Mathematics Curriculum Map

Sussex Montessori School

The mathematics curriculum is built around several research-based curriculum and standards documents including:

First State Montessori Academy Curriculum

Montessori Mathematics Curriculum Albums

The National Common Core Standards

National Council of Teachers of Mathematics

Investigations 3/ Connections Mathematics

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Introduction to the Mathematics Curriculum Framework

In the Mathematics Curriculum, the teacher must be knowledgeable about the "Processes and Proficiencies" and have tools to assess when students demonstrate these proficiencies as they work within the various mathematical strands. Teachers also need a clear understanding of the knowledge goals for mathematical thinking within each of strands. The Montessori Mathematics Curriculum Framework provides teachers with the goals for mathematics at each multi-age stage of development (5-7, 7-9, and 9-12-year-old). Early in the school year, teachers use a variety of assessments to determine where children are on the learning continuum in each area. The Curriculum Framework provides the teacher with instructional strategies that are used in small group and individual lessons/units using hands-on *Montessori materials*, *TERC Investigations 3 Mathematics*, and other resources to meet the individual instructional needs of the child. These lessons provide opportunities for teachers to observe children and to evaluate their progress towards the goals for learning across each strand of the mathematics curriculum as well as their understanding and demonstration of the processes and proficiencies. The Framework provides various formative and summative assessment tools for teachers to confirm their observations, and to make adjustments to instruction as a result of those observations. These tools include daily observations, teacher designed assessments, and summative assessments. DIBELS Math assessments are used to identify children who may need to be more closely followed and monitored in the RTI model of assessment/instruction.

The development of the child in Mathematics is embedded within the context of a classroom that supports the best educational practices. It is generally accepted that the workforce of the future will require skills such as creative and innovative thinking, comfort with ideas and abstraction, along with a global worldview and vibrant imagination. Research (Adams, 2005) shows that children develop these skills in classrooms designed to promote intrinsic motivation; to provide choice, time for focus and deep study in areas of interest; to allow opportunities to experiment and discover, and to develop a focus on "What did you learn?" rather than "How well did you do?" The overall Montessori Program is designed to support the following:

- A focus on **big ideas and essential questions** with **extended work periods** to allow for **depth of understanding** and **habits of mind.**

- **Child-centered inclusive** learning environments that utilize **differentiated instruction** and **flexible grouping** to meet individual children's learning needs.
- **Classroom-based assessment** and observation that **informs instructional decision making** as the basis for **RTI.**
- **Hands-on interactive** curricular materials and classroom environment supporting children developing from **concrete to abstract thinking.**
- **Academic development** supported by an emphasis on the **social/emotional development** of the child within a **multi-age community of learners.**
- **Collaborative learning** and **community service** leading to mutual respect of others and the development of the child's **global perspective.**

References

Adams, K. (2005). "Sources of innovation and creativity". A paper commissioned by the National Center on Education and the Economy. Accessed 10/26/2011 at http://www.fpspi.org/Pdf/InnovCreaitivy.pdf.

Montessori, Maria. (1991). *The Advanced Montessori Method: Scientific pedagogy as applied to the education of children from seven to eleven years.* Oxford, England: Clio Press.

Montessori, Maria. (1973). *The Advanced Montessori Method: Volume 2.* New York: Schocken Books.

Mathematics Processes and Proficiencies

This table presents the overall set of 'habits of mind" that contribute to the development of mathematical processes and proficiencies. This table provides an over-arching guide to these processes and proficiencies that will be focused on by SMS teachers through the use of Montessori Math materials and the *Investigations 3/ Connections* Math units.

Mathematically proficient students:

Make sense of problems and persevere in solving them.	**Reason abstractly and quantitatively.**	**Construct viable arguments and critique the reasoning of others.**	**Model with mathematics.**
Explain to themselves the meaning of a problem. Look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution. Plan a solution pathway rather than simply jumping into a solution attempt. Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. Monitor and evaluate their progress and change course if necessary. Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of	Make sense of quantities and their relationships in problem situations. Bring two complementary abilities to bear on problems involving quantitative relationships: The ability to *decontextualize*, to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents The ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Create a coherent representation of the problem at hand,	Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Analyze situations by breaking them into cases. Recognize and use counterexamples. Justify their conclusions, communicate them to others, and respond to the arguments of others. Make plausible arguments that take into account the context from which the data arose, reasoning inductively. Compare the effectiveness of two plausible arguments.	Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Apply what they know. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. Identify important quantities in a practical situation. Map relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. Analyze relationships mathematically to draw conclusions. Interpret their mathematical

important features and relationships, graph data. Search for regularity or trends. Check their answers to problems using a different method. Ask themselves, "Does this make sense?" Understand the approaches of others to solving complex problems. Identify correspondences between different approaches. Use concrete objects or pictures to help conceptualize and solve a problem. (younger students) Transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need, depending on the context of the problem. (older students)	considering the units involved. Attend to the meaning of quantities, not just how to compute them. Know and flexibly use different properties of operations and objects.	Distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Listen to or read the arguments of others, decide whether they make sense. Ask useful questions to clarify or improve arguments. Construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. (younger students) Determine domains to which an argument applies. (older students)	results in the context of the situation. Reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Mathematically proficient students:			
Use Appropriate Tools Strategically. Consider the available tools when solving a mathematical problem. These tools might	**Attend to precision.** Communicate precisely to others. Use clear definitions in discussion with others and in their own	**ok for and make use of structure.** Look closely to discern a pattern or structure. Young students might notice that three and seven more is the	**Look for and express regularity in repeated reasoning.** Notice if calculations are repeated, and look both for general methods and for

include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Develop familiarity with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. Detect possible errors by strategically using estimation and other mathematical knowledge. Know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Identify relevant external	reasoning. State the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Use care to correctly specify units of measure, and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently. Express numerical answers with a degree of precision appropriate for the problem context. Give carefully formulated explanations to each other. (elementary school) Examine claims and make explicit use of definitions. (high school)	same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. Older students can look at the expression $2x + 9x + 14$ and see the 14 as 2×7 and the 9 as $2 + 7$. Recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. Consider an overview and be able to shift perspective. See complicated things as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. Apply what they know. Maintain oversight of the problem-solving process, while attending to the details. Evaluate the reasonableness of their intermediate results.

mathematical resources, such as digital content located on a website, and use them to pose or solve problems. Use technological tools to explore and deepen their understanding of concepts.			

Curriculum Resources and Materials

There are three resources that will be the foundation for the math curriculum at FSMA.

- Montessori Math Lessons
- TERC Investigations 3 (Grades K-5)
- Connected Mathematics (Grade 6)

Montessori Math Lessons are part of a larger integrated curriculum. This integrated curriculum is founded on the teaching of five "Great Lessons." Relevant to the Math Curriculum articulated in this document is the fifth Great lesson, "The History of Mathematics," also called "The Story of Numbers," which focuses on learning about the numeric system of early civilizations and continuing by looking at the different number systems that have been used and culminates with a study of the decimal system used today.

The Fifth Great Lesson: The Story of Numbers leads to the study of:

- Mathematics: operations, fractions, decimals, multiples, squares, cubes, percentages, ration, probability, intro to algebra
- Numbers: origins of numbers and systems, bases, types of numbers, scientific notation, mathematicians
- Geometry: congruency, similarity, nomenclature of lines, angels, shapes, solids, measurement and theorems
- Application: story problems, measurement, estimation, graphs, patterning, rounding, money concepts.

***Montessori Math in the Lower Elementary Classrooms*:** The lower elementary Montessori classroom is full of ongoing discoveries. Spurred on by the telling of the fifth Great Lesson, "the Story of Numbers," children are motivated to

learn about their own number system and uncover the mysteries as did those who came before. The absorbent mind of the early childhood has given way to a reasoning mind which enjoys learning about natural truths and laws of nature. The mathematical facts learned in the younger grades are now tested to see if there are rules and laws to be discovered and manipulated. Patterns are sought as the child seeks to discover the empirical truths of the universe through the use of concrete Montessori math materials. It is now that children are able to use their imaginations to see beyond the immediate. They are able to see beyond the concrete reputations and imagine higher place values in the decimal system.

Montessori math in the upper Elementary Classroom: The inquisitiveness of the upper elementary Montessori student is astounding. The beauty of the advanced squaring and cubing materials beckons like beacons, inviting the students to come explore and learn with them. They dive into the study of fractions and decimals, eager to move beyond to more complex mathematics, geometry, and, algebra. While the concrete materials are still in place, the need for repetition is gone. "Show me. Then, show me more" is the litany of the upper elementary Montessori math students. Upper elementary students move quickly form the concrete experience abstract thought. They are eager to test their knowledge with pencil and paper and need, at times, a gentle reminder to return to the materials as a way of building neurological pathways.

TERC Investigations 3 (k-5th grade) helps students develop a strong conceptual foundation and skills based on that foundation. Each curriculum unit focuses on an area of content and provides opportunities for student to develop and practice e ideas across a variety of activities and contexts that build on each other. The units also address the learning needs of real students in a wide range of classrooms and communities. there are six major goals of the curriculum:

- Support student to make sense of mathematics and learn that they can be mathematical thinking
- Focus on computational fluency with whole numbers as a major goal of elementary grades
- Provide substantive work in important areas of mathematics – rational numbers, geometry, measurement, data, and earl algebra – and connections among them.
- Emphasize reasoning about mathematical ideas
- Communicate mathematics content and pedagogy to teachers
- Engage the range of learners in understanding mathematics

Underlying these goals are three guiding principles that are closely connected to the Montessori philosophy:

1. Students have mathematical ideas. The curriculum must support all students in developing and expanding those ideas.
2. Teachers are engaged in ongoing earning about mathematics content and how students learn mathematics.
3. Teachers collaborate with the students and curriculum materials to create the curriculum as enacted in the classroom. The curriculum must support teachers in implementing it in a way that accommodates the needs of their particular students.

Connected Math (6th grade) is a math curriculum designed for students in grades 6-8. It is a natural extension of the TERC Investigations. Each grade level of the curriculum is a full-year program and covers numbers, algebra, geometry/measurement, probability, and statistics. The curriculum uses an investigative approach, and students utilize interactive problems and everyday situation to learn math concepts.

SMS Mathematics Assessments

The following tables outline the mathematics assessments that will allow the Sussex Montessori School teacher to collect both formative (F) and summative data (S) on students' progress in mathematics in each age grouping.

K/1 Mathematics Assessments

Common Core Domain	Smarter Balance	Third Period of Montessori Lesson: Observation		TERC Investigations3/ Connections (6th grade)			DIBELS Math 3 time a year
		Anecdotal Records	Journals	Assessment Checklists	Portfolios	Embedded Benchmark Assessments	
Mathematics Processes and Proficiencies	N/A	X	X	X	X	X	
Counting and Cardinality	N/A	X	X	X	X	X	X
Numbers and Operations in Base 10	N/A	X	X	X	X	X	X
Numbers and Operations Fractions	N/A	X	X	X	X	X	
Operations/Algebraic Thinking	N/A	X	X	X	X	X	X
Geometry	N/A	X	X	X	X	X	X
Measurement and Data	N/A	X	X	X	X	X	X

2/3 Mathematics Assessments

Common Core Domain	Smarter Balance	Third Period of Montessori Lesson: Observation		TERC Investigations3/ Connections (6th grade)			DIBELS Math 3 time a year
		Anecdotal Records	Journals	Assessment Checklists	Portfolios	Embedded Benchmark Assessments	
Mathematics Processes and Proficiencies	X	X	X	X	X	X	
Numbers and Operations in Base 10	X	X	X	X	X	X	X
Numbers and Operations Fractions	X	X	X	X	X	X	
Operations/Algebraic Thinking	X	X	X	X	X	X	X
Geometry	X	X	X	X	X	X	X
Measurement and Data	X	X	X	X	X	X	X

4/5/6 Mathematics Assessments

Common Core Domain	Smarter Balance	Third Period of Montessori Lesson: Observation		TERC Investigations3/ Connections (6th grade)			DIBELS Math 3 time a year
		Anecdotal Records	Journals	Assessment Checklists	Portfolios	Embedded Benchmark Assessments	
Mathematics Processes and Proficiencies	X	X	X	X	X	X	
Numbers and Operations in Base 10	X	X	X	X	X	X	X
Numbers and Operations Fractions	X	X	X	X	X	X	X
Operations/Algebraic Thinking	X	X	X	X	X	X	X
Geometry	X	X	X	X	X	X	X
Ratios and Proportional Relationships (6th)		X	X	X	X	X	X
Measurement and Data	X	X	X	X	X	X	X
Statistics and Probability (6th)	X	X	X	X	X	X	X

Sussex Montessori School
Mathematics Curriculum
Kindergarten

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations 3* Grade: K

Unit One: Counting Timeline: 16 Sessions

Unit Description: In this unit children will explore counting objects, connect number names to the written number, and use numbers to organize and label sets.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. K.CC.B.4c Understand that each successive	Count objects and represent quantities with numbers, names, and numerals. Compare and order quantities. Counting is cumulative. Counting tells how many are in a set. There is a unique vocabulary that helps describe quantities. Represent quantities with pictures, numbers, objects, and/or words. Use attributes of objects to sort them according to	Essential Questions: How can numbers be counted, read, and written? How does counting tell how many? How can numbers be represented? What are strategies that we can use to keep track of quantities when we count? What is the unique vocabulary related to quantity? **Learning Targets:** Students will: • use strategies to accurately count and keep track of quantities up to 10. • count and compare quantities up to 10.	**Montessori Materials** Cards and Counters Spindle Box (for review) Number Rods Golden Bead Material Bead Cabinet Teens Board Tens Board Hundred Board **Investigations Unit 1 – Counting People, Sorting Buttons** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks

number name refers to a quantity that is one larger. K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	how they are alike or different.	• represent quantities with pictures, numbers, objects, or words.	<u>Summative</u> • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Two: Counting and Measurement Timeline: 22 Sessions

Unit Description: In this unit, students will further develop counting skills, connect number names and numerals, and develop visual images of quantities up to 10. They will compare and order two or more amounts and describe and measure the length of objects by direct comparison.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they	Understanding length Understanding weight Counting and representing quantities	<u>Essential Questions:</u> How can we make equivalent sets? How can we use numbers to represent quantities? How can we determine what has more? What is length? What is weight? <u>Learning Targets:</u> Students will: • count and count out a set of objects up to 10 objects. • Students will compare two quantities up to 10 to determine which is greater. • Describe length and decide which of two objects is longer.	**<u>Montessori Materials</u>** Cards and Counters Number Rods Golden Bead Material Bead Cabinet Teens Board Tens Board Hundred Board **Investigations Unit 2 – Counting Quantities, Comparing Lengths** **<u>Assessments:</u>** <u>Formative</u> • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks <u>Summative</u>

were counted. K.CC.B.4c Understand that each successive number name refers to a quantity that is one larger. K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals. K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category			• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

and sort the categories by count.			

