2.1 Conditional Statements

What you should learn

GOAL 1 Recognize and analyze a conditional statement.

GOAL 2 Write postulates about points, lines, and planes using conditional statements.

Why you should learn it

▼ Point, line, and plane postulates help you analyze **real-life** objects, such as the research buggy below and in **Ex. 54**.

REAL LIFE

Coastal Research Amphibious Buggy

GOAL 1 RECOGNIZING CONDITIONAL STATEMENTS

In this lesson you will study a type of logical statement called a conditional statement. A **conditional statement** has two parts, a *hypothesis* and a *conclusion*. When the statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

$$\text{If } \underbrace{\text{it is noon in Georgia}}_{\text{Hypothesis}}, \text{ then } \underbrace{\text{it is 9 A.M. in California}}_{\text{Conclusion}}.$$

EXAMPLE 1 *Rewriting in If-Then Form*

Rewrite the conditional statement in *if-then form.*

a. Two points are collinear if they lie on the same line.

b. All sharks have a boneless skeleton.

c. A number divisible by 9 is also divisible by 3.

SOLUTION

a. If two points lie on the same line, then they are collinear.

b. If a fish is a shark, then it has a boneless skeleton.

c. If a number is divisible by 9, then it is divisible by 3.

. . . .

Conditional statements can be either true or false. To show that a conditional statement is true, you must present an argument that the conclusion follows for *all* cases that fulfill the hypothesis. To show that a conditional statement is false, describe a single counterexample that shows the statement is not always true.

EXAMPLE 2 *Writing a Counterexample*

Write a counterexample to show that the following conditional statement is false.

$$\text{If } x^2 = 16, \text{ then } x = 4.$$

SOLUTION

As a counterexample, let $x = -4$. The hypothesis is true, because $(-4)^2 = 16$. However, the conclusion is false. This implies that the given conditional statement is false.

The **converse** of a conditional statement is formed by switching the hypothesis and conclusion. Here is an example.

Statement: If you see lightning, then you hear thunder.

Converse: If you hear thunder, then you see lightning.

EXAMPLE 3 *Writing the Converse of a Conditional Statement*

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Write the converse of the following conditional statement.

Statement: If two segments are congruent, then they have the same length.

SOLUTION

Converse: If two segments have the same length, then they are congruent.

. . . .

A statement can be altered by **negation**, that is, by writing the negative of the statement. Here are some examples.

STATEMENT	NEGATION
$m\angle A = 30°$	$m\angle A \neq 30°$
$\angle A$ is acute.	$\angle A$ is not acute.

When you negate the hypothesis and conclusion of a conditional statement, you form the **inverse**. When you negate the hypothesis and conclusion of the converse of a conditional statement, you form the **contrapositive**.

Original	If $m\angle A = 30°$, then $\angle A$ is acute.
Inverse	If $m\angle A \neq 30°$, then $\angle A$ is not acute.
Converse	If $\angle A$ is acute, then $m\angle A = 30°$.
Contrapositive	If $\angle A$ is not acute, then $m\angle A \neq 30°$.

Both false **Both true**

When two statements are both true or both false, they are called **equivalent statements**. A conditional statement is equivalent to its contrapositive. Similarly, the inverse and converse of any conditional statement are equivalent. This is shown in the table above.

FOCUS ON APPLICATIONS

REAL LIFE **CROCUS** There are some exceptions to the statement in Example 4. For instance, crocuses can bloom when snow is on the ground.

EXAMPLE 4 *Writing an Inverse, Converse, and Contrapositive*

Write the (**a**) inverse, (**b**) converse, and (**c**) contrapositive of the statement.

If there is snow on the ground, then flowers are not in bloom.

SOLUTION

a. Inverse: If there is no snow on the ground, then flowers are in bloom.

b. Converse: If flowers are not in bloom, then there is snow on the ground.

c. Contrapositive: If flowers are in bloom, then there is no snow on the ground.

GOAL 2 Using Point, Line, and Plane Postulates

In Chapter 1, you studied four postulates.

Ruler Postulate	(Lesson 1.3, page 17)
Segment Addition Postulate	(Lesson 1.3, page 18)
Protractor Postulate	(Lesson 1.4, page 27)
Angle Addition Postulate	(Lesson 1.4, page 27)

Remember that postulates are assumed to be true—they form the foundation on which other statements (called *theorems*) are built.

Student Help

Study Tip
There is a list of all the postulates in this course at the end of the book beginning on page 827.

Point, Line, and Plane Postulates

Postulate 5 Through any two points there exists exactly one line.

Postulate 6 A line contains at least two points.

Postulate 7 If two lines intersect, then their intersection is exactly one point.

Postulate 8 Through any three noncollinear points there exists exactly one plane.

Postulate 9 A plane contains at least three noncollinear points.

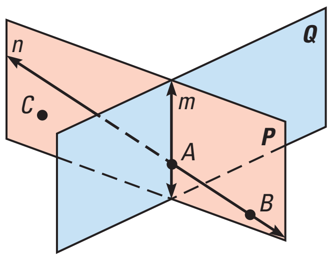
Postulate 10 If two points lie in a plane, then the line containing them lies in the plane.

Postulate 11 If two planes intersect, then their intersection is a line.

Example 5 *Identifying Postulates*

Logical Reasoning

Use the diagram at the right to give examples of Postulates 5 through 11.



Solution

a. Postulate 5: There is exactly one line (line n) that passes through the points A and B.

b. Postulate 6: Line n contains at least two points. For instance, line n contains the points A and B.

c. Postulate 7: Lines m and n intersect at point A.

d. Postulate 8: Plane P passes through the noncollinear points A, B, and C.

e. Postulate 9: Plane P contains at least three noncollinear points, A, B, and C.

f. Postulate 10: Points A and B lie in plane P. So, line n, which contains points A and B, also lies in plane P.

g. Postulate 11: Planes P and Q intersect. So, they intersect in a line, labeled in the diagram as line m.

EXAMPLE 6 *Rewriting a Postulate*

a. Rewrite Postulate 5 in if-then form.

b. Write the inverse, converse, and contrapositive of Postulate 5.

SOLUTION

a. Postulate 5 can be rewritten in if-then form as follows:

> If two points are distinct, then there is exactly one line that passes through them.

b. **Inverse:** If two points are not distinct, then it is not true that there is exactly one line that passes through them.

Converse: If exactly one line passes through two points, then the two points are distinct.

Contrapositive: If it is not true that exactly one line passes through two points, then the two points are not distinct.

EXAMPLE 7 *Using Postulates and Counterexamples*

Logical Reasoning

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

a. A line can be in more than one plane.

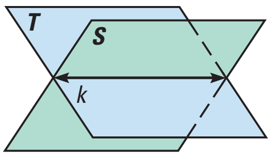
b. Four noncollinear points are always coplanar.

c. Two nonintersecting lines can be noncoplanar.

SOLUTION

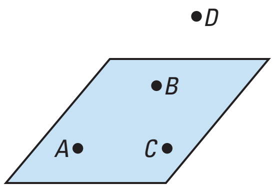
a. In the diagram at the right, line k is in plane S and line k is in plane T.

So, it is *true* that a line can be in more than one plane.



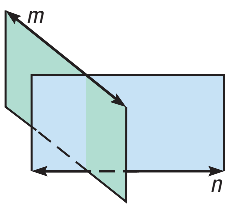
b. Consider the points A, B, C, and D at the right. The points A, B, and C lie in a plane, but there is no plane that contains all four points.

So, as shown in the counterexample at the right, it is *false* that four noncollinear points are always coplanar.



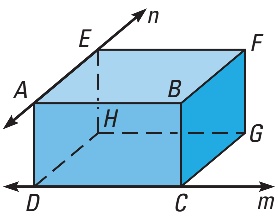
c. In the diagram at the right, line m and line n are nonintersecting and are also noncoplanar.

So, it is *true* that two nonintersecting lines can be noncoplanar.



STUDENT HELP

Study Tip
A box can be used to help visualize points and lines in space. For instance, the diagram shows that $\overleftrightarrow{AE}$ and $\overleftrightarrow{DC}$ are noncoplanar.



GUIDED PRACTICE

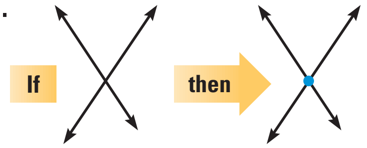
Vocabulary Check ✓

1. The ___?___ of a conditional statement is found by switching the hypothesis and conclusion.

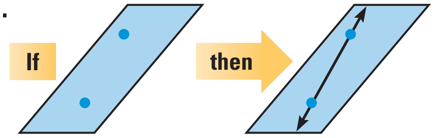
Concept Check ✓

2. State the postulate described in each diagram.

a.



b.



Skill Check ✓

3. Write the hypothesis and conclusion of the statement, "If the dew point equals the air temperature, then it will rain."

In Exercises 4 and 5, write the statement in if-then form.

4. When threatened, the African ball python protects itself by coiling into a ball with its head in the middle.

5. The measure of a right angle is 90°.

6. Write the inverse, converse, and contrapositive of the conditional statement, "If a cactus is of the *cereus* variety, then its flowers open at night."

Decide whether the statement is *true* or *false*. Make a sketch to help you decide.

7. Through three noncollinear points there exists exactly one line.

8. If a line and a plane intersect, and the line does not lie in the plane, then their intersection is a point.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

9. An object weighs one ton if it weighs 2000 pounds.

10. An object weighs 16 ounces if it weighs one pound.

11. Three points are collinear if they lie on the same line.

12. Blue trunkfish live in the waters of a coral reef.

13. Hagfish live in salt water.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 9–13
Example 2: Exs. 14–17
Example 3: Exs. 18–21
Example 4: Exs. 46–52
Example 5: Exs. 25–34
Example 6: Exs. 22–24
Example 7: Exs. 35–38

ANALYZING STATEMENTS **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

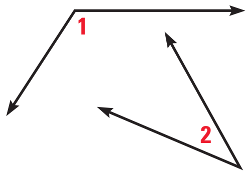
14. A point may lie in more than one plane.

