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Algebra I • Module 1

Relationships Between Quantities and Reasoning with Equations and Their Graphs

OVERVIEW

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to nonlinear equations and their graphs. Students formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Algebra I: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise (**A-CED.A.2**). As they graph, they reason abstractly and quantitatively as well as choose and interpret units to solve problems related to the graphs they create (**N-Q.A.1, N-Q.A.2, N-Q.A.3**).

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions (**6.EE.A.3, 6.EE.A.4, 7.EE.A.1, 8.EE.A.1**). Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves (**A-SSE.A.2**). They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions (**A-APR.A.1**). The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations (**6.EE.B.5, 6.EE.B.7, 7.EE.B.4, 8.EE.C.7**) and systems of linear equations (**8.EE.C.8**). Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and nonlinear equations (**A-REI.A.1, A-REI.B.3, A-CED.A.4**). Students take their experience solving systems of linear equations further as they prove the validity of the addition method, learn a formal definition for the graph of an equation and use it to explain the reasoning of solving systems graphically, and represent the solution to systems of linear inequalities graphically (**A-CED.A.3, A-REI.C.5, A-REI.C.6, A-REI.D.10, A-REI.D.12**).

In Topic D, students are formally introduced to the modeling cycle (see page 61 of the CCLS) through problems that can be solved by creating equations and inequalities in one variable, systems of equations, and graphing (**N-Q.A.1, A-SSE.A.1, A-CED.A.1, A-CED.A.2, A-REI.B.3**). The End-of-Module Assessment follows Topic D.

Focus Standards

Reason quantitatively and use units to solve problems.

- N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
- N-Q.A.2²** Define appropriate quantities for the purpose of descriptive modeling.*
- N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

Interpret the structure of expressions.

- A-SSE.A.1** Interpret expressions that represent a quantity in terms of its context.*
- Interpret parts of an expression, such as terms, factors, and coefficients.
 - Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)n$ as the product of P and a factor not depending on P .*
- A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Perform arithmetic operations on polynomials.

- A-APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create equations that describe numbers or relationships.

- A-CED.A.1³** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**
- A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- A-CED.A.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**

²This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.

³In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

- A-CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .*[★]

Understand solving equations as a process of reasoning and explain the reasoning.

- A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

- A-REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

- A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.C.6**⁴ Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

- A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A-REI.D.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

⁴Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

Foundational Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

- 6.NS.C.7** Understand ordering and absolute value of rational numbers.
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*
 - Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that -3°C is warmer than -7°C .*

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*
- 6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

- 6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
- 6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Use properties of operations to generate equivalent expressions.

- 7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- 7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
- 7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
 - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Work with radicals and integer exponents.

- 8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*
- 8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Analyze and solve linear equations and pairs of simultaneous linear equations.

- 8.EE.C.7** Solve linear equations in one variable.
- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
 - Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain better understanding of the problem.
- MP.2** **Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.
- MP.3** **Construct viable arguments and critique the reasoning of others.** Students reason about solving equations using *if-then* moves based on equivalent expressions and properties of equality and inequality. They analyze when an *if-then* move is not reversible.
- MP.4** **Model with mathematics.** Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.
- MP.6** **Attend to precision.** Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (e.g., squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.

- MP.7 Look for and make use of structure.** Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.
- MP.8 Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b = cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.

Terminology

New or Recently Introduced Terms

- **Algebraic Expression** (An *algebraic expression* is either: (1) a numerical symbol or a variable symbol or (2) the result of placing previously generated algebraic expressions into the two blanks of one of the four operators ($(_) + (_)$, $(_) - (_)$, $(_) \times (_)$, $(_) \div (_)$) or into the base blank of an exponentiation with an exponent that is a rational number.)
- **Constant Term of a Polynomial in Standard Form** (The *constant term* is the value of the numerical expression found by substituting 0 into all the variable symbols of the polynomial, namely a_0 .)
- **Degree of a Monomial** (The *degree of a nonzero monomial* is the sum of the exponents of the variable symbols that appear in the monomial.)
- **Degree of a Polynomial in Standard Form** (The *degree of a polynomial in standard form* is the highest degree of the terms in the polynomial, namely n .)
- **Equivalent Algebraic Expressions** (Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression.)
- **Equivalent Numerical Expressions** (Two numerical expressions are *equivalent* if they evaluate to the same number.)
- **Graph of an Equation in Two Variables** (The set of all points in the coordinate plane that are solutions to an equation in two variables is called the *graph of the equation*.)
- **Leading Term and Leading Coefficient of a Polynomial in Standard Form** (The term $a_n x^n$ is called the *leading term*, and a_n is called the *leading coefficient*.)
- **Monomial** (A *monomial* is a polynomial expression generated using only the multiplication operator ($_ \times _$). Monomials are products whose factors are numerical expressions or variable symbols.)
- **Numerical Expression** (A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols) and that evaluates to a single number.)
- **Numerical Symbol** (A *numerical symbol* is a symbol that represents a specific number.)

- **Piecewise Linear Function** (Given a finite number of non-overlapping intervals on the real number line, a *(real) piecewise linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)
- **Polynomial Expression** (A *polynomial expression* is either: (1) a numerical expression or a variable symbol or (2) the result of placing two previously generated polynomial expressions into the blanks of the addition operator ($_ + _$) or the multiplication operator ($_ \times _$.)
- **Solution** (A *solution* to an equation with one variable is a number in the domain of the variable that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence.)
- **Solution Set** (The set of solutions of an equation is called its *solution set*.)
- **Standard Form of a Polynomial Expression in One Variable** (A polynomial expression with one variable symbol x is in *standard form* if it is expressed as $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where n is a nonnegative integer, and $a_0, a_1, a_2, \dots, a_n$ are constant coefficients with $a_n \neq 0$. A polynomial expression in x that is in standard form is often called a *polynomial in x* .)
- **Variable Symbol** (A *variable symbol* is a symbol that is a placeholder for a number. It is possible that a question may restrict the type of number that a placeholder might permit, maybe integers only or a positive real number, for instance.)
- **Zero Product Property** (The *Zero Product Property* states that given real numbers, a and b , if $a \cdot b = 0$ then either $a = 0$ or $b = 0$, or both a and $b = 0$.)

Familiar Terms and Symbols⁵

- Equation
- Formula
- Identity
- Inequality
- Linear Function
- Properties of Equality
- Properties of Inequality
- Solve
- System of Equations
- Term

Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities

⁵These are terms and symbols students have seen previously.

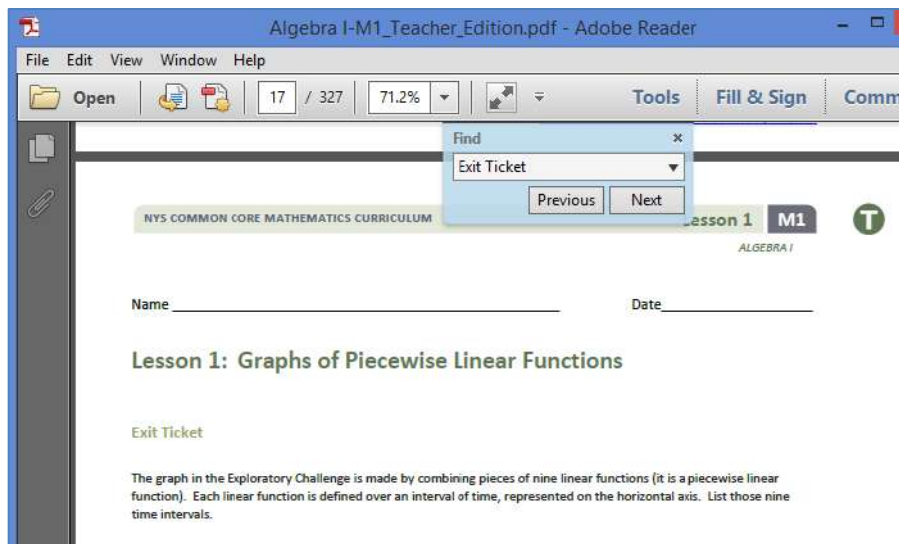
Preparing to Teach a Module

Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in *A Story of Functions* can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module.