Unit Three: 2-D Geometry Timeline: 12 Sessions
Unit Description: In this unit students will explore 2-D shapes including squares, circles, triangles, rectangles, hexagons.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. K.G.A.2 Correctly name shapes regardless of their orientations or overall size. K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). K.G.B.4 Analyze and compare two- and three dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). K.G.B.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Describing, identifying, and comparing 2-D shapes Composing and decomposing 2-D shapes	Essential Questions: What are 2-D shapes? What are attributes? What are the attributes of curved shapes? What are the attributes of rectangles, squares, triangles, and hexagons? What shapes can I make with other shapes? What shapes do I see around me? Learning Targets Students will: • Identify and describe the overall size, shape, and features of familiar 2-D shapes. • Make 2-D shapes. • Combine smaller shapes to make larger shapes.	**Montessori Materials** Geometry Cabinet Constructive Triangle Boxes Insets 3-Part Cards **Investigations Unit 3 – Make a Shape, Fill a Hexagon** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery.

K.G.B.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*		• Find combinations of shapes that fill a region. • Describe 2-D shapes by their attributes. • Identify 2-D shapes in their environments.	• Problem-based interactive Learning activities • Performance assessment

Unit Four: Counting and Measurement 2 **Timeline: 23 Sessions**

Unit Description: In this unit, students will focus on counting and representing sets of up to 15 objects, using counting skills, decomposing numbers in many ways, and beginning to make sense of the operations of addition and subtraction.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted.	Counting and representing quantities Comparing and ordering quantities Collecting, representing, describing, and interpreting data	Essential Questions: How do we represent quantities over 10? How do we make an equivalent set and represent the quantity for a given number? How can we keep track of growing sets of objects? Learning Targets: Students will: • Count and count out a set of up to 15 objects. • Make a set of a given size. • Use numbers to represent measurements and quantities. • Record an arrangement of a quantity.	**Montessori Materials** Golden Bead Material Bead Cabinet Teens Board Tens Board Hundred Board A variety of counters **Investigations Unit 4 – Collect, Count, and Measure** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative • Montessori Three-Period Lesson including

The number of objects is the same regardless of their arrangement or the order in which they were counted. K.CC.B.4c Understand that each successive number name refers to a quantity that is one larger. K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.		• Establish one-to-one correspondence between equal groups.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). K.OA.A.5 Fluently add and subtract within 5. K.MD.A.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.			

Unit Five: 3-D Geometry Timeline: 10 Sessions

Unit Description: In this Unit, students will focus on identifying, describing, and comparing attributes of 3-D shapes and naming, constructing, and composing and decomposing 3-D shapes from and into smaller shapes.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. K.G.A.2 Correctly name shapes regardless of their orientations or overall size. K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). K.G.B.4 Analyze and compare two- and three dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). K.G.B.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Describing, identifying, and comparing 3-D shapes Composing and decomposing 3-D shapes Comparing and contrasting 2-D and 3-D shapes	Essential Questions: What are the attributes of cones, cylinders, spheres, prisms, cubes, ellipsoids, and ovoids? How do 3-D shapes relate to real-world objects? How are 2-D and 3-D shapes alike and different? How can 3-D shapes be used to make different 3-D shapes? Learning Targets: Students will: • Identify and describe the overall size, shape, and features of familiar 3-D shapes. • Make 3-D shapes. • Combine shapes to make 3-D shapes. • Understand words that describe relative position.	**Montessori Materials** Golden bead material Geometry Cabinet Insets Geometric Solids 3-Part Cards **Investigations Unit 5 – Build a Block, Build a Wall** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery.

K.G.B.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*			• Problem-based interactive Learning activities • Performance assessment

Unit Six: Addition, Subtraction, and the Number System 1 Timeline: 20 Sessions
Unit Description: In this unit, students have repeated opportunities and experiences with join two or more amounts, remove and amount from the whole, and think of a number as being composed of two parts.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. K.CC.B.4c Understand that each successive	Understanding, representing, and solving addition and subtraction problems. Combining two numbers (0-10), with totals to 20. Representing and solving addition and subtraction story problems with result unknown. Decomposing numbers to 6 into two or more addends. Using numbers, pictures, words, and/or addition/subtraction notation to represent a solution to a problem	Essential Questions: How do we accurately count and keep track of quantities up to 20? How can we use numbers, pictures, words, and/or addition notation to represent a quantity? How can we compare quantities to 20 to determine which is greater? Learning Targets Students will: • Students will write numbers to 10. • Figure out what is one more or lone less than a number. • Represent and solve addition story problems within 10. • Decompose a number into two addends in more than one way.	**Montessori Materials** 100 Board Bead bars Addition strip board Addition finger charts Subtraction strip board Subtraction finger charts Bead Cabinet A variety of counting objects **Investigations Unit 6 – How Many Now?** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative

number name refers to a quantity that is one larger. K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).			• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

K.OA.A.5 Fluently add and subtract within 5.			

Unit Seven: Modeling with Data **Timeline: 15 Sessions**

Unit Description: In this unit, student will describe attributes of objects and data, use this information to sort, classify, count, order, compare, and represent dates, and use this date to model real-world problems with mathematics.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. K.CC.B.4c Understand that each successive number name refers to a quantity that is one larger.	Sorting and classifying Collecting, representing, describing, and interpreting data Comparing and ordering quantities Counting and representing quantities There are various ways to collect, represent, describe and interpret data.	Essential Questions How can we count and keep track of quantities? Can we find a total of up to 6 small quantities? Can we count by groups of 2? Can we count by groups of 10? Can we establish the one-to-one correspondence between a set of data and a representation of this data set? Learning Targets Students will: • Sort a set of objects by a given attribute and order the groups based on the number in each. • Use data to represent and solve a real-world problem.	**Montessori Materials** Attribute blocks 100 board Golden bead material **Investigations Unit 7 – How Many Noses? How Many Eyes?** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery.

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.			• Problem-based interactive Learning activities • Performance assessment

Unit Eight: Addition, Subtraction, and the Number System 2 **Timeline: 22 Sessions**
Unit Description:

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
K.CC.A.1 Count to 100 by ones and by tens. K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.B.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.B.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	Understanding, representing, and solving addition and subtraction problems Understanding that the value of a number is dependent on its position in a number	Essential Questions Can we make a set and represent the quantity equivalent to a given expression? Can we recognize, identify, and write the teen numbers? How can use a group of ten ones and some number of ones to represent a teen number? Can we count groups of 10? How can we use addition notation to represent the teen numbers as 10 plus some number of ones? Learning Targets: Students will: • Write numbers to 20. • Count by 1s and 10s to 100; when counting by 1s, start from a number other than 1.	**Montessori Materials** Golden bead material Stamp Game Hundred Board Million Cube Teens board Tens board Addition strip board Addition finger charts Teacher-made materials including 10-frames **Investigations Unit 8 – Ten Frames and Teen Numbers** **Assessments:** Formative • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks Summative

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). K.OA.A.4 For any number from 1 to 9, find the		• Add and subtract fluently within 5. • Figure out a missing addend when the sum is 10. • Represent the teen numbers as ten 1s and some number of 1s. • Represent and solve subtraction story problems within 10, with result unknown.	• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. K.OA.A.5 Fluently add and subtract within 5. K.NBT. A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.			

Sussex Montessori School

Mathematics Curriculum

1st Grade

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations* 3 Grade: 1

Unit One: Number and Operations **Timeline: 20 Sessions**

Unit Description: In this unit students extend their understanding of counting and the number sequence, and build a strong foundation for their work with place value and the operations of addition and subtraction.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.B.3 Apply properties of operations as strategies to add and subtract. Examples: If 8+3=11 is known, then 3+8=11 is also known. (Commutative property of addition.) To add 2+6+4, the second two numbers can be added to make a ten, so 2+6+4=2+10=12. (Associative property of addition.) 1.OA.B.4 Understand subtraction as an unknown-addend problem. For example, subtract 10-8 by finding the number that makes 10 when added to 8.	Counting forward and backward can begin from any number. Number lines can be used as a tool for counting. Counting tells how many are in a set and numbers represent a set of objects. A teen number is a group of ten ones and some number of ones The first numeral in a 2-digit number designates the number of tens in a given quantity. The first digit of a 2-digit number designates the number of groups of 10 and the second digit	**Essential Questions** How can numbers to 1,000 be read and written? What does each digit in a 2-digit number designate? How can we use symbols compare two 2-digit quantities? How can we count by groups of 2, 5, or 10? How can we use the 100 board to find patterns of numbers? **Learning Targets** Students will: • Understand ten ones as one ten, and a teen numbers as one ten	**Montessori Materials** 100 board Colored bead bars Addition and subtraction snake games Teacher-made materials for greater than, less than, and equal to Teacher-made materials for missing addends and subtrahends Bead cabinet Teens board Tens Board Number lines **Investigations Unit 1 – Building Numbers and Solving Story Problems**

1.OA.C.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+ 4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7by creating the known equivalent 6+6+1=12+1=13). 1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6=6, 7=8-1,5+2=2+5, 4+1=5+2. 1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the	designates the number of ones Two quantities can be compared to see which is greater. A 100 chart is a representation of the counting numbers 1 to 100.	and some number of ones. • Rote count, read, and write numbers to 1,000. • Understand that the multiples of 10 through 90 refer to 1 – 9 tens and 0 ones. • Use a numeral to represent a number of objects organized into tens and ones and, given a numeral, represent it with tens and ones. • Use standard notation (<, >) to represent the comparison of two 2-digit numbers. • Add or subtract 10 to/from any 2-digit number.	**Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance • assessment

unknown number that makes the equation true in each of the equations 8+?=11, 5=□-3, 6+6=□ 1.NBT.A.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. 1.NBT.B.2a 10 can be thought of as a bundle of ten ones—called a "ten." 1.NBT.B.2b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. 1.NBT.B.2c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols 7, =, and 6. 1.NBT.C.4			

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.NBT.C.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.			

Unit Two: 2-D Geometry **Timeline: 12 Sessions**

Unit Description: In this unit, students will focus on careful observation, description, and comparison of two-dimensional and three-dimensional geometric shapes.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD.A.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. 1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks. 1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	2- and 3-D shapes have common attributes. Geometric language helps us describe geometric shapes. Shapes can be composed of or decomposed into different shapes. There are many types of quadrilaterals.	Essential Questions What language can we use to describe 2- and 3-D shapes? What are the attributes of 2-D shapes? What are the attributes of 3-D shapes? How can we use 2-D shapes to create other 2-D shapes? What are the common attributes of 2- and 3-D shapes? What is a quadrilateral? **Learning Targets** Students will: • Compose and decompose shapes in different ways.	**Montessori Materials** Geometry cabinet Attribute blocks Constructive Triangle boxes Solid geometric shapes 3-part cards Insets **Investigations Unit 2 – Comparing and Combining Shapes** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-

1.G.A.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. 1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. 1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols 7, =, and 6.		• Build and draw familiar 2-D shapes. • Use geometric language to describe and identify important attributes, and use those attributes to sort familiar 2-D shapes. • Use geometric language to describe and identify defining attributes of familiar 3-D shapes • Compose 3-D shapes. • Match a 2-D representation of a 3-D shape to the outline of one of its faces.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks.			

Unit Three: Addition, Subtraction, and the Number System 2 Timeline: 26 Sessions

Unit Description: In this unit, student will focus on counting on and back as a strategy for adding/subtracting, on composing and decomposing numbers into two or more parts, on adding more than two numbers, on expanding understanding of addition and subtraction notation, and on counting and comparing larger quantities.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.B.3 Apply properties of operations as strategies to add and subtract. Examples: If 8+3=11is known, then 3+8=11 is also known. (Commutative property of addition.) To add 2+6+4, the second two numbers can be added to make a ten, so 2+6+=2+10=12. (Associative property of addition.)	The equals sign represents equivalence and is used to show equivalent expressions. Counting on or counting back can be used as a strategy for adding or subtracting two numbers. The order of addends does not affect the total. A 100 chart is a representation of the counting numbers from 1 to 100. A teen number is a group of ten and some number of ones.	**Essential Questions** Does the order of addends matter to the sum? How does counting on or counting back help in solving an addition or subtraction problem? How can we determine equivalence? How can number patterns help us? **Learning Targets** Students will: • Find at least 5 solutions to put together/take apart problem with both addends unknown. • Solve story problems with 3 addends.	**Montessori Materials** Addition and subtraction snake games Colored bead bars Golden bead material Teacher-made materials for greater than/less than Number lines **Investigations Unit 3 – How Many of Each? How Many in All?** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative**

1.OA.B.4 Understand subtraction as an unknown-addend problem. For example, subtract 10-8 by finding the number that makes 10 when added to 8. 1.OA.C.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1= 13). 1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6=6, 7=8-1,5+2=2+5, 4+1=5+2. 1.OA.D.8		• Represent numbers with equivalent expressions. • Find at least 5 solutions to a put together/take apart problem with both addends unknown. • Solve story problems with 3 addends. • Represent numbers with equivalent expressions. • Understand that you can count on/back to add/subtract. • Rote count, read, and write numbers to 1,000.	• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8+? =11, 5=□-3, 6+6=□. 1.NBT.A.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. 1.NBT.B.2.a 10 can be thought of as a bundle of ten ones—called a "ten." 1.NBT. B.2b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. 1.NBT. B.2c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the			

symbols 7, =, and 6. 1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks.			

Unit Four: Measurement and Fractions Timeline: 14 Sessions

Unit Description: In this unit, students will focus on developing accurate techniques for linear measurement, solving comparison problems about length, and on ideas about time and fractions as equal parts of a whole.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, equations with a symbol for the unknown number to represent the problem. 1.OA.B.3 Apply properties of operations as strategies to add and subtract. Examples: If 8+3=11 is known, then 3+8=11 is also known (Commutative property of addition.) To add 2+6+4, the second two numbers can be added to make a ten, so 2+6+4=2+10=12. (Associative property of addition.) 1.OA.B.4 Understand subtraction as an unknown-addend problem. For example, subtract 10-8 by finding the number that makes 10 when added to 8. 1.OA.C.6	A whole can be partitioned into equal parts. Equal parts of a whole can be described as halves, fourths or quarters. When a whole is cut into fractional pieces, the pieces are smaller. Length can be quantified by repeating identical, multiple units from one end of an object to the other with no gaps or overlaps. Inch tiles can be used to measure objects. Measurements of the same lengths are the same when they are measured twice or by	**Essential Questions** How can we solve comparison story problems with the difference unknown? How can we use numbers, pictures, words, and/or notation to represent a solution to a problem? How can we describe equal parts of a whole? How can we accurately measure an object or length? Will two people measuring the same length or object get the same measurement? **Learning Targets** Students will: • Solve comparison problems with the difference unknown (how many	**Montessori Materials** Fraction Insets Fraction box Bead box Teacher-made materials for missing addends and subtrahends Teacher-made materials for greater than/less than Measurement materials Clock matching materials **Investigations Unit 4 – Fish Lengths and Fraction Rugs** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+ 4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7by creating the known equivalent 6+6+1=12+1=13). 1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD.A.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. 1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks.	different people using the same unit of measure.	more, and how many fewer). • Solve comparison problems with the difference unknown (how many more, and how many fewer). • Solve comparison problems with the difference unknown (how many more, and how many fewer). • Compare the lengths of two objects indirectly by using a third length. • Demonstrate accurate measuring techniques when measuring an object or distance with multiples units. These techniques include starting at the beginning, ending at the end, leaving no gaps or overlaps,	**<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. 1.G.A.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.		measuring in a straight line, and keeping track of the number of units. • Tell time to the hour and half hour. • Understand that halves or fourths (quarters) apply to wholes divided into two (four) equal parts; partition circles and rectangles into two and four equal parts.	