15. If x^4 equals 81, then x must equal 3.

16. If it is snowing, then the temperature is below freezing.

17. If four points are collinear, then they are coplanar.

WRITING CONVERSES **Write the converse of the statement.**



18. If $\angle 1$ measures 123°, then $\angle 1$ is obtuse.

19. If $\angle 2$ measures 38°, then $\angle 2$ is acute.

20. I will go to the mall if it is not raining.

21. I will go to the movies if it is raining.

REWRITING POSTULATES **Rewrite the postulate in if-then form. Then write the inverse, converse, and contrapositive of the conditional statement.**

22. A line contains at least two points.

23. Through any three noncollinear points there exists exactly one plane.

24. A plane contains at least three noncollinear points.

ILLUSTRATING POSTULATES **Fill in the blank. Then draw a sketch that helps illustrate your answer.**

25. If two lines intersect, then their intersection is __?__ point(s).

26. Through any __?__ points there exists exactly one line.

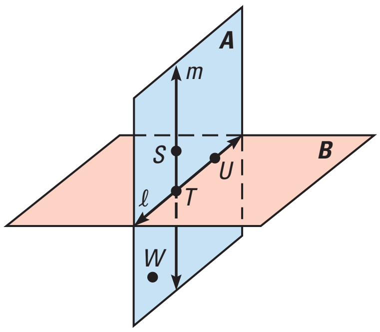
27. If two points lie in a plane, then the __?__ containing them lies in the plane.

28. If two planes intersect, then their intersection is __?__.

STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 29–34.

LINKING POSTULATES **Use the diagram to state the postulate(s) that verifies the truth of the statement.**



29. The points U and T lie on line ℓ.

30. Line ℓ contains points U and T.

31. The points W, S, and T lie in plane A.

32. The points S and T lie in plane A. Therefore, line m lies in plane A.

33. The planes A and B intersect in line ℓ.

34. Lines m and ℓ intersect at point T.

USING POSTULATES **In Exercises 35–38, state the postulate that shows that the statement is false.**

35. A line contains only one point.

36. Two planes intersect in exactly one point.

37. Three points, A, B, and C, are noncollinear, and two planes, M and N, each contain points A, B, and C.

38. Two points, P and Q, are collinear and two different lines, $\overleftrightarrow{RS}$ and $\overleftrightarrow{XY}$, each pass through points P and Q.

39. *Writing* Give an example of a true conditional statement with a true converse.

POINTS AND LINES IN SPACE **Think of the intersection of the ceiling and the front wall of your classroom as line *k*. Think of the center of the floor as point *A* and the center of the ceiling as point *B*.**

40. Is there more than one line that contains both points A and B?

41. Is there more than one plane that contains both points A and B?

42. Is there a plane that contains line k and point A?

43. Is there a plane that contains points A, B, and a point on the front wall?

USING ALGEBRA **Find the inverse, converse, and contrapositive of the statement.**

44. If $x = y$, then $5x = 5y$.

45. $6x - 6 = x + 14$ if $x = 4$.

QUOTES OF WISDOM **Rewrite the statement in if-then form. Then (a) determine the hypothesis and conclusion, and (b) find the inverse of the conditional statement.**

46. "If you tell the truth, you don't have to remember anything." — Mark Twain

47. "One can never consent to creep when one feels the impulse to soar." — Helen Keller

48. "Freedom is not worth having if it does not include the freedom to make mistakes." — Mahatma Ghandi

49. "Early to bed and early to rise, makes a man healthy, wealthy, and wise." — Benjamin Franklin

FOCUS ON CAREERS

REAL LIFE

ADVERTISING COPYWRITER

Advertising copywriters write the advertisements you see and hear everyday. These ads appear in many forms including Internet home pages.

INTERNET CAREER LINK
www.mcdougallittell.com

ADVERTISING **In Exercises 50–52, use the following advertising slogan: "You want a great selection of used cars? Come and see Bargain Bob's Used Cars!"**

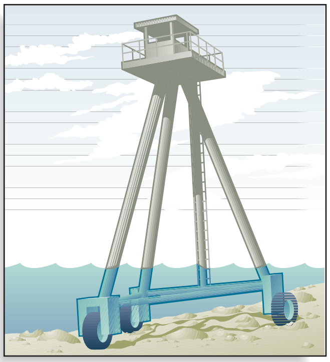
50. Write the slogan in if-then form. What are the hypothesis and conclusion of the conditional statement?

51. Write the inverse, converse, and contrapositive of the conditional statement.

52. *Writing* Find a real-life advertisement or slogan similar to the one given. Then repeat Exercises 50 and 51 using the advertisement or slogan.

53. **TECHNOLOGY** Use geometry software to draw a segment with endpoints A and C. Draw a third point B not on $\overline{AC}$. Measure $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Move B closer to $\overline{AC}$ and observe the measures of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

54. **RESEARCH BUGGY** The diagram at the right shows the 35 foot tall Coastal Research Amphibious Buggy, also known as CRAB. This vehicle moves along the ocean floor collecting data that are used to make an accurate map of the ocean floor. Using the postulates you have learned, make a conjecture about why the CRAB was built with three legs instead of four.



Test Preparation

55. MULTIPLE CHOICE Use the conditional statement "If the measure of an angle is 44°, then the angle is acute" to decide which of the following are true.

I. The statement is true.
II. The converse of the statement is true.
III. The contrapositive of the statement is true.

(A) I only (B) II only (C) I and II (D) I and III (E) I, II, and III

56. MULTIPLE CHOICE Which one of the following statements is *not* true?

(A) If $x = 2$, then $x^2 = 4$.

(B) If $x = -2$, then $x^2 = 4$.

(C) If $x^3 = -8$, then $x = -2$.

(D) If $x^2 = 4$, then $x = 2$.

(E) If $x = -2$, then $x^3 = -8$.

★ **Challenge**

MAKING A CONJECTURE **Sketch a line *k* and a point *P* not on line *k*. Make a conjecture about how many planes can be drawn through line *k* and point *P*, and then answer the following questions.**

57. Which postulate allows you to state that there are two points, R and S, on line k?

58. Which postulate allows you to conclude that exactly one plane X can be drawn to contain points P, R, and S?

59. Which postulate guarantees that line k is contained in plane X?

60. Was your conjecture correct?

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

DRAWING ANGLES **Plot the points in a coordinate plane. Then classify $\angle ABC$.** (Review 1.4 for 2.2)

61. $A(0, 7)$, $B(2, 2)$, $C(6, -1)$

62. $A(-1, 0)$, $B(-6, 4)$, $C(-6, -1)$

63. $A(1, 3)$, $B(1, -5)$, $C(-5, -5)$

64. $A(-3, -1)$, $B(2, 5)$, $C(3, -2)$

FINDING THE MIDPOINT **Find the coordinates of the midpoint of the segment joining the two points.** (Review 1.5)

65. $A(-2, 8)$, $B(4, -12)$

66. $A(8, 8)$, $B(-6, 1)$

67. $A(-7, -4)$, $B(4, 7)$

68. $A(0, -9)$, $B(-8, 5)$

69. $A(1, 4)$, $B(11, -6)$

70. $A(-10, -10)$, $B(2, 12)$

FINDING PERIMETER AND AREA **Find the area and perimeter (or circumference) of the figure described. (Use $\pi \approx 3.14$ when necessary.)** (Review 1.7 for 2.2)

71. circle, radius = 6 m

72. square, side = 11 cm

73. square, side = 38.75 mm

74. circle, diameter = 23 ft

2.2 Definitions and Biconditional Statements

What you should learn

GOAL 1 Recognize and use definitions.

GOAL 2 Recognize and use biconditional statements.

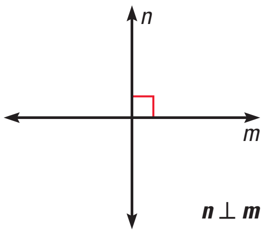
Why you should learn it

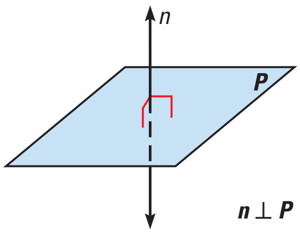
▼ You can use biconditional statements to help analyze geographic relations, such as whether three cities in Florida lie on the same line, as in **Ex. 50**.

GOAL 1 RECOGNIZING AND USING DEFINITIONS

In Lesson 1.2 you learned that a *definition* uses known words to describe a new word. Here are two examples.

Two lines are called **perpendicular lines** if they intersect to form a right angle. A **line perpendicular to a plane** is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it. The symbol ⊥ is read as "is perpendicular to."

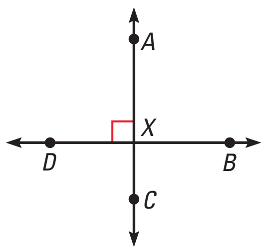




All definitions can be interpreted "forward" and "backward." For instance, the definition of perpendicular lines means (1) if two lines are perpendicular, then they intersect to form a right angle, *and* (2) if two lines intersect to form a right angle, then they are perpendicular.

EXAMPLE 1 *Using Definitions*

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. Points D, X, and B are collinear.

b. $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{DB}$.

c. $\angle AXB$ is adjacent to $\angle CXD$.

SOLUTION

a. This statement is true. Two or more points are *collinear* if they lie on the same line. The points D, X, and B all lie on line $\overleftrightarrow{DB}$ so they are collinear.

b. This statement is true. The right angle symbol in the diagram indicates that the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ intersect to form a right angle. So, the lines are perpendicular.

c. This statement is false. By definition, adjacent angles must share a common side. Because $\angle AXB$ and $\angle CXD$ do not share a common side, they are not adjacent.

STUDENT HELP

Study Tip
When a conditional statement contains the word "if," the hypothesis does not always follow the "if." This is shown in the "only-if" statement at the right.

GOAL 2 USING BICONDITIONAL STATEMENTS

Conditional statements are not always written in if-then form. Another common form of a conditional statement is *only-if* form. Here is an example.

It is Saturday, only if **I am working at the restaurant**.

Hypothesis **Conclusion**

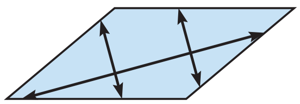
You can rewrite this conditional statement in if-then form as follows:

If **it is Saturday**, then **I am working at the restaurant**.

A **biconditional statement** is a statement that contains the phrase "if and only if." Writing a biconditional statement is equivalent to writing a conditional statement *and* its converse.

EXAMPLE 2 *Rewriting a Biconditional Statement*

The biconditional statement below can be rewritten as a conditional statement and its converse.



Three lines are coplanar if and only if they lie in the same plane.

Conditional statement: If three lines are coplanar, then they lie in the same plane.

Converse: If three lines lie in the same plane, then they are coplanar.