Step 1: Get a preview of the plot.

- A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?
- B: Preview the module’s Exit Tickets to see the trajectory of the module’s mathematics and the nature of the work students are expected to be able to do.

Note: When studying a PDF file, enter “Exit Ticket” into the search feature to navigate from one Exit Ticket to the next.



Step 2: Dig into the details.

- A: Dig into a careful reading of the Module Overview. While reading the narrative, liberally reference the lessons and Topic Overviews to clarify the meaning of the text—the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts.
- B: Having thoroughly investigated the Module Overview, read through the Student Outcomes of each lesson (in order) to further discern the plot of the module. How do the topics flow and tell a coherent story? How do the outcomes move students to new understandings?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models presented in the module to explain the thinking involved. Again, liberally reference the lessons to anticipate how students who are learning with the curriculum might respond.

Preparing to Teach a Lesson

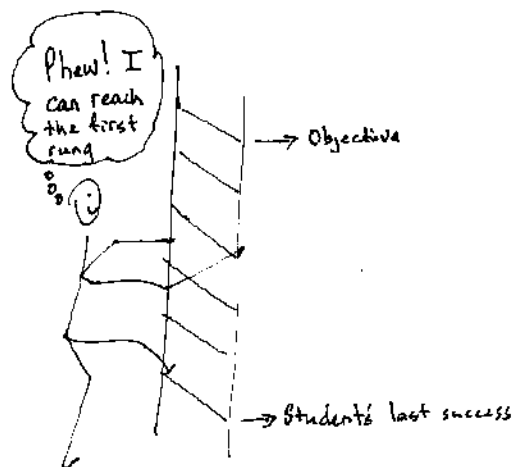
A three-step process is suggested to prepare a lesson. It is understood that at times teachers may need to make adjustments (customizations) to lessons to fit the time constraints and unique needs of their students. The recommended planning process is outlined below. Note: The ladder of Step 2 is a metaphor for the teaching sequence. The sequence can be seen not only at the macro level in the role that this lesson plays in the overall story, but also at the lesson level, where each rung in the ladder represents the next step in understanding or the next skill needed to reach the objective. To reach the objective, or the top of the ladder, all students must be able to access the first rung and each successive rung.

Step 1: Discern the plot.

- A: Briefly review the module’s Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.
- B: Read the Topic Overview related to the lesson, and then review the Student Outcome(s) and Exit Ticket of each lesson in the topic.
- C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

- A: Work through the lesson, answering and completing each question, example, exercise, and challenge.
- B: Analyze and write notes on the new complexities or new concepts introduced with each question or problem posed; these notes on the sequence of new complexities and concepts are the rungs of the ladder.
- C: Anticipate where students might struggle, and write a note about the potential cause of the struggle.
- D: Answer the Closing questions, always anticipating how students will respond.



Step 3: Hone the lesson.

Lessons may need to be customized if the class period is not long enough to do all of what is presented and/or if students lack prerequisite skills and understanding to move through the entire lesson in the time allotted. A suggestion for customizing the lesson is to first decide upon and designate each question, example, exercise, or challenge as either “Must Do” or “Could Do.”

- A: Select “Must Do” dialogue, questions, and problems that meet the Student Outcome(s) while still providing a coherent experience for students; reference the ladder. The expectation should be that the majority of the class will be able to complete the “Must Do” portions of the lesson within the allocated time. While choosing the “Must Do” portions of the lesson, keep in mind the need for a balance of dialogue and conceptual questioning, application problems, and abstract problems, and a balance between students using pictorial/graphical representations and abstract representations. Highlight dialogue to be included in the delivery of instruction so that students have a chance to articulate and consolidate understanding as they move through the lesson.

- B: “Must Do” portions might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on the anticipated difficulties, the remedial work might take on different forms as suggested in the chart below.

Anticipated Difficulty	“Must Do” Remedial Problem Suggestion
The first problem of the lesson is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct students to complete those first problems to empower them to begin the lesson.
There is too big of a jump in complexity between two problems.	Provide a problem or set of problems that bridge student understanding from one problem to the next.
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the lesson, do a quick, engaging fluency exercise. ⁶ Before beginning any fluency activity for the first time, assess that students have conceptual understanding of the problems in the set and that they are poised for success with the easiest problem in the set.
More work is needed at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies.
More work is needed at the abstract level.	Add a set of abstract problems to be completed toward the end of the lesson.

- C: “Could Do” problems are for students who work with greater fluency and understanding and can, therefore, complete more work within a given time frame.
- D: At times, a particularly complex problem might be designated as a “Challenge!” problem to provide to advanced students. Consider creating the opportunity for students to share their “Challenge!” solutions with the class at a weekly session or on video.
- E: If the lesson is customized, be sure to carefully select Closing questions that reflect such decisions, and adjust the Exit Ticket if necessary.

⁶Look for fluency suggestions at www.eureka-math.org.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.A.1, N-Q.A.2, N-Q.A.3, A-APR.A.1, A-SSE.A.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	N-Q.A.1, A-SSE.A.1, A-SSE.A.2, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.A.1, A-REI.C.5, A-REI.C.6, A-REI.D.10, A-REI.D.12

Name _____

Date _____

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

- a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

- b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.
- c. If he drives directly back to his house after the grocery store, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.
- a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.
- b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.
- c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.
- a. What quantities could the following expressions represent in terms of the problem context?

$$m + n$$

$$mB + nK$$

$$\frac{mB+nK}{m+n}$$

- b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x + 3)(3x + 8) - 3x(x + 3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

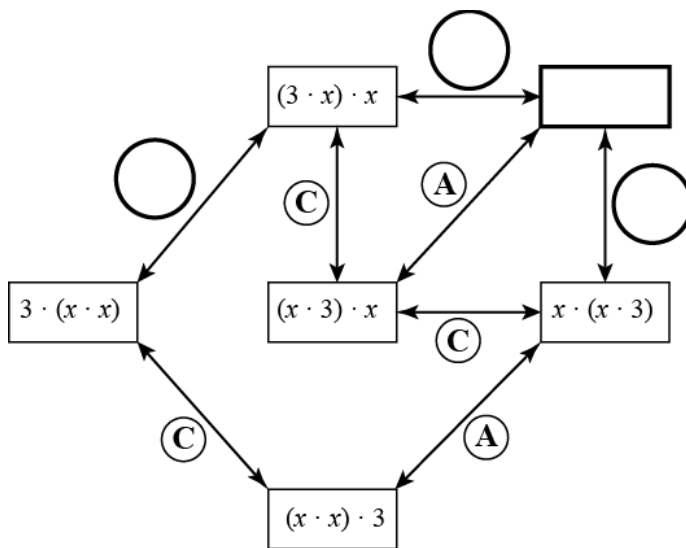
5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1 + ((2 + 3) \cdot 4)$ is one such expression.
- a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

- b. In both of your expressions, replace 1 with a , 2 with b , 3 with c , and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:
 - (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for a , b , c , and d into each expression, the expressions evaluate to **different numbers**, and
 - (2) Find four different, nonzero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



- Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
- Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10 , he says, is 28,130. He doesn't understand why this rule is true.
- What is the product of the polynomial $2x^3 + 8x^2 + x + 3$ times the polynomial x ?
 - Use part (a) as a hint. Explain why the rule Ahmed learned is true.
- 8.
- Find the following products:
 - $(x - 1)(x + 1)$
 - $(x - 1)(x^2 + x + 1)$
 - $(x - 1)(x^3 + x^2 + x + 1)$
 - $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

v. $(x - 1)(x^n + x^{n-1} + \dots + x^3 + x^2 + x + 1)$

- b. Substitute $x = 10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.
- c. If we substituted $x = 10$ into the product $(x - 2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?
- d. Multiply $(x - 2)$ and $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$, and express your answer in standard form.

Substitute $x = 10$ into your answer, and see if you obtain the same result that you obtained in part (c).

e. Francois says $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ must equal $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ because when $x = 10$, multiplying by $x - 9$ is the same as multiplying by 1.

i. Multiply $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$.

ii. Put $x = 10$ into your answer.

Is it the same as $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ with $x = 10$?

iii. Was Francois right?

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a N-Q.A.1 N-Q.A.2	Student is unable to respond to question. OR Student provides a minimal attempt to create an incorrect graph.	Student creates a graph that reflects something related to the problem, but the axes do not depict the correct units of distance from the house on the y -axis and a measurement of time on the x -axis, or the graph indicates significant errors in calculations or reasoning.	Student creates axes that depict distance from the house on the y -axis and some measurement of time on the x -axis, and the graph represents a reflection of what occurred but with errors in calculations, missing or erroneous axis labels, or choice of units that makes the graph difficult to obtain information from.	Student creates and labels the y -axis to represent distance from the house in miles and an x -axis to represent time (in minutes past 1:00 p.m.) and creates a graph based on solid reasoning and correct calculations.

	<p>b</p> <p>N-Q.A.1</p>	<p>Student answers incorrectly with no evidence of reasoning to support the answer. OR Student leaves item blank.</p>	<p>Student answers incorrectly but demonstrates some reasoning in explaining the answer.</p>	<p>Student answers 1:21 p.m. but does not either refer to a correct graph or provide sound reasoning to support the answer. OR Student answers incorrectly because either the graph in part (a) is incorrect, and the graph is referenced or because a minor calculation error is made, but sound reasoning is used.</p>	<p>Student answers 1:21 p.m. and either refers to a correct graph from part (a) or provides reasoning and calculations to explain the answer.</p>
	<p>c</p> <p>N-Q.A.1</p>	<p>Student answers incorrectly with no evidence of reasoning to support the answer. OR Student leaves item blank.</p>	<p>Student answers incorrectly but demonstrates some reasoning in explaining the answer.</p>	<p>Student answers 12 miles but does not either refer to the work in part (a) or provide sound reasoning in support of the answer. OR Student answers incorrectly because either the work in part (a) is referenced, but the work is incorrect or because a minor calculation error is made but sound reasoning is used.</p>	<p>Student answers 12 miles and either references correct work from part (a) or provides reasoning and calculations to support the answer.</p>

2	a	<p>Student leaves the question blank.</p> <p>OR</p> <p>Student provides an answer that reflects no or very little reasoning.</p>	<p>Student either begins with an assumption that is not based on the evidence of water being used at a rate of approximately 10 liters/second at noon.</p> <p>OR</p> <p>Student uses poor reasoning in extending that reading to consider total use across 24 hours.</p>	<p>Student answers beginning with the idea that water is being used at a rate of approximately 10 liters/second at noon but makes an error in the calculations to extend and combine that rate to consider usage across 24 hours.</p> <p>OR</p> <p>Student does not defend the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.</p>	<p>Student answers beginning with the idea that water is being used at a rate of approximately 10 liters/second at noon and makes correct calculations to extend and combine that rate to consider usage across 24 hours.</p> <p>AND</p> <p>Student defends the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.</p>
	b	<p>Student leaves the question blank.</p> <p>OR</p> <p>Student provides an answer that reflects no or very little reasoning.</p>	<p>Student provides an answer that is outside of the range from <i>to the nearest ten</i> to <i>to the nearest hundred</i>.</p> <p>OR</p> <p>Student provides an answer that is within the range but is not supported by an explanation.</p>	<p>Student answer ranges from <i>to the nearest ten</i> to <i>to the nearest hundred</i> but is not well supported by sound reasoning.</p> <p>OR</p> <p>Student answer contains an error in the way the explanation is written, even if it is clear what the student means to say.</p>	<p>Student answer ranges from <i>to the nearest ten</i> to <i>to the nearest hundred</i> and is supported by correct reasoning that is expressed accurately.</p>

	c N-Q.A.3	Student leaves the question blank. OR Student provides an answer that reflects no or very little reasoning.	Student answer is not in the range of 6 to 48 checks but provides some reasoning to justify the choice. OR Student answer is in that range, perhaps written in the form of <i>every x minutes</i> or <i>every x hours</i> but is not supported by an explanation with solid reasoning.	Student answer is in the range of 6 to 48 checks but is only given in the form of x checks per minute or x checks per hour; the answer is well supported by a written explanation. OR Student answer is given in terms of number of checks but is not well supported by a written explanation.	Student answer is in the range of 6 to 48 checks, and student provides solid reasoning to support the answer.
3	a A-SSE.A.1a A-SSE.A.1b	Student either does not answer. OR Student answers incorrectly for all three expressions.	Student answers one or two of the three correctly but leaves the other one blank or makes a gross error in describing what it represents.	Student answers two of the three correctly and makes a reasonable attempt at describing what the other one represents.	Student answers all three correctly.
	b A-SSE.A.1a A-SSE.A.1b	Student either does not answer. OR Student answers incorrectly for all three parts of the question.	Student understands that the expressions represent a portion of the orders for each color but mis-assigns the colors and/or incorrectly determines which one is larger.	Student understands that the expressions represent a portion of the orders for each color and correctly determines which one is larger but has errors in the way the answer is worded or does not provide support for why $\frac{n}{m+n}$ is larger.	Student understands that the expressions represent a portion of the orders for each color, correctly determines which one is larger, and provides a well-written explanation for why.

4	A-SSE.A.1b A-SSE.A.2	Student leaves the question blank. OR Student is unable to rewrite the expression successfully, even by multiplying out the factors first.	Student gets to the correct rewritten expression of $8x + 24$ but does so by multiplying out the factors first. OR Student does not show the work needed to demonstrate how $8x + 24$ is determined.	Student attempts to use structure to rewrite the expression as described, showing the process, but student makes errors in the process.	Student correctly uses the process described to arrive at $8x + 24$ without multiplying out linear factors and demonstrates the steps for doing so.
5	a–b A-SSE.A.2	Student is unable to respond to many of the questions. OR Student leaves several items blank.	Student is only able to come up with one option for part (a) and, therefore, has only partial work for part (b). OR Student answers Yes for the question about equivalent expressions.	Student successfully answers part (a) and identifies that the expressions created in part (b) are not equivalent, but there are minor errors in the answering of the remaining questions.	Student answers all four parts correctly and completely.
6	a A-SSE.A.2	Student leaves at least three items blank. OR Student answers at least three items incorrectly.	Student answers one or two items incorrectly or leaves one or more items blank.	Student completes circling task correctly and provides a correct ordering of symbols in the box, but the answer does not use parentheses or multiplication dots.	Student completes all four item correctly, including exact placement of parentheses and symbols for the box: $x \cdot (3 \cdot x)$.
	b A-SSE.A.2	Student does not complete either proof successfully.	Student attempts both proofs but makes minor errors in both. OR Student only completes one proof, with or without errors.	Student attempts both proofs but makes an error in one of them.	Student completes both proofs correctly, and the two proofs are different from one another.

7	a A-APR.A.1	Student leaves the question blank or demonstrates no understanding of multiplication of polynomials.	Student makes more than one error in his multiplication but demonstrates some understanding of multiplication of polynomials.	Student makes a minor error in the multiplication.	Student multiplies correctly and expresses the resulting polynomial as a sum of monomials.
	b A-APR.A.1	Student leaves the question blank or does not demonstrate a level of thinking that is higher than what is given in the problem’s description of Ahmed’s thinking.	Student uses language that does not indicate an understanding of base x and/or the place value system. Student may use language such as <i>shifting</i> or <i>moving</i> .	Student makes only minor errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x .	Student makes no errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x .
8	a–c A-APR.A.1	Student shows limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x .	Student makes multiple errors but shows some understanding of polynomial multiplication. Student may not combine like terms to present the product as the sum of monomials.	Student makes one or two minor errors but demonstrates knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also shows understanding of evaluating a polynomial for the given value of x .	Student completes all products correctly, expressing each as a sum of monomials with like terms collected and evaluated correctly when x is 10.
	d A-APR.A.1	Student shows limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x .	Student makes multiple errors but shows some understanding of polynomial multiplication. Student may combine like terms to present the product as the sum of monomials. Student get an incorrect result when evaluating with $x = 10$.	Student makes one or two minor errors but demonstrates knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also shows understanding of evaluating a polynomial for the given value of x .	Student correctly multiplies the polynomials and expresses the product as a polynomial in standard form. Student correctly evaluates with a value of 10 and answers Yes.