Unit Five: Addition, Subtraction and the Number System 3 Timeline: 10 Sessions

Unit Description: In this unit, student focus on developing fluency with addition and subtraction with 10, including the 2-addend combinations of 10, understanding the meaning of the equal sign as a symbol of equivalence, and finding an unknown addend or unknown change.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.B.3 Apply properties of operations as strategies to add and subtract. Examples: If 8+3=11 is known, then 3+8=11 is also known. (Commutative property of addition.) To add 2+6+4, the second two numbers can be added to make a ten, so 2+6+4=2+10=12. (Associative property of addition.) 1.OA.B.4 Understand subtraction as an unknown-addend problem. For example, subtract 10-8 by finding the number that makes 10 when added to 8. 1.OA.C.5 Relate counting to addition and	The equal sign represents equivalence. A teen number is a group of ten plus some number of ones. There are many strategies for solving put together/take apart story problems. Numbers, pictures, words, and/or notation may be used to represent a solution to a problem.	**Essential Questions** How can we determine equivalence? How can we determine if equations are true or false? How can we use 5 + 5 to reason about other combinations of 10? Does order matter in addition? How can we solve problems with one, tow, or more addends unknown? How can we show that all possible two-addend combinations of a number have been found? How can we use numbers, pictures, words, and/or notation to represent a solution to a problem?	**Montessori Materials** Teacher-made materials for missing addends and subtrahends Addition and subtraction snake games Bead bars **Investigations Unit 5 – Number Games and Crayon Problems** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-

subtraction (e.g., by counting on 2 to add 2). 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1= 13). 1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6=6, 7=8-1,5+2=2+5, 4+1=5+2. 1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8+? =11, 5=□-3, 6+6=□. 1.NBT. B.2b		**<u>Learning Targets</u>** Students will: • Fluency with addition and subtraction within 10. • Solve a put together/take apart problem with one addend unknown. • Solve add to and take from problems with unknown change. • Understand the meaning of the equal sign. • Determine the unknown in an addition or subtraction equation relating 3 numbers (e.g., 5+? =8).	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.			

Unit Six: Modeling with Data **Timeline: 12 Sessions**
Montessori Materials:
Investigations Unit 6 – Would You Rather Be an Eagle or a Whale?
Unit Description: IN this unit, student focus on collecting, recording, representing, describing, and comparing date in two and three categories, and on conducting data investigations.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-	Numbers, pictures, words, and/or notation can be used to represent a solution to a problem. The sum of the responses in each data category must equal the total responses collected.	**Essential Questions** How can counting by 10s help me? How can we make sense of and compare different data representations? How can we keep track of data collected? How can we represent and compare data? How can we make a comparative statement (more than/fewer than/same as) about a data representation? **Learning Targets** Students will: • Represent and describe a set	**Montessori Materials** Teacher-made materials Attribute blocks **Investigations Unit 6 – Would You Rather Be an Eagle or a Whale?** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-Period Lesson including

1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1= 13). 1.NBT. B.2a 10 can be thought of as a bundle of ten ones—called a "ten." 1.NBT. B.2c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).		of data with two or three categories (e.g., how many are in each group, which group has more/how many more, and how many people responded to the survey). • Solve comparison story problems with bigger or smaller unknown.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Seven: Addition, Subtraction, and the Number System 4 Timeline: 24 Sessions

Unit Description: In this unit, students will focus on counting by numbers other than 1 with an emphasis on groups of 10, on adding and subtracting 10 from a 2-digit number, on subtracting a multiple of 10 from a multiple of 10, and on representing 2-digt number with tens and ones.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.C.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1= 13). 1.OA.D.8	A quantity can be described by its number on ones, tens, etc. The first digit of a 2-digit number changes when 10 is added or subtracted and the second digit remains the same. The first digit in a 2-digit number determines the number of groups of 10 and the second number determines the number on ones. Multiples of 10 (up to 90) can be represented as groups of ten and no ones.	**Essential Questions** How can we tell the number on tens in a 2-digit number? How can I record what I have counted? How can groups of 2, 5, and/or 10 be easily counted? **Learning Targets** Students will: • Understand that the multiples of 10 through 90 refer to 1–9 tens and 0 ones. • Use a numeral to represent a number of objects organized into tens and ones and, given a numeral, represent it with tens and ones.	**Montessori Materials** Tens board Tens board 100 Board Gold bead material **Investigations Unit 7 – How Many Tens? How Many Ones?** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-Period Lesson including

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8+? =11, 5=□-3, 6+6=□. 1.NBT.A.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. 1.NBT.B.2a 10 can be thought of as a bundle of ten ones—called a "ten." 1.NBT.B.2c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols 7, =, and 6. 1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a		• Subtract multiples of 10 from multiples of 10 using concrete models that represent tens and ones. • Use standard notation (<, >) to represent the comparison of two 2-digit numbers. • Add or subtract 10 to/from any 2-digit number. • Add within 100 using concrete models that represent tens and ones.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.NBT.C.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.			

Unit Eight: 3-D Geometry **Timeline: 9 Sessions**
Montessori Materials:
Unit Description: In this unit, students will focus on observing, describing, comparing, and building 3-D shapes and on developing vocabulary for naming and describing defining attributes of 2-D and 3-D shapes.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks. 1.G.A.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.	Clocks allow us to tell time. 3-D shapes can be identified by their attributes. 3-D shapes can be combined to create other 3-D shapes.	**Essential Questions** How can we combine smaller 3-D shapes to compose a larger 3-D structure? How can we describe 3-D shapes? How can a shapes attributes help us identify 3-D shapes? How can we tell time? How can clocks help us tell time? How can we determine the length of something? How can attributes help us identify a 3-D shape? **Learning Targets** Students will:	**Montessori Materials** Teacher-made clock materials Geometric cabinet Geometric solids 3-part geometry cards **Investigations Unit 8 – Blocks and Buildings** **Assessments** **Formative** • Student Exercises • Peer questioning • Classroom Discussion • Problem Solving • Challenges • Exit Tickets • Vocabulary checks **Summative** • Montessori Three-Period Lesson including

		• Use geometric language to describe and identify defining attributes of familiar 3-D shapes. • Match a 2-D representation of a 3-D shape to the outline of one of its faces. • Compose 3-D shapes. • Compare the lengths of two objects indirectly by using a third length. • Demonstrate accurate measuring techniques when measuring an object or distance with multiple units. • Tell time to the hour. • Tell time to the half hour.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Sussex Montessori School

Mathematics Curriculum

2nd Grade

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations 3* Grade: 2

Unit One: Addition, Subtraction, and the Number System 1 Timeline: 20 Sessions

Unit Description: In this unit students focus on adding and subtracting single-digit numbers, especially on adding numbers in any order; shifting from counting by 1s to counting by groups, particularly groups of tens and ones, which lays the foundation for students' work with place value and the base-10 number system; and developing and refining strategies for solving a variety of addition and subtraction problems.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	There is more than one way to count a quantity. Numbers, symbols, pictures, and/or words can be used to represent a solution to a problem. Identifying and using patterns can help us count, read, and write numbers to 100 and beyond. The first digit of a 2-digit number designates the number of groups of 10 and the second digit designates the number of ones.	**Essential Questions** How can we count coins? How can we equivalent expressions for a number? How can we solve problems about 10s and 1s? How can we count groups of 2, 5, or 10? Does order matter in addition? Does order matter in subtraction? How can using known facts to add two or more numbers. How do clocks tell us time? **Learning Targets**	**Montessori Materials** 100 board 3-part coin quantity cards 3-part time cards Stamp game Place value cards Bead frame Bead Cabinet **Investigations Unit 1 – Building Numbers and Solving Story Problems** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. 2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally Spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. 2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? 2.G.A.1		Students will: • Solve a comparison story problem with the difference unknown. • Solve put together/take apart story problems with the total unknown, and add to and take from story problems with the result unknown. • Use known combinations to add several numbers in any order. • Recognize and identify coins and their values.	• Exit Tickets • 3-Period Lesson **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.			

Unit Two: 2-D Geometry **Timeline: 12 Sessions**

Unit Description: In this unit students focus on observing and describing defining attributes of 2-D and 3-D shapes (e.g., number and shape of faces, number and length of sides, and number of angles and vertices), and using those attributes as they sort, construct, draw, and compare shapes.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. 2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal	Geometry has its own vocabulary. A 3-D shapes attributes help us identify it. Quadrilaterals have four sides and four angles. Regular polygons can be named and sorted by their number of sides. Different rectangular arrays can be made with the same number of tiles. Fractions are equal parts of a whole.	**Essential Questions** How can we categorize shapes based on their attributes? How can we determine a half, third, or fourth of a region? Can halves or fourths of the same shape look different? How can we develop fluency with doubles facts within 20? What is the relationship between doubles and near doubles? What is the relationship between quadrilaterals, rectangles, and squares? **Learning Targets** Students will: • Identify defining attributes	**Montessori Materials** Geometric cabinet Geometric solids Constructive triangle boxes Fraction skittles Fraction insets Fraction box **Investigations Unit 2 – Comparing and Combining Shapes** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson

shares of identical wholes need not have the same shape.		of 2-D and 3-D shapes (number and shape of faces, number and length of sides, number of angles and vertices) and draw shapes with those attributes. • Make a rectangle out of same size squares and specify the number of rows and the number of squares in each row. • Recognize that [halves, thirds, fourths] of the same whole can look different. • Partition 2-D shapes into halves, thirds and fourths and name the regions.	**Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Three: Addition, Subtraction, and the Number System 2 Timeline: 26 Sessions

Unit Description: In this unit students focus on the place value of 2-digit numbers, and operating on those numbers within 100. Students come to see 100 as ten 10s and multiples of 100 as being made up of some number of hundreds.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.B.5	Different combinations of a 2-digit number, using only tens and ones, can represent the same number, (e.g. 4 tens and 6 ones, 3 tens and 16 ones, etc.). An equation can represent a 2-digit number as the sum of multiples of ten and some number of ones (e.g., 22-20+2, 22=10+10+2). Adding two-digit numbers Sums can be represented as lengths on a number line Numbers can be grouped and added in any order	**Essential Questions** How can 100 charts help us reason about the magnitude and relationship of numbers? What is the relationship between 1, 10, and 100? How can we use standard notation (<, >) to express the relationship between two quantities? How can we identify a quantity given a number of tens and ones? What happens to the tens place when a multiple of 10 is added or subtracted? **Learning Targets** Students will: • Solve a put together/take	**Montessori Materials** 100 Board Teens Board Tens Board **Investigations Unit 3 – How Many of Each? How Many in All?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson **Summative** • Montessori Three-Period Lesson including

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. 2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. 2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally Spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. 2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using	Regrouping Subtracting two-digit numbers Differences can be calculated using a number line Using addition to check subtraction	apart story problem with both addends unknown, and find all the possible combinations. • Solve a put together/take apart story problem with one addend unknown. • Solve two-step story problems about money. • Solve story problems with an unknown change. • Solve story problems with an unknown start. • Solve a put together/take apart story problem with both addends unknown, and find all the possible combinations.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?			

Unit Four: Measurement and Fractions Timeline: 14 Sessions

Unit Description: In this unit students focus on sorting and classifying categorical data; ordering numerical data; and collecting and representing categorical and numerical data using a variety of representations: student-generated representations, picture graphs, bar graphs, Venn diagrams, cube towers, and line plots.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s. 2.MD.D.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the	Data can be represented on a picture graph, bar graph, or line plot. An equation can show that the sum of the responses in each data category equals the total responses collected.	**Essential Questions** How can we represent data on a picture graph, bar graph, or line plot? How can we gather data? How can we record data? How can we interpret data from graphs? **Learning Targets** Students will: • Organize a set of data into up to four categories. • Create, describe, and interpret a variety of data representations, including picture graphs and bar graphs. • Order, represent, and	**Montessori Materials** Stamp game **Investigations Unit 4 – Fish Lengths and Fraction Rugs** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson **Summative** • Montessori Three-Period Lesson including introduction, practice,

horizontal scale is marked off in whole-number units. 2.MD.D.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take apart, and compare problems using information presented in a bar graph.		describe a set of numerical data.	and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Five: Addition, Subtraction and the Number System 3 Timeline: 10 Sessions
Unit Description: In this unit students focus on the place value of 3-digit numbers and operating on numbers within 100. Students come to see 100 as 10 tens and multiples of 100 as being made up of some number of hundreds.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.A.3	Three-digit numbers: place value Finding missing parts Using models to solve problems	**Essential Questions** How can using cubes or a number line show the relationship between adding (or subtracting) 9 and addition (or subtracting) 10 to/from a number? How can we solve 2-step problems? How can we solve story problems that involve comparison and finding the difference? What combinations of coins equal $1.00? What strategies can we use to add three-digit numbers? What strategies can we use to subtract three-digit numbers?	**Montessori Materials** Teacher-made materials Golden Beads Place value cards Stamp Game **Investigations Unit 5 – Number Games and Crayon Problems** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson **Summative** • Montessori Three-

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. 2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. 2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally Spaced points corresponding to the		What strategies and techniques are used to add mentally? What strategies are used to subtract mentally? What techniques are used to subtract three-digit numbers? When is counting on a helpful Strategy? When is counting back a helpful strategy? **<u>Learning Targets</u>** Students will: • Solve a 2-step story problem that involves finding the difference between a 2-digit number and 100. • Solve comparison story problems with a bigger unknown. • Read, write, count and compare numbers to 1,000.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. 2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?		• Count by 5s, 10s, and 100s within 1,000. • Read, write, count and compare numbers to 1,000. • Add fluently within 100. • Add/subtract 10 or 100 to/from numbers within 1,000.	

Unit Six: Modeling with Data Timeline: 12 Sessions

Unit Description: In this unit students focus on developing strategies for accurately measuring length with nonstandard and standard units (e.g., craft sticks, cubes, inches, feet, yards, centimeters, and meters) and tools (e.g., inch-brick measuring tools, rulers, yardsticks, and meter sticks) and for considering the relationship between different units and tools (e.g., the larger the unit, the smaller the count will be).

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.MD.A.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 2.MD.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.	Identifying contexts for measurement Comparing lengths Accurately using measuring tools Inches, feet, yards Centimeters, meters Length and width Representing and describing a set of measurement data in a table and on a line plot	**Essential Questions** How can we measure and compare lengths? Can we use nonstandard units to measure length? How can we use subtraction to compare lengths? Why do we have standard measurement units for length? How can the length of one object be used to measure another? What strategies can we use to add or subtract measurements in the same unit? How can we represent measurement data in a table or on a line plot?	**Montessori Materials** No specific Montessori materials necessary **Investigations Unit 6 – Would You Rather Be an Eagle or a Whale?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson **Summative** • Montessori Three-Period Lesson including

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. 2.MD.B.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. 2.MD.D.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.		**<u>Learning Targets</u>** Students will: • Recognize that, when measuring the same length, larger units yield smaller counts (and vice versa). • Estimate and measure lengths in inches, feet, centimeters, and meters. • Solve comparison and other story problems about lengths. • Represent measurement data on a line plot.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Seven: Addition, Subtraction, and the Number System 4 Timeline: 24 Sessions
Unit Description: The focus of this unit is on working with equal groups as the foundation of multiplication by investigating even and odd numbers and by representing equal groups with arrays and tables.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s. 2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Adding two-digit numbers Sums can be represented as lengths on a number line Numbers can be grouped and added in any order Regrouping Subtracting two-digit numbers Differences can be calculated using a number line Using addition to check Subtraction Any number that can be divided into groups of two can also be divided into two equal groups.	**<u>Essential Questions</u>** What is an array? What is the relationship between a number of equal groups and their total? Can all numbers be made into two equal parts? How are multiplication and division related? How can skip counting help us solve math problems? **<u>Learning Targets</u>** Students will: • Numbers can and cannot be made into groups of two or two equal groups. • Understand that any number that can be divided into groups of	**<u>Montessori Materials</u>** Bead cabinet Multiplication board Multiplication ticket box Multiplication finger charts Pythagorean board Division board Division ticket box Division finger charts Cards and counters **Investigations Unit 7 – How Many Tens? How Many Ones?** **<u>Assessments</u>** **<u>Formative</u>** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets • Vocabulary checks • Problem Solving Challenges • Exit Tickets

		two can also be divided into two equal groups (and vice versa). • Characterize even and odd numbers as those that do or do not make groups of two (partners) and two equal groups (teams). • Represent an even number as the sum of two equal addends and an odd number as the sum of two equal addends plus 1. • Consider whether observations about even or odd numbers apply to all even numbers or all odd numbers. • Add equal groups. • Use an equation to model adding equal groups. • Describe the relationship between a	• 3-Period Lesson **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

		number of equal groups and their total. • Represent multiplicative relationships with tables. • Compare situations that look different but have the same • equal group structure. • Solve problems involving equal groups and the total number of objects.	

Unit Eight: Addition, Subtraction, and the Number System 5 Timeline: 20 Sessions
Unit Description: This unit focuses on developing and achieving fluency with subtraction within 100, and on achieving fluency with addition and subtraction facts within 20

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	Fluency with addition and subtraction within 20 Using standard notation to represent addition and subtraction situations Coin equivalences for $1.00 Strategies for subtracting 2-digit numbers Place value Expanded notation Rounding to the nearest 10 and 100 Adding and subtracting three-digit numbers accurately	**Essential Questions:** How can we demonstrate fluency with addition and subtraction within 20. How can knowing doubles help us solve addition or subtraction problems? How can we solve make-change story problems? Where do we start when we add three-digit numbers? **Learning Targets:** Students will: • Solve a comparison story problem with a smaller unknown. • Fluently subtract 2-digit numbers.	**Montessori Materials** Money Roll-out Three-part money cards Coin amount matching cards Making change matching cards Stamp game Teacher-made materials Golden beads Place value cards Stamp game **Investigations Unit 8 – Enough for the class? Enough for the grade?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Quick Check sheets

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. 2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. 2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally Spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.		• Fluently add and subtract within 20. • Represent and solve addition and subtraction problems with 3-digit numbers. • Name, notate, and tell time to the nearest 5 minutes using analog and digital formats and associate A.M. and P.M. with time of day.	• Vocabulary checks • Problem Solving Challenges • Exit Tickets • 3-Period Lesson **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

2.MD.C.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. 2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?			

Sussex Montessori School

Mathematics Curriculum

3rd Grade

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations* 3 Grade: 3

Unit One: Understanding Equal Groups, Multiplication and Division I **Timeline: 23 Sessions**

Unit Description: In this unit students focus on understanding the meaning of multiplication, modeling multiplication using arrays, and understanding the inverse relationship between multiplication and division.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7. 3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8. 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and	Multiplication is combining equal groups. Represent and solve problems using multiplication and division. Multiply and divide within 100. Solve problems using the four operations, and identify patterns in arithmetic. Geometric measurement: Understand the concept of area and relate area to multiplication and addition.	**Essential Questions** What is the relationship among skip counting, repeated addition, and multiplication? Does doubling (or halving) on factor in a multiplication expression double (or halve) the product? How can we model multiplication situations with arrays? What is the square of a number? What is a prime number? **Learning Targets** Students will: • Demonstrate an understanding of multiplication and	**Montessori Materials** Multiplication board, tickets, and finger charts Division board, tickets, and finger charts Bead cabinet Teacher-made materials Geometry cabinet **Investigations Unit 1 – Understanding Equal Groups** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions

equations with a symbol for the unknown number to represent the problem. 3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8×?=48, 5=?÷3, 6×6=?. 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6×4=24 is known, then 4×6=24 is also known. (Commutative property of multiplication.) 3×5×2 can be found by 3×5=15, then 15×2=30, or by 5×2=10, then 3×10=30. (Associative property of multiplication.) Knowing that 8×5=40 and 8×2=16, one can find 8×7as 8×(5+2)=(8×5)+(8×2)=40+16=56. (Distributive property.) 3.OA.B.6 Understand division as an unknown-factor problem. For example, find 32÷8 by finding the number that makes 32 when multiplied by 8. 3.OA.C.7		division as involving equal groups. • Solve multiplication and related division problems by using skip counting or known multiplication facts. • Interpret and use multiplication and division notation. • Demonstrate fluency with multiplication facts ×1, ×2, ×5, and ×10. • Solve multiplication and division word problems and write equations to represent the problems. • Solve multi-step problems involving multiplication and addition.	• Vocabulary checks • Problem Solving Challenges • Exit Tickets **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8×5=40, one knows 40÷5=8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. 3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. 3.MD.C.7 Relate area to the operations of multiplication and addition. 3.MD.C.7.b Multiply side lengths to find areas of rectangles with whole number side lengths in the context			

of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. 3.MD.C.7.c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b+c is the sum of a×b and a×c. Use area models to represent the distributive property in mathematical reasoning.			

Unit Two: Graphs and Line Plots, Modeling with Data Timeline: 15 Sessions

Unit Description: In this unit students focus on using bar graphs, pictographs, and line plots to represent, describe, and compare categorical and numerical data. They solve one- and two-step "how many more" and "how many less" problems using information presented in the graphs. Students also generate measurement data in inches, half-inches, feet, and yards.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar Graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. 3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	Represent and interpret data Reading and interpreting bar graphs and pictographs Using data to compare groups Feet and inches Measure to the nearest half inch Using and interpreting a scale on a bar graph or pictograph with intervals larger than 1	**Essential Questions** What is the relationship between feet and inches? Can we combine feet and inches to get a total measurement? How can we organize and summarize collected data? What is categorical data? When should we use a pictograph, line plot, or bar graph to organize data? **Learning Targets** Students will: • Organize, represent, and describe categorical data, choosing categories that help	**Montessori Materials** Teacher-made materials for measurement and graphing, but no specific Montessori materials **Investigations Unit 2 – Graphs and Line Plots** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

		make sense of the data. • Make a line plot for a set of measurement data, with a scale that includes inches and half inches. • Generate measurement data by measuring lengths to the half inch. • Make and interpret bar graph and a pictograph, including use of scales greater than 1. • Describe and summarize a set of data, describing concentrations of data and what those concentrations mean in terms of the situation the data represent.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Three: Addition, Subtraction, and the Number System 1 Timeline: 25 Sessions

Unit Description: In this unit focuses on understanding and extending knowledge of place value and the number system to 1,000, and adding and subtracting accurately and efficiently. Students use a place value context to represent numbers as hundreds, tens, and ones, and find equivalent ways to use 100s, 10s, and 1s to represent a given number.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. 3.NBT.A.1 Use knowledge of place value to read, write, sequence, and round numbers up to 1,000. 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	Use place-value understanding and properties of operations to perform multi-digit arithmetic. Solve problems using the four operations, and identify patterns in arithmetic. Use place value understanding and properties of operations to perform multi-digit arithmetic. Solve problems involving measurement and estimation of intervals of times, liquid volumes, and masses of objects. Using expanded notation.	**Essential Questions** What is the relationship between 1, 10, 100, and 100? How many groups of 10s are in a 3-digit number (e.g., there are 27 tens in 276)? How can place-value understanding help us to round whole numbers to the nearest ten or hundred? How can number lines be used to represent solutions to comparison problems? **Learning Targets** Students will: • Use knowledge of place value to read, write, sequence, and round numbers up to 1,000.	**Montessori Materials** Golden bead materials Stamp games Bead frames Place value/expanded notation cards Bead cabinet Number lines Problem tickets Clock materials **Investigations Unit 3 – Travel Stories and Collections** **Assessments** **Formative** • Student Exercises

	Using place value to determine the size of any number to 1,000.	• Solve addition problems with 3-digit numbers (up to 400) by using strategies that involve breaking each number apart by place, or by adding one number in parts. • Solve subtraction problems with 2- and 3-digit numbers (up to 300) by using strategies that involve either subtracting one number in parts, adding up, or subtracting back. • Tell time to the nearest minute.	• Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Four: Perimeter, Area, and Polygons, 2-D Geometry and Measurement Timeline: 17 Sessions

Unit Description: In this unit students focus on understanding and finding perimeter and area using standard units of measurement, and on classifying 2-D figures. Students use standard measurement tools to measure the length of objects and the distance around 2-dimensional figures (perimeter). They use square units to measure the amount of space a given object covers (area). Students build on work connecting arrays and multiplication, and find area by multiplying length and width.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. 3.MD.C.5.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. 3.MD.C.5.b A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 3.MD.C.7 Relate area to the operations of multiplication and addition.	Geometric measurement: Understand the concept of area and relate area to multiplication and addition. Recognize perimeter as an attribute of plane figures, and distinguish between linear and area measurements. Rectangles can have the same perimeter and different areas of the same area and different perimeters. Reason with shapes and their attributes.	**Essential Questions** How can we find perimeter? Can different shapes have the same perimeter? How can we find perimeter of an irregular shape? How can we find an unknown side length if we know the perimeter and some side lengths? What are some possible sources of measurement error? **Learning Targets** Students will: • Measure and find the perimeter of 2-D figures using U.S. standard and metric units.	**Montessori Materials** Geometric cabinet Measurement materials **Investigations Unit 4 – Perimeter, Area, and Polygons** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

3.MD.C.7.d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. 3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. 3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.		• Find the area of 2-D figures using U.S. standard and metric units. • Categorize quadrilaterals, including squares, rhombuses and rectangles, based on their attributes.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Five: Cube Patterns, Arrays, and Multiples of 10, Multiplication and Division II **Timeline: 17 Sessions**

Unit Description: In this unit students focus on the meaning and structure of multiplication and division and the relationship between them, solving multiplication and division problems, multiplying by multiples of 10, and learning the remaining multiplication facts to 10 × 10.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7. 3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8. 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Division can be represented as equal groups. Distributive property Represent and solve problems using multiplication and division. Represent and compare multiplication problems with pictures, diagrams, or models. Multiply and divide within 100. Solve problems using the four operations, and identify patterns in arithmetic. Use place value understanding and	**Essential Questions** What are the multiples of 3, 4, and 6? What is the relationship between multiplication and division? How can knowing the multiplication facts be used to solve division problems? What happens to a product when we double one factor in a multiplication expression? What do parentheses in a problem mean? **Learning Targets** Students will: • Solve division problems (2-digit number divided by a single-digit number).	**Montessori Materials** Bead Cabinet Multiplication board, tickets, and finger charts Division board, tickets, and finger charts Place value/expanded notation cards **Investigations Unit 5 – Cube Patterns, Arrays, and Multiples of 10** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets

3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8×?=48, 5=?÷3, 6×6=?. 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6×4=24 is known, then 4×6=24 is also known. (Commutative property of multiplication.) 3×5×2 can be found by 3×5=15, then 15×2=30, or by 5×2=10, then 3×10=30. (Associative property of multiplication.) Knowing that 8×5=40 and 8×2=16, one can find 8×7as 8×(5+2)=(8×5)+(8×2)=40+16=56. (Distributive property.) 3.OA.B.6 Understand division as an unknown-factor problem. For example, find 32÷8 by finding the number that makes 32 when multiplied by 8. 3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that	properties of operations to perform multi-digit arithmetic. Use place-value understanding and properties of operations to perform multi-digit arithmetic. Geometric measurement: Understand the concept of area and relate area to multiplication and addition.	• Solve multiplication and division word problems and write equations to represent the problems. • Multiply a single-digit number by a multiple of 10, up to 90. • Represent and explain the relationship between multiplication and division. • Solve multi-step problems involving multiplication and addition. • Demonstrate fluency with multiplication facts to 10×10.	**<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

8×5=40, one knows 40÷5=8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. 3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. 3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations. 3.MD.C.7.b Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. 3.MD.C.7.c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b+c is the sum of a×b and a×c.			

Use area models to represent the distributive property in mathematical reasoning.			

Unit Six: Fair Shares and Fractions on Number Lines Timeline: 13 Sessions

Unit Description: I this unit students focus on understanding the meaning of fractions as numbers and as equal parts of a whole; reasoning about equivalent fractions; comparing fractions; and using notation to model fractions and fraction relationships. Students build their knowledge of fractions and fraction equivalents as they represent and compare fractional quantities, including fractions greater than one.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a part of size 1/b. 3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. 3.NF.A.2. a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. 3.NF.A.2. b Represent a fraction a/b on a number line diagram by marking off a length 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	Develop an understanding of fractions as numbers. Fractional parts are constructed of unit fractions. A unit fraction is a number represented on a number line. The size of a fraction is determined by the size of the whole. Different-shaped pieces are the same fraction of a whole. Represent and interpret data. Reason with shapes and their attributes.	**Essential Questions** How can we represent fractional parts of an area? How can fractions be represented on a number line? How can representations help us to visualize how fractions relate to each other? **Learning Targets** Students will: • Partition a quantity into equal parts, and name those parts as fractions. • Represent fractions as numbers on a number line. • Compare fractions with the same numerator or	**Montessori Materials** Fraction skittles Fraction box Fraction insets Number lines Geometric cabinet **Investigations Unit 6 – Fair Shares and Fractions on Number Lines** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets

3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. 3.NF.A.3.a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. 3.NF.A.3.b Recognize and generate simple equivalent fractions, e.g., 1/2=2/4, 4/6=2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. 3.NF.A.3.c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3=3/1; recognize that 6/1=6; locate 4/4 and 1 at the same point of a number line diagram. 3.NF.A.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the		same denominator by reasoning about their size. • Identify equivalent fractions. • Measure to the nearest fourth inch and represent measurement data to the nearest fourth inch on a line plot.	**Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. 3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. 3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4of the area of the shape.			