. . . .

A biconditional statement can be either true or false. To be true, *both* the conditional statement and its converse must be true. This means that a true biconditional statement is true both "forward" and "backward." All definitions can be written as true biconditional statements.

EXAMPLE 3 *Analyzing a Biconditional Statement*

Using Algebra

Consider the following statement: $x = 3$ if and only if $x^2 = 9$.

a. Is this a biconditional statement?

b. Is the statement true?

SOLUTION

a. The statement is biconditional because it contains "if and only if."

b. The statement can be rewritten as the following statement and its converse.

Conditional statement: If $x = 3$, then $x^2 = 9$.

Converse: If $x^2 = 9$, then $x = 3$.

▶ The first of these statements is true, but the second is false. So, the biconditional statement is false.

Logical Reasoning

EXAMPLE 4 *Writing a Biconditional Statement*

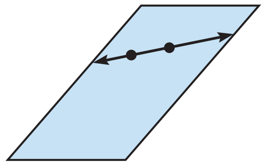
Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If two points lie in a plane, then the line containing them lies in the plane.

b. If a number ends in 0, then the number is divisible by 5.

SOLUTION

a. Converse: If a line containing two points lies in a plane, then the points lie in the plane.



The converse is true, as shown in the diagram. So, it can be combined with the original statement to form the true biconditional statement written below.

Biconditional statement: Two points lie in a plane if and only if the line containing them lies in the plane.

b. Converse: If a number is divisible by 5, then the number ends in 0.

The converse is false. As a counterexample, consider the number 15. It is divisible by 5, but it does not end in 0, as shown at the right.

$10 \div 5 = 2$
▸ **$15 \div 5 = 3$**
$20 \div 5 = 4$

. . . .

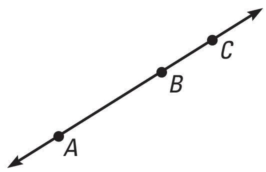
Knowing how to use true biconditional statements is an important tool for reasoning in geometry. For instance, if you can write a true biconditional statement, then you can use the conditional statement or the converse to justify an argument.

EXAMPLE 5 *Writing a Postulate as a Biconditional*

The second part of the Segment Addition Postulate is the converse of the first part. Combine the statements to form a true biconditional statement.

SOLUTION

The first part of the Segment Addition Postulate can be written as follows:



If B lies between points A and C, then $AB + BC = AC$.

The converse of this is as follows:

If $AB + BC = AC$, then B lies between A and C.

Combining these statements produces the following true biconditional statement:

Point B lies between points A and C if and only if $AB + BC = AC$.

STUDENT HELP

Study Tip
Unlike definitions, not all postulates can be written as true biconditional statements.

Guided Practice

Vocabulary Check ✓

1. Describe in your own words what a *true biconditional statement* is.

Concept Check ✓

2. **Error Analysis** What is wrong with Jared's argument below?

~~The statements "I eat cereal only if it is morning" and "If I eat cereal, then it is morning" are not equivalent.~~

Skill Check ✓

Tell whether the statement is a biconditional.

3. I will work after school only if I have the time.
4. An angle is called a right angle if and only if it measures 90°.
5. Two segments are congruent if and only if they have the same length.

Rewrite the biconditional statement as a conditional statement and its converse.

6. The ceiling fan runs if and only if the light switch is on.
7. You scored a touchdown if and only if the football crossed the goal line.
8. The expression $3x + 4$ is equal to 10 if and only if x is 2.

Windows **Decide whether the statement about the window shown is true. Explain your answer using the definitions you have learned.**

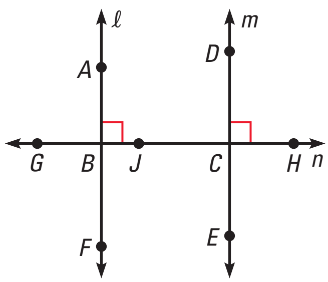
9. The points D, E, and F are collinear.
10. $m\angle CBA = 90°$
11. $\angle DBA$ and $\angle EBC$ are not complementary.
12. $\overline{DE} \perp \overline{AC}$

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 805.

Perpendicular Lines **Use the diagram to determine whether the statement is *true* or *false*.**



13. Points A, F, and G are collinear.
14. $\angle DCJ$ and $\angle DCH$ are supplementary.
15. $\overline{DC}$ is perpendicular to line ℓ.
16. $\overline{FB}$ is perpendicular to line n.
17. $\angle FBJ$ and $\angle JBA$ are complementary.
18. Line m bisects $\angle JCH$.
19. $\angle ABJ$ and $\angle DCH$ are supplementary.

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–19
Example 2: Exs. 20–23
Example 3: Exs. 28–31
Example 4: Exs. 32–37
Example 5: Exs. 44–46

BICONDITIONAL STATEMENTS **Rewrite the biconditional statement as a conditional statement and its converse.**

20. Two angles are congruent if and only if they have the same measure.

21. A ray bisects an angle if and only if it divides the angle into two congruent angles.

22. Two lines are perpendicular if and only if they intersect to form right angles.

23. A point is a midpoint of a segment if and only if it divides the segment into two congruent segments.

FINDING COUNTEREXAMPLES **Give a counterexample that demonstrates that the converse of the statement is false.**

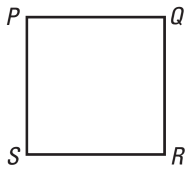
24. If an angle measures 94°, then it is obtuse.

25. If two angles measure 42° and 48°, then they are complementary.

26. If Terry lives in Tampa, then she lives in Florida.

27. If a polygon is a square, then it has four sides.

ANALYZING BICONDITIONAL STATEMENTS **Determine whether the biconditional statement about the diagram is *true* or *false*. If false, provide a counterexample.**



28. $\overline{SR}$ is perpendicular to $\overline{QR}$ if and only if $\angle SRQ$ measures 90°.

29. PQ and PS are equal if and only if PQ and PS are both 8 centimeters.

30. $\angle PQR$ and $\angle QRS$ are supplementary if and only if $m\angle PQR = m\angle QRS = 90°$.

31. $\angle PSR$ measures 90° if and only if $\angle PSR$ is a right angle.

FOCUS ON APPLICATIONS

SNOW LEOPARDS The pale coat of the snow leopard, as mentioned in Ex. 37, allows the animal to blend in with the snow 3960 meters (13,000 feet) high in the mountains of Central Asia.

REWRITING STATEMENTS **Rewrite the true statement in if-then form and write the converse. If the converse is true, combine it with the if-then statement to form a true biconditional statement. If the converse is false, provide a counterexample.**

32. Adjacent angles share a common side.

33. Two circles have the same circumference if they have the same diameter.

34. The perimeter of a triangle is the sum of the lengths of its sides.

35. All leopards have spots.

36. Panthers live in the forest.

37. A leopard is a snow leopard if the leopard has pale gray fur.

USING ALGEBRA **Determine whether the statement can be combined with its converse to form a true biconditional.**

38. If $3u + 2 = u + 12$, then $u = 5$.

39. If $v = 1$, then $9v - 4v = 2v + 3v$.

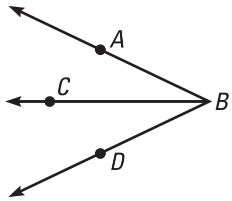
40. If $w^2 - 10 = w + 2$, then $w = 4$.

41. If $x^3 - 27 = 0$, then $x = 3$.

42. If $y = -3$, then $y^2 = 9$.

43. If $z = 3$, then $7 + 18z = 5z + 7 + 13z$.

44. REWRITING A POSTULATE Write the converse of the Angle Addition Postulate and decide whether the converse is *true* or *false*. If true, write the postulate as a true biconditional. If false, provide a counterexample.



Angle Addition Postulate: If C is in the interior of $\angle ABD$, then $m\angle ABC + m\angle CBD = m\angle ABD$.

45. *Writing* Give an example of a true biconditional statement.

46. MUSICAL GROUPS The table shows four different groups, along with the number of instrumentalists in each group. Write your own definitions of the musical groups and verify that they are true biconditional statements by writing each definition "forward" and "backward." The first one is started for you.

Sample: A musical group is a *piano trio* if and only if it contains exactly one pianist, one violinist, and one cellist.

Musical group	Pianist	Violinist	Cellist	Violist
Piano trio	1	1	1	—
String quartet	—	2	1	1
String quintet	—	2	1	2
Piano quintet	1	2	1	1

TECHNOLOGY In Exercises 47–49, use geometry software to complete the statement.

47. If the sides of a square are doubled, then the area is ___?___.

48. If the sides of a square are doubled, then the perimeter is ___?___.

49. Decide whether the statements in Exercises 47 and 48 can be written as true biconditionals. If not, provide a counterexample.

50. AIR DISTANCES The air distance between Jacksonville, Florida, and Merritt Island, Florida, is 148 miles and the air distance between Merritt Island and Fort Pierce, Florida, is 70 miles. Given that the air distance between Jacksonville and Fort Pierce is 218 miles, does Merritt Island fall on the line connecting Jacksonville and Fort Pierce?

FOCUS ON APPLICATIONS

REAL LIFE WINDS AT SEA Along with wind speed, sailors need to know the direction of the wind. Flags, also known as telltales, help sailors determine wind direction.

INTERNET APPLICATION LINK www.mcdougallittell.com

WINDS AT SEA Use the portion of the Beaufort wind scale table shown to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

51. A storm is a hurricane if and only if the winds of the storm measure 64 knots or greater.

52. Winds at sea are classified as a strong gale if and only if the winds measure 34–40 knots.

53. Winds are classified as 10 on the Beaufort scale if and only if the winds measure 41–55 knots.

Beaufort Wind Scale for Open Sea

Number	Knots	Description
8	34–40	gale winds
9	41–47	strong gale
10	48–55	storm
11	56–63	violent storm
12	64+	hurricane

Test Preparation

54. MULTIPLE CHOICE Which one of the following statements cannot be written as a true biconditional statement?

(A) Any angle that measures between 90° and 180° is obtuse.

(B) $2x - 5 = x + 1$ only if $x = 6$.

(C) Any angle that measures between 0° and 90° is acute.

(D) If two angles measure 110° and 70°, then they are supplementary.

(E) If the sum of the measures of two angles equals 180°, then they are supplementary.

55. MULTIPLE CHOICE Which of the following statements about the conditional statement "If two lines intersect to form a right angle, then they are perpendicular" is true?

I. The converse is true.
II. The statement can be written as a true biconditional.
III. The statement is false.

(A) I only (B) I and II only (C) II and III only
(D) III only (E) I, II, and III

★ Challenge

WRITING STATEMENTS In Exercises 56 and 57, determine (a) whether the contrapositive of the true statement is *true* or *false* and (b) whether the true statement can be written as a true biconditional.

56. If I am in Des Moines, then I am in the capital of Iowa.

57. If two angles measure 10° and 80°, then they are complementary.

58. LOGICAL REASONING You are given that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional? Explain. (*Hint:* Use your results from Exercises 56 and 57.)

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

STUDYING ANGLES Find the measures of a complement and a supplement of the angle. (Review 1.6 for 2.3)

59. 87° **60.** 73° **61.** 14° **62.** 29°

FINDING PERIMETER AND AREA Find the area and perimeter, or circumference of the figure described. (Use $\pi \approx 3.14$ when necessary.) (Review 1.7 for 2.3)

63. rectangle: $w = 3$ ft, $l = 12$ ft **64.** rectangle: $w = 7$ cm, $l = 10$ cm

65. circle: $r = 8$ in. **66.** square: $s = 6$ m

CONDITIONAL STATEMENTS Write the converse of the statement. (Review 2.1 for 2.3)

67. If the sides of a rectangle are all congruent, then the rectangle is a square.

68. If $8x + 1 = 3x + 16$, then $x = 3$.

2.4 Reasoning with Properties from Algebra

What you should learn

GOAL 1 Use properties from algebra.

GOAL 2 Use properties of length and measure to justify segment and angle relationships, such as the angles at the turns of a racetrack, as in **Example 5** and **Ex. 28**.

Why you should learn it

▼ Using algebraic properties helps you when rewriting a formula, such as the formula for an athlete's target heart rate in **Example 3**.

REAL LIFE

GOAL 1 USING PROPERTIES FROM ALGEBRA

Many properties from algebra concern the equality of real numbers. Several of these are summarized in the following list.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

ADDITION PROPERTY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY	If $a = b$, then $ac = bc$.
DIVISION PROPERTY	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
REFLEXIVE PROPERTY	For any real number a, $a = a$.
SYMMETRIC PROPERTY	If $a = b$, then $b = a$.
TRANSITIVE PROPERTY	If $a = b$ and $b = c$, then $a = c$.
SUBSTITUTION PROPERTY	If $a = b$, then a can be substituted for b in any equation or expression.

Properties of equality along with other properties from algebra, such as the *distributive property*,

$$a(b + c) = ab + ac$$

can be used to solve equations. For instance, you can use the subtraction property of equality to solve the equation $x + 3 = 7$. By subtracting 3 from each side of the equation, you obtain $x = 4$.

EXAMPLE 1 *Writing Reasons*

Using Algebra

Solve $5x - 18 = 3x + 2$ and write a reason for each step.

SOLUTION

$5x - 18 = 3x + 2$	**Given**
$2x - 18 = 2$	**Subtraction property of equality**
$2x = 20$	**Addition property of equality**
$x = 10$	**Division property of equality**

EXAMPLE 2 *Writing Reasons*

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

Solve $55z - 3(9z + 12) = -64$ and write a reason for each step.

SOLUTION

$55z - 3(9z + 12) = -64$	**Given**
$55z - 27z - 36 = -64$	**Distributive property**
$28z - 36 = -64$	**Simplify.**
$28z = -28$	**Addition property of equality**
$z = -1$	**Division property of equality**

EXAMPLE 3 *Using Properties in Real Life*

REAL LIFE **FITNESS** Before exercising, you should find your target heart rate. This is the rate at which you achieve an effective workout while not placing too much strain on your heart. Your target heart rate r (in beats per minute) can be determined from your age a (in years) using the equation $a = 220 - \frac{10}{7}r$.

a. Solve the formula for r and write a reason for each step.

b. Use the result to find the target heart rate for a 16 year old.

c. Find the target heart rate for the following ages: 20, 30, 40, 50, and 60. What happens to the target heart rate as a person gets older?

SOLUTION

a.

$a = 220 - \frac{10}{7}r$	**Given**
$a + \frac{10}{7}r = 220$	**Addition property of equality**
$\frac{10}{7}r = 220 - a$	**Subtraction property of equality**
$r = \frac{7}{10}(220 - a)$	**Multiplication property of equality**

b. Using $a = 16$, the target heart rate is:

$r = \frac{7}{10}(220 - a)$	**Given**
$r = \frac{7}{10}(220 - 16)$	**Substitute 16 for *a*.**
$r = 142.8$	**Simplify.**

▶ The target heart rate for a 16 year old is about 143 beats per minute.

c. From the table, the target heart rate appears to decrease as a person ages.

Age	20	30	40	50	60
Rate	140	133	126	119	112

GOAL 2 Using Properties of Length and Measure

The algebraic properties of equality can be used in geometry.

CONCEPT SUMMARY — PROPERTIES OF EQUALITY

	SEGMENT LENGTH	ANGLE MEASURE
REFLEXIVE	For any segment AB, $AB = AB$.	For any angle A, $m\angle A = m\angle A$.
SYMMETRIC	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
TRANSITIVE	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 *Using Properties of Length*

Logical Reasoning

In the diagram, $AB = CD$. The argument below shows that $AC = BD$.

$AB = CD$	Given
$AB + BC = BC + CD$	Addition property of equality
$AC = AB + BC$	Segment Addition Postulate
$BD = BC + CD$	Segment Addition Postulate
$AC = BD$	Substitution property of equality

EXAMPLE 5 *Using Properties of Measure*

AUTO RACING The Talladega Superspeedway racetrack in Alabama has four banked turns, which are described in the diagram at the left. Use the given information about the maximum banking angle of the four turns to find $m\angle 4$.

$m\angle 1 + m\angle 2 = 66°$

$m\angle 1 + m\angle 2 + m\angle 3 = 99°$

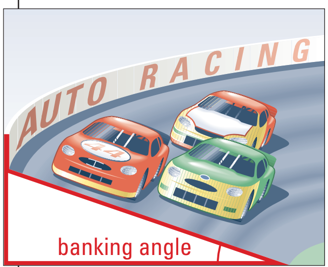
$m\angle 3 = m\angle 1$

$m\angle 1 = m\angle 4$

SOLUTION

$m\angle 1 + m\angle 2 = 66°$	Given
$m\angle 1 + m\angle 2 + m\angle 3 = 99°$	Given
$66° + m\angle 3 = 99°$	Substitution property of equality
$m\angle 3 = 33°$	Subtraction property of equality
$m\angle 3 = m\angle 1,\ m\angle 1 = m\angle 4$	Given
$m\angle 3 = m\angle 4$	Transitive property of equality
$m\angle 4 = 33°$	Substitution property of equality

FOCUS ON APPLICATIONS



REAL LIFE — AUTO RACING Banked turns help the cars travel around the track at high speeds. The angles provide an inward force that helps keep the cars from flying off the track.

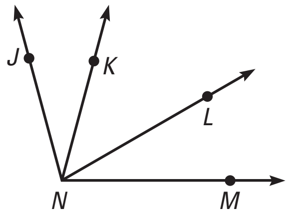
GUIDED PRACTICE

Vocabulary Check ✓

1. Name the property that makes the following statement true: "If $m\angle 3 = m\angle 5$, then $m\angle 5 = m\angle 3$."

Concept Check ✓

Use the diagram at the right.



2. Explain how the addition property of equality supports this statement: "If $m\angle JNK = m\angle LNM$, then $m\angle JNL = m\angle KNM$."

3. Explain how the subtraction property of equality supports this statement: "If $m\angle JNL = m\angle KNM$, then $m\angle JNK = m\angle LNM$."

Skill Check ✓

In Exercises 4–8, match the conditional statement with the property of equality.

4. If $JK = PQ$ and $PQ = ST$, then $JK = ST$.

5. If $m\angle S = 30°$, then $5° + m\angle S = 35°$.

6. If $ST = 2$ and $SU = ST + 3$, then $SU = 5$.

7. If $m\angle K = 45°$, then $3(m\angle K) = 135°$.

8. If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$.

A. Addition property

B. Substitution property

C. Transitive property

D. Symmetric property

E. Multiplication property

9. **WIND-CHILL FACTOR** If the wind is blowing at 20 miles per hour, you can find the wind-chill temperature W (in degrees Fahrenheit) by using the equation $W = 1.42T - 38.5$, where T is the actual temperature (in degrees Fahrenheit). Solve this equation for T and write a reason for each step. What is the actual temperature if the wind chill temperature is $-24.3°$F and the wind is blowing at 20 miles per hour?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

COMPLETING STATEMENTS **In Exercises 10–14, use the property to complete the statement.**

10. Symmetric property of equality: If $m\angle A = m\angle B$, then __?__.

11. Transitive property of equality: If $BC = CD$ and $CD = EF$, then __?__.

12. Substitution property of equality: If $LK + JM = 12$ and $LK = 2$, then __?__.

13. Subtraction property of equality: If $PQ + ST = RS + ST$, then __?__.

14. Division property of equality: If $3(m\angle A) = 90°$, then $m\angle A =$ __?__.

15. Copy and complete the argument below, giving a reason for each step.

$2(3x + 1) = 5x + 14$	**Given**
$6x + 2 = 5x + 14$	__?__
$x + 2 = 14$	__?__
$x = 12$	__?__

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 10–23
Example 2: Exs. 15–23
Example 3: Exs. 29–31
Example 4: Exs. 24–27
Example 5: Ex. 28

SOLVING EQUATIONS In Exercises 16–23, solve the equation and state a reason for each step.

16. $p - 1 = 6$

17. $q + 9 = 13$

18. $2r - 7 = 9$

19. $7s + 20 = 4s - 13$

20. $3(2t + 9) = 30$

21. $-2(-w + 3) = 15$

22. $26u + 4(12u - 5) = 128$

23. $3(4v - 1) - 8v = 17$

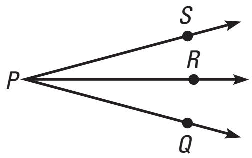
24. **LOGICAL REASONING** In the diagram, $m\angle RPQ = m\angle RPS$. Verify each step in the argument that shows $m\angle SPQ = 2(m\angle RPQ)$.

$m\angle RPQ = m\angle RPS$

$m\angle SPQ = m\angle RPQ + m\angle RPS$

$m\angle SPQ = m\angle RPQ + m\angle RPQ$

$m\angle SPQ = 2(m\angle RPQ)$



25. **LOGICAL REASONING** In the diagram, $m\angle ABF = m\angle BCG$ and $m\angle ABF = 90°$. Verify each step in the argument that shows $\overleftrightarrow{GK} \perp \overleftrightarrow{AD}$.

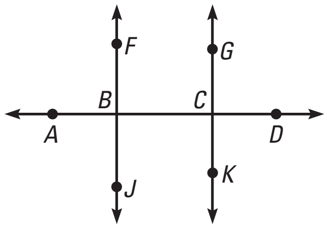
$m\angle ABF = 90°$

$m\angle ABF = m\angle BCG$

$m\angle BCG = 90°$

$\angle BCG$ is a right angle.

$\overleftrightarrow{GK} \perp \overleftrightarrow{AD}$



DEVELOPING ARGUMENTS In Exercises 26 and 27, give an argument for the statement, including a reason for each step.

26. If $\angle 1$ and $\angle 2$ are right angles, then they are supplementary.

27. If B lies between A and C and $AB = 3$ and $BC = 8$, then $AC = 11$.

FOCUS ON PEOPLE

REAL LIFE **BILL ELLIOTT** holds the qualifying record at Daytona International Speedway with a speed of 210.364 miles per hour.

28. **AUTO RACING** Some facts about the maximum banking angles of Daytona International Speedway at corners 1, 2, 3, and 4 are at the right. Find $m\angle 3$. Explain your steps. (Banked corners are described on page 98.)

$m\angle 1 + m\angle 3 + m\angle 4 = 93°$

$m\angle 2 + m\angle 4 = 62°$

$m\angle 2 = m\angle 3$

$m\angle 1 = m\angle 2$

PAY RAISES In Exercises 29–31, suppose you receive a raise at work. You can calculate your percent increase by using the pay raise formula $c(r + 1) = n$, where c is your current wage (in dollars per hour), r is your percent increase (as a decimal), and n is your new wage (in dollars per hour).

29. Solve the formula for r and write a reason for each step.

30. Use the result from Exercise 29 to find your percent increase if your current wage is \$10.00 and your new wage will be \$10.80.

31. Suppose Donald gets a 6% pay raise and his new wage is \$12.72. Find Donald's old wage. Explain the steps you used to find your answer.

Test Preparation

32. MULTI-STEP PROBLEM State a reason that makes the statement true.

a. If $4(x - 5 + 2x) = 0.5(12x - 16)$, then $4x - 20 + 8x = 6x - 8$.

b. If $4x - 20 + 8x = 6x - 8$, then $12x - 20 = 6x - 8$.

c. If $12x - 20 = 6x - 8$, then $6x - 20 = -8$.

d. If $6x - 20 = -8$, then $6x = 12$.

e. If $6x = 12$, then $x = 2$.

f. *Writing* Use parts (a) through (e) to provide an argument for "If $4(x - 5 + 2x) = 0.5(12x - 16)$, then $x = 2$."

★ Challenge

DETERMINING PROPERTIES Decide whether the relationship is *reflexive*, *symmetric*, or *transitive*. When the relationship does not have any of these properties, give a counterexample.

33. Set: students in a geometry class
Relationship: "earned the same grade as"
Example: Jim earned the same grade as Mario.

34. Set: letters of the alphabet
Relationship: "comes after"
Example: H comes after G.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

USING THE DISTANCE FORMULA Find the distance between the two points. Round your result to two decimal places. (Review 1.3 for 2.5)

35. $A(4, 5), B(-3, -2)$ **36.** $E(-7, 6), F(2, 0)$ **37.** $J(1, 1), K(-1, 11)$

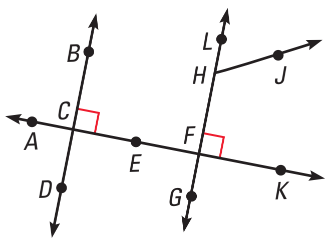
38. $P(8, -4), Q(1, -4)$ **39.** $S(9, -1), T(2, -6)$ **40.** $V(7, 10), W(1, 5)$

DETERMINING ENDPOINTS In Exercises 41–44, you are given an endpoint and the midpoint of a line segment. Find the coordinates of the other endpoint. Each midpoint is denoted by $M(x, y)$. (Review 1.5 for 2.5)

41. $B(5, 7)$, $M(-1, 0)$ **42.** $C(-4, -5)$, $M(3, -6)$ **43.** $F(0, 9)$, $M(6, -2)$ **44.** $Q(-1, 14)$, $M(2, 7)$

45. Given that $m\angle A = 48°$, what are the measures of a complement and a supplement of $\angle A$? (Review 1.6)

ANALYZING STATEMENTS Use the diagram shown at the right to determine whether the statement is *true* or *false*. (Review 2.2)



46. Points G, L, and J are collinear.

47. $\overline{BC} \perp \overline{FG}$

48. $\angle ECB \cong \angle ACD$

49. $\angle JHL$ and $\angle JHF$ are complementary.

50. $\overleftrightarrow{AK} \perp \overleftrightarrow{BD}$

2.5 Proving Statements about Segments

What you should learn

GOAL 1 Justify statements about congruent segments.

GOAL 2 Write reasons for steps in a proof.

Why you should learn it

▼ Properties of congruence allow you to justify segment relationships in **real life**, such as the segments in the trestle bridge shown in Exs. 3–5.

REAL LIFE

GOAL 1 PROPERTIES OF CONGRUENT SEGMENTS

A true statement that follows as a result of other true statements is called a **theorem**. All theorems must be proved. You can prove a theorem using a *two-column proof*. A **two-column proof** has numbered statements and reasons that show the logical order of an argument.

THEOREM

THEOREM 2.1 ***Properties of Segment Congruence***

Segment congruence is reflexive, symmetric, and transitive.
Here are some examples:

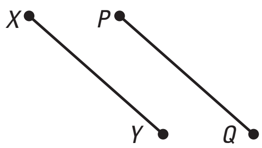
REFLEXIVE For any segment AB, $\overline{AB} \cong \overline{AB}$.

SYMMETRIC If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

TRANSITIVE If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

EXAMPLE 1 *Symmetric Property of Segment Congruence*

You can prove the Symmetric Property of Segment Congruence as follows.



GIVEN ▶ $\overline{PQ} \cong \overline{XY}$

PROVE ▶ $\overline{XY} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{XY}$	**1.** Given
2. $PQ = XY$	**2.** Definition of congruent segments
3. $XY = PQ$	**3.** Symmetric property of equality
4. $\overline{XY} \cong \overline{PQ}$	**4.** Definition of congruent segments

You are asked to complete proofs for the Reflexive and Transitive Properties of Segment Congruence in Exercises 6 and 7.

. . . .

A proof can be written in paragraph form, called **paragraph proof**. Here is a paragraph proof for the Symmetric Property of Segment Congruence.

Paragraph Proof You are given that $\overline{PQ} \cong \overline{XY}$. By the definition of congruent segments, $PQ = XY$. By the symmetric property of equality, $XY = PQ$. Therefore, by the definition of congruent segments, it follows that $\overline{XY} \cong \overline{PQ}$.

STUDENT HELP

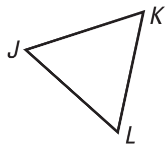
Study Tip
When writing a reason for a step in a proof, you must use one of the following: given information, a definition, a property, a postulate, or a previously proven theorem.

GOAL 2 USING CONGRUENCE OF SEGMENTS

EXAMPLE 2 *Using Congruence*

Proof

Use the diagram and the given information to complete the missing steps and reasons in the proof.



GIVEN ▶ $LK = 5$, $JK = 5$, $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\overline{LK} \cong \overline{JL}$

Statements	Reasons
1. ___**a.**___	**1.** Given
2. ___**b.**___	**2.** Given
3. $LK = JK$	**3.** Transitive property of equality
4. $\overline{LK} \cong \overline{JK}$	**4.** ___**c.**___
5. $\overline{JK} \cong \overline{JL}$	**5.** Given
6. ___**d.**___	**6.** Transitive Property of Congruence

SOLUTION

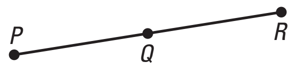
a. $LK = 5$ **b.** $JK = 5$ **c.** Definition of congruent segments **d.** $\overline{LK} \cong \overline{JL}$

EXAMPLE 3 *Using Segment Relationships*

Proof

In the diagram, Q is the midpoint of $\overline{PR}$.

Show that PQ and QR are each equal to $\frac{1}{2}PR$.



SOLUTION

Decide what you know and what you need to prove. Then write the proof.

GIVEN ▶ Q is the midpoint of $\overline{PR}$.

PROVE ▶ $PQ = \frac{1}{2}PR$ and $QR = \frac{1}{2}PR$.

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	**1.** Given
2. $PQ = QR$	**2.** Definition of midpoint
3. $PQ + QR = PR$	**3.** Segment Addition Postulate
4. $PQ + PQ = PR$	**4.** Substitution property of equality
5. $2 \cdot PQ = PR$	**5.** Distributive property
6. $PQ = \frac{1}{2}PR$	**6.** Division property of equality
7. $QR = \frac{1}{2}PR$	**7.** Substitution property of equality

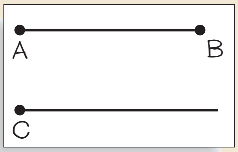
STUDENT HELP

Study Tip
The distributive property can be used to simplify a sum, as in Step 5 of the proof. You can think of $PQ + PQ$ as follows: $1(PQ) + 1(PQ) = (1 + 1)(PQ) = 2 \cdot PQ$.

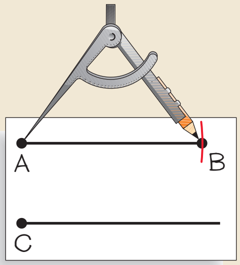
ACTIVITY

Construction — Copy a Segment

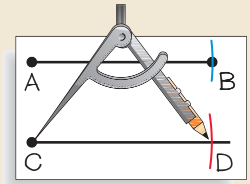
Use the following steps to construct a segment that is congruent to $\overline{AB}$.



1 Use a straightedge to draw a segment longer than $\overline{AB}$. Label the point C on the new segment.



2 Set your compass at the length of $\overline{AB}$.



3 Place the compass point at C and mark a second point, D, on the new segment. $\overline{CD}$ is congruent to $\overline{AB}$.

