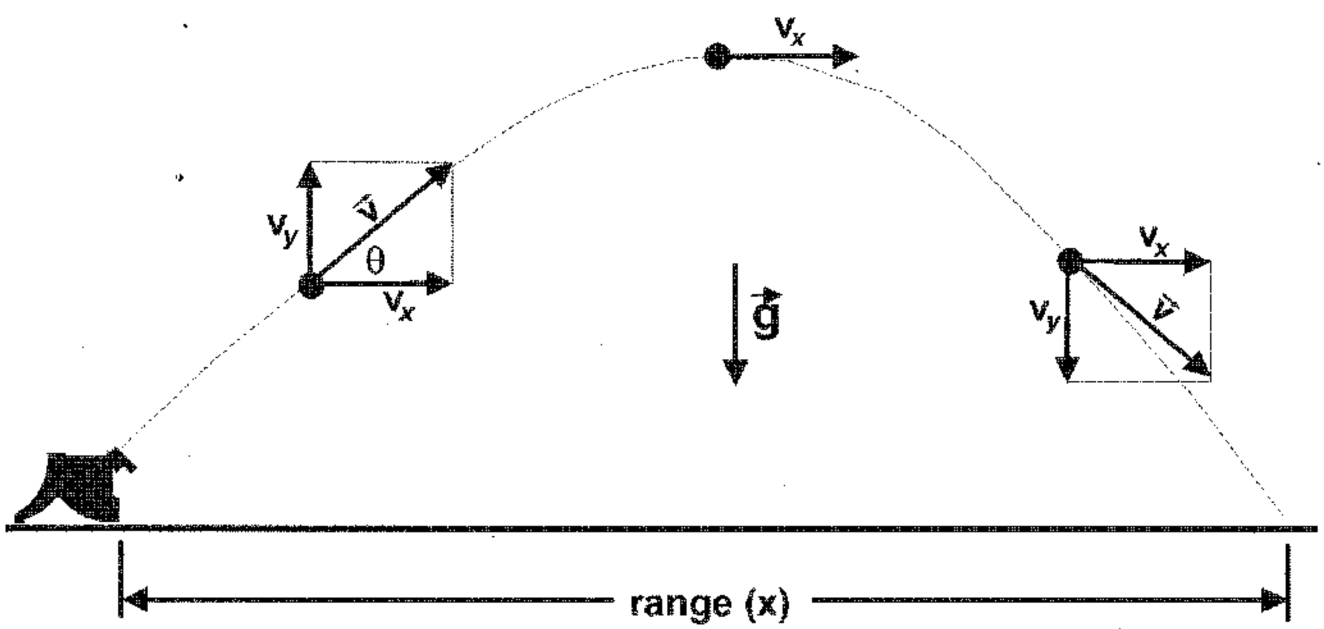
In this Investigation, you will find and test a model that will predict the range of the marble from the initial velocity and launch angle.



You have learned that the motion of any object moving through the air affected only by gravity is an example of **projectile motion**. Examples of projectile motion include a basketball thrown toward a hoop, a car driven off a cliff by a stunt person, and a marble launched from the CPO marble launcher. Projectile motion is also called two-dimensional motion because it depends on two components: vertical and horizontal. In this Investigation, you will determine a mathematical model (the **range equation**) that predicts the range of the marble given launch angle and initial velocity.

1 Analyzing the motion of the marble in two dimensions

How can you predict the range of the marble? Since gravity pulls *down* and not sideways, the motion of the marble must be separated into *components*. It makes sense to pick one component (y) in the vertical direction aligned with gravity. The other component (x) is then chosen to be in the horizontal direction, *perpendicular* to the force of gravity. The diagram below shows the velocity of the marble (v) at three points in its trajectory, resolved into x and y components, v_x and v_y.



a. Use the diagram above to explain why projectiles travel in a curved path called a *trajectory*.

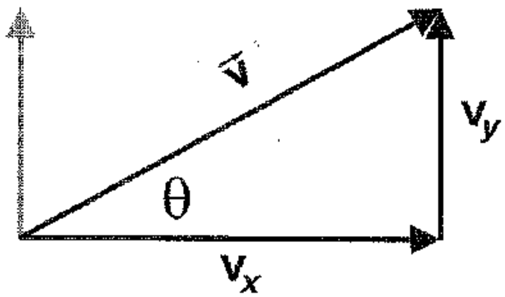
b. How does the marble's velocity in x change over the time of the flight? How does its velocity in y change over the time of flight?

2 Understanding the velocity equations

C-1

The object of this Investigation is to find and test a model that will predict the range of the marble from the initial velocity and launch angle—the range equation.

a. The first step is to separate the velocity of the marble into x and y components. Use the triangle formed by velocities (at right) to express v_x and v_y in terms of the initial velocity, v, and the sine and cosine of the launch angle, θ.



When the initial velocity is separated into x and y components, Equations 1a-2b give the relationships between the motion variables *separately* for x and y. In these equations the subscript i refers to the *initial* values at launch.

Equations 1a and 1b are for the marble's velocity while equations 2a and 2b are for the marble's position.

Equation 1a $$v_x = v_{xi} + a_x t$$

Equation 1b $$v_y = v_{yi} + a_y t$$

Equation 2a $$x = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Equation 2b $$y = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

b. Since gravity does not pull up or sideways, one of the accelerations (a_x, a_y) is $-g$, and the other is zero. For the first approximation, the range (x) is defined so the initial x and y positions are also zero. Rewrite Equations 1a-2b leaving out terms that are zero and substituting your previous results for v_{xi} and v_{yi}. $V_x = V\cos\theta$ $\quad V_{oy} = V\sin\theta$

Equation 1a: $V\cos\theta = V_x$

Equation 1b: $V_y = V\sin\theta - gt$

Equation 2a: $\Delta x = Vt\cos\theta$

Equation 2b: $\Delta y = Vt\sin\theta - \frac{1}{2}gt$

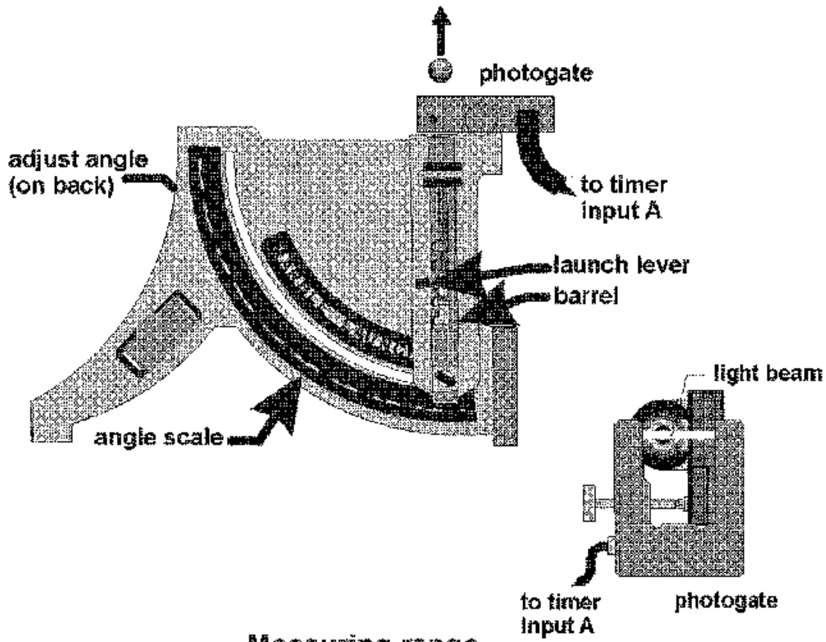
c. The purpose of this exercise is to find a theory that predicts where the marble will land (the range, x) given the initial velocity and launch angle. This problem can be solved in several steps. First, assume that gravity acts only on the y component of velocity. Solve for the *time* it takes the marble to reach its maximum height (where $v_y = 0$). $t = \frac{V\sin\theta}{g}$

d. Since gravity does not pull sideways, the x component of the marble's velocity is not affected and remains constant. Use Equation 2a to calculate the range (x) from v_{xi} and the total time of flight (this will be the range equation). $\Delta x = \frac{2V^2\sin\theta\cos\theta}{g}$