Unit Seven: How Many Miles? Addition, Subtraction, and the Number System 2 Timeline: 18 Sessions

Unit Description: In this unit students focus on understanding the operations of addition and subtraction, and adding and subtracting fluently. Students add multiples of 10 and 100 to, and subtract them from, 3-digit numbers. They use multiples of 100 as landmarks as they solve addition and subtraction problems with 3-digit numbers, including problems that involve liquid volume and mass.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	Solve problems using the four operations, and identify patterns in arithmetic. Solve addition problems with more than 2 addends. Use place-value understanding and properties of operations to perform multi-digit arithmetic. Use place value understanding and properties of operations to perform multi-digit arithmetic. Solve problems involving measurement and estimation of intervals of times, liquid volumes,	**Essential Questions** How can we solve multi-step word problems? How can we solve addition problems with more than 2 addends? How can we solve addition problems with 3- or 4-digit numbers? How do we know when to measure in volume, mass, or length? **Learning Targets** Students will: • Solve addition and subtraction problems involving masses or volumes.	**Montessori Materials** Stamp game Bead frame Teacher-made or purchased problems Measurement materials **Investigations Unit 7 – How Many Miles?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets

	masses of objects, and money.	• Estimate and measure liquid volume and mass using standard units. • Solve 3-digit addition problems using at least one strategy fluently. • Solve 3-digit subtraction problems fluently.	**Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Eight: Large Numbers and Multi-Step Problems: Multiplication and Division III Timeline: 16 Sessions
Unit Description: In this unit students focus on solving multiplication and division problems, learning the division facts, identifying arithmetic patterns, and solving multi-step problems. Students develop strategies to solve multiplication and division problems, including problems with remainders.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56÷8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8. 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations	Represent and solve problems using multiplication and division. Multiply and divide within 100. Solve problems using the four operations, and identify patterns in arithmetic. Solve problems involving measurement and estimation of intervals of times, liquid volumes, and masses of objects. Geometric measurement: Understand the concept of area and relate area to multiplication and addition.	**Essential Questions** Why is it helpful to internalize division facts? How do we know when to measure in volume, mass, or length? What is the relationship between perimeter and area? **Learning Targets** Students will: • Solve multiplication and division problems within 100. • Demonstrate an understanding of multiplication and division as involving equal groups.	**Montessori Materials** Multiplication board, tickets, and finger charts Division board, tickets, and finger charts Stamp games Bead frames Geometric cabinet **Investigations Unit 8 – Large Numbers and Multi-Step Problems** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks

8×?=48, 5=?÷3, 6×6=?. 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6×4=24 is known, then 4×6=24 is also known. (Commutative property of multiplication.) 3×5×2 can be found by 3×5=15, then 15×2=30, or by 5×2=10, then 3×10=30. (Associative property of multiplication.) Knowing that 8×5=40 and 8×2=16, one can find 8×7as 8×(5+2)=(8×5)+(8×2)=40+16=56. (Distributive property.) 3.OA.B.6 Understand division as an unknown-factor problem. For example, find 32÷8 by finding the number that makes 32 when multiplied by 8. 3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8×5=40, one knows 40÷5=8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. 3.OA.D.8	Use tables to identify and interpret arithmetic patterns.	• Demonstrate fluency with the division facts. • Solve multi-step problems involving more than one operation. • Solve multiplication and division problems involving masses or volumes. • Find the area of a rectangular array by breaking it apart (using the distributive property).	• Problem Solving Challenges • Exit Tickets **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. 3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. 3.MD.C.7.c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b+c is the sum of a×b and a×c.			

Use area models to represent the distributive property in mathematical reasoning.			

Sussex Montessori School

Mathematics Curriculum

4th Grade

***Curricu**lum Framework for Mathematics*

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations 3* Grade: 4

Unit One: Multiplication and Division I Timeline: 12 Sessions

Unit Description: In this unit students focus on using arrays and multiplicative comparison problems to understand multiplication, and gaining familiarity with factors and multiples. Students use arrays to model multiplication situations and to find factors and identify prime numbers to 100.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given	Determining multiples and factors Factors of 100 Using known factors to find related factors for a given number (e.g., if 4 × 25 = 100, then 8 × 25 = 200) Using representations to show that a factor of a number is also a factor of its multiples (e.g., if 25 is a factor of 100, then 25 is also a factor of 300) Representing multiplicative comparison problems with multiplication or division equations	**Essential Questions:** What is a factor? What is a multiple? What are prime numbers? How can we find factors using arrays? **Learning Targets:** Students will: • Use multiplication to solve multiplicative comparison problems. • Determine whether numbers up to 100 are prime or composite. • Find factors of numbers up	**Montessori Materials** Stamp game Checkerboard Pegboard **Investigations Unit 1 – Arrays, Factors, and Multiplicative Comparison** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative**

whole number in the range 1–100 is prime or composite. 4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		to 100 and recognize multiples of 1-digit numbers.	• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Two: Modeling with Data **Timeline: 11 Sessions**

Unit Description: In this unit students focus on using line plots to represent, describe, and compare measurement data; on modeling real-world problems with mathematics; and on constructing arguments based on data. Students measure and compare the heights of first and fourth graders. They collect measurement data of their choosing, and use line plots to represent and analyze the data.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... 4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 4.MD.B.4	Organizing ordered, numerical data to describe a data set Using a line plot to represent ordered, numerical data Representing two sets of data in order to compare them Considering how well a data representation communicates information to an audience Using a line plot to represent measurement data that includes fractions Describing the shape of a data set	**Essential Questions** How can we develop and revise a data question? How can we use a line plot to represent measurement data? How can we describe a data set? How can we represent a data set? How can we collect and accurately record data? **Learning Targets** Students will: • Use a line plot to organize, represent, and analyze measurement data about two groups in order to compare the two groups.	**Montessori Materials** Teacher-made materials to augment what is commercially available **Investigations Unit 2 – Generating and Representing Measurement Data** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	Describing what is typical about the data set as a whole Describing and interpreting data that compare two groups Comparing two sets of data using the shape of the data Developing arguments based on data Recording and keeping track of data Developing and revising a data question	• Use a line plot to organize, represent, and analyze measurement data about two groups in order to compare the two groups. • Design a data question that involves measurement to compare two groups. • Use a line plot to represent measurement data that includes fractions.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Three: Multiplication and Division 2 **Timeline: 18 Sessions**

Unit Description: In this unit students focus on solving multiplication problems with 2-digit numbers, understanding the meaning and structure of, and the relationship between, multiplication and division, and using that understanding to solve multiplication and division problems. Students use marked and unmarked arrays to represent multiplication strategies that involve breaking numbers apart.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using	Solving division problems by making groups of the divisor Using the relationship between multiplication and division to solve division problems Using known multiples of a number to find other multiples of that number Finding multiples of 2-digit numbers Understanding the effect of multiplying by a multiple of 10 Determining the effect on the product when one factor is doubled and one factor is halved	**Essential Questions:** How can we use the relationship between multiplication and division to solve division problems? How can we use pictures, diagrams, or models to represent multiplication problems? How can we use known multiples of a number to find other multiples of that number? How can we find multiples of 2-digit numbers? **Learning Targets** Students will: • Multiply a 2-digit number by 1-digit and small 2-digit numbers (e.g., 12, 15, 20), using	**Montessori Materials** Stamp Game Checkerboard Million Cube **Investigations Unit 3 – Multiple Towers and Cluster Problems** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		strategies that involve breaking the numbers apart. • Multiply a number by a multiple of 10. • Solve division problems (2-digit and small 3-digit numbers divided by 1-digit numbers), including some that result in a remainder.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Four: 2-D Geometry and Measurement **Timeline: 20 Sessions**

Unit Description: In this unit, students focus on linear measurement, sorting and classifying polygons, measuring angles, and finding symmetry and area.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the	Using U.S. standard units to measure lengths longer than the measuring tool Measuring accurately Using U.S. standard and metric units to accurately measure length Estimating lengths in common units (centimeter, inch, foot, yard, meter) Identifying measurement equivalents Converting measurements from larger units to smaller units Finding perimeter using standard units	**Essential Questions:** What is perimeter? What is area? How can we find the area of a rectangle? What are polygons? How can we make polygons? How can we find the area of polygons? What are the parts of a line? How can we distinguish angles? How do we use a protractor? How can create specific angles? What is the line of symmetry? **Learning Targets**	**Montessori Materials** Montessori protractor Box of Sticks **Investigations Unit 4 – Measuring and Classifying Shapes** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice,

relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... 4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 4.MD.A.3	Using a generalizable method to determine the perimeter of a rectangle Identifying right, acute, and obtuse angles Identifying and creating 90-degree angles Using known angles to build and find the measure of other angles Drawing angles of a specific measure Understanding the relationship between the degree measure of an angle and rotation in a circular arc Measuring angles using a protractor Finding the area of symmetrical shapes Dividing irregular polygons into two shapes that have	Students will: • Convert linear measurements from a larger unit to a smaller unit. • Draw and identify lines and angles, including parallel and perpendicular lines, and classify polygons by properties of their sides and angles. • Add or subtract angles to determine the size of angles. • Use a protractor to measure angles and sketch angles of specific sizes. • Identify lines of symmetry in polygons.	and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. 4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: 4.MD.C.5.a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. 4.MD.C.5.b An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 4.MD.C.6	equal area Using symmetry and congruence to identify equal areas Finding the area of polygons using square units Finding the area of polygons by decomposing shapes Finding the area of a rectangle Using a generalizable method to determine the area of a rectangle Determining an unknown dimension of a rectangle when one dimension and the area are known Defining polygons as closed figures with line segments as sides that come together at points called vertices		

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.A.3 Recognize a line of symmetry for a two-	Identifying geometric figures including: points, lines, rays, line segments, and parallel and perpendicular lines Identifying shapes with parallel or perpendicular sides Combining polygons to make new polygons Recognizing number of sides as a descriptor of various polygons Classifying polygons by attribute, including number and relative length of sides, size of angles, and absence or presence of parallel or perpendicular sides Determining lines of symmetry in a two-dimensional figure		

dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Making designs with mirror symmetry		

Unit Five: Addition, Subtraction, and Number System **Timeline: 19 Sessions**

Unit Description: In this unit students focus on understanding the meaning of addition and subtraction, understanding the base-10 number system with numbers to 1,000,000, and adding and subtracting multidigit numbers fluently, including with the U.S. standard algorithms.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. 4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the	Reading, writing, and sequencing numbers to 10,000 Understanding the structure of 10,000 and its equivalence to one thousand 10s, one hundred 100s, and ten 1,000s Writing multidigit numbers using expanded form Using place-value understanding to round numbers to any place Adding 3- and 4-digit numbers fluently Using clear and concise notation to record addition strategies	**Essential Questions:** How can we represent addition and subtraction on a number line? How can we find combinations of 3-digit numbers that add up to 1,000? How can we determine what subtraction strategy to use to solve a problem? **Learning Targets** Students will: • Read, write, and compare numbers up to 1,000,000 and round them to any place. • Fluently solve multidigit addition and subtraction problems using a variety of	**Montessori Materials** Million Cube Checkerboard Place Value cards Stamp game **Investigations Unit 5 – Large Numbers and Landmarks** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

digits in each place, using > , =, and < symbols to record the results of comparisons. 4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place. 4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm. 4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Understanding the steps and notation of the U.S. standard algorithm for addition Adding using the U.S. standard algorithm for addition Solving subtraction problems using different strategies Understanding the meaning of the steps and notation of the U.S. standard algorithm for subtraction Using story contexts and representations to support explanations about related subtraction expressions	strategies, including the U.S. standard algorithm. • Use addition and subtraction to solve word problems involving measurement.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Six: Fractions and Decimals Timeline: 24 Sessions

Unit Description: In this unit students focus on understanding the meaning of fractions and decimals; and comparing fractions and decimals including finding equivalents. Students use contexts and representations such as rectangles (an area model) and number lines (a linear model) to add, subtract, and multiply fractions.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. 4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole.	Finding fractional parts of a rectangle Interpreting the meaning of the numerator and the denominator of a fraction Representing fractions greater than 1 Reading and writing tenths and hundredths Representing tenths and hundredths as parts of an area Identifying equivalent fractions and explaining why they are equivalent Identifying relationships between unit fractions when one denominator is a multiple of the other	**Essential Questions:** How can we represent fractions greater than 1? How can we represent fractions on a number line? How can we identify equivalent fractions? What is the relationship between fractions and decimals? How can we add and subtract fractions and mixed numbers? How do we multiply fractions by a whole number? **Learning Targets** Students will: • Identify equivalent fractions	**Montessori Materials** Fraction material squares Fraction box Fraction cabinet Decimal box materials **Investigations Unit 6 – Fraction Cards and Decimal Grids** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative**

Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. 4.NF.B.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b. 4.NF.B.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. 4.NF.B.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 +1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. 4.NF.B.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. 4.NF.B.3.d	(e.g., halves and fourths, thirds and sixths) Comparing the same fractional parts of different-sized wholes Comparing and ordering fractions and justifying their order through reasoning about fraction equivalencies and relationships Representing fractions on a number line Comparing fractions to the landmarks 0, 1/2, 1, and 2 Comparing and ordering decimals and justifying their order through reasoning about representations and the meaning of the numbers	and explain why they are equivalent. • Compare fractions with like and unlike denominators. • Add and subtract fractions and mixed numbers with like denominators. • Multiply a fraction by a whole number. • Read, write, and compare decimals in tenths and hundredths. • Add tenths and hundredths. • Represent data on a line plot including fourths and eighths.	• Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. 4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. 4.NF.B.4. a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (¼), recording the conclusion by the equation 5/4 = 5 × (¼). 4.NF.B.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? 4.NF.C.5			

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. 4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. 4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. 4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement			

quantities using diagrams such as number line diagrams that feature a measurement scale. 4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.			

