Interactive Mathematics Program Curriculum Framework

School: First State Military Academy **Curricular Tool:** IMP **Grade or Course** Year 1 (grade 9)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Unit One: Patterns **Timeline: 13 days**			
Interpret expressions that represent a quantity in terms of its context. **CC.A-SSE.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **CC.F-IF.1** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* **CC.F-IF.3** Write a function that describes a relationship between two quantities. **CC.F-BF.1** Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.F-BF.1a** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **CC.F-BF.2** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**CC.G-CO.1**	Patterns emphasizes extended, open-ended exploration and the search for patterns. Important mathematics introduced or reviewed in Patterns includes In-Out tables, functions, variables, positive and negative numbers, and basic geometry concepts related to polygons. Proof, another major theme, is developed as part of the larger theme of reasoning and explaining. Students' ability to create and understand proofs will develop over their four years in IMP; their work in this unit is an important start. This unit focuses on several mathematical ideas: • Finding, analyzing, and generalizing geometric and numeric patterns • Analyzing and creating In-Out tables • Using variables in a variety of ways, including to express generalizations • Developing and using general principles for working with variables, including the distributive property • Working with order-of-operations rules for arithmetic • Using a concrete model to understand and do arithmetic with positive and negative integers • Applying algebraic ideas, including In-Out tables, in geometric settings • Developing proofs concerning consecutive sums and other topics	Can students use variables and algebraic expressions to represent concrete situations, generalize results, and describe functions? Can students use different representations of functions—symbolic, graphical, situational, and numerical—and understanding the connections between these representations? Can students use function notation? Can students model, and computing with signed numbers? Can students solve equations using trial and error?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Unit Two: Game of Pig **Timeline: 17 days**			
Represent data with plots on the real number line (dot plots, histograms, and box plots). **CC.S-ID.1** Distinguish between correlation and causation. **CC.S-ID.9** Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.S-IC.2** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **CC.S-CP.1** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **CC.S-CP.2** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **CC.S-CP.3** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. **CC.S-CP.6** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.* **CC.S-CP.7** *– supplementary lesson is being developed by the publisher* *Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B),*	As an introduction to the probability concepts and skills needed to analyze the game of Pig, students work on a variety of problems involving chance occurrences. Through these experiences, they develop an understanding of the concept of expected value and learn to calculate expected value using an area model. They also encounter some real-life "games," such as buying insurance and playing the lottery, and discover that in such situations, expected value may not be the sole criterion for making a decision. In the unit activities, students explore these important mathematical ideas: • Learning what constitutes a "complete strategy" for a game and developing and analyzing strategies • Calculating probabilities as fractions, decimals, and percents by emphasizing equally likely outcomes and by constructing mathematical models, including area models and tree diagrams • Determining whether events are independent • Using the idea of "in the long run" to develop the concept of expected value and calculating and interpreting expected values • Solving problems involving conditional probability • Making and interpreting frequency bar graphs • Using simulations to estimate probabilities and compare strategies • Comparing the theoretical analysis of a	Can students apply basic methods for calculating probabilities? Can students construct area models and tree diagrams? Can students distinguish between theoretical and experimental probabilities? Can students plan and carry out simulations? Can students collect and analyze data? Can students construct frequency bar graphs? Can students calculate, and interpret expected value? Can students apply the concept of expected value to real-world situations?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
and interpret the answer in terms of the model. **CC.S-CP.8** - *unit supplement to be developed* Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **CC.S-MD.1** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **CC.S-MD.2** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **CC.S-MD.3** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **CC.S-MD.5** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **CC.S-MD.5a** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* **CC.S-MD.5b** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **CC.S-MD.6** Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). **CC.S-MD.7**	situation with experimental results • Examining how the number of trials in a simulation affects the results		

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Unit Three: The Overland Trail **Timeline: 18 days**			
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **CC.N-Q.1** Define appropriate quantities for the purpose of descriptive modeling. **CC.N-Q.2** Interpret parts of an expression, such as terms, factors, and coefficients. **CC.A-SSE.1a** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. **CC.A-SSE.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **CC.A-SSE.3** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CC.A-CED.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **CC.A-REI.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.A-REI.3** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.A-**	Building on students' work in *Patterns*, this unit develops the central mathematical idea of functions and their representations. Students will move among the following four "faces" of functions: situations, graphs, tables and rules. The focus of this unit is on linear functions. Students will use starting values and rate of change to characterize linear functions, build In-Out tables, draw graphs, and write equations to represent specific contexts. They will use tables, graphs, and symbols to solve linear equations and systems of linear equations. They will fit lines to real data and use graphs and symbols representing these lines to solve problems in the context of the unit. The main concepts and skills that students will encounter and practice during the course of this unit can be summarized by category. **Constraints and Decision Making** • Creating examples that fit a set of constraints • Finding numbers that fit several conditions • Using tables of information and lines of best fit to make predictions and estimates • Working with mean and median **Algorithms, Variables, and Notation** • Strengthening understanding of the distributive property • Developing numeric algorithms for	Can students interpret graphs and use graphs to represent situations? Can students relate graphs to their equations, with emphasis on linear relationships? Can students solve pairs of linear equations by graphing? Can students fit equations to data, both with and without graphing calculators? Can students develop and use principles for equivalent expressions, including the distributive property? Can students use the distributive property? Can students apply principles for equivalent equations to solve equations? Can students solve linear equations in one variable? Do students understand relationships between the algebraic expression defining a linear function and the graph of that function?	All assessments are listed at the end of the curriculum map.

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REI.10 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **CC.A-REI.11** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.F-IF.2** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **CC.F-IF.6** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.F-IF.7**	problem situations • Expressing algorithms in words and symbols • Interpreting algebraic expressions in words using summary phrases • Developing meaningful algebraic expressions **Basics of Graphing** • Reviewing the coordinate system • Interpreting graphs intuitively and using graphs intuitively to represent situations • Making graphs from tabular information • Quantifying graphs with appropriate scales • Using graphs to represent two-variable equations and data sets • Using multiple representations—graphs, tables, and algebraic relationships—to describe situations **Linear Equations, Graphs, and Situations** • Finding and interpreting lines of best fit intuitively • Seeing the role of constant rate in linear situations • Using rates and starting values, or other data points, to create equations for straight lines • Laying the groundwork for the concept of slope • Using the point of intersection of two graphs to find values that satisfies two conditions • Solving linear equations for one variable in terms of another • Solving problems involving two linear conditions		

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Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.F-IF.7b** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.F-LE.1a** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **CC.F-LE.1b** Interpret the parameters in a linear or exponential function in terms of a context. **CC.F-LE.5** Represent data on two quantitative variables on a scatter plot and describe how the variables are related. **CC.S-ID.6** Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.* **CC.S-ID.6a** *Informally assess the fit of a model function by plotting and analyzing residuals.* **CC.S-ID.6b** – *unit supplement to be developed* Fit a linear function for scatter plots that suggest a linear association. **CC.S-ID.6c** Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data. **CC.S-ID.7** *Compute (using technology) and interpret the correlation coefficient of a linear fit.* ***CC.S-ID.8*** *– supplementary lesson is being developed by the publisher*	• Solving linear equations in one variable **Graphs and Technology** • Making and interpreting graphs on a graphing calculator • Using the zoom and trace features to get information from a graphing calculator		

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Unit Four: The Pit and the Pendulum **Timeline: 18 day (90 minute blocks)**			
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **CC.N-Q.3** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* **CC.A-SSE.1b** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.F-IF.7** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.F-IF.7b** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, k $f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **CC.F-BF.3** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table). **CC.F-LE.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **CC.S-ID.2** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **CC.S-ID.3**	This unit draws on and extends students' work in the first three units. It blends scientific experiments with the statistical concepts of normal distribution and standard deviation and the algebra of functions and graphs. The main concepts and skills that students will encounter and practice during the course of this unit are summarized below. References to graphing calculators should be understood to include other technology that might be available. **Experiments and Data** • Planning and performing controlled scientific experiments • Working with the concept of period • Recognizing and accommodating for the phenomenon of measurement variation • Collecting and analyzing data • Expressing experimental results and other data using frequency bar graphs **Statistics** • Recognizing the normal distribution as a model for certain kinds of data • Making area estimates to understand the normal distribution • Developing concepts of data spread, especially standard deviation • Working with symmetry and concavity in connection with the normal distribution and standard deviation • Applying standard deviation and the normal distribution in problem contexts • Distinguishing between population standard deviation and sample • Calculating the mean and standard deviation of data sets, both by hand and	• Can students describe normal distributions and their properties? • Can students use mean and standard deviation? • Can students use normal distribution, mean, and standard deviation?	All assessments are listed at the end of the curriculum map.

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Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. **CC.S-ID.4** Understand that statistics is a process for making inferences about population parameters based on a random sample from that population. **CC.S-IC.1** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **CC.S-MD.4**	with calculators • Using standard deviation to decide whether a variation in experimental results is significant **Functions and Graphs** • Using function notation • Using graphing calculators to explore the graphs of various functions • Fitting a function to data using a graphing calculator • Making predictions based on curve-fitting		
Unit Five: Shadows **Timeline: 17 days**			
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* **CC.A-CED.4** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**CC.G-CO.1** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a rigid motion on a figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **CC.G-CO.6** Explain using rigid motions the meaning of congruence for triangles as the equality of all corresponding pairs of sides and all corresponding pairs of angles. **CC.G-CO.7**	The concept of similarity is the central theme of this unit. Through this concept, students explore the following important ideas from geometry and algebra. **Similarity and Congruence** • Developing intuitive ideas about the meaning of "same shape" and learning the formal definitions of similar and congruent • Discovering the special properties of triangles in connection with similarity, as well as other features of triangles as special polygons • Understanding the role of similarity in defining the trigonometric functions of sine, cosine and tangent **Proportional Reasoning and the Algebra of Proportions** • Understanding the meaning of	Do students understand the meaning of angles and angle measurement? Can students apply the relationships among angles of polygons, including angle-sum formulas? Can students apply criteria for similarity and congruence? Can students use properties of similar polygons to solve real-world problems? Can students use similarity to define right-triangle trigonometric functions? Can students apply right-	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence. **CC.G-CO.8** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **CC.G-CO.9** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **CC.G-CO.10** – *supplementary lessons are being developed by the publisher to cover theorems not already included in the curriculum.* Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangle are parallelograms with congruent diagonals.* **CC.G-CO.11** – *supplementary lessons are being developed by the publisher to cover theorems not already included in the curriculum.* Verify experimentally the properties of dilations: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **CC.G-SRT.1a** Verify experimentally the properties of dilations: The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **CC.G-SRT.1b** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar;	proportionality in connection with similarity • Developing equations of proportionality from situations involving similar figures • Understanding the role of proportionality in nongeometric situations • Developing techniques for solving equations involving fractional expressions **Polygons and Angles** • Developing angle sum formulas for triangles and other polygons • Discovering the properties of angles formed by a transversal across parallel lines • Discovering the triangle inequality and investigating its extension to polygons **Logical Reasoning and Proof** • Working with the concept of counterexample in understanding the criteria for similarity • Proving conjectures about vertical and polygon angle sums • Understanding the role of the parallel postulate in proofs **Right Triangles and Trigonometry** • Learning standard terminology for triangles, including hypotenuse, leg, opposite side, and adjacent side • Learning the right triangle definitions of *sine*, *cosine*, and *tangent* • Using sine, cosine, and tangent to solve real-world problems **Experiments and Data Analysis** • Planning and carrying out controlled experiments • Collecting and analyzing data	triangle trigonometry to real-world problems? Do students understand the meaning of angles and their measurement? Do students recognize relationships among angles of polygons, including angle-sum formulas? Can students define and apply properties of similarity and congruence? Can students use properties of similar polygons to solve real-world problems? Can students use similarity to define right-triangle trigonometric functions? Can students apply right-triangle trigonometry to real-world problems?	

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides. **CC.G-SRT.2** Use the properties of similarity transformations to establish the AA criterion for similarity of triangles. **CC.G-SRT.3** Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.* **CC.G-SRT.4** Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures. **CC.G-SRT.5** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.G-SRT.6** Explain and use the relationship between the sine and cosine of complementary angles. **CC.G-SRT.7** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CC.G-SRT.8** Find the point on a directed line segment between two given points that divide the segment in a given ratio. **CC.G-GPE.6**	• Identifying key features in graphs of data **Mathematical Modeling** • Using a geometric diagram to represent a real-world situation • Using scale drawings to solve problems • Applying properties of similar triangles to real-world situations • Exploring how models provide insight in a variety of situations		

Assessment Opportunities in this Unit:

End-of-Unit Assessments:
Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

On-Going Assessments:
Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills. Students' written and oral work provides many opportunities for teachers to gather this information.

- *Presentations on Calculator Exploration:* These presentations will give you information on how comfortable students are with calculators and open-ended investigation.

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- *Pulling Out Rules:* This activity will help you gauge how well students understand the basic ideas of In-Out tables and evaluate their ability in writing rules to describe tables.
- *You're the Chef:* This summary activity will tell you how well students understand the arithmetic of positive and negative integers.
- *Presentations on Consecutive Sums:* These presentations will indicate how students are developing in their ability to conduct independent mathematical investigations.
- *An Angular Summary:* This activity will help you gauge students' understanding of the sum of the angles in a polygon and related formulas.
- *Border Varieties:* This activity will reflect students' understanding of the use of variables.
- *Pig Strategies:* This activity will help you gauge how well students understand the rules of Pig and assess their comfort level with the idea of strategy.
- *0 to 1, or Never to Always:* This activity will illustrate students' grasp of the 0-to-1 scale for probability.
- *Two-Dice Sums and Products:* This activity will show how well students understand and can work with two-dimensional area models.
- *Spinner Give and Take:* This activity can provide a baseline of students' initial understanding of the meaning of "the long run," in preparation for work with expected value.
- *Spins and Draws:* This activity will tell you how well students understand and can work with expected value.
- *A Fair Deal for the Carrier?:* This activity will inform you about students' ability to find probabilities in two-stage situations.
- *Little Pig Strategies:* This activity will tell you how well prepared students are for the detailed analysis of Little Pig.
- *The Best Little Pig:* This activity will inform you of students' grasp of the big picture in the analysis of Little Pig.
- *Creating Families:* This assignment will give you information on how well students can deal with verbal constraints.
- *Laced Travelers:* This activity will tell you whether students can put arithmetic processes into words.
- *Ox Expressions at Home:* This assignment will help you assess how well students understand meaningful algebraic expressions
- *Graph Sketches:* This activity will give you a sense of how well students understand graphs.
- *Who Will Make It?* This activity can help you gauge students' ability to make meaningful inferences from graphs.
- *All Four, One--Linear Functions:* This assignment will give you information about students' understanding of the connections among different ways to represent a situation.
- *Straight Line Reflections:* This activity will give you a sense of how well students understand concepts related to straight-line graphs.
- *More Fair Share for Hired Hands:* This assignment can provide information on student understanding of the connection between graphs and equations.
- *Family Comparisons by Algebra:* This activity will help you evaluate students' ability to represent situations using equations and their facility with solving linear equations.
- *Initial Experiments:* This activity will tell you how well students understand the idea of isolating a single variable.
- *Pulse Analysis:* This assignment will tell you about students' understanding of mean and frequency bar graphs.
- *Kai and Mai Spread Data:* This activity will give you a baseline of information about students' understanding of data spread.
- *Penny Weight Revisited:* This activity will guide you in determining students' intuitive understanding of standard deviation.
- *Pendulum Conclusions:* This assignment will tell you how well students can reason using the concept of standard deviation.
- *Graphing Summary:* This activity will give you information on what students know about the shape of graphs of various functions.
- *Mathematics and Science:* This assignment will give you insight into what students see as the key ideas of the unit.
- *Shadow Data Gathering* and *Working with Shadow Data:* These activities, which ask students to set up and conduct controlled experiments (as in the unit *The Pit and the Pendulum*), will provide evidence of their understanding of the unit problems.
- *Similar Problems:* This assignment will provide evidence of students' ability to write and solve proportions derived from similar figures.
- *Angles and Counterexamples:* This activity will help you assess students' ability to create and solve linear equations derived from a geometric context and their developing understanding of similarity.
- *Angles, Angles, Angles:* This assignment will give you information on students' knowledge of facts about angles created by intersecting lines (including

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transversals of parallel lines) and interior angles of polygons.

- *Mirror Madness:* This activity will tell you whether students can use the reflective property of mirrors along with the concept of similarity to do indirect measurement.
- *A Shadow of a Doubt:* This activity will provide evidence about whether students understand the general solution to the lamp shadow problem.
- *The Tree and the Pendulum:* This assignment will illustrate students' ability to use trigonometry to do indirect measurement.
- *A Bright, Sunny Day:* This activity will provide evidence of students' understanding of the general solution to the sun shadow problem.

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

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Interactive Mathematics Program Curriculum Framework

School: First State Military Academy **Curricular Tool:** IMP **Grade or Course:** Year 2 (grade 10)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit One: Do Bees Build it Best? **Timeline: 20 days**			
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **CC.N-Q.3** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.A-REI.2** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.A-REI.4b** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.* **CC.G-SRT.4** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CC.G-SRT.8** Prove the Laws of Sines and Cosines and use them to solve problems. **CC.G-SRT.10**	The regular form of a honeycomb is striking. Viewed end on, honeycomb cells resemble the hexagonal tiles on a bathroom floor. But a honeycomb is a three-dimensional object, a collection of right hexagonal prisms. Why do bees build their honeycombs this way? Concepts of measurement—especially area, surface area, and volume—are the mathematical focus of this unit. The main concepts and skills that students will encounter and practice during the unit are summarized by category here. ***Area*** • Understanding the role of units in measuring area • Establishing standard units for area, especially those based on units of length • Recognizing that a figure's perimeter alone does not determine its area • Discovering formulas for the areas of rectangles, triangles, parallelograms, and trapezoids • Establishing that a square has the greatest area of all rectangles with a fixed perimeter • Developing a formula for the area of a regular polygon with a given perimeter in terms of the number of sides • Discovering that for a fixed perimeter, the more sides a regular polygon has, the greater its area • Discovering that the ratio of the areas of similar figures is equal to the square of the ratio of their corresponding linear dimensions	Can students measure area using both standard and nonstandard units? Can students use several methods for finding areas of polygons, including development of formulas for area of triangles, rectangles, parallelograms, trapezoids, and regular polygons? Can students find surface area and volume for three-dimensional solids, including prisms and cylinders? Can students apply the Pythagorean theorem? Can students prove the Pythagorean theorem? Can students maximize area for a given perimeter? Do students understand the relationship between the areas and volumes of similar figures? Can students create successful tessellations? Can students apply right triangle trigonometry to area and perimeter problems?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **CC.G-SRT.11** Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **CC.G-GