Chapter 29:Nuclear Physics

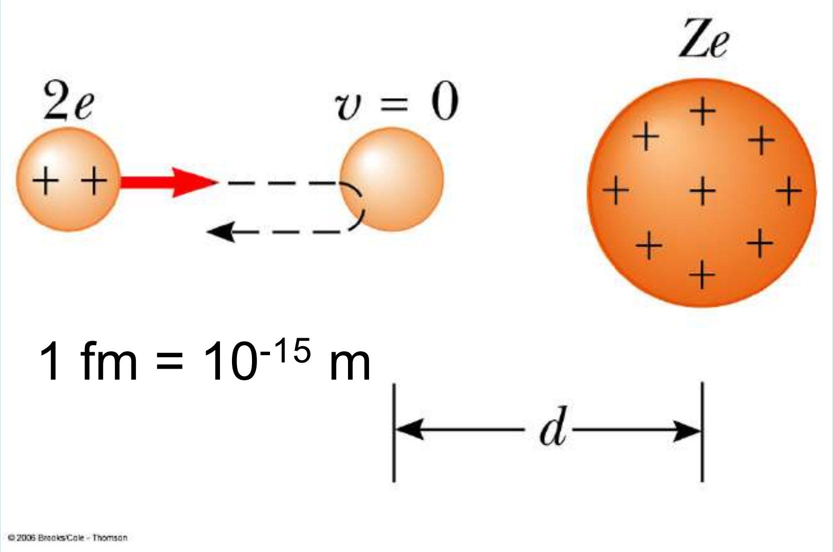
Homework : Read and understand the lecture note.

Some Properties of Nuclei

- Some terminology
  - Atomic number Z : number of protons in the nucleus
    Neutron number N : number of neutrons in the nucleus
    Mass number A : number of nucleons (protons and neutrons) in the nucleus
  - The symbol used to represent the nucleus of an atom is ${}^{A}_{Z}X$.
  - An isotope of an element has the same Z value but different N and A values.
- Charge and mass
  - Proton charge +e : 1.602×10^{-19} C
  - Electron charge –e : -1.602×10^{-19} C
  - Unified mass unit u : the mass of one atom of the isotope ^{12}C = 12 u
    1 u $= 1.660 \times 10^{-27}$ kg = 931.5 MeV/c^2
  - Electron mass : 5.486×10^{-4} u = 0.511 MeV/c^2
  - Proton mass : 1.6726×10^{-27} kg = 1.007 u = 938.3 MeV/c^2
  - Neutron mass : 1.6750×10^{-27} kg = 1.009 u = 939.6 MeV/c^2

Some Properties of Nuclei

- Size of nuclei
  - How close an α particle can approach to a nucleus of charge Ze?



$$\frac{1}{2}mv^2 = k_e\frac{q_1q_2}{r} = k_e\frac{(2e)(Ze)}{d}$$

⇒ Rutherford's estimate

$$d = \frac{4k_eZe^2}{mv^2} \approx 3.2\times 10^{-14}\ \text{m} = 32\ \text{fm for gold nucleus}$$

20 fm for silver

Approximately most nuclei are spherical and have an average radius r : $r = r_0A^{1/3}$

⇒

All nuclei have nearly the same density.

- Nuclear stability
  - The force that bind nucleon together (strong force) is stronger than the Coulomb force – this gives stability to nuclei.
  - Light nuclei are most stable if N=Z, while heavy nuclei are more stable if N>Z.

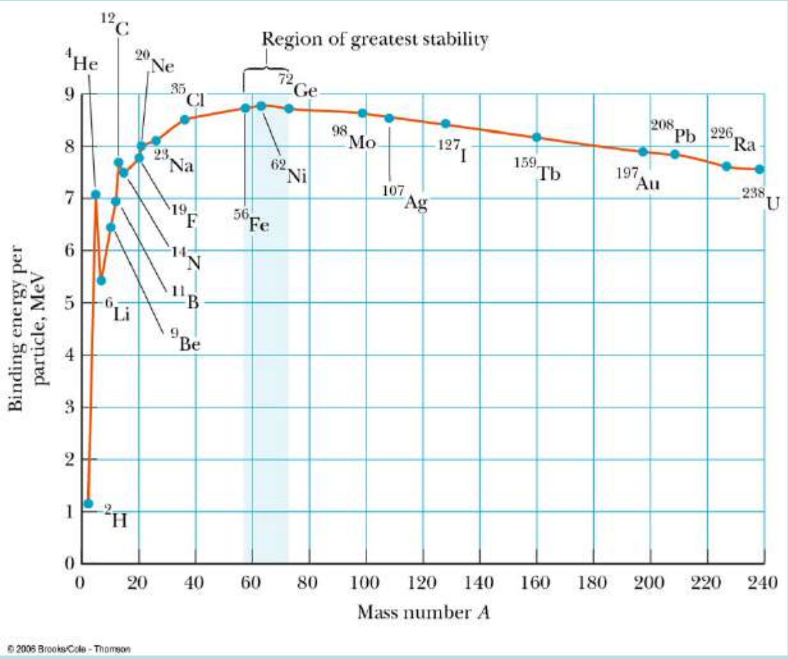
Binding Energy

- Binding energy
  - The total mass of a nucleus is always less than the sum of the masses of its nucleons. Therefore the total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons. This difference is called binding energy.
  - Binding energy of deuteron – a bound system of a neutron and a proton (also the nucleus of deuterium)

$$\Delta m = (m_p + m_n) - m_d = (1.007825\,\mathrm{u} + 1.008665\,\mathrm{u}) - 2.014102\,\mathrm{u} = 0.002388\,\mathrm{u}$$

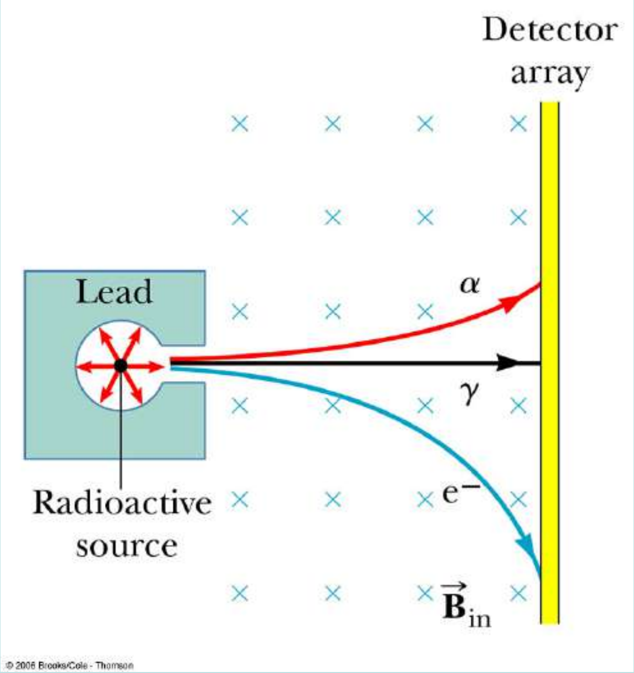
$$E_b = (0.002388\,\mathrm{u})(931.5\,\mathrm{MeV})/(1\,\mathrm{u}) = 2.224\,\mathrm{MeV}$$

  - Binding energy per nucleon peaks at about A=60. This means the elements around this peak are more stable. The average binding energy per nucleon is 8 MeV.



Radio Activities

- Types of radiation emitted from a radio active substance
  - Alpha (α) (nucleus of ${}^{4}_{2}He$)
  - Electron (e^-) or positron (e^+) (anti-electron)
  - Gamma ray (γ)



- Decay constant and half-life
  - Observations established that if a radioactive sample contains N radioactive nuclei at some instance, the number of nuclei, ΔN, that decay in a short time interval Δt is proportional to N.

N decreases

$$\frac{\Delta N}{\Delta t} \propto N \Rightarrow \Delta N = -\lambda N \Delta t$$

decay constant

$$\Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$ exponential decay

  - The decay rate or activity R of a sample is defined as the number of decays per second:

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N$$

Radio Activities

❑ Decay constant and half-life (cont'd)

- Exponential decay and half-life

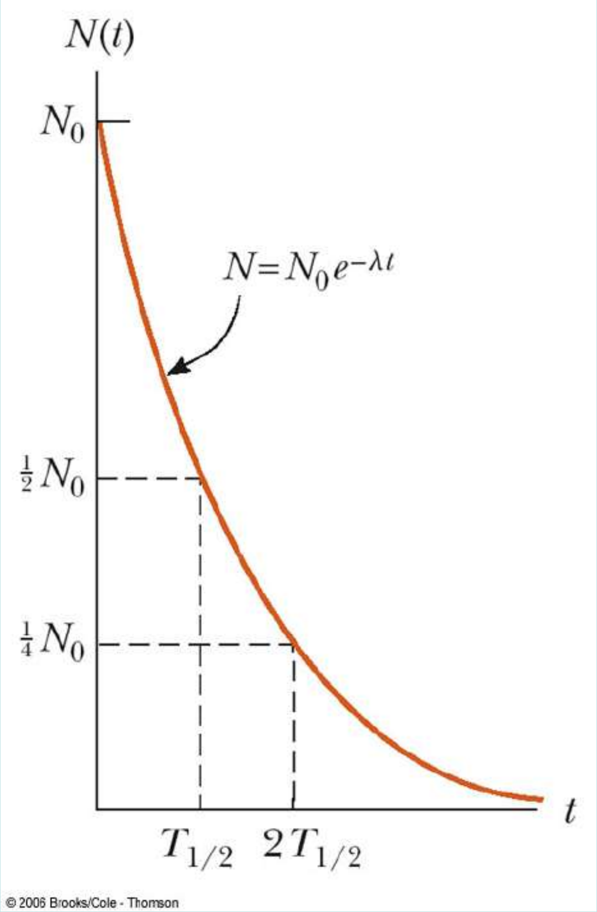
$$\boxed{N = N_0 e^{-\lambda t}}$$ exponential decay

- The half-life $T_{1/2}$ of a radio active substance is the time it takes for half of a given number of radioactive nuclei to decay.

$$\boxed{N = N_0 \left(\frac{1}{2}\right)^n \quad n = \frac{t}{T_{1/2}}}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$



- Units of activity R (curie and becquerel)

$$1\,\mathrm{Ci} \equiv 3.7 \times 10^{10}\ \mathrm{decays/s} \qquad 1\,\mathrm{Bq} = 1\ \mathrm{decay/s}\quad \text{(SI unit)}$$

Radio Activities

- Example 29.3: Activity of radium
  - The half-life of the radioactive nucleus $^{226}_{88}Ra$ is 1.6x10^3 yr. If a sample contains 3.00x10^{16} such nuclei, determine the followings:

(a) the initial activity in curies

$$T_{1/2} = (1.6\times 10^3 \text{ yr})(3.156\times 10^7 \text{ s/yr}) = 5.0\times 10^{10} \text{ s}$$

$$\lambda = 0.693/T_{1/2} = 1.4\times 10^{-11} \text{ s}^{-1}$$

$$R_0 = \lambda N_0 = 4.2\times 10^5 \text{ decays/s} = 1.1\times 10^{-5} \text{ Ci} = 11\ \mu\text{Ci}$$

(b) the number of radium nuclei remaining after 4.8x10^3 yr

$$n = \frac{4.8\times 10^3 \text{ yr}}{1.6\times 10^3 \text{ yr/half-life}} = 3.0 \text{ half-lives}$$

$$N = N_0 (1/2)^n \rightarrow N = (3.0\times 10^{16} \text{ nuclei})(1/2)^{3.0} = 3.8\times 10^{15} \text{ nuclei}$$

(c) the activity at this later time

$$R = \lambda N = 5.3\times 10^4 \text{ decays/s} = 1.4\ \mu\text{Ci}$$

Radio Activities

❑ Example 29.4: Radon gas

- Radon ${}^{222}_{86}Rn$ is a radioactive gas that can be trapped in the basements of homes, and its presence in high concentrations is a known health hazard. radon has a half-life of 3.83 days. A gas sample contains $4.00\text{x}10^8$ radon atoms initially.

(a) How many atoms will remain after 14.0 days have passed if no more radon leaks in?

$$\lambda = 0.693 / T_{1/2} = 0.181\,\text{day}^{-1}$$

$$N = N_0 e^{-\lambda t} = (4.00\times 10^8 \text{ atoms}) e^{-(0.181\,\text{day}^{-1})(14.0\text{ days})} = 3.17\times 10^7 \text{ atoms}$$

(b) What is the activity of the radon sample after 14.0 days?

$$\lambda = (0.181\,\text{day}^{-1})(1\,\text{day}/8.64\times 10^4\text{ s}) = 2.09\times 10^{-6}\text{ s}^{-1}$$

$$R = \lambda N = 66.3\text{ decays/s} = 66.3\text{ Bq}$$

(c) How much time must pass before 99% of the sample has decayed?

$$\ln(N) = \ln(N_0 e^{-\lambda t}) = \ln(N_0) + \ln(e^{-\lambda t}) = \ln(N_0) - \lambda t$$

$$t = \frac{\ln(N_0) - \ln(N)}{\lambda} = \frac{\ln(N_0 / N)}{\lambda} = \frac{\ln(N_0 /(0.01 N_0))}{2.09\times 10^{-6}\text{ s}^{-1}} = 2.20\times 10^6\text{ s} = 25.5\text{ days}$$

Decay Processes

❑ Alpha decay

- If a nucleus emits an alpha particle ${}^{4}_{2}He$, it loses two protons and two neutrons. So the reaction can be written symbolically as:

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

X : parent nucleus, Y : daughter nucleus

- Two examples:

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{2}_{4}He$$

half-life : $4.47\text{x}10^{9}$ years

$${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

half-life : $1.60\text{x}10^{3}$ years

- For alpha emission to take place, the mass of the parent must be greater than the combined mass of the daughter and the alpha particle. The excess mass is converted to kinetic energy of the daughter nucleus and the alpha particle.
- Since momentum is conserved, two particles in the final state carry the same momentum in the opposite direction if they are produced by the parent nucleus at rest. As the kinetic energy $KE=p^2/(2m)$, the heavier particle carries more energy.

Decay Processes

- Alpha decay (cont'd)
  - Example 29.5 : Decaying radium

Calculate the amount of energy liberated in the decay:

$${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

$$m_d + m_\alpha = 222.017571\,\mathrm{u} + 4.002602\,\mathrm{u} = 226.020173\,\mathrm{u}$$

$$\Delta m = M_p - (m_d + m_\alpha) = 226.025402\,\mathrm{u} - 226.020173\,\mathrm{u} = 0.005229\,\mathrm{u}.$$

$$E = (0.005229\ \mathrm{u})(931.494\,\mathrm{MeV/u}) = 4.871\,\mathrm{MeV}$$

Decay Processes

- Beta decay

• If a nucleus emits a β particle, the daughter nucleus has the same number of nucleons as the parent nucleus but the atomic number is changed by 1. So the reaction can be written symbolically as:

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e$$

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + e^{+} + \nu_e$$

• An example:

$$^{14}_{6}C \rightarrow ^{14}_{7}N + e^{-} + \bar{\nu}_e$$

In this case the electron comes from the decay of neutron:

$$^{1}_{0}n \rightarrow ^{1}_{1}p + e^{-} + \bar{\nu}_e$$

• Example 29.6 : Beta decay of carbon-14

$$\Delta m = m_C - m_N = 14.003242\,\mathrm{u} - 14.003074\,\mathrm{u} = 0.000168\,\mathrm{u}$$

$$E = (0.000168\,\mathrm{u})(931.494\,\mathrm{MeV}) = 0.156\,\mathrm{MeV}$$

Decay Processes

- Gamma decay
  - Often a nucleus that undergoes radioactive decay is left in an excited energy state. the nucleus can then undergoes a second decay to a lower energy state by emitting one or more photons (called gamma rays).

$$^{12}_{5}B \rightarrow ^{12}_{6}C^{*} + e^{-} + \overline{\nu}_{e}$$

$$\hookrightarrow ^{12}_{6}C + \gamma$$

- Practical uses of radio activity (See the textbook for detains)
  - Carbon dating
  - Smoke detector
  - Radon detection

Nuclear Reactions

❑ Nuclear reactions

- The structure of nuclei can be changed by bombarding them with energetic particles. Such changes are called nuclear reactions.
- First person who observed a nuclear reaction in the following process was Rutherford. He found that protons were released when alpha particles were allowed to collide with nitrogen atoms:

$$^4_2He + ^{14}_7N \rightarrow X + ^1_1H$$

By balancing atomic numbers and mass numbers, we can conclude that the known nucleus X is in fact isotope of oxygen:

$$^4_2He + ^{14}_7N \rightarrow ^{17}_8O + ^1_1H$$

❑ Example 29.8 : Discovery of neutron by Chadwick (1932)

Reaction used: $^4_2He + ^9_4Be \rightarrow ^{12}_6C + ^A_ZX$

$$4 + 9 = 12 + A \rightarrow A = 1$$

$$2 + 4 = 6 + Z \rightarrow Z = 0$$

⇨ $$^4_2He + ^9_4Be \rightarrow ^{12}_6C + ^1_0n$$

Nuclear Reactions

❑ Q values

- Consider the nuclear reaction: $^{2}_{1}H + ^{14}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$

  initial total mass m_i : $2.014102\,\mathrm{u} + 14.003074\,\mathrm{u} = 16.017176\,\mathrm{u}$

  final total mass m_f : $12.000000\,\mathrm{u} + 4.023602\,\mathrm{u} = 16.002602\,\mathrm{u}$

  mass difference Δm: $\Delta m = m_f - m_i = -0.014574\,\mathrm{u}$

  The negative mass difference comes from the fact that part of the initial mass energy is converted into kinetic energy. The Q value is defined as : $Q \equiv -\Delta m$

  If the Q value is positive, the reaction is said to be exothermic reaction.

- Consider the nuclear reaction: $^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H$

$$Q = m_i - m_f = -0.001282\,\mathrm{u} = -1.194\,\mathrm{MeV}$$ endothermic reaction

A careful analysis of this reaction reveals that, even if the incoming alpha particle has kinetic energy of 1.194 MeV is not enough to have this reaction happen because, although the energy is conserved, the momentum is not. The incoming particle needs at least kinetic energy of $KE_{\min} = (1 + m/M)|Q|$. (m/M: mass of incoming/target particle).

Threshold energy