Unit Seven: Multiplication and Division 3 **Timeline: 18 Sessions**

Unit Description: In this unit students focus on the operations of multiplication and division, including problems involving converting measurements. Students refine their strategies for solving multiplication problems with two 2-digit numbers and with a 4-digit number and a 1-digit number, and they use the relationship between multiplication and division to develop and practice strategies for solving division problems with up to a 4-digit dividend and a 1-digit divisor.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.NBT.B.5 Multiply a whole number of up to four	Identifying factors of a number Solving division problems by breaking the problem into parts Representing a division problem with pictures or diagrams, including arrays, and pictures of groups Solving multiplication problems by breaking a problem into smaller parts and combining the sub-products Dividing a 4-digit number by a 1-digit number Breaking apart, reordering, rounding, or changing	**Essential Questions:** How can we use pictures or diagrams, including arrays and pictures of groups to represent a multiplication problem? How can we estimate answers to division problems? How can we use multiples of 10 to solve division problems? How can we use breaking apart, reordering, rounding, or changing numbers mentally to determine a reasonable estimate? **Learning Targets** Students will: • Multiply two 2-digit numbers and up to a 4-digit number by a 1-digit number.	**`Montessori Materials** Peg board Stamp game Division with hierarchical materials (Test tube division) **Investigations Unit 7: How many Packages and Groups?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets

digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for	numbers mentally to determine a reasonable estimate Using four operations to solve word problems involving measurement Using a story problem represented by a multiplication expression to keep track of parts of the problem Converting measurements in larger units to smaller units Making tables of equivalent measurements Using multiplication to convert measurements Using the four operations to solve word problems involving measurements Solving multi-step problems	• Solve division problems with up to 4-digit dividends and 1-digit divisors. • Solve measurement and conversion problems.	**Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... 4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Writing equations using a letter for the unknown quantity		

Unit Eight: Analyzing Patterns and Rules **Timeline: 10 Sessions**

Unit Description: In this students unit focus on solving multi-step problems, generating and analyzing patterns, and using symbolic notation to model situations. Two contexts (Penny Jars and Windows and Towers) are used that include both additive and multiplicative situations.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	Using tables to model situations Using symbolic notation to model situations Using letters in equations to represent unspecified quantities Generating number patterns and identifying features of the pattern Articulating a rule that describes a number pattern Comparing situations represented by arithmetic sequences Analyzing arithmetic patterns to solve problems	**Essential Questions:** What are the features of a pattern? How can we develop a rule that describes a pattern? **Learning Targets** Students will: • Model the mathematics of a situation with tables and with mathematical notation, including using letters to represent unspecified quantities. • Solve multi-step word problems using the four operations. • Generate a number pattern that follows a given rule and analyze features of the pattern	**Montessori Materials** Teacher-made materials to augment what is commercially available **Investigations Unit: Penny Jars and Towers** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice,

	Representing and solving multi-step problems involving more than one operation	in order to solve problems.	and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Sussex Montessori School

Mathematics Curriculum

5th Grade

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations 3* Grade: 5

Unit One: Multiplication and Division 1 Timeline: 19 Sessions

Unit Description: In this unit students focus on the operations of multiplication and division. Students refine their strategies for solving multiplication problems with 2-digit numbers, and use the relationship between multiplication and division to develop and practice strategies for solving division problems. They use order of operations to solve computation problems

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 +921, without having to calculate the indicated sum or product. 5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is Multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	Using arrays to represent multiplication Determining whether one number is a factor or multiple of another Identifying prime, square, even, and odd numbers Using properties (even, odd, prime, square) and relationships (factor, multiple) of numbers to solve problems Identifying and learning multiplication facts not yet fluently known Solving 2-digit by 2-digit multiplication problems	**Essential Questions:** How can knowing the properties of numbers help us solve problems? What strategies can we use to solve 2-digit by 2-digit multiplication problems? How do we multiply by 10 and what happens to our product when we do? What is the order of operations? Why is it important to know? What happens if the order is not followed? How can we use a picture or diagram to represent a division problem?	**Montessori Materials** Stamp Game Checker Board Materials **Investigations Unit 1 – Puzzles, Clusters, and Towers** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Describing and comparing strategies used to solve multiplication problems Creating a story problem represented by a multiplication expression Multiplying fluently by multiples of 10 Comparing multiplication problems to determine which product is greater Estimating the product of two numbers Breaking apart multiplication problems efficiently Using clear and concise notation Solving problems using the order of operations Writing and interpreting expressions involving grouping symbols	How can I use the multiplication strategies I know to estimate a product of two 2-digit numbers? How can knowing multiples of ten help me solve division problems? **<u>Learning Targets</u>** Students will: • Solve 2-digit by 2-digit multiplication problems efficiently. • Solve division problems with 1-digit and 2-digit divisors. • Use the order of operations to solve computation problems.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	Representing a division problem with a picture or diagram Creating a story problem represented by a division expression Describing and comparing strategies used to solve division problems Using knowledge of multiples of 10 to solve division problems Using and interpreting notation that represents division, and relating division and multiplication notations (e.g., 170 ÷ 15 = _____, and _____ × 15 = 170) Solving division problems with 2-digit divisors Making sense of remainders in terms of problem contexts		

	Solving a division problem by breaking the dividend into parts Comparing division problems to determine which quotient is greater		

Unit Two: 3-D Geometry and Measurement **Timeline: 12 Sessions**

Unit Description: In this unit students focus on the structure and volume of three-dimensional (3-D) shapes, specifically on rectangular prisms and solids composed of rectangular prisms. Students build models and patterns for boxes that hold quantities of cubes and calculate the volume of these boxes, using a cube as a unit of measure.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. 5.MD.C.3a A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. 5.MD.C.3b A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	Decomposing 3-D shapes and then recombining them to make a given solid Determining the number of cubes that will fit into the box made by a given pattern Developing a strategy for determining the volume of rectangular prisms Designing patterns for boxes with given dimensions Considering how the dimensions of a box change when the volume is changed (doubled or halved)	**Essential Questions:** What are the important attributes of 3-D shapes and how can we use them to create new 3-D shapes? How do we find the volume of a rectangular prism? How does volume change when we join rectangular prisms? How can we measure a large space – what unit of measure should we use? **Learning Targets** Students will: • Find the volume of rectangular prisms, including the use of volume formulas.	**Montessori Materials** Volume materials Geometric solids **Investigations Unit 2 – Prisms and Solids** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice,

5.MD.C.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. 5.MD.C.5b Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. 5.MD.C.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	Organizing rectangular packages to fit in rectangular boxes Using formulas to find the volume of rectangular prisms Finding the volume of a solid composed of two rectangular prisms Designing a box that can be completely filled with several differently-shaped rectangular packages Determining the volume, in cubic centimeters, of a small rectangular prism Constructing units of volume—cubic centimeter, cubic inch, cubic foot, cubic yard (optional), cubic meter Choosing an appropriate unit of volume to measure a large space	• Find the volume of a solid composed of two rectangular prisms. • Use standard units to measure volume.	and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	Describing and defending measurement methods Finding the volume of a large space using cubic meters		

Unit Three: Rational Numbers: Addition and Subtraction 1 Timeline: 19 Sessions

Unit Description: In this unit students focus on deepening and extending students' understanding of fractions and equivalent fractions and representing fractions using an area model (rectangles), a rotation model (a clock), and a linear model (number lines). They use these understandings to add and subtract fractions and mixed numbers.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 +921, without having to calculate the indicated sum or product. 5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/ d = (ad + bc)/bd.) 5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators,	Finding fractional parts of a whole or of a group Identifying fraction equivalents through reasoning about representations and relationships Representing fractions on different-sized rectangles Ordering fractions and justifying their order through reasoning about fraction equivalents and relationships Comparing fractions to the landmarks 0, 1/2, and 1 Finding and comparing fractional parts of a whole or a group	**Essential Questions:** How can we represent fractions of different-sized rectangles? How can we find fractional units for a whole? How can we show fractional parts on a number line? What are common denominators? How can we add and subtract fractions with unlike denominators? How can we use our fraction knowledge to estimate sums and differences of fractions? **Learning Targets** Students will:	**Montessori Materials** Fractions cabinet Fraction circles box Decimal box materials **Investigations Unit 3 – Rectangles, Clocks, and Tracks** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < 1/2. 5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	Comparing fractional parts of different-sized wholes Using equivalent fractions to solve problems Comparing fractions on a number line Finding fractional parts of the rotation around a circle Adding fractions by using a rotation model Representing fractions on a number line Finding combinations of fractions with sums between 0 and 2 Adding and subtracting fractions by using a number line Adding and subtracting fractions through reasoning about	• Add fractions with unlike denominators. • Subtract fractions with unlike denominators. • Represent data including fractions on a line plot and solve addition and subtraction problems about data.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	fraction equivalents and relationships Using common denominators to add and subtract fractions Adding and subtracting fractions and mixed numbers Finding general rules for adding and subtracting fractions Making a line plot to display a data set of measurements involving fractions Using addition and subtraction of fractions to solve problems involving information given in line plots Using benchmark fractions to estimate sums and differences		

Unit Four: Multiplication and Division 2 **Timeline: 17 Sessions**

Unit Description: In this unit students focus on the operations of multiplication and division. Students refine their strategies for solving multiplication problems fluently, including using the U.S. standard algorithm. Students continue using the relationship between multiplication and division to efficiently solve division problems with 4-digit dividends and 2-digit divisors.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. 5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is Multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.B.5	Solving multiplication problems fluently Describing and comparing strategies used to solve multidigit multiplication problems Estimating answers to multiplication problems Understanding the U.S. standard algorithm for multiplication Multiplying using the U.S. standard algorithm for multiplication Using clear and concise notation Solving multi-step word problems	**Essential Questions:** How can we find the product of two multi-digit numbers? How can we use our known multiplication strategies to estimate the product in multiplication problems? How can we deduce what operations(s) are necessary in a multi-step word problem? How can we use diagrams or pictures to represent division problems? **Learning Targets** Students will: • Fluently solve multi-digit multiplication problems using a variety of strategies	**Montessori Materials** Division with hierarchical materials (Test Tube Division) Place value cards Checkerboard materials **Investigations Unit 4 –How Many People and Teams?** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

Fluently multiply multi-digit whole numbers using the standard algorithm. 5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Using all four operations to solve problems Solving division problems with a 2-digit divisor efficiently Representing a division problem with a picture or diagram Creating a story context for a division expression Describing and comparing strategies used to solve division problems	including the standard algorithm. • Solve division problems with up to 4-digit dividends and 2-digit divisors efficiently. .	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Five: Analyzing Patterns and Rules **Timeline: 14 Sessions**

Unit Description: In this unit students focus on using coordinate graphs, ordered pairs, tables, and symbolic notation to model real world and mathematical situations. Students analyze arithmetic patterns in tables and the shapes of graphs to describe and compare these situations. Students work both with situations that follow patterns, allowing predictions of future values (e.g., how the area of a square varies as the length of a side increases) and situations based on data (e.g., temperature over time).

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 +921, without having to calculate the indicated sum or product. 5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	Understanding the meaning of points on a coordinate graph Plotting points on a coordinate grid Generating ordered pairs and recording them in a table Identifying points in a graph with corresponding ordered pairs in a table Identifying the x- and y-coordinates of a point on a coordinate grid Comparing situations by describing differences in their graphs	**Essential Questions:** What are the x- and y-coordinates of a point on a coordinate grid? What are the points on a coordinate graph? How can we record generated ordered pairs on a table? How can we then identify these pairs as points on a graph? How can we think through and articulate a rule that describes a numerical pattern? How can we use tables or graphs or compare two situations of sets of data? **Learning Targets** Students will:	**Montessori Materials** Teacher-made materials to extend commercially available products **Investigations Unit 5 – Temperature, Height, and Growth** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). 5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Articulating a rule that describes a numerical pattern Describing the relationship between two varying quantities related by a rule (e.g., age and height) Using a numerical relationship generated by a given rule to solve problems in that context Writing an equation that describes the relationship between two varying quantities Analyzing and comparing numerical patterns generated by different rules Interpreting values of the points on a coordinate grid in the context of the situation	• Use tables to record ordered pairs and construct coordinate graphs to represent the relationship between x-coordinates and y-coordinates. • Determine what values are represented by points on a coordinate grid. • Represent real world and mathematical problems by graphing points in the coordinate plane and interpret the graph in the context of the situation. • Use tables and graphs to compare two situations governed by rules that generate numerical patterns.	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	Interpreting a numerical pattern in a table in terms of the situation it represents Interpreting the shape of a graph in terms of the situation the graph represents Using symbolic letter notation to represent the value of one varying quantity in terms of another		

Unit Six: Rational Numbers: Addition and Subtraction 2 Timeline: 17 Sessions

Unit Description: In this unit students focus on deepening and extending students' understanding of decimals and the base-10 number system. Students represent decimals on grids and number lines. They use their understanding of decimals to compare, add, and subtract decimals.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.A.3 Read, write, and compare decimals to thousandths. 5.NBT. A.3a Read and write decimals to thousandths using base-ten numerals, number names, and Expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). 5.NBT. A.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 5.NBT.A.4	Identifying everyday uses of fractions and decimals Representing decimals as parts of an area Reading and writing tenths, hundredths, and thousandths Identifying decimal and fraction equivalents Representing decimals on a number line Rounding decimals to the nearest one, tenth, or hundredth Writing decimals in expanded form Comparing decimals to thousandths	**Essential Questions:** How do we use decimals and fractions everyday? How are decimals and fractions related? How can we show decimals on a number line? How do we write decimals in expanded form? How do we add and subtract decimals with fidelity to place value? **Learning Targets** Students will: • Write, compare, and round decimals to thousandths. • Add and subtract decimals.	**Montessori Materials** Fraction cabinet Decimal board materials Million Cube **Investigations Unit 6 – Between 0 and 1** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

Use place value understanding to round decimals to any place. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Ordering decimals and justifying their order through reasoning about decimal representations, equivalents, and relationships Comparing decimals to the landmarks 0, 1/2, and 1 Estimating sums and differences of decimals Using representations to add and subtract tenths, hundredths, and thousandths Adding and subtracting decimals through reasoning about place value, equivalents, and representations Comparing sums and differences of decimal numbers to determine which is greater		introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Unit Seven: Rational Numbers: Multiplication and Davison Timeline: 26 Sessions