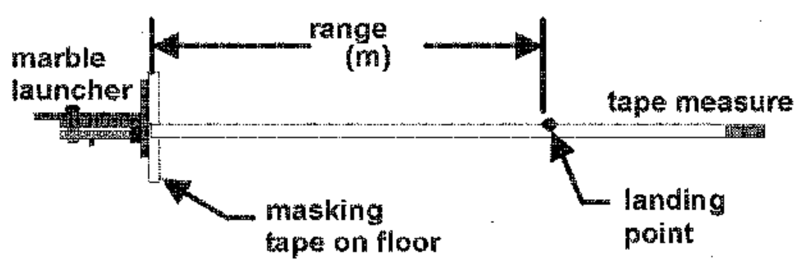
Setting up the marble launcher

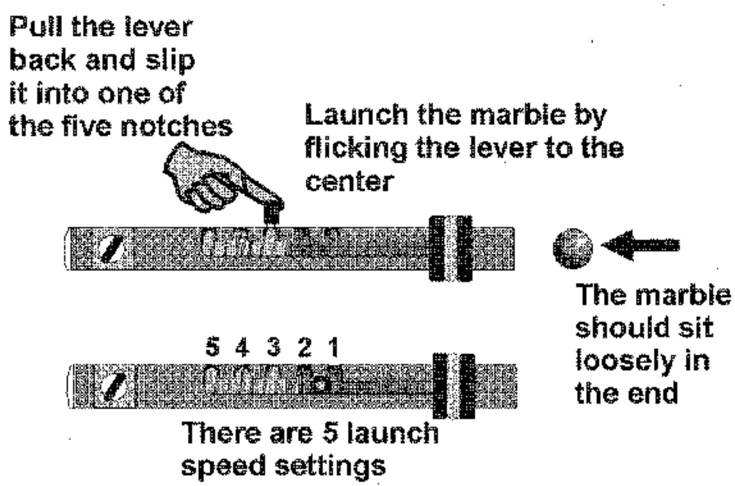
1. Attach one photogate to the marble launcher so that the marble breaks the light beam as it comes out of the barrel. Put the timer in interval mode, and connect the photogate to input A. The launcher can launch at angles from 0 (horizontal) to 90 degrees (vertical). For this experiment, you will use angles from 0 to 80 degrees.

   The photogate attaches to the tab on the end of the wood piece that supports the barrel. Be sure the light beam crosses the center of the barrel.
2. Use a strip of masking tape on the floor to make sure that the marble launcher is consistently placed in the same location. A tape measure laid along the floor provides a good range reference..
3. To launch marbles, pull the launching lever back and slip it sideways into one of the slots. Put a marble in the end of the barrel and the marble launcher is ready to launch.
4. There are five notches that change the compression on the spring and give different launch speeds. In this experiment, you will change the launch speed setting for different launches.



Measuring range





SAFETY RULES:
- **Never launch marbles at people.**
- **Wear safety glasses or other eye protection when launching marbles.**
- **Launch only the black plastic marbles that come with the marble launcher.**

4 Doing the experiment

1. To check your theory, a spread of data for different *launch angles* and *initial velocities* will be needed. For each spring setting you should have five different launch angles from 20 degrees to 80 degrees. To cover both variables (angle and speed), at least 20 data points are needed.
2. A minimum of two people are needed per launcher. One person launches the marbles and the other person/people watches where they land.
3. Use *only* the black plastic marbles provided by CPO.
4. Record the spring setting, launch angle, time from photogate A, and measured range for each launch. It often takes several launches with the same setup to locate the landing point precisely. For each setup, you may need to run several trials until the measured range is consistent within 5 cm.
5. Calculate the initial velocity by dividing the distance traveled (width of marble = 0.019 m) by the time at photogate A. Record initial velocities in the table. *A larger version of the table is found on your answer sheet.*

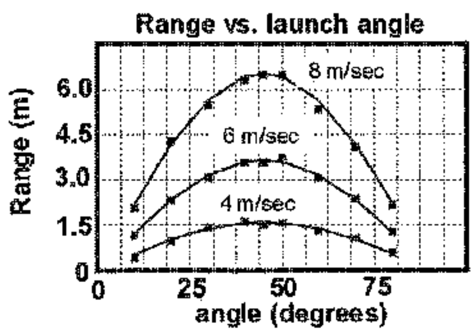
Spring setting (1 - 5):				
Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

5 Comparing theory predictions to measured data

Use the table to help you compare your measured data to theory predictions using your range equation. Fill in several launch angles for each initial velocity. *A larger table is found on your answer sheet.*

Initial velocity (m/sec)		Launch angles (degrees)					
	x (predicted)						
	x (measured)						

a. Make a graph showing the range vs. launch angle for several different initial velocities. The graph at right is one example of how this graph could look. You could also choose other ways to graph the data. Plot the measured points as unconnected dots and the theoretical values as solid lines since the theory predicts the speed of the marble at all points.



b. How does your theory compare with your measurements? In particular, is there a consistent deviation between theory and experiment? The word "consistent" means the difference between the theoretical and experimental data seems to depend on something in the experiment and is not random. For example, a consistent deviation would occur if the measured range for small angles is *always* smaller than predicted by theory *regardless of the velocity.* Explain how the consistent deviations you found are affected by velocity and angle.

c. Consistent deviations indicate that something is missing from the theory. What is missing, and why does it have the observed effect on your results?