New York State Common Core

9-12 GRADE

Mathematics Curriculum

A Story of Functions:
A Curriculum Overview for Grades 9-12

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Introduction

The Common Core Learning Standards (CCLS) define progressions of learning that develop the major content of school mathematics over grades Pre-K through 12. When those standards are further connected to each other within a grade and throughout a sequence of lessons, a coherent story emerges of mathematics as an elegant subject in which the collective body of knowledge results from reasoning from a cohesive set of principles. The word *story* in the title *A Story of Functions* is meant to capture this notion of coherence as students study functions and model with them.

This document provides an overview of the academic year for Grades 9 through 12, beginning with a curriculum map and followed by detailed grade level descriptions. Courses for Algebra I, Geometry, and Algebra II were designed in accordance with PARCC Model Content Frameworks for High School Mathematics.[1] The courses outlined in this document were informed by, but are not identical to, Appendix A of the Common Core State Standards.[2] A Precalculus course is provided as a fourth course.

Each course description begins with a list of the modules that comprise the instruction of the course. The list is followed by five sections of information:

- Summary of Year, which describes the focus of the course[3]
- Recommended Fluencies for the course, as stated in the PARCC Model Content Frameworks for High School Mathematics (Note that this information is not available for Precalculus.)
- CCLS Major Emphasis Clusters for the course, as stated in the PARCC Model Content Frameworks for High School Mathematics (Note that this information is not available for Precalculus.)
- Rationale for the Module Sequence of the course
- Alignment Chart of the course standards

[1] http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsNovember2012V3_FINAL.pdf
[2] http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
[3] Text in the summary paragraphs and Rationale for Module Sequencing for Algebra I, Geometry, and Algebra II were informed by, but are not identical to, Appendix A of the Common Cores State Standards.

Key for reading this document:

(★) According to the CCLS, "Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★)." Opportunities for modeling are woven throughout all four courses and are also indicated by (★) in this document.

(+) The CCLS notes, "Standards beginning with the (+) symbol form a starting point for fourth year courses in Precalculus and in Probability and Statistics." A few key (+) standards are included in the Geometry and Algebra II courses to provide coherence to the curriculum. They can be used to effectively extend a topic (e.g., G-GMD.2 as an extension of G-GMD.1) or to introduce a theme/concept that will be fully covered in the Precalculus course. ***Note: None of the (+) standard in the Geometry or Algebra II course will be assessed on the Regents Exam in those courses.*** All (+) standards are in the Precalculus course where they are assessed.

Timeline

The curriculum map on the next page shows the approximate number of instructional days designated for each module of each grade. The number of instructional days and dates will vary due to different school calendars, school holidays, snow days, and especially student needs.

To accommodate the January and June Regents Exam periods, the modules are based 150 instructional days instead of 180. The remaining 30 days takes into consideration 15 days of test administration and at least 10 days for review. ***Note: For the first administration of the Regents Exams, there will be less than 150 instructional days because the Regents Exams are given early (impacting Algebra I and Geometry in the 2013–2014 school year and all courses in the 2014–2015 school year).***

Curriculum Map

	Grade 9 -- Algebra I	Grade 10 -- Geometry	Grade 11 -- Algebra II	Grade 12 -- Precalculus	
20 days	M1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)	M1: Congruence, Proof, and Constructions (45 days)	M1: Polynomial, Rational, and Radical Relationships (45 days)	M1: Complex Numbers and Transformations (40 days)	20 days
20 days					20 days
20 days	M2: Descriptive Statistics (25 days)	M2: Similarity, Proof, and Trigonometry (45 days)	M2: Trigonometric Functions (20 days)	M2: Vectors and Matrices (40 days)	20 days
20 days	M3: Linear and Exponential Functions		M3: Functions (45 days)		20 days
	State Examinations	State Examinations	State Examinations	State Examinations	
20 days	(35 days)	M3: Extending to Three Dimensions (10 days)		M3: Rational and Exponential Functions (25 days)	20 days
20 days	M4: Polynomial and Quadratic Expressions, Equations and Functions (30 days)	M4: Connecting Algebra and Geometry through Coordinates (25 days)	M4: Inferences and Conclusions from Data (40 days)	M4: Trigonometry (20 days)	20 days
20 days					20 days
20 days	M5: A Synthesis of Modeling with Equations and Functions (20 days)	M5: Circles with and Without Coordinates (25 days)		M5: Probability and Statistics (25 days)	20 days
20 days	Review and Examinations	Review and Examinations	Review and Examinations	Review and Examinations	20 days

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

Standards for Mathematical Practice

The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

MP.1 **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others so solving complex problems and identify correspondences between different approaches.

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and the ability to *contextualize*—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the 6 units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (exemplified in Topic D).

MP.3 **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the argument (exemplified in Topics A and E).

MP.4 **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students, who can apply what they know, are comfortable making assumptions and approximations to simplify a complicated situation and realize that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using tools, such as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions (exemplified in Topics C and F).

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y (exemplified in Topic B).

MP.8 **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results (exemplified in Topic G).

Sequence of Algebra I Modules Aligned with the Standards

Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs

Module 2: Descriptive Statistics

Module 3: Linear and Exponential Functions

Module 4: Polynomial and Quadratic Expressions, Equations and Functions

Module 5: A Synthesis of Modeling with Equations and Functions

Summary of Year

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The modules deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra I

- Solving characteristic problems involving the analytic geometry of lines, including, writing the equation of a line given a point and a slope.
- Adding, subtracting and multiplying polynomials.
- Transforming expressions and chunking (seeing the parts of an expression as a single object) as used in factoring, completing the square, and other algebraic calculations.

CCLS Major Emphasis Clusters
Seeing Structure in Expressions • Interpret the structure of expressions Arithmetic with Polynomials and Rational Expressions • Perform arithmetic operations on polynomials Creating Equations • Create equations that describe numbers or relationships Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Represent and solve equations and inequalities graphically Interpreting Functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context Interpreting Categorical and Quantitative Data • Interpret linear models

Rationale for Module Sequence in Algebra I

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They learn the terminology specific to polynomials and understand that polynomials form a system analogous to the integers.

Module 2: This module builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model. With linear models, they plot and analyze residuals to informally assess the goodness of fit.

Module 3: In earlier grades, students defined, evaluated, and compared functions in modeling relationships between quantities. In this module, students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on their understanding of integer exponents to consider exponential functions with integer domains. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function.

Module 4: In this module, students build on their knowledge from Module 3. Students strengthen their ability to discern structure in polynomial expressions. They create and solve equations involving quadratic and cubic expressions. In this module's modeling applications, students reason abstractly and quantitatively in interpreting parts of an expression that represent a quantity in terms of its context; they also learn to make sense of problems and persevere in solving them by choosing or producing equivalent forms of an expression (e.g., completing the square in a quadratic expression to reveal a maximum value). Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They learn through repeated reasoning to anticipate the graph of a quadratic function by interpreting the structure

Sequence of Geometry Modules Aligned with the Standards

Module 1: Congruence, Proof, and Constructions
Module 2: Similarity, Proof, and Trigonometry
Module 3: Extending to Three Dimensions
Module 4: Connecting Algebra and Geometry through Coordinates
Module 5: Circles with and Without Coordinates

Summary of Year

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria.
- Using coordinates to establish geometric results.
- Calculating length and angle measures.
- Using geometric representations as a modeling tool.
- Using construction tools, physical and computational to draft models of geometric phenomenon.

CCLS Major Emphasis Clusters
Congruence • Understand congruence in terms of rigid motions • Prove geometric theorems Similarity, Right Triangles, and Trigonometry • Understand similarity in terms of similarity transformations • Prove theorems using similarity • Define trigonometric ratios and solve problems involving right triangles Expressing Geometric Properties with Equations • Use coordinates to prove simple geometric theorems algebraically Modeling with Geometry • Apply geometric concepts in modeling situations

Rationale for Module Sequence in Geometry

Module 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions—translations, reflections, and rotations—and have strategically applied a rigid motion to informally show that two triangles are congruent. In this module, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They build upon this familiar foundation of triangle congruence to develop formal proof techniques. Students make conjectures and construct viable arguments to prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They construct figures by manipulating appropriate geometric tools (compass, ruler, protractor, etc.) and justify why their written instructions produce the desired figure.

Module 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, make sense of and persevere in solving similarity problems, and apply similarity to right triangles to prove the Pythagorean Theorem. Students attend to precision in showing that trigonometric ratios are well defined, and apply trigonometric ratios to find missing measures of general (not necessarily right) triangles. Students model and make sense out of indirect measurement problems and geometry problems that involve ratios or rates.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

Module 4: Building on their work with the Pythagorean Theorem in 8^{th} grade to find distances, students analyze geometric relationships in the context of a rectangular coordinate system, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, relating back to work done in the first module. Students attend to precision as they connect the geometric and algebraic definitions of parabola. They solve design problems by representing figures in the coordinate plane, and in doing so, they leverage their knowledge from synthetic geometry by combining it with the solving power of algebra inherent in analytic geometry.

Module 5: In this module, students prove and apply basic theorems about circles, such as: a tangent line is perpendicular to a radius theorem, the inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations. Students visualize, with the aid of appropriate software tools, changes to a three-dimensional model by exploring the consequences of varying parameters in the model.

Sequence of Algebra II Modules Aligned with the Standards

Module 1: Polynomial, Rational, and Radical Relationships
Module 2: Trigonometric Functions
Module 3: Functions
Module 4: Inferences and Conclusions from Data

Summary of Year

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra II

- Divide polynomials with remainder by inspection in simple cases.
- See structure in expressions and use this structure to rewrite expressions (e.g., factoring, grouping).
- Translate between recursive definitions and closed forms for problems involving sequences and series.

CCLS Major Emphasis Clusters
The Real Number System • Extend the properties of exponents to rational exponents Seeing Structure in Expressions • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems Arithmetic with Polynomials and Rational Expressions • Understand the relationship between zeros and factors of polynomials Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Represent and solve equations and inequalities graphically Interpreting Functions • Interpret functions that arise in applications in terms of the context Building Functions • Build a function that models a relationship between two quantities Making Inferences and Justifying Conclusions • Make inferences and justify conclusions from sample surveys, experiments and observational studies

Rationale for Module Sequence in Algebra II

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 1: In this module, students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect the structure inherent in multi-digit whole number multiplication with multiplication of polynomials and similarly connect division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials. Through regularity in repeated reasoning, they make connections between zeros of polynomials and solutions of polynomial equations. Students analyze the key features of a graph or table of a polynomial function and relate those features back to the two quantities in the problem that the function is modeling. A theme of this module is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Module 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students extend trigonometric functions to all (or most) real numbers. To reinforce their understanding of these functions, students begin building fluency with the values of sine, cosine, and tangent at $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, etc. Students make sense of periodic phenomena as they model with trigonometric functions.

Module 3: In this module, students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore (with appropriate tools) the effects of transformations on graphs of diverse functions, including functions arising in an application. They notice, by looking for general methods in repeated calculations, that the transformations on a graph always have the same effect regardless of the type of the underlying function. These observations lead to students to conjecture and construct general principles about how the graph of a function changes after applying a function transformation to that function. Students identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as, "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions,*" is at the heart of this module. In particular, through repeated opportunities in working through the modeling cycle (see page 61 of the CCLS), students acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations.

Module 4: In this module, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, experiments, and simulations, and the role that randomness and careful design play in the conclusions that can be drawn. Students create theoretical and experimental probability models following the modeling cycle (see page 61 of CCLS). They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Sequence of Precalculus Modules Aligned with the Standards

Module 1: Complex Numbers and Transformations

Module 2: Vectors and Matrices

Module 3: Rational and Exponential Functions

Module 4: Trigonometry

Module 5: Probability and Statistics

Summary of Year

Extending their understanding of complex numbers to points in the complex plane, students come to understand that multiplying a given set of points by a complex number amounts to rotating and dilating those points in the complex plane about zero. Matrices are studied as tools for performing rotations and reflections of the coordinate plane, as well as for solving systems of linear equations. Inverse functions are explored as students study the relationship between exponential and logarithmic functions and restrict the domain of the trigonometric functions to allow for their inverses. The year concludes with a capstone module on modeling with probability and statistics. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Rationale for Module Sequence in Precalculus

In Algebra II, students extended their understanding of number to include complex numbers as they studied polynomials with complex zeros. In Module 1, students graph complex numbers in the complex plane and translate between rectangular and polar forms of complex numbers. In particular, through repeated reasoning, they come to realize that multiplying a given set of points by a complex number amounts to rotating and dilating those points in the complex plane around the point zero. Thinking of a complex number, $a + bi$, once again as a point (a,b) in the coordinate plane, students investigate how multiplying by a complex number can be thought of as a map from the coordinate plane to itself. That study, in turn, leads to matrix notation and a natural definition for multiplying a vector by a matrix:

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i \text{ is equivalent to } \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix}=\begin{pmatrix} ac-bd \\ ad+bc \end{pmatrix}.$$

Thus, students discern structure in the operations with matrices and vectors by comparing them to arithmetic with complex numbers.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students began the study of transformations in Grade 8, and precisely defined rigid motions in the plane in terms of angles, circles, perpendicular lines, parallel lines, and segments in Geometry. In this module, students precisely define rotations, reflections and dilations in the coordinate plane using 2×2 matrices (and translations by vector addition). These well-defined definitions of transformations of the coordinate plane shed light on how geometry software and video games efficiently perform rigid motion calculations.

In the first module, students viewed matrices as tools for performing rotations and reflections of the coordinate plane. In Module 2, they move beyond this viewpoint to study matrices and vectors as objects in their own right. Students interpret the properties and operations of matrices to learn multiple ways to solve problems with them, including solving systems of linear equations. They construct viable arguments using matrices to once again derive equations for conic sections, this time by translating and rotating the locus of points into a “standard” position using matrix operations. (For example, applying rigid motions to move the directrix of a parabola to one of the coordinate axes.)

Students study rational and exponential functions in Module 3. They graph rational functions by extending what they learned about graphing polynomials functions. Students, through repeatedly exploiting the relationship between exponential and logarithmic functions, learn the meaning of inverse functions. Additionally, students learn to explicitly build composite functions to model relationships between two quantities. In particular, they analyze the composite of two functions in describing the relationship of three or more quantities in modeling activities in this module.

In Module 4, students visualize graphs of trigonometric functions with the aid of appropriate software and interpret how a family of graphs defined by varying a parameter in a given function changes based upon that parameter. They analyze symmetry and periodicity of trigonometric functions. They extend their knowledge of inverse functions to trigonometric functions by restricting domains to create the inverses, and apply inverse functions to