You will practice copying a segment in Exercises 12–15. It is an important construction because copying a segment is used in many constructions throughout this course.

GUIDED PRACTICE

Vocabulary Check ✓

1. An example of the Symmetric Property of Segment Congruence is "If $\overline{AB} \cong$ __?__, then $\overline{CD} \cong$ __?__."

Concept Check ✓

2. **ERROR ANALYSIS** In the diagram below, $\overline{CB} \cong \overline{SR}$ and $\overline{CB} \cong \overline{QR}$. Explain what is wrong with Michael's argument.

~~Because $\overline{CB} \cong \overline{SR}$ and $\overline{CB} \cong \overline{QR}$, then $\overline{CB} \cong \overline{AC}$ by the Transitive Property of Segment Congruence.~~

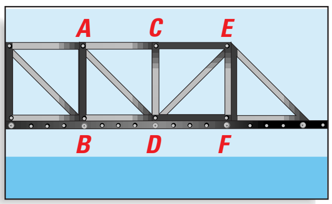
Skill Check ✓

BRIDGES **The diagram below shows a portion of a trestle bridge, where $\overline{BF} \perp \overline{CD}$ and D is the midpoint of $\overline{BF}$.**

3. Give a reason why $\overline{BD}$ and $\overline{FD}$ are congruent.

4. Are $\angle CDE$ and $\angle FDE$ complementary? Explain.

5. If $\overline{CE}$ and $\overline{BD}$ are congruent, explain why $\overline{CE}$ and $\overline{FD}$ are congruent.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

PROVING THEOREM 2.1 **Copy and complete the proof for two of the cases of the Properties of Segment Congruence Theorem.**

6. Reflexive Property of Segment Congruence

GIVEN ▶ EF is a line segment

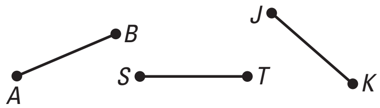
PROVE ▶ $\overline{EF} \cong \overline{EF}$

Statements	Reasons
1. $EF = EF$	**1.** ___?___
2. ___?___	**2.** Definition of congruent segments

7. Transitive Property of Segment Congruence

GIVEN ▶ $\overline{AB} \cong \overline{JK}$, $\overline{JK} \cong \overline{ST}$

PROVE ▶ $\overline{AB} \cong \overline{ST}$



Statements	Reasons
1. $\overline{AB} \cong \overline{JK}$, $\overline{JK} \cong \overline{ST}$	**1.** ___?___
2. $AB = JK$, $JK = ST$	**2.** ___?___
3. $AB = ST$	**3.** ___?___
4. $\overline{AB} \cong \overline{ST}$	**4.** ___?___

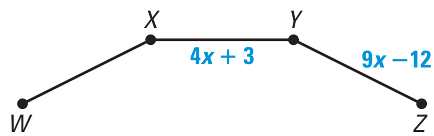
USING ALGEBRA **Solve for the variable using the given information. Explain your steps.**

8. GIVEN ▶ $\overline{AB} \cong \overline{BC}$, $\overline{CD} \cong \overline{BC}$

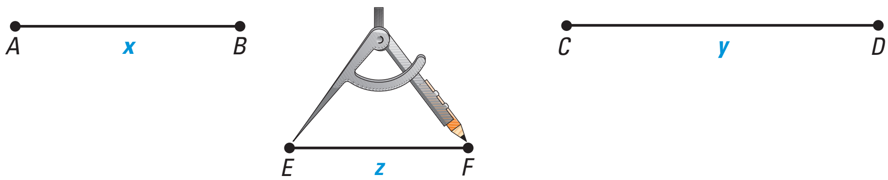
9. GIVEN ▶ $PR = 46$

10. GIVEN ▶ $\overline{ST} \cong \overline{SR}$, $\overline{QR} \cong \overline{SR}$

11. GIVEN ▶ $\overline{XY} \cong \overline{WX}$, $\overline{YZ} \cong \overline{WX}$



CONSTRUCTION **In Exercises 12–15, use the segments, along with a straightedge and compass, to construct a segment with the given length.**

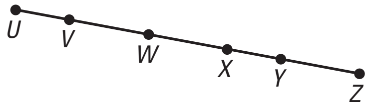


12. $x + y$ **13.** $y - z$ **14.** $3x - z$ **15.** $z + y - 2x$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 6, 7
Example 2: Exs. 16–18
Example 3: Exs. 16–18

16. **DEVELOPING PROOF** Write a complete proof by rearranging the reasons listed on the pieces of paper.



GIVEN ▶ $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{WX}$, $\overline{WX} \cong \overline{YZ}$

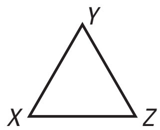
PROVE ▶ $\overline{UW} \cong \overline{XZ}$

Statements	Reasons
1. $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{WX}$, $\overline{WX} \cong \overline{YZ}$	Transitive Property of Segment Congruence
2. $\overline{VW} \cong \overline{YZ}$	Addition property of equality
3. $UV = XY$, $VW = YZ$	Definition of congruent segments
4. $UV + VW = XY + YZ$	Given
5. $UV + VW = UW$, $XY + YZ = XZ$	Segment Addition Postulate
6. $UW = XZ$	Definition of congruent segments
7. $\overline{UW} \cong \overline{XZ}$	Substitution property of equality

TWO-COLUMN PROOF **Write a two-column proof.**

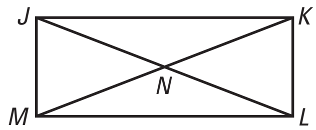
17. **GIVEN** ▶ $XY = 8$, $XZ = 8$, $\overline{XY} \cong \overline{ZY}$

PROVE ▶ $\overline{XZ} \cong \overline{ZY}$

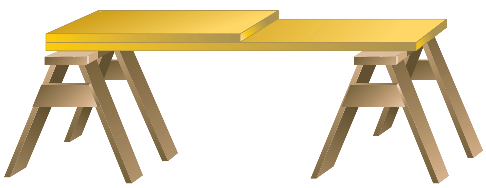


18. **GIVEN** ▶ $\overline{NK} \cong \overline{NL}$, $NK = 13$

PROVE ▶ $NL = 13$



19. **CARPENTRY** You need to cut ten wood planks that are the same size. You measure and cut the first plank. You cut the second piece, using the first plank as a guide, as in the diagram below. The first plank is put aside and the second plank is used to cut a third plank. You follow this pattern for the rest of the planks. Is the last plank the same length as the first plank? Explain.

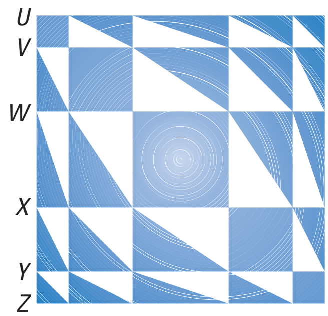


FOCUS ON CAREERS

REAL LIFE **CARPENTRY** For many projects, carpenters need boards that are all the same length. For instance, equally-sized boards in the house frame above insure stability.

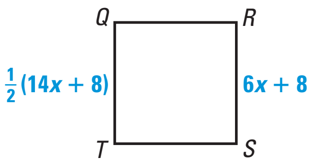
INTERNET **CAREER LINK** www.mcdougallittell.com

20. **OPTICAL ILLUSION** To create the illusion, a special grid was used. In the grid, corresponding row heights are the same measure. For instance, $\overline{UV}$ and $\overline{ZY}$ are congruent. You decide to make this design yourself. You draw the grid, but you need to make sure that the row heights are the same. You measure $\overline{UV}$, $\overline{UW}$, $\overline{ZY}$, and $\overline{ZX}$. You find that $\overline{UV} \cong \overline{ZY}$ and $\overline{UW} \cong \overline{ZX}$. Write an argument that allows you to conclude that $\overline{VW} \cong \overline{YX}$.



Test Preparation

21. MULTIPLE CHOICE In $QRST$, $\overline{QT} \cong \overline{TS}$ and $\overline{RS} \cong \overline{TS}$. What is x?



Ⓐ 1 Ⓑ 4 Ⓒ 12

Ⓓ 16 Ⓔ 32

22. MULTIPLE CHOICE In the figure shown below, $\overline{WX} \cong \overline{YZ}$. What is the length of $\overline{XZ}$?

Ⓐ 25 Ⓑ 34 Ⓒ 59 Ⓓ 60 Ⓔ 84

★ Challenge

REPRESENTING SEGMENT LENGTHS **In Exercises 23–26, suppose point *T* is the midpoint of $\overline{RS}$ and point *W* is the midpoint of $\overline{RT}$. If $\overline{XY} \cong \overline{RT}$ and $\overline{TS}$ has a length of *z*, write the length of the segment in terms of *z*.**

23. $\overline{RT}$ **24.** $\overline{XY}$ **25.** $\overline{RW}$ **26.** $\overline{WT}$

27. CRITICAL THINKING Suppose M is the midpoint of $\overline{AB}$, P is the midpoint of $\overline{AM}$, and Q is the midpoint of $\overline{PM}$. If a and b are the coordinates of points A and B on a number line, find the coordinates of P and Q in terms of a and b.

EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

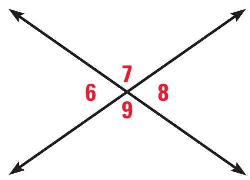
FINDING COUNTEREXAMPLES **Find a counterexample that shows the statement is false.** (Review 1.1)

28. For every number n, $2^n > n + 1$.

29. The sum of an even number and an odd number is always even.

30. If a number is divisible by 5, then it is divisible by 10.

FINDING ANGLE MEASURES **In Exercises 31–34, use the diagram to find the angle measure.** (Review 1.6 for 2.6)



31. If $m\angle 6 = 64°$, then $m\angle 7 =$ __?__.

32. If $m\angle 8 = 70°$, then $m\angle 6 =$ __?__.

33. If $m\angle 9 = 115°$, then $m\angle 8 =$ __?__.

34. If $m\angle 7 = 108°$, then $m\angle 8 =$ __?__.

35. Write the contrapositive of the conditional statement, "If Matthew wins this wrestling match, then he will win first place." (Review 2.1)

36. Is the converse of a true conditional statement always true? Explain. (Review 2.1)

USING SYMBOLIC NOTATION **Let *p* be "the car is in the garage" and let *q* be "Mark is home." Write the statement in words and symbols.** (Review 2.3)

37. The conditional statement $p \rightarrow q$

38. The converse of $p \rightarrow q$

39. The inverse of $p \rightarrow q$

40. The contrapositive of $p \rightarrow q$

2.6 Proving Statements about Angles

GOAL 1 Congruence of Angles

In Lesson 2.5, you proved segment relationships. In this lesson, you will prove statements about angles.

THEOREM

THEOREM 2.2 ***Properties of Angle Congruence***

Angle congruence is reflexive, symmetric, and transitive.

Here are some examples.

REFLEXIVE For any angle A, $\angle A \cong \angle A$.

SYMMETRIC If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

TRANSITIVE If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

The Transitive Property of Angle Congruence is proven in Example 1. The Reflexive and Symmetric Properties are left for you to prove in Exercises 10 and 11.

What you should learn

GOAL 1 Use angle congruence properties.

GOAL 2 Prove properties about special pairs of angles.

Why you should learn it

▼ Properties of special pairs of angles help you determine angles in wood-working projects, such as the corners in the piece of furniture below and in the picture frame in **Ex. 30**.

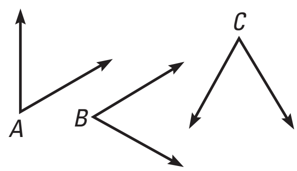
REAL LIFE

EXAMPLE 1 *Transitive Property of Angle Congruence*

Prove the Transitive Property of Congruence for angles.

Solution

To prove the Transitive Property of Congruence for angles, begin by drawing three congruent angles. Label the vertices as A, B, and C.



GIVEN ▶ $\angle A \cong \angle B$,
$\angle B \cong \angle C$

PROVE ▶ $\angle A \cong \angle C$

Statements	Reasons
1. $\angle A \cong \angle B$, $\angle B \cong \angle C$	**1.** Given
2. $m\angle A = m\angle B$	**2.** Definition of congruent angles
3. $m\angle B = m\angle C$	**3.** Definition of congruent angles
4. $m\angle A = m\angle C$	**4.** Transitive property of equality
5. $\angle A \cong \angle C$	**5.** Definition of congruent angles

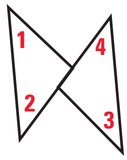
Proof

EXAMPLE 2 *Using the Transitive Property*

This two-column proof uses the Transitive Property.

GIVEN ▶ $m\angle 3 = 40°$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$

PROVE ▶ $m\angle 1 = 40°$



Statements	Reasons
1. $m\angle 3 = 40°$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Transitive Property of Congruence
3. $m\angle 1 = m\angle 3$	**3.** Definition of congruent angles
4. $m\angle 1 = 40°$	**4.** Substitution property of equality

THEOREM

THEOREM 2.3 ***Right Angle Congruence Theorem***

All right angles are congruent.

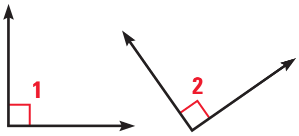
Proof

EXAMPLE 3 *Proving Theorem 2.3*

You can prove Theorem 2.3 as shown.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles

PROVE ▶ $\angle 1 \cong \angle 2$

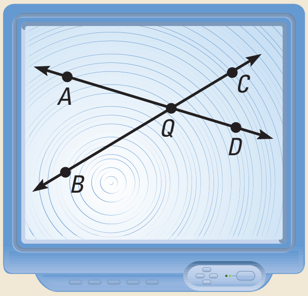


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles	**1.** Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive property of equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

ACTIVITY Using Technology

Investigating Supplementary Angles

Use geometry software to draw and label two intersecting lines.



1. What do you notice about the measures of $\angle AQB$ and $\angle AQC$? $\angle AQC$ and $\angle CQD$? $\angle AQB$ and $\angle CQD$?
2. Rotate $\overleftrightarrow{BC}$ to a different position. Do the angles retain the same relationship?
3. Make a conjecture about two angles supplementary to the same angle.

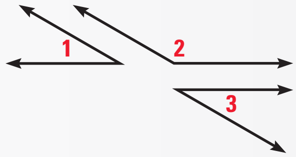
GOAL 2 Properties of Special Pairs of Angles

THEOREMS

THEOREM 2.4 ***Congruent Supplements Theorem***

If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.

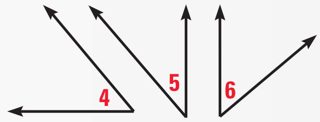
If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$.



THEOREM 2.5 ***Congruent Complements Theorem***

If two angles are complementary to the same angle (or to congruent angles) then the two angles are congruent.

If $m\angle 4 + m\angle 5 = 90°$ and $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.

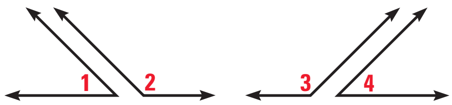


EXAMPLE 4 *Proving Theorem 2.4*

Proof

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements, $\angle 3$ and $\angle 4$ are supplements, $\angle 1 \cong \angle 4$

PROVE ▶ $\angle 2 \cong \angle 3$

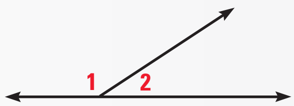


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplements, $\angle 3$ and $\angle 4$ are supplements, $\angle 1 \cong \angle 4$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**3.** Transitive property of equality
4. $m\angle 1 = m\angle 4$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	**5.** Substitution property of equality
6. $m\angle 2 = m\angle 3$	**6.** Subtraction property of equality
7. $\angle 2 \cong \angle 3$	**7.** Definition of congruent angles

POSTULATE

POSTULATE 12 ***Linear Pair Postulate***

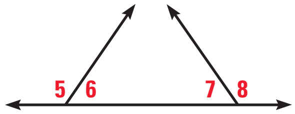
If two angles form a linear pair, then they are supplementary.



$m\angle 1 + m\angle 2 = 180°$

EXAMPLE 5 *Using Linear Pairs*

In the diagram, $m\angle 8 = m\angle 5$ and $m\angle 5 = 125°$. Explain how to show $m\angle 7 = 55°$.



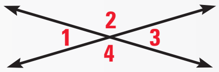
SOLUTION

Using the transitive property of equality, $m\angle 8 = 125°$. The diagram shows $m\angle 7 + m\angle 8 = 180°$. Substitute $125°$ for $m\angle 8$ to show $m\angle 7 = 55°$.

THEOREM

THEOREM 2.6 ***Vertical Angles Theorem***

Vertical angles are congruent.



$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

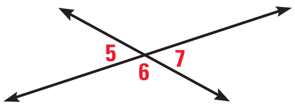
STUDENT HELP

Study Tip
Remember that previously proven theorems can be used as reasons in a proof, as in **Step 3** of the proof at the right.

EXAMPLE 6 *Proving Theorem 2.6*

GIVEN ▶ $\angle 5$ and $\angle 6$ are a linear pair, $\angle 6$ and $\angle 7$ are a linear pair

PROVE ▶ $\angle 5 \cong \angle 7$



Statements	Reasons
1. $\angle 5$ and $\angle 6$ are a linear pair, $\angle 6$ and $\angle 7$ are a linear pair	**1.** Given
2. $\angle 5$ and $\angle 6$ are supplementary, $\angle 6$ and $\angle 7$ are supplementary	**2.** Linear Pair Postulate
3. $\angle 5 \cong \angle 7$	**3.** Congruent Supplements Theorem

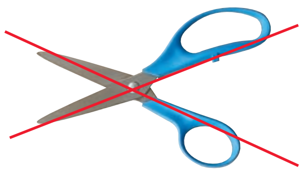
GUIDED PRACTICE

Vocabulary Check ✓

1. "If $\angle CDE \cong$ __?__ and $\angle QRS \cong \angle XYZ$, then $\angle CDE \cong \angle XYZ$," is an example of the __?__ Property of Angle Congruence.

Concept Check ✓

2. To close the blades of the scissors, you close the handles. Will the angle formed by the blades be the same as the angle formed by the handles? Explain.



Skill Check ✓

3. By the Transitive Property of Congruence, if $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then __?__ $\cong \angle C$.

In Exercises 4–9, $\angle 1$ and $\angle 3$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 2$ are vertical angles. Is the statement true?

4. $\angle 1 \cong \angle 3$ **5.** $\angle 1 \cong \angle 2$ **6.** $\angle 1 \cong \angle 4$

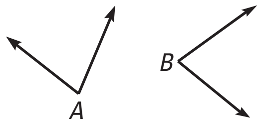
7. $\angle 3 \cong \angle 2$ **8.** $\angle 3 \cong \angle 4$ **9.** $m\angle 2 + m\angle 3 = 180°$

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 806.

10. **Proving Theorem 2.2** Copy and complete the proof of the Symmetric Property of Congruence for angles.



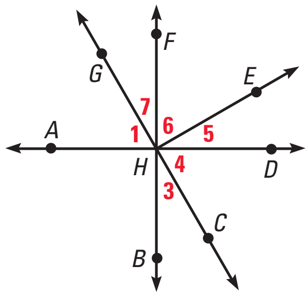
GIVEN ▶ $\angle A \cong \angle B$

PROVE ▶ $\angle B \cong \angle A$

Statements	Reasons
1. $\angle A \cong \angle B$	**1.** ___?___
2. ___?___	**2.** Definition of congruent angles
3. $m\angle B = m\angle A$	**3.** ___?___
4. $\angle B \cong \angle A$	**4.** ___?___

11. **Proving Theorem 2.2** Write a two-column proof for the Reflexive Property of Congruence for angles.

Finding Angles **In Exercises 12–17, complete the statement given that $m\angle EHC = m\angle DHB = m\angle AHB = 90°$**



12. If $m\angle 7 = 28°$, then $m\angle 3 =$ ___?___.

13. If $m\angle EHB = 121°$, then $m\angle 7 =$ ___?___.

14. If $m\angle 3 = 34°$, then $m\angle 5 =$ ___?___.

15. If $m\angle GHB = 158°$, then $m\angle FHC =$ ___?___.

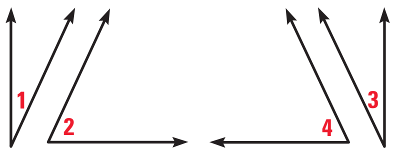
16. If $m\angle 7 = 31°$, then $m\angle 6 =$ ___?___.