	<p>e</p> <p>A-APR.A.1</p>	<p>Student is unable to demonstrate an understanding that part (iii) is <i>No</i> and/or demonstrates limited or no understanding of polynomial multiplication.</p>	<p>Student may make some errors as he multiplies the polynomials and expresses the product as a sum of monomials. Student may make some errors in the calculation of the value of the polynomial when x is 10. Student incorrectly answers part (iii) or applies incorrect reasoning.</p>	<p>Student may make minor errors in multiplying the polynomials and expressing the product as a sum of monomials. Student may make minor errors in calculating the value of the polynomial when x is 10. Student explains that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explains that the two expressions are not algebraically equivalent.</p>	<p>Student correctly multiplies the polynomials and expresses the product as a sum of monomials with like terms collected. Student correctly calculates the value of the polynomial when x is 10. Student explains that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explains that the two expressions are not algebraically equivalent.</p>
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Name _____

Date _____

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late, and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

- a. Draw a graph that shows the distance Jacob’s car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).



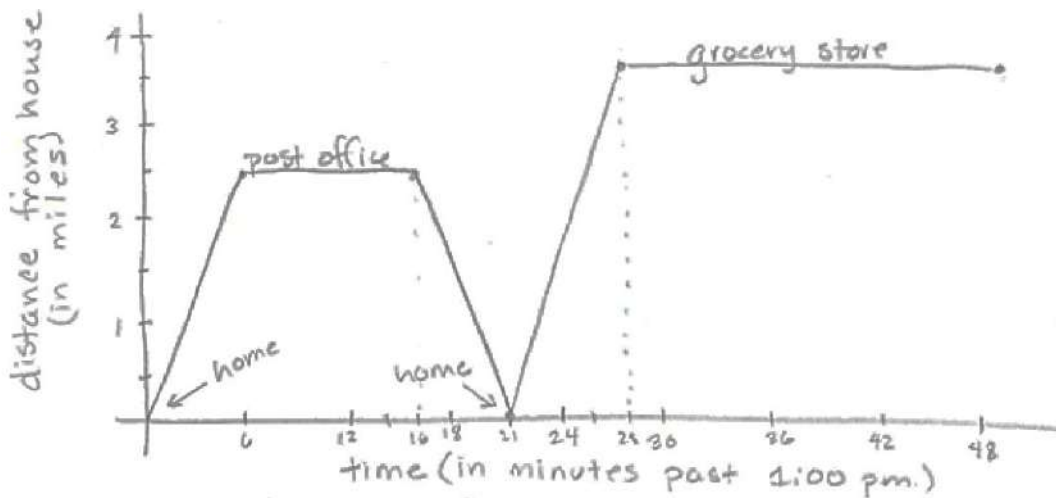
$$25 \frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.5 \text{ miles from house to post office}$$

$$30 \frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 6 \text{ miles from post office to store}$$

$$6 \text{ miles} - 2.5 \text{ miles} = 3.5 \text{ miles from home to store}$$

$$6 \text{ miles in } 12 \text{ minutes is } 1 \text{ mile in } 2 \text{ minutes}$$

so 2.5 miles takes 5 minutes and 3.5 miles takes 7 minutes

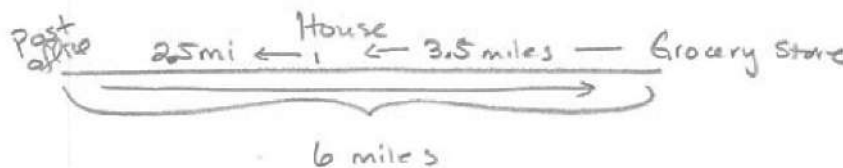


- b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

It is 1:21 p.m. The graph shows the time as 21 minutes past 1:00 p.m. He spent 6 minutes getting to the post office, 10 minutes at the post office, and 5 minutes getting from the post office to the point of passing by his house. You know it took 5 minutes for the last part because he traveled 30 miles per hour and went 2.5 miles.

$$2.5 \text{ miles} \times \frac{60 \text{ minutes}}{30 \text{ miles}} = 5 \text{ minutes}$$

- c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.



12 miles.

$$2.5 \text{ miles} + 6 \text{ miles} + 3.5 \text{ miles} = 12 \text{ miles}$$

You know it is 2.5 miles from the house to the post office because

$$25 \frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.5 \text{ miles.}$$

You know it is 6 miles from the post office to the store because

$$30 \frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 6 \text{ miles.}$$

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the ones place on the water meter display changes too rapidly to read the digit and that the digit in the tens place changes every second or so.
- a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.

$$10 \frac{\text{liters}}{\text{second}} \times 60 \frac{\text{seconds}}{\text{minute}} \times 60 \frac{\text{minutes}}{\text{hour}} \times 18 \text{ hours} = 648\,000 \text{ liters}$$

Since water is probably only used from about 5:00 a.m. to 11:00 p.m., I did not multiply by 24 hours, but by 18 hours instead.

- b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

It can be reported within ± 10 liters, since he can read the 10's place, but it is changing by a 10 during the second he reads it.

- c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage rate with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

24 checks. Every hour would be good to show the peaks in usage during morning and evening hours from those in the apartments. And it might also show that businesses stop using it after business hours. It would depend on what portion of the building is business vs. apartments.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

- a. What quantities could the following expressions represent in terms of the problem context?

$m + n$ *Total number of ink orders over a one-month period.*

$mB + nK$ *Total number of ink orders over a one-month period.*

$\frac{mB+nK}{m+n}$ *Total number of ink orders over a one-month period.*

- b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

and explain which expression must be greater using those interpretations.

$\frac{m}{m+n}$ is the fraction of orders that are for blue ink.

$\frac{n}{m+n}$ is the fraction of orders that are for black ink.

$\frac{n}{m+n}$ would be bigger, 2 times as big as $\frac{m}{m+n}$ because they ordered twice as many orders for black ink than for blue ink.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x + 3)(3x + 8) - 3x(x + 3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

$$((3x + 8) - 3x) \cdot (x + 3)$$

$$8(x + 3)$$

$$8x + 24$$

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1 + ((2 + 3) \cdot 4)$ is one such expression.
- a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

$$(1 + 2) \cdot (3 + 4) = 21$$

$$((2 + 4) + 1) \cdot 3 = 21$$

- b. In both of your expressions, replace 1 with a , 2 with b , 3 with c , and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

$$(a + b) \cdot (c + d) = ac + ad + bc + bd$$

$$((b + d) + a) \cdot c = ac + bc + dc$$

Are your algebraic expressions equivalent? Circle: Yes

No

- If they are equivalent, prove that they are using the properties of operations.
 - If not, provide **two** examples:
- (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for a , b , c , and d into each expression, the expressions evaluate to **different numbers**, and

$$a = 5 \quad b = 10 \quad c = 20 \quad d = 30$$

$$(5 + 10) \cdot (20 + 30) = 750$$

$$((10 + 30) + 5) \cdot 20 = 900$$

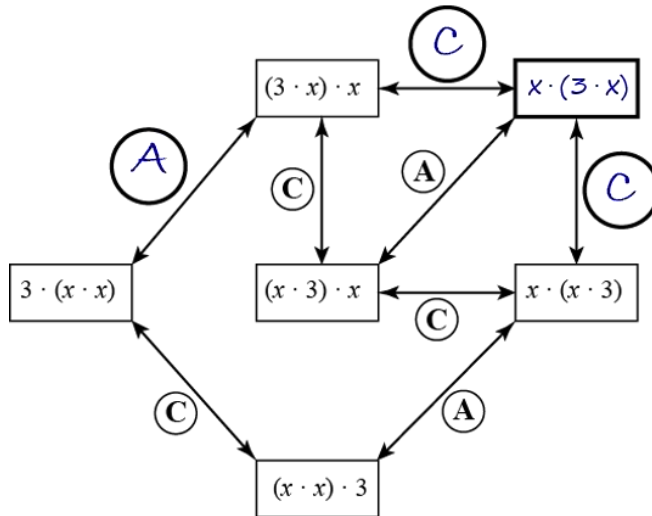
- (2) Find four different, nonzero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

$$5, 6, 11, 7 \quad (ac + ad + bc + bd) \text{ needs to equal } (ac + bc + dc);$$

$$(5 + 6) \cdot (11 + 7) = 11 \cdot 18 = 198 \quad \text{so, } (ad + bd) \text{ needs to equal } (dc);$$

$$((6 + 7) + 5) \cdot 11 = 18 \cdot 11 = 198 \quad \text{so, } (a + b) \text{ needs to equal } c.$$

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



- Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
- Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

- $(x \cdot x) \cdot 3 = x \cdot (x \cdot 3)$ *by associative property*
 $x \cdot (x \cdot 3) = x \cdot (3 \cdot x)$ *by commutative property*
 $x \cdot (3 \cdot x) = (3 \cdot x) \cdot x$ *by commutative property*
- $(x \cdot x) \cdot 3 = 3 \cdot (x \cdot x)$ *by commutative property*
 $3 \cdot (x \cdot x) = (3 \cdot x) \cdot x$ *by associative property*

7. Ahmed learned: “To multiply a whole number by ten, just place a zero at the end of the number.” For example, 2813×10 , he says, is 28,130. He doesn’t understand why this *rule* is true.

- a. What is the product of the polynomial $2x^3 + 8x^2 + x + 3$ times the polynomial x ?

$$2x^4 + 8x^3 + x^2 + 3x$$

- b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

When you multiply by the same number as the base, it creates a new number where each digit in the original number is now one place value higher so that there is nothing left (no numbers) to represent the ones digit, which leads to a trailing “0” in the ones digit.

8.

- a. Find the following products:

i. $(x - 1)(x + 1)$

$$x^2 + x - x - 1$$

$$x^2 - 1$$

ii. $(x - 1)(x^2 + x + 1)$

$$x^3 + x^2 + x - x^2 - x - 1$$

$$x^3 - 1$$

iii. $(x - 1)(x^3 + x^2 + x + 1)$

$$x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$$

$$x^4 - 1$$

iv. $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

$$x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$$

$$x^5 - 1$$

$$v. \quad (x - 1)(x^n + x^{n-1} + \dots + x^3 + x^2 + x + 1)$$

$$x^{n+1} - 1$$

- b. Substitute $x = 10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

$$i. \quad (10 - 1) \cdot (10 + 1) = (100 - 1)$$

$$9 \cdot (11) = 99$$

$$ii. \quad (10 - 1) \cdot (100 + 10 + 1) = (1000 - 1)$$

$$9 \cdot (111) = 999$$

$$iii. \quad (10 - 1) \cdot (1000 + 100 + 10 + 1) = (10000 - 1)$$

$$9 \cdot (1111) = 9,999$$

$$iv. \quad (10 - 1) \cdot (10000 + 1000 + 100 + 10 + 1) = (100000 - 1)$$

$$9 \cdot (11111) = 99,999$$

- c. If we substituted $x = 10$ into the product $(x - 2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

$$8 \cdot (11\,111\,111) = 88\,888\,888$$

- d. Multiply $(x - 2)$ and $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and express your answer in standard form.

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 2x^7 - 2x^6 - 2x^5 - 2x^4 - 2x^3 - 2x^2 - 2x - 2$$

$$x^8 - x^7 - x^6 - x^5 - x^4 - x^3 - x^2 - x - 2$$

Substitute $x = 10$ into your answer and see if you obtain the same result as you obtained in part (c).

$$10^8 - 10^7 - 10^6 - 10^5 - 10^4 - 10^3 - 10^2 - 10 - 2 = 88\,888\,888$$

Yes, I get the same answer.

e. Francois says $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ must equal $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ because when $x = 10$, multiplying by $x - 9$ is the same as multiplying by 1.

i. Multiply $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$.

$$x^8 - 8x^7 - 8x^6 - 8x^5 - 8x^4 - 8x^3 - 8x^2 - 8x - 9$$

ii. Put $x = 10$ into your answer.

$$100\,000\,000 - 80\,000\,000 - 8\,000\,000 - 800\,000 - 80\,000 - 8\,000 - 800 - 80 - 9$$

$$100\,000\,000 - 88\,888\,889 = 11\,111\,111$$

Is it the same as $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ with $x = 10$?

Yes.

iii. Was Francois right?

No, just because it is true when x is 10, doesn't make it true for all real x . The two expressions are not algebraically equivalent.

Name _____

Date _____

1. Solve the following equations for x . Write your answer in set notation.

a. $3x - 5 = 16$

b. $3(x + 3) - 5 = 16$

c. $3(2x - 3) - 5 = 16$

d. $6(x + 3) - 10 = 32$

e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

2. Let c and d be real numbers.
- If $c = 42 + d$ is true, then which is greater: c or d , or are you not able to tell? Explain how you know your choice is correct.

 - If $c = 42 - d$ is true, then which is greater: c or d , or are you not able to tell? Explain how you know your choice is correct.

3. If $a < 0$ and $c > b$, circle the expression that is greater:

$$a(b - c) \quad \text{or} \quad a(c - b)$$

Use the properties of inequalities to explain your choice.

4. Solve for x in each of the equations or inequalities below, and name the property and/or properties used:

a. $\frac{3}{4}x = 9$

b. $10 + 3x = 5x$

c. $a + x = b$

d. $cx = d$

e. $\frac{1}{2}x - g < m$

f. $q + 5x = 7x - r$

g. $\frac{3}{4}(x + 2) = 6(x + 12)$

h. $3(5 - 5x) > 5x$

5. The equation $3x + 4 = 5x - 4$ has the solution set $\{4\}$.

a. Explain why the equation $(3x + 4) + 4 = (5x - 4) + 4$ also has the solution set $\{4\}$.

- b. In part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

- c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

- d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is $\{4\}$ to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m , the number of minutes the customer used that month.

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc., were not allowed.

Student tickets cost \$5 each, and adult tickets cost \$10 each. A total of \$4,500 was collected. 700 tickets were sold.

- Write a system of equations that can be used to find the number of student tickets, s , and the number of adult tickets, a , that were sold at the playoff game.
- Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?
- Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?

8. Alexis is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria approximately every 20 minutes. However, for the purposes of the model, Alexis assumes that each bacterium subdivides into two new bacteria exactly every 20 minutes.
- a. Create a table that shows the total number of bacteria in the Petri dish at $\frac{1}{3}$ hour intervals for 2 hours starting with time 0 to represent 12:00 noon.
- b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.
- c. If Alexis starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.

- d. For her experiment, Alexis plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexis expect to find in the Petri dish right after she adds the anti-bacterial chemical?
9. Jack is 27 years older than Susan. In 5 years, he will be 4 times as old as she is.
- a. Find the present ages of Jack and Susan.
- b. What calculations would you do to check if your answer is correct?

10.

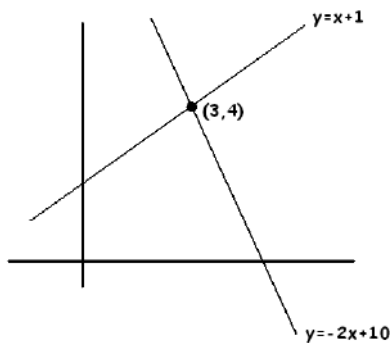
a. Find the product: $(x^2 - x + 1)(2x^2 + 3x + 2)$.

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution $x = 3, y = 4$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: _____

Equation B2: _____

- b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

- c. What is the solution to the system given in part (b)?

- d. For any real number m , the line $y = m(x - 3) + 4$ passes through the point $(3, 4)$.

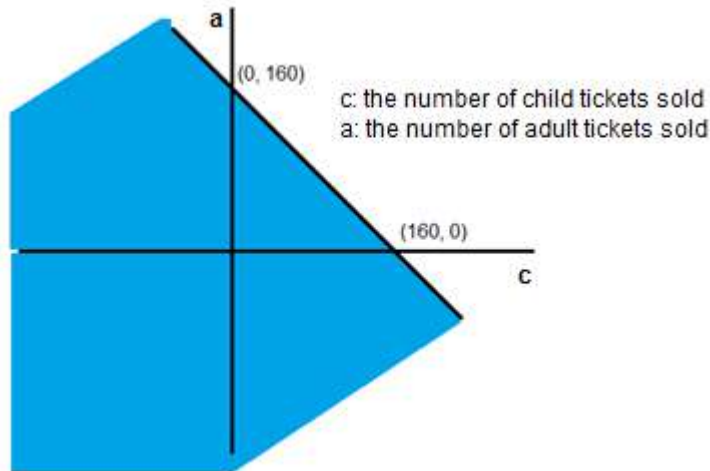
Is it certain, then, that the system of equations

Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3, y = 4$? Explain.

12. The local theater in Jamie’s home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



- a. Venus objected and said there was more than one reason that Jamie’s thinking was flawed. What reasons could Venus be thinking of?

- b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.
- c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a–d A-REI.A.1	Student gives a short incorrect answer or leaves the question blank.	Student shows at least one correct step, but the solution is incorrect.	Student solves the equation correctly (every step that is shown is correct) but does not express the answer as a solution set.	Student solves the equation correctly (every step that is shown is correct) and expresses the answer as a solution set.
	e A-SSE.A.1b A-REI.B.3	Student does not answer or answers incorrectly with something other than (b) and (d).	Student answers (b) and (d) but does not demonstrate solid reasoning in the explanation.	Student answers (b) and (d) but makes minor misstatements in the explanation.	Student answers (b) and (d) and articulates solid reasoning in the explanation.
2	a A-CED.A.3	Student responds incorrectly or leaves the question blank.	Student responds correctly that (c) must be greater but does not use solid reasoning to explain the answer.	Student responds correctly that (c) must be greater but gives an incomplete or slightly incorrect explanation of why.	Student responds correctly that (c) must be greater and supports the statement with solid, well-expressed reasoning.
	b A-CED.A.3	Student responds incorrectly or leaves the question blank.	Student responds correctly that there is no way to tell but does not use solid reasoning to explain the answer.	Student responds correctly that there is no way to tell but gives an incomplete or slightly incorrect explanation of why.	Student responds correctly that there is no way to tell and supports the statement with solid, well-expressed reasoning.

3	A-SSE.A.1b	Student responds incorrectly or leaves the question blank.	Student responds correctly by circling the expression on the left but does not use solid reasoning to explain the answer.	Student responds correctly by circling the expression on the left but gives limited explanation or does not use the properties of inequality in the explanation.	Student responds correctly by circling the expression on the left and gives a complete explanation that uses the properties of inequality.
4	a–h A-REI.A.1 A-REI.B.3	Student answers incorrectly with no correct steps shown.	Student answers incorrectly but has one or more correct steps.	Student answers correctly but does not correctly identify the property or properties used.	Student answers correctly and correctly identifies the property or properties used.
5	a A-REI.A.1	Student does not answer or demonstrates incorrect reasoning throughout.	Student demonstrates only limited reasoning.	Student demonstrates solid reasoning but falls short of a complete answer or makes a minor misstatement in the answer.	Student answer is complete and demonstrates solid reasoning throughout.
	b A-REI.A.1	Student does not answer or does not demonstrate understanding of what the question is asking.	Student makes more than one misstatement in the definition.	Student provides a mostly correct definition with a minor misstatement.	Student answers completely and uses a correct definition without error or misstatement.
	c A-REI.A.1	Student makes mistakes in both verifications and demonstrates incorrect reasoning or leaves the question blank.	Student conducts both verifications but falls short of articulating reasoning to answer the question.	Student conducts both verifications and articulates valid reasoning to answer the question but makes a minor error in the verification or a minor misstatement in the explanation.	Student conducts both verifications without error and articulates valid reasoning to answer the question.
	d A-REI.A.1	Student answers incorrectly or does not answer.	Student identifies one or both solutions but is unable to convey how the solutions could be found using the fact that 4 is a solution to the original equation.	Student identifies only one solution correctly but articulates the reasoning of using the solution to the original equation to find the solution to the new equation.	Student identifies both solutions correctly and articulates the reasoning of using the solution to the original equation to find the solution to the new equation.

6	A-CED.A.4	Student does not answer or shows no evidence of reasoning.	Student makes more than one error in the solution process but shows some evidence of reasoning.	Student answer shows valid steps but with one minor error.	Student answers correctly.
7	a–c A-CED.A.3	Student is unable to answer any portion correctly.	Student answers one part correctly or shows some evidence of reasoning in more than one part.	Student shows solid evidence of reasoning in every part but may make minor errors.	Student answers every part correctly and demonstrates and expresses valid reasoning throughout.
8	a A-CED.A.2	Student provides no table or a table with multiple incorrect entries.	Student provides a data table that is incomplete or has more than one minor error.	Student provides a data table that is complete but may have one error or slightly inaccurate headings.	Student provides a data table that is complete and correct with correct headings.
	b A-CED.A.2	Student provides no equation or an equation that does not represent exponential growth.	Student provides an incorrect equation but one that models exponential growth.	Student provides a correct answer in the form of $T = B(2)^{3h}$.	Student provides a correct answer in the form of $T = B8^h$ or in more than one form, such as $T = B(2)^{3h}$ and $T = B8^h$.
	c A-CED.A.2	Student provides no graph or a grossly inaccurate graph.	Student provides a graph with an inaccurate shape but provides some evidence of reasoning in labeling the axes and/or data points.	Student creates a graph with correct general shape but may leave off or make an error on one or two axes or data points.	Student creates a complete graph with correctly labeled axes and correctly labeled data points (or a data table) showing the values for $h = 0, 1, 2, 3, 4$.
	d A-CED.A.2	Student provides no answer or an incorrect answer with no evidence of reasoning in arriving at the answer.	Student provides limited evidence of reasoning and an incorrect answer.	Student answers that 409.6 bacteria are alive.	Student answers that 410, or about 410, bacteria are alive.

9	a A-CED.A.1	Student writes incorrect equations or does not provide equations.	Student answers are incorrect, but at least one of the equations is correct. Student makes a gross error in the solution, makes more than one minor error in the solution process, or has one of the two equations incorrect.	Both equations are correct, but student makes a minor mistake in finding the solution.	Both equations are correct and student solves them correctly to arrive at the answer that Jack is 31 and Susan is 4.
	b A-REI.B.3	Student does not answer or gives a completely incorrect answer.	Student articulates only one of the calculations correctly.	Student articulates the two calculations but with a minor misstatement in one of the descriptions.	Student articulates both calculations correctly.
10	a–b A-APR.A.1	Student work is blank or demonstrates no understanding of multiplication of polynomials, nor how to apply part (a) to arrive at an answer for part (b).	Student makes more than one error in the multiplication but demonstrates some understanding of multiplication of polynomials. Student may not be able to garner or apply information from part (a) to use in answering part (b) correctly.	Student demonstrates the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part (a) to solve part (b). There may be minor errors.	Student demonstrates the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part (a) to solve part (b) as 91(232).
11	a A-REI.C.6	Student is unable to demonstrate the understanding that two equations with (3, 4) as a solution are needed.	Student provides two equations that have (3, 4) as a solution (or attempts to provide such equations) but makes one or more errors. Student may provide an equation with a negative slope.	Student shows one minor error in the answer but attempts to provide two equations both containing (3, 4) as a solution and both with positive slope.	Student provides two equations both containing (3, 4) as a solution and both with positive slope.
	b A-REI.C.6	Student is unable to identify the multiple correctly.	Student identifies the multiple as 3.	N/A	Student correctly identifies the multiple as 2.

	c A-REI.C.6	Student is unable to demonstrate even a partial understanding of how to find the solution to the system.	Student shows some reasoning required to find the solution but makes multiple errors.	Student makes a minor error in finding the solution point.	Student successfully identifies the solution point as (3, 4).
	d A-REI.C.5 A-REI.C.6 A-REI.D.10	Student is unable to answer or to support the answer with any solid reasoning.	Student concludes yes or no but is only able to express limited reasoning in support of the answer.	Student correctly explains that all the systems have the solution point (3, 4) but incorrectly assumes this is true for all cases of m .	Student correctly explains that while in most cases this is true, if $m = 1$, the two lines are coinciding lines, resulting in a solution set consisting of all the points on the line.
12	a MP.2 A-REI.D.12	Student is unable to articulate any sound reasons.	Student is only able to articulate one sound reason.	Student provides two sound reasons but makes minor errors in the expression of reasoning.	Student is able to articulate at least two valid reasons. Valid reasons include the following: the graph assumes x could be less than zero, the graph assumes y could be less than zero, the graph assumes a and b could be non-whole numbers, the graph assumes 160 children could attend with no adults.
	b A-CED.A.2 A-REI.D.10 A-REI.D.12	Student is unable to communicate a relevant requirement of the solution set.	Student provides a verbal description that lacks precision and accuracy but demonstrates some reasoning about the solution within the context of the problem.	Student makes minor errors in communicating the idea that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.	Student communicates effectively that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.