Unit Description: In this unit students focus on multiplying and dividing rational numbers, which includes extending students' understanding of the meaning of those operations and of place value. Students use contexts and representations (fraction bars, arrays, and grids) to solve problems involving multiplication and division of fractions and decimals. Students also apply their understandings of multiplication and division to solve measurement conversion problems.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is Multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to	Using a representation to multiply a fraction and a whole number Extending understanding of the operation of multiplication to include fractions Writing multiplication equations for multiplying a fraction and a whole number Writing and interpreting multiplication equations involving a fraction and a whole number Using a representation and reasoning to multiply a whole number by a fraction or mixed number	**Essential Questions:** How do we multiply a fraction by a whole number? What can we estimate about the product of two factors based on what we know about each factor? How can we represent a fractional part of a quantity? How can we use arrays to represent the multiplication of fractions? What representations can we use to solve problems requiring the division of a whole number by a fraction or a fraction by a whole number?	**Montessori Materials** Peg board Stamp game Checkerboard Place value cards Division with hierarchical materials (Test tube division) **Investigations Unit 7 – Races, Arrays, and Grids** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks

a written method and explain the reasoning used. 5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? 5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. 5.NF.B.4a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a	Comparing the size of the product to the size of one factor based on the size of the other factor Multiplying a fraction or mixed number and a whole number Multiplying a fraction by a fraction Representing a fractional part of a fractional quantity Using arrays to represent multiplication of fractions Understanding the relationship between the denominators and numerators of the factors and the denominator and numerator of the product Using representations to solve problems involving dividing a whole number by a unit fraction and	What is the relationship between fractions and division? How can we find equivalencies between fractions and decimals? **Learning Targets** Students will: • Multiply fractions, mixed numbers, and whole numbers. • Compare the size of the factors and the size of the product and explain their relationship. • Divide a unit fraction by a whole number and a whole number by a unit fraction. • Recognize and use place value relationships to explain patterns when multiplying and	• Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

story context for this equation. Do the same with (2/3) × (4/5) =8/15. (In general, (a/b) × (c/d) = ac/bd.) 5.NF.B.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5.NF.B.5 Interpret multiplication as scaling (resizing), by: 5.NF.B.5a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. 5.NF.B.5b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and	dividing a unit fraction by a whole number Using reasoning, and the relationship between division and multiplication, to solve division problems involving whole numbers and unit fractions Solving problems that involve dividing a whole number by a whole number resulting in a fraction or a mixed number Interpreting fractions as division Identifying decimal and fraction equivalents Interpreting the meaning of digits in a decimal number Using representations and reasoning to multiply whole numbers by powers	dividing by powers of 10, including placement of the decimal point. • Multiply and divide decimals to hundredths. • Solve division problems with two whole numbers resulting in a fraction or a mixed number. • Solve measurement conversion problems including multi-step word problems.	

relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1. 5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) 5.NF.B.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. 5.NF.B.7b	of 10 (including 1, 0.1, and 0.01) Explaining the patterns in the placement of the decimal point when a decimal is multiplied by a power of 10 Estimating products of decimals Multiplying decimals to hundredths through reasoning about place value and multiplication Writing a strategy for multiplying decimals Using representations and reasoning to divide whole numbers by powers of 10 (including 1, 0.1, and 0.01) Explaining the patterns in the placement of the decimal point when a decimal is divided by a power of 10		

Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. 5.NF.B.7c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? 5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	Estimating quotients of decimals Dividing decimals to hundredths through reasoning about place value and division Converting U.S. standard and metric measurements Solving multi-step word problems involving measurement		

Unit Eight: 2-D Geometry and Measurement **Timeline: 10 Sessions**

Unit Description: In this unit students focus on classifying triangles and quadrilaterals based on their properties and on using patterns to describe how the perimeters and areas of rectangles change when the dimensions of the rectangle change. Students examine how categories of polygons are related and how a figure can belong to more than one category.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Instructional Materials/ Assessments
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. 5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/ d = (ad + bc)/bd.) 5.NF.B.4b	Comparing the perimeters and areas of rectangles when the dimensions are multiplied by given amounts Using numerical and/or geometric patterns to describe how the perimeters and areas of rectangles change when the dimensions change Using representations to explain how perimeters and areas of rectangles change Creating different rectangles with the same area but different perimeters Creating different rectangles with the same	**Essential questions** Can a polygon belong to more than one category? What happens to the perimeter and area when dimensions of a rectangle are multiplied by a given amount? Can rectangles with the same area have different perimeters? **Learning Targets** Students will: • Classify polygons by their attributes and know that some quadrilaterals can be classified in more than one way. • Identify and explain numerical patterns when comparing	**Montessori Materials** Geometry cabinet Box of Sticks Teacher-made extension materials **Investigations Unit 8 – Properties of Polygons** **Assessments:** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. 5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.	perimeter but different areas Describing the shapes of rectangles that have the same area or the same perimeter Identifying attributes of polygons Classifying triangles by the sizes of their angles and the lengths of their sides Using attributes to classify quadrilaterals Identifying the properties of categories of quadrilaterals Recognizing that a polygon can belong to more than one category	perimeters and areas of related rectangles.	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Sussex Montessori School

Mathematics Curriculum

6th Grade

Curriculum Framework for Mathematics

School: Sussex Montessori School Curricular Resources: Montessori Materials and Lessons / *Investigations 3* Grade: 6

Unit One: Factors and Multiples Timeline: 22 Lessons

Unit Description: Through this unit, students will understand relationships among factors, multiples, divisors, and products. They will understand why two expressions are equivalent.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Assessments
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. ***6.EE.A.2.B*** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **6.EE.A.2.C**	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.	**Essential Questions:** How can you find all the factors (or divisors) of a number? What information about a number can you find by looking at its factors? If you know one factor of a number, how can you find another factor of the number? How do you know when you have found all of the factors of a number? How can you find the least common multiple of two or more numbers? How can you decide when finding common factors is useful in solving a problem?	**Montessori Materials:** Pegboard Teacher-made extension materials **Connected Mathematics Project: Prime Time** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. **6.EE.A.4** The idea that equivalence means two expressions give the same outputs is first highlighted in Investigation 4 of *Prime Time*. Featuring applications of the distributive property, the most common principle for generating equivalent expressions, Problem 4.3 of that investigation states the property in generality with letter names for variables *a(b + c) = a(b) + a(c)*. **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to	Exponential notation Sum, product, factor Distributive property Writing numeric and symbolic expressions and sentences to represent the operations required to solve problems The mathematical meaning of *term* Order of operations Reasoning quantitatively about inequalities	How can you find the greatest common factor of two numbers? How can you find the prime factorization of a number? How many unique prime factorizations of a number are there? How can the prime factorization of a number be used to find the LCM and GCF of two or more numbers? What characteristics of numbers, such as factors and multiples did you use to answer the questions? What special numbers, such as prime numbers, composite numbers, and square numbers, did you use? How do you decide whether a number is even or odd?	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

determine whether a given number in a specified set makes an equation or inequality true.		How is the Distributive Property used to create equivalent expressions? How is finding the area of a rectangle related to the Distributive Property? How do you decide the order when you work on number sentences with more than one operation? How do you decide what operations are needed in a given situation? **<u>Learning Targets</u>** Students will: • If a number *N* can be written as a product of two whole numbers, $N = a \times b$, then *a* and *b* are factors of *N*. Multiples of a can be found using the expression $a \times$ (some whole number), such as $2a$, $3a$, $4a$, etc. Some numbers can be	

		expressed in exponential notation, such as a^2, a^3, a4, etc. • When all factors of a number are broken down into prime numbers, you have a unique prime factorization. Finding the prime factorization of two numbers can be useful in finding the least common multiple and greatest common factor of the numbers. • When calculating the value of an expression, the operations have to be performed in a conventional order, the order of operations. • Sometimes a numerical expression can be written in different ways but the expressions are equivalent because the value is the same.	

		Properties of operations, including the Distributive Property, are essential tools for writing equivalent expressions.	

Unit Two: Estimating and Ordering Rational Numbers Timeline: 25 Lessons

Unit Description: In this unit, students **u**nderstand fractions and decimals as numbers that can be located on the number line, compared, counted, partitioned, and
Decomposed. They understand ratios as comparisons. They understand equivalence of fractions and ratios, and use equivalence to solve problems.

Standards Alignment	**Unit Concept/Big Ideas**	**Essential Questions/ Learning Targets**	**Assessments**
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.	**Essential Questions:** How can we order and compare rational numbers on a number line? How can we find decimal equivalents? How can we know which values from a specified set, if any, make the equation or inequality true? **Learning Targets:** Students will: • Rational numbers can be written in fraction form or decimal form and can be represented as points or distances on a number line. The absolute value of a number is its distance from 0 on the number	**Montessori Materials:** Fraction cabinet Fraction box Decimal checkerboard Teacher-made extension materials **Connected Mathematics Project: Comparing Bits and Pieces** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges

specified set makes an equation or inequality true. **6.EE.B.8** Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **6.EE.B.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables and graphs. **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **6.RP.A.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a* : *b* with *b* ≠ 0, and use	Order of operations Exponential notation Equivalence Inequity Ordering and comparing rational numbers on a number line Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	line. A number-line representation is useful for ordering and comparing rational numbers. • Benchmarks are useful for estimating values of fractions and decimals. • Ratios are comparisons between two numbers. You can scale ratios to make equivalent ratios. Percents are ratios where 100 parts represent the whole. • A rate is a particular kind of ratio, where the amounts compared are in different units. A unit rate is a ratio in which one of the quantities being compared has a value of 1. • Fractions and decimals can be renamed or repartitioned to find equivalent fractions or	• Exit Tickets **<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

rate language in the context of a ratio relationship. **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.7** Understand ordering and absolute value of rational numbers		decimals. Equivalence is useful for moving between fraction and decimal representations and for solving problems. • Equivalent ratios represent the same relationship between quantities.	

Unit Three: Understanding Fraction Operations Timeline: 20 Lessons

Unit Description: In this unit, students will understand that estimation can be used as a tool in a variety of situations including checking answers and making decisions, and develop strategies for estimating results of arithmetic operations. They will revisit and continue to develop meanings for the four arithmetic operations and skill at using algorithms for each. They will use variables to represent unknown values and equations to represent relationships.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Assessments
6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **6.EE.A.3**	Use benchmarks and other strategies to estimate results of operations with fractions. Use estimates to check the reasonableness of exact computations Give various reasons to estimate and identify when a situation calls for an overestimate or an underestimate. Use estimates and exact solutions to make decisions. Determine when addition, subtraction, multiplication, or division is the appropriate	**Essential Questions:** What are some strategies for estimating the sums of fractions? What strategies can you use to multiply all combinations of factors including whole numbers, fractions, and mixed numbers? What are some strategies for adding and subtracting fractions? How do you know if your estimate is an underestimate or overestimate? What information does an underestimate or overestimate tell you?	**Montessori Materials:** Pegboard Teacher-made extension materials **Connected Mathematics Project: Let's Be Rational** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including

Apply the properties of operations to generate equivalent expressions. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	operation to solve a problem. Develop ways to model sums, differences, products, and quotients with areas, fraction strips, and number lines. Use knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying, and dividing fractions. Write fact families with fractions to show the inverse relationship between addition and subtraction, and between multiplication and division. Compare and contrast dividing a whole number by a fraction to dividing a fraction by a whole number. Recognize that when you	What are some strategies for adding and subtracting mixed numbers? How does an area model relate to multiplying fractions? How can you use number properties and equivalent fractions to multiply rational numbers? What does it mean to divide a fraction by a fraction? What strategies help you divide a fraction by a fraction? What does it mean to divide a whole number or mixed number by a fraction? What strategies help you divide a whole number or mixed number by a fraction? What does it mean to divide a fraction by a whole number? What strategies help you divide a fraction by a whole number?	introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	multiply or divide a fraction, your answer might be less than or more than the numbers you started with. Solve real-world problems using arithmetic operations on fractions Represent unknown real-world and abstract values with variables. Write equations (or number sentences) to represent relationships among real-world and abstract values. Use fact families to solve for unknown values	What is an efficient algorithm for division problems involving fractions and mixed numbers? How do fact families help you solve equations such as 4/5 − N = 3/8 ? How do fact families help you solve equations such as 2/9 ÷ N = 2 /3? How do you know when a particular operation is called for to solve a problem? How do you represent the problem with a number sentence? **<u>Learning Targets:</u>** Students will: • Understand that estimation as a tool for a variety of situations and develop strategies for estimating results of arithmetic operations.	

		• Understand that estimation as a tool for a variety of situations and develop strategies for estimating results of arithmetic operations. • Revisit and develop meanings for the four arithmetic operations and skill at using algorithms for each. • Understand that variables can represent unknown values and equations to represent relationships. • Use variables to represent unknown values and equations to represent relationships.	

Unit Four: Two-Dimensional Measurement	**Timeline: 23 Lessons**

Unit Description: In this unit, students will understand that perimeter is a measure of linear units needed to surround a two-dimensional shape and that area is a measure of square units needed to cover a two-dimensional shape. They will understand that the linear measurements of the base, height, and slanted height of parallelograms and triangles are essential to finding the area and perimeter of these shapes. Additionally, they will understand that the surface area of a three-dimensional shape is the sum of the areas of each two-dimensional surface of the shape and that the volume of a rectangular prism is a measure in cubic units of the capacity of the prism.

Standards Alignment	**Unit Concept/Big Ideas**	**Essential Questions/ Learning Targets**	**Assessments**
6.EE.A.2.A Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* **6.EE.A.2.C** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. **6.EE.A.4**	Deepen the understanding of area and perimeter of rectangular and nonrectangular shapes. Relate area to covering a figure. Relate perimeter to surrounding a figure. Analyze what it means to measure area and perimeter. Develop and use formulas for calculating area and perimeter. Develop techniques for estimating the area and perimeter of an irregular figure.	**Essential Questions:** What are the formulas for finding the area and perimeter of a rectangle? Explain why they work. For a fixed area, what are the shape and perimeter of the rectangles with the greatest and least perimeters? For a fixed perimeter, what are the shape and area of the rectangles the greatest and least area? What is a formula for finding the area of a triangle? Does it make any difference which side is used as the base when finding the area of a triangle?	**Montessori Materials:** Box of Sticks Geometric solids Geometry cabinet Teacher-made extension materials **Connected Mathematics Project: Prime Time** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets

The idea that equivalence means two expressions give the same outputs is first highlighted in Investigation 4 of *Prime Time*. Featuring applications of the distributive property, the most common principle for generating equivalent expressions, Problem 4.3 of that investigation states the property in generality with letter names for variables *a(b + c) = a(b) + a(c)*. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **6.G.A.1**	Explore relationships between perimeter and area, including that one can vary considerably while the other stays fixed. Visually represent relationships between perimeter and area on a graph. Solve problems involving area and perimeter of rectangles. Analyze how the area of a triangle and the area of a parallelogram are related to each other and to the area of a rectangle. Recognize that a triangle can be thought of as half of a rectangle whose sides are equal to the base and height of the triangle. Recognize that a parallelogram can be decomposed into two	What can you say is true and what can you say is not true about triangles that have the same base and height? What conditions for a triangle produce triangles that have the same area? Do they have the same shape? Explain. What is a strategy for finding the area of a parallelogram? Explain why the strategy works. What can you say about two parallelograms that have the same base and height? Under what conditions will two or more parallelograms have the same area? Do these parallelograms have the same shape? Explain. How can you find the area of a polygon drawn on a coordinate graph? On grid paper? What is a strategy for finding the surface area of a rectangular prism? Explain why the strategy works.	**<u>Summative</u>** • Montessori Three-Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets	triangles. Thus the area of a parallelogram is twice the area of a triangle with the same base and height as the parallelogram. Know that the choice of base of a triangle (or parallelogram) is arbitrary but that the choice of the base determines the height. Recognize that there are many triangles (or parallelograms) that can be drawn with the same base and height. Develop formulas and strategies, stated in words or symbols, for finding the area and perimeter of triangles and parallelograms. Find the side lengths and area of polygons on a coordinate grid.	What is a strategy for finding the volume of a rectangular prism? Explain why the strategy works. What is a strategy for finding the surface area of three-dimensional object? Explain why the strategy works. **<u>Learning Targets:</u>** Students will: • Understand area and perimeter as a measure. • Perimeter is a measure of linear units needed to surround a two-dimensional shape and that area is a measure of square units needed to cover a two-dimensional shape. • A fixed number of area units can be enclosed by many different perimeters, and a fixed number of perimeter	

made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Solve problems involving area and perimeter of parallelograms and triangles. Solve problems involving area and perimeter of polygons by composing into rectangles or decomposing into triangles. Extend the understanding of the volume of rectangular prisms. Relate volume to filling a three-dimensional figure. Extend understanding of the strategies for finding the volume of rectangular prisms to accommodate fractional side lengths. Relate finding area of two-dimensional shapes to finding the surface area of three-dimensional objects.	units can enclose many different areas. • Formulas for the area and perimeter of a rectangle can help you solve problems by reasoning about the relationship between values. . • Understand area and perimeter of parallelograms and triangles. • Linear measurements of the base, height, and slanted height of parallelograms and triangles are essential to finding the area and perimeter of these shapes. • The area of a triangle and the area of a parallelogram are related to each other and to the area of a rectangle.	

	Develop strategies for finding the surface area of three-dimensional objects made from rectangles and triangles. Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.	• There are many triangles (and parallelograms) that can be drawn with the same base and height. • Polygons and irregular figures can be decomposed into triangles and rectangles to find the area of the figures. • Understand the surface area and volume of a three-dimensional shape. • The volume of a prism is a measure in cubic units of the capacity of the prism and can be thought of as multiplying a base layer of unit cubes by the number of layers needed to fill the prism.	

		• Surface areas of three-dimensional solids can be found by adding the areas of the faces.	

Unit Five: Computing with Decimals and Percents Timeline: 23 Lessons

Unit Description: In this unit, students will understand that estimation can be used as a tool in a variety of situations, including as a way to check answers and make decisions. They will revisit and continue to develop meanings for the four arithmetic operations on rational numbers, and practice using algorithms to operate on decimals. Students will use variables to represent unknown values and number sentences to represent relationships between values and develop understanding of percents through various contexts, such as sales tax, tips, discounts, and percent increases

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Assessments
6.EE.A.2.A Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **6.EE.A.2.A** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.*	Use estimates to solve problems and check answers. Recognize when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem. Use place value to develop understanding of algorithms and to relate operations with decimals to the same operations with fractions. Extend understanding of multiplication and division of multidigit whole numbers.	**Essential Questions:** What signals in a real- world problem tell you which operation to use? When you work with decimal computations, what strategies can you use to estimate the results? How can you express a unit rate as a decimal and use it to solve problems? How do you subtract one decimal number from another? Do fact families apply to operations with decimal numbers? How do you find the product of any two decimal numbers?	**Montessori Materials:** Decimal checkerboard Fraction cabinet Fraction circles box **Connected Mathematics Project: Decimal Ops** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship	Develop standard algorithms for multiplying and dividing decimals with the aid of, at most, paper and pencil Find a repeating or terminating decimal equivalent to a given fraction Solve problems using arithmetic operations on decimals, including finding unit rates. Write number sentences to represent relationships between both real-world and abstract values. Use fact families to write and solve equivalent number sentences. Use multiplication sentences to check division sentences. Develop models for percent problems.	What algorithm can be used to find any decimal product? How can a decimal division problem be written in equivalent fraction and whole number form? How can you carry out a decimal division using a method similar to long division of whole numbers? How can you complete a long division problem that doesn't give a whole number quotient? That is, how do you express remainders in decimal form? How do you find the tax and the total cost of an item from a given selling price and tax rate? How do you find the base price from a given tax rate and amount? How do you find the tip and the total cost of a restaurant meal from a given meal price and tip rate? How do you find the meal	Period Lesson including introduction, practice, and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.A.Cb** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **6.RP.A.3C** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times	Write and solve number sentences involving percents. Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.	price from a given tip percent and amount? How do you find the discount and the total cost of an item from a given selling price and discount rate? How do you find the base price from a given discount rate and amount? How can you express a change in a given amount as a percent change? How do you decide which operations to perform when a problem involves decimals and percents? **<u>Learning Targets:</u>** Students will: • Understand that estimation can be used as a tool in a variety of situations to solve problems. • Estimation is an important part of reasoning quantitatively. It helps	

the quantity); solve problems involving finding the whole, given a part and the percent. **6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm.		you make sense of a situation, allows you to recognize errors, and complements other problem solving skills. • Use variables to represent unknown values and number sentences to represent relationships between values. • Writing number sentences to represent relationships between both real- world and abstract values contributes to an initial understanding of algebra. • Fact families can be used to write and solve equivalent number sentences. • Use variables to represent unknown values and number sentences to represent	

		relationships between values. • Writing number sentences to represent relationships between both real- world and abstract values contributes to an initial understanding of algebra. • Fact families can be used to write and solve equivalent number sentences. • Develop understanding of percents through various contexts. • Using models for percent helps you to develop the meaning of percent and to solve problems involving sales tax, tips, discounts, and percent increases.	

Unit Six: Introducing Algebra Timeline: 23 Lessons
Unit Description: In this unit, students will develop understanding of variables and how they are related and of expressions and equations.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Assessments
6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distance between points with the same first coordinate or the same second coordinate. **6.EE.A.2.A** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* ***6.EE.A.2.B*** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a*	Explore problem situations that involve variables and relationships. Identify the dependent and independent variables and describe how they are related in a situation. Interpret the "stories" told by patterns in tables and coordinate graphs of numeric (*x*, *y*) data. Represent the pattern of change that relates two variables in words, data tables, graphs, and equations. Investigate situations that change over time. Examine increasing and decreasing patterns of change.	**Essential Questions:** How can you construct a graph from a table of data that depicts change over time? How is the pattern of change represented in the graph? What are the advantages and disadvantages of tables and graphs in representing and describing the patterns of change in a variable over time? Which representation of data – table, graph, or written notes- seems to better show patterns of change in distance over time, and why? How do you calculate average speed for a trip? How do a table and graph of (time, distance) data show speed? How do you analyze and compare the relationship	**Montessori Materials:** Teacher-made extension materials **Connected Mathematics Project: Variables and Patterns** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice, and assessment of the %

product of two factors; view (8 + 7) as both a single entity and a sum of two terms. **6.EE.A.2.C** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$, and $px = q$ for cases in which p and q and x are all nonnegative rational numbers. **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms	Compare linear and nonlinear patterns of change by using tables or graphs. Use tables, graphs, and equations to find the value of a variable given the value of the associated variable. Explore relationships that require graphing in all four quadrants. Describe advantages and disadvantages of using words, tables, graphs, and equations to represent patterns of change relating two variables and make connections across those representations Write an equation to express the relationship between two variables in one and two operations: $y=mx$, $y=b+x$, and $y=b+mx$	between variables given in different representations? How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data? How are the variables*, tour income* and *tour profit,* related to each other? How do you plot data points with one or both coordinates negative? When the relationship between dependent and independent variables is displayed in a graph, what can you learn about the relationship from a rising graph, a level graph, and a falling graph? In what kinds of situations will the equation between dependent and independent	of concept mastery. • Problem-based interactive Learning activities • Performance assessment

of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **6.RP.A.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a* : *b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.A.3A** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **6.RP.A.3B** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be*	Calculate average speed and show how it is reflected in a table or graph and vice versa. Recognize and express direct proportionality relationships with a unit rate ($y=mx$) and represent these relationships in rate tables and graphs. Solve problems that involve variables. Develop understanding of expressions and equations. Use properties of operations, including the Distributive Property and the Order of Operations, to write equivalent expressions for the dependent variable in terms of the independent variable. Use tables, graphs, or properties of numbers	variables be in the form y=x+k? y=x–k?y=kx? y=x/k? What can you tell about the relationship between dependent and independent variables in an equation of the form y = mx? How is that relationship shown in a table and a graph of sample (x, y) values? Why is the point (1, m) on every graph? How do you calculate values of y from an equation like y = 3x + 5 when values of x are given? How about y = 5 + 3x? When do you need such equations that involve two operations? When an equation relating two variables involves two or more operations, how do you use the equation to find values of the dependent variable from given values of the independent variable? Is it possible to have two different, but equivalent,	

mowed in 35 hours? At what rate were lawns being mowed? **6.RP.A.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.B** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3°C > -7 °C to express the fact that -3°C is warmer than -7°C*. **6.NS.C.6C** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars*.	such as the Distributive Property to show that two expressions are equivalent. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. Interpret and evaluate expressions in which letters stand for numbers and apply the Order of Operations as needed. Recognize that equations are statements of equivalence between two expressions. Solve linear equations of the forms $y=ax$, $y=b+x$, and $y=b+ax$ using numeric guess and check, tables of (x,y) values, and graphs or fact families.	expressions for a given situation? Explain. What does it mean to say that two algebraic expressions are equivalent? How can expressions such as $3x + 7x$ *or* $3(x+2)$ be written in equivalent form? What strategies can you use to solve equations in the forms $x + a = b$, $x - a = b$, $ax = b$, and $x \div a = b$ $(a \neq 0)$? How can you represent and find solutions for inequalities? **<u>Learning Targets:</u>** Students will: • Develop understanding of variables and how they are related. • Develop understanding of expressions and equations	

6.EE.A.1, Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or,	Write an inequality and associate it with an equation to find solutions and graph the solutions on a number line.	• Understand and use the process of statistical investigation. • The process of statistical investigation involves posing questions, collecting and analyzing data, and interpreting answers. • Understand the role of multiple representations of data distributions. • Finding measures of center or variability and graphing data are useful for summarizing the information in a variable data set. Visual representations of a data set can help you interpret the measures of center and spread and relate this to the overall shape of the representation.	

depending on the purpose at hand, any number in a specified set. **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.		• Distinguish data and data types. • The answers to a statistical question are called data. Data can be either numerical or categorical. • Understand that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread). • There are several ways to try to say what is typical of a set of data; in each case a single number, called a measure of center, summarizes the data. Because various measures of center are calculated differently, they respond differently to changes in the data or to unusual data values.	

		• The variability of a set of data can be measured, interpreted, and compared with the variability of other data sets. Measures of variability tell you how spread out the data are in relation to each other or to the center.	

Unit Seven: Statistics and Data Analysis Timeline: 23 Lessons
Unit Description: In this unit, students understand and use the process of statistical investigation, distinguish data and data types, and display data with multiple representations.

Standards Alignment	Unit Concept/Big Ideas	Essential Questions/ Learning Targets	Assessments
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2.A** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* ***6.EE.A.2.B*** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a*	Understand and use the process of statistical investigation. Ask questions, collect and analyze data, and interpret data to answer questions. Describe data with respect to its shape, center, and variability or spread. Construct and use simple surveys as a method of collecting data. Distinguish data and data types. Recognize that data consist of counts or measurements of a variable, or an	**Essential Questions:** What are "data"? How do you represent data using a frequency table or a line plot? How can you compare two distributions of data? What are the measures of central tendency and variability (or spread)? How do you compare and use mode and range? How do you identify and use the median? How can you compare two distributions of data using the medians? How do you go about finding a number that is a good estimate of typical household size based on the given data? How do you interpret, compute, and use the mean?	**Montessori Materials:** Pegboard Teacher-made extension materials **Connected Mathematics Project: Data About Us** **Assessments** **Formative** • Student Exercises • Peer Questioning • Classroom Discussions • Vocabulary checks • Problem Solving Challenges • Exit Tickets **Summative** • Montessori Three-Period Lesson including introduction, practice,

product of two factors; view (8 + 7) as both a single entity and a sum of two terms. **6.EE.A.2.C** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. **6.EE.A.4** The idea that equivalence means two expressions give the same outputs is first highlighted in Investigation 4 of *Prime Time*. Featuring applications of the distributive property, the most common principle for generating equivalent expressions, Problem 4.3 of that investigation states the property in generality with letter names for variables *a(b + c) = a(b) + a(c)*.	attribute; these observations comprise a distribution of data values. Distinguish between categorical data and numerical data, and identify which graphs and statistics can be used to represent each kind of data. Display data with multiple representations. Organize and represent data using tables, dot plots, line plots, ordered-value bar graphs, frequency bar graphs, histograms, and box-and-whisker plots. Make informed decisions about which graphs or tables can be used to display a particular set of data.	How do the median and the mean respond to the data in a distribution? How do you choose which measure of center to use when describing what is typical? How do you distinguish different types of data? What statistics are used with different types of data? What information does the interquartile range provide about how data vary in a distribution? How is the interquartile range used to make comparisons among distributions? What information does the mean absolute deviation provide about how data vary in a distribution? How can you use a histogram to help you interpret data? How can you interpret data using a box-and-whisker plot?	and assessment of the % of concept mastery. • Problem-based interactive Learning activities • Performance assessment

	Recognize that a graph shows the overall shape of a distribution, whether the data values are symmetrical around a central value, and whether the graph contains any unusual characteristics such as gaps, clusters, or outliers Recognize that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread). Distinguish between and compute measures of central tendency (mean, median, and mode) and measures of spread (range, interquartile range (IQR), and mean absolute deviation (MAD)).	How can you compare and contrast data represented by dot plots, histograms, and box plots? **<u>Learning Targets:</u>** Students will: • Understand relationships among factors, multiples, divisors, and products. • If a number *N* can be written as a product of two whole numbers, $N = a \times b$, then *a* and *b* are factors of *N*. Multiples of a can be found using the expression $a \times$ (some whole number), such as $2a$, $3a$, $4a$, etc. Some numbers can be expressed in exponential notation, such as a2, a3, a4, etc. • When all factors of a number are broken down into prime	

	Identify how the median and mean respond to changes in the data values of a distribution. Relate the choice of measures of central tendency and variability to the shape of the distribution and the context. Describe the amount of variability in a distribution by noting whether the data values cluster in one or more areas or are fairly spread out. Use measures of center and spread to compare data distributions.	numbers, you have a unique prime factorization. Finding the prime factorization of two numbers can be useful in finding the least common multiple and greatest common factor of the numbers and in classifying numbers as prime, composite, even, odd, or square. • Understand why two expressions are equivalent. • When calculating the value of an expression, the operations have to be performed in a conventional order, the order of operations. • Sometimes a numerical expression can be written in different ways but the expressions are equivalent because the	

		value is the same. Properties of operations, including the Distributive Property, are essential tools for writing equivalent expressions.	