17. If $m\angle GHD = 119°$, then $m\angle 4 =$ ___?___.

18. **Proving Theorem 2.5** Copy and complete the proof of the Congruent Complements Theorem.

GIVEN ▶ $\angle 1$ and $\angle 2$ are complements,
$\angle 3$ and $\angle 4$ are complements,
$\angle 2 \cong \angle 4$

PROVE ▶ $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complements, $\angle 3$ and $\angle 4$ are complements, $\angle 2 \cong \angle 4$	**1.** ___?___
2. ___?___, ___?___	**2.** Def. of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**3.** Transitive property of equality
4. $m\angle 2 = m\angle 4$	**4.** ___?___
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**5.** ___?___
6. $m\angle 1 = m\angle 3$	**6.** ___?___
7. ___?___	**7.** Definition of congruent angles

Student Help

HOMEWORK HELP
Example 1: Exs. 10, 11
Example 2: Exs. 12–17
Example 3: Exs. 12–17
Example 4: Exs. 19–22
Example 5: Exs. 23–28
Example 6: Exs. 23–28

FINDING CONGRUENT ANGLES **Make a sketch using the given information. Then, state all of the pairs of congruent angles.**

19. $\angle 1$ and $\angle 2$ are a linear pair. $\angle 2$ and $\angle 3$ are a linear pair. $\angle 3$ and $\angle 4$ are a linear pair.

20. $\angle XYZ$ and $\angle VYW$ are vertical angles. $\angle XYZ$ and $\angle ZYW$ are supplementary. $\angle VYW$ and $\angle XYV$ are supplementary.

21. $\angle 1$ and $\angle 3$ are complementary. $\angle 4$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 2$ are vertical angles.

22. $\angle ABC$ and $\angle CBD$ are adjacent, complementary angles. $\angle CBD$ and $\angle DBF$ are adjacent, complementary angles.

STUDENT HELP

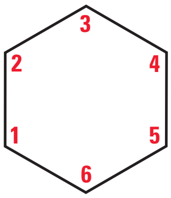
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 23–26.

WRITING PROOFS **Write a two-column proof.**

23. GIVEN ▶ $m\angle 3 = 120°$, $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 4$

PROVE ▶ $m\angle 1 = 120°$

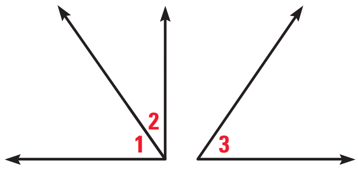
Plan for Proof First show that $\angle 1 \cong \angle 3$. Then use transitivity to show that $m\angle 1 = 120°$.



24. GIVEN ▶ $\angle 3$ and $\angle 2$ are complementary, $m\angle 1 + m\angle 2 = 90°$

PROVE ▶ $\angle 3 \cong \angle 1$

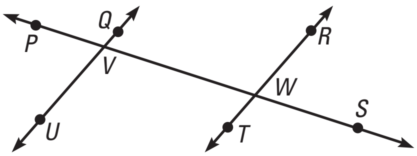
Plan for Proof First show that $\angle 1$ and $\angle 2$ are complementary. Then show that $\angle 3 \cong \angle 1$.



25. GIVEN ▶ $\angle QVW$ and $\angle RWV$ are supplementary

PROVE ▶ $\angle QVP \cong \angle RWV$

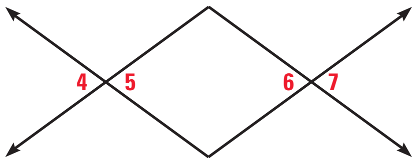
Plan for Proof First show that $\angle QVP$ and $\angle QVW$ are supplementary. Then show that $\angle QVP \cong \angle RWV$.



26. GIVEN ▶ $\angle 5 \cong \angle 6$

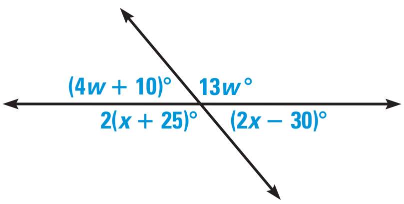
PROVE ▶ $\angle 4 \cong \angle 7$

Plan for Proof First show that $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$. Then use transitivity to show that $\angle 4 \cong \angle 7$.

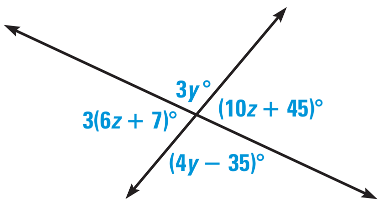


USING ALGEBRA **In Exercises 27 and 28, solve for each variable. Explain your reasoning.**

27.



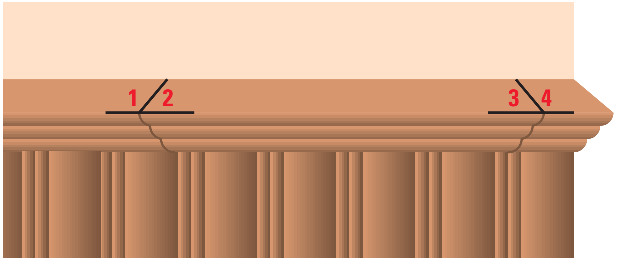
28.



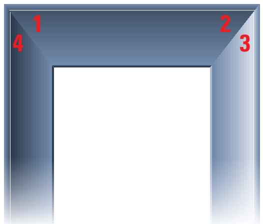
FOCUS ON APPLICATIONS

REAL LIFE **MITER BOX** This box has slotted sides to guide a saw when making angled cuts.

29. **WALL TRIM** A chair rail is a type of wall trim that is placed about three feet above the floor to protect the walls. Part of the chair rail below has been replaced because it was damaged. The edges of the replacement piece were angled for a better fit. In the diagram, $\angle 1$ and $\angle 2$ are supplementary, $\angle 3$ and $\angle 4$ are supplementary, and $\angle 2$ and $\angle 3$ each have measures of 50°. Is $\angle 1 \cong \angle 4$? Explain.



30. **PICTURE FRAMES** Suppose you are making a picture frame, as shown at the right. The corners are all right angles, and $m\angle 1 = m\angle 2 = 52°$. Is $\angle 4 \cong \angle 3$? Explain why or why not.



31. *Writing* Describe some instances of mitered, or angled, corners in the real world.

32. **TECHNOLOGY** Use geometry software to draw two overlapping right angles with a common vertex. Observe the measures of the three angles as one right angle is rotated about the other. What theorem does this illustrate?

Test Preparation

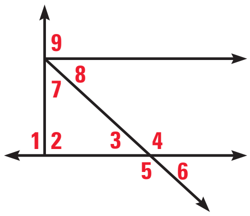
QUANTITATIVE COMPARISON **Choose the statement that is true about the diagram. In the diagram, $\angle 9$ is a right angle and $m\angle 3 = 42°$.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship can't be determined from the given information.



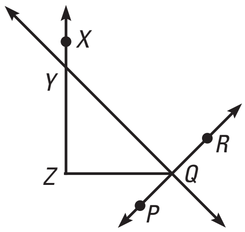
	Column A	Column B
33.	$m\angle 3 + m\angle 4$	$m\angle 1 + m\angle 2$
34.	$m\angle 3 + m\angle 6$	$m\angle 7 + m\angle 8$
35.	$m\angle 5$	$3(m\angle 3)$
36.	$m\angle 7 + m\angle 8$	$m\angle 9$

★ Challenge

37. **PROOF** Write a two-column proof.

GIVEN ▶ $m\angle ZYQ = 45°$, $m\angle ZQP = 45°$

PROVE ▶ $\angle ZQR \cong \angle XYQ$



MIXED REVIEW

FINDING ANGLE MEASURES **In Exercises 38–40, the measure of $\angle 1$ and the relationship of $\angle 1$ to $\angle 2$ is given. Find $m\angle 2$.** (Review 1.6 for 3.1)

38. $m\angle 1 = 62°$, complementary to $\angle 2$

39. $m\angle 1 = 8°$, supplementary to $\angle 2$

40. $m\angle 1 = 47°$, complementary to $\angle 2$

41. **PERPENDICULAR LINES** The definition of perpendicular lines states that if two lines are perpendicular, then they intersect to form a right angle. Is the converse true? Explain. (Review 2.2 for 3.1)

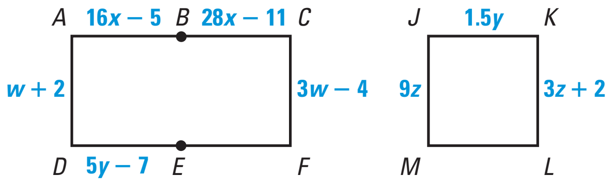
USING ALGEBRA **Use the diagram and the given information to solve for the variable.** (Review 2.5)

42. $\overline{AD} \cong \overline{EF}$, $\overline{EF} \cong \overline{CF}$

43. $\overline{AB} \cong \overline{EF}$, $\overline{EF} \cong \overline{BC}$

44. $\overline{DE} \cong \overline{EF}$, $\overline{EF} \cong \overline{JK}$

45. $\overline{JM} \cong \overline{ML}$, $\overline{ML} \cong \overline{KL}$



QUIZ 2

Self-Test for Lessons 2.4–2.6

Solve the equation and state a reason for each step. (Lesson 2.4)

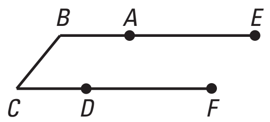
1. $x - 3 = 7$ **2.** $x + 8 = 27$ **3.** $2x - 5 = 13$

4. $2x + 20 = 4x - 12$ **5.** $3(3x - 7) = 6$ **6.** $-2(-2x + 4) = 16$

PROOF **In Exercises 7 and 8 write a two column proof.** (Lesson 2.5)

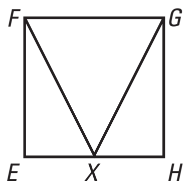
7. **GIVEN** ▶ $\overline{BA} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$, $\overline{AE} \cong \overline{DF}$

PROVE ▶ $\overline{BE} \cong \overline{CF}$



8. **GIVEN** ▶ $\overline{EH} \cong \overline{GH}$, $\overline{FG} \cong \overline{GH}$

PROVE ▶ $\overline{FG} \cong \overline{EH}$



9. **ASTRONOMY** While looking through a telescope one night, you begin looking due east. You rotate the telescope straight upward until you spot a comet. The telescope forms a 142° angle with due east, as shown. What is the angle of inclination of the telescope from due west? (Lesson 2.6)