	<p>c</p> <p>A-CED.A.2 A-REI.C.6</p>	<p>Student is unable to demonstrate any substantive understanding in how to create the equations and solve the system of equations.</p>	<p>Student makes multiple errors in the equations and/or solving process but demonstrates some understanding of how to create equations to represent a context and/or solve the system of equations.</p>	<p>Student makes minor errors in the equations but solves the system accurately, or the student creates the correct equations but makes a minor error in solving the system of equations.</p>	<p>Student correctly writes the equations to represent the system. Student solves the system accurately and summarizes by defining or describing the values of the variable in the context of the problem (i.e., that there are 100 adult tickets and 44 child tickets sold.)</p>
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Name _____

Date _____

1. Solve the following equations for x . Write your answer in set notation.

a. $3x - 5 = 16$

$3x = 21$

$x = 7$

Solution set: $\{7\}$

b. $3(x + 3) - 5 = 16$

$3x + 9 - 5 = 16$

$3x = 12$

$x = 4$

Solution set: $\{4\}$

c. $3(2x - 3) - 5 = 16$

$6x - 9 - 5 = 16$

$6x - 14 = 16$

$6x = 30$

$x = 5$

Solution set: $\{5\}$

d. $6(x + 3) - 10 = 32$

$6x + 18 - 10 = 32$

$6x = 24$

$x = 4$

Solution set: $\{4\}$

- e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

Problems (b) and (d) have the same solution set. The expressions on each side of the equal sign for (d) are twice those for (b). So, if (left side) = (right side) is true for only some x -values, then $2(\text{left side}) = 2(\text{right side})$ will be true for exactly the same x -values. Or simply, applying the multiplicative property of equality does not change the solution set.

2. Let c and d be real numbers.

- a. If $c = 42 + d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

c must be greater because c is always 42 more than d .

- b. If $c = 42 - d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

There is no way to tell. We only know that the sum of c and d is 42. If d were 10, c would be 32 and, therefore, greater than d . But if d were 40, c would be 2 and, therefore, less than d .

3. If $a < 0$ and $c > b$, circle the expression that is greater:

$a(b - c)$ or $a(c - b)$

Use the properties of inequalities to explain your choice.

*Since $c > b$,
it follows that $0 > b - c$,
and since $a < 0$, a is negative,
and the product of two negatives will be
a positive.*

*Since $c > b$,
it follows that $c - b > 0$.
so $(c - b)$ is positive. And since a is
negative, the product of
 $a \cdot (c - b) < a \cdot (b - c)$.*

4. Solve for x in each of the equations or inequalities below and name the property and/or properties used:

a. $\frac{3}{4}x = 9$

$$x = 9 \cdot \left(\frac{4}{3}\right)$$

$$x = 12$$

Multiplication property of equality

b. $10 + 3x = 5x$

$$10 = 2x$$

$$5 = x$$

Addition property of equality

Multiplication property of equality

c. $a + x = b$

$$x = b - a$$

Addition property of equality

d. $cx = d$

$$x = \frac{d}{c}, c \neq 0$$

Multiplication property of equality

e. $\frac{1}{2}x - g < m$

$$\frac{1}{2}x < m + g$$

$$x < 2 \cdot (m + g)$$

Addition property of equality

Multiplication property of equality

f. $q + 5x = 7x - r$

$$q + r = 2x$$

$$\frac{(q+r)}{2} = x$$

Addition property of equality

Multiplication property of equality

g. $\frac{3}{4}(x + 2) = 6(x + 12)$

$$3 \cdot (x + 2) = 24 \cdot (x + 12)$$

Multiplication property of equality

$$3x + 6 = 24x + 288$$

Distributive property

$$-\frac{282}{21} = x$$

Addition property of equality and multiplication

$$-\frac{94}{7} = x$$

Property of equality

$$-\frac{94}{7} = x$$

h. $3(5 - 5x) > 5x$

$$15 - 15x > 5x$$

Distributive property

$$15 > 20x$$

Addition property of inequality

$$\frac{3}{4} > x$$

Multiplication property of equality

5. The equation, $3x + 4 = 5x - 4$, has the solution set $\{4\}$.

a. Explain why the equation, $(3x + 4) + 4 = (5x - 4) + 4$, also has the solution set $\{4\}$.

Since the new equation can be created by applying the addition property of equality, the solution set does not change.

OR

Each side of this equation is 4 more than the sides of the original equation. Whatever value(s) make $3x + 4 = 5x - 4$ true would also make 4 more than $3x + 4$ equal to 4 more than $5x - 4$.

- b. In part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent algebraic expressions? Describe why changing an expression on one side of an equation to an equivalent expression leaves the solution set unchanged?

Algebraic expressions are equivalent if (possibly repeated) use of the distributive, associative, and commutative properties and/or the properties of rational exponents can be applied to one expression to convert it to the other expression.

When two expressions are equivalent, assigning the same value to x in both expressions will give an equivalent numerical expression, which then evaluates to the same number. Therefore, changing the expression to something equivalent will not change the truth value of the equation once values are assigned to x .

- c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

$(3 \cdot 4 + 4)^2 = (5 \cdot 4 - 4)^2$ gives $16^2 = 16^2$, which is true.

$(3 \cdot 0 + 4)^2 = (5 \cdot 0 - 4)^2$ gives $4^2 = (-4)^2$, which is true.

But, $(3 \cdot 0 + 4) = (5 \cdot 0 - 4)$ gives $4 = -4$, which is false.

When both sides are squared, you might introduce new numbers to the solution set because statements like $4 = -4$ are false, but statements like $4^2 = (-4)^2$ are true.

- d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is $\{4\}$ to find the solution set to this new equation.

Since the original equation $3x + 4 = 5x - 4$ was true when $x = 4$, the new equation $3x^2 + 4 = 5x^2 - 4$ should be true when $x^2 = 4$. And, $x^2 = 4$ when $x = 2$, so the solution set to the new equation is $\{-2, 2\}$.

6. The Zonda Information and Telephone Company calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m , the number of minutes the customer used that month.

$$C = b + bt + rm + rmt$$

$$C - b - bt = m \cdot (r + rt)$$

$$\frac{C - b - bt}{r + rt} = m$$

$$t \neq -1$$

$$r \neq 0$$

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc., were not allowed.

Student tickets cost \$5 each, and adult tickets cost \$10 each. A total of \$4,500 was collected. 700 tickets were sold.

- a. Write a system of equations that can be used to find the number of student tickets, s , and the number of adult tickets, a , that were sold at the playoff game.

$$5s + 10a = 4500$$

$$s + a = 700$$

- b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?

$$700 \times \$10 = \$7000$$

$$\$7000 - \$4500 = \$2500 \text{ more}$$

- c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?

First solve for a and s

$$5s + 10a = 4500$$

$$-5s - 5a = -3500$$

$$5a = 1000$$

$$a = 200$$

$$s = 500$$

$$\$5 \cdot (500) + \$15 \cdot (200) = \$5500$$

$$\$1,000 \text{ more}$$

OR

$$\$5 \text{ more per adult ticket } (200 \cdot \$5 = \$1000 \text{ more})$$

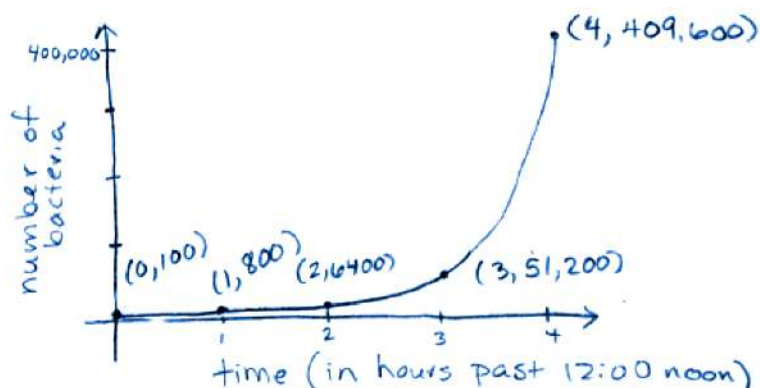
8. Alexis is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria approximately every 20 minutes. However, for the purposes of the model, Alexis assumes that each bacterium subdivides into two new bacteria exactly every 20 minutes.
- a. Create a table that shows the total number of bacteria in the Petri dish at $\frac{1}{3}$ hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

Time	Number of Bacteria
0	B
$\frac{1}{3}$ hour	$2B$
$\frac{2}{3}$ hour	$4B$
1 hour	$8B$
$1\frac{1}{3}$ hour	$16B$
$1\frac{2}{3}$ hour	$32B$
2 hour	$64B$

- b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.

$$T = B \cdot (2)^{3h} \text{ or } T = B \cdot 8^h$$

- c. If Alexis starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.



- d. For her experiment, Alexis plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexis expect to find in the Petri dish right after she adds the anti-bacterial chemical?

$$(1 - 0.999) \cdot 409600 = 409.6$$

about 410 live bacteria

9. Jack is 27 years older than Susan. In 5 years time, he will be 4 times as old as she is.
- a. Find the present ages of Jack and Susan.

$$J = S + 27$$

$$J + 5 = 4 \cdot (S + 5)$$

$$S + 27 + 5 = 4S + 20$$

$$S + 32 = 4S + 20$$

$$12 = 3S$$

$$S = 4$$

$$J = 4 + 27$$

$$J = 31$$

Jack is 31 and Susan is 4.

- b. What calculations would you do to check if your answer is correct?

Is Jack's age – Susan's age = 27?

Add 5 years to Jack's and Susan's ages, and see if that makes Jack 4 times as old as Susan.

10.

- a. Find the product:
- $(x^2 - x + 1)(2x^2 + 3x + 2)$

$$\begin{array}{r} 2x^4 + 3x^3 + 2x^2 - 2x^3 - 3x^2 - 2x + 2x^2 + 3x + 2 \\ 2x^4 + x^3 + x^2 + x + 2 \end{array}$$

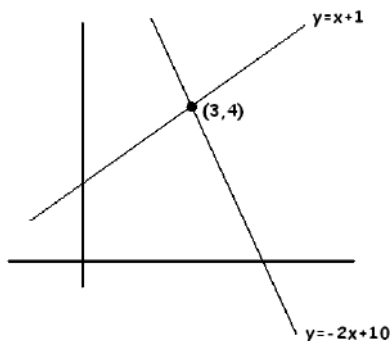
- b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

$$\begin{array}{r} (100 - 10 + 1) \cdot (200 + 30 + 2) \\ (91) \cdot (232) \end{array}$$

11. Consider the following system of equations with the solution $x = 3, y = 4$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



- a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: $y = \frac{4}{3}x$

Equation B2: $y = x + 1$

- b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

2 times A2 was added to A1.

- c. What is the solution to the system given in part (b)?

(3,4)

- d. For any real number m , the line $y = m(x - 3) + 4$ passes through the point $(3,4)$.

Is it certain then that the system of equations:

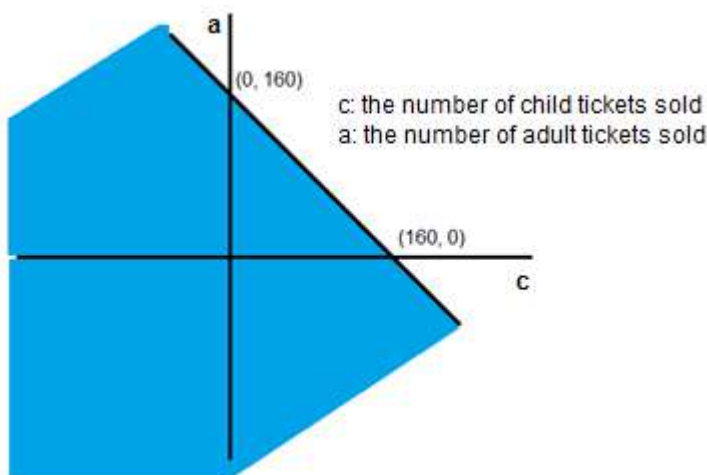
Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3, y = 4$? Explain.

No. If $m = 1$, then the two lines have the same slope. Both lines pass through the point $(3,4)$, and the lines are parallel; therefore, they coincide. There are infinite solutions. The solution set is all the points on the line. Any other nonzero value of m would create a system with the only solution of $(3,4)$.

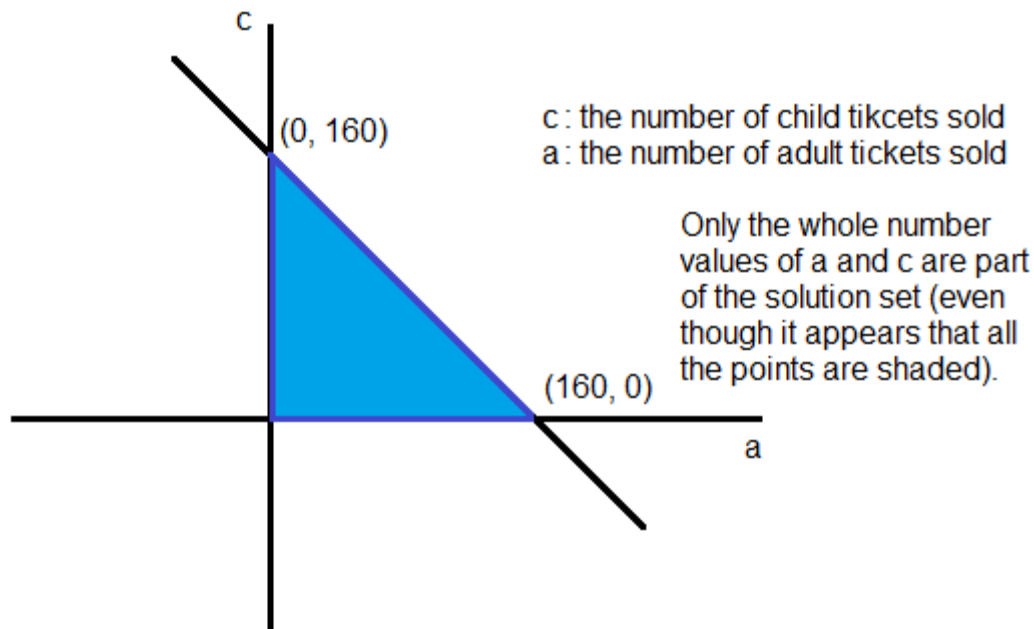
12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



- a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?
- The graph implies that the number of tickets sold could be a fractional amount, but really it only makes sense to sell whole number tickets. x and y must be whole numbers.*
 - The graph also shows that negative ticket amounts could be sold, which does not make sense.*

- b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

The system would be $\begin{cases} a + c \leq 160 \\ a \geq 0 \\ c \geq 0 \end{cases}$ where a and c are whole numbers.



- c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

a : the number of adult tickets sold (must be a whole number)

c : the number of child tickets sold (must be a whole number)

$$\begin{cases} 9a + 6c = 1164 \\ a + c = 144 \end{cases}$$

$$9a + 6c = 1164$$

$$-6a - 6c = -864$$

$$3a = 300$$

$$a = 100, c = 44$$

In all, 100 adult tickets and 44 child tickets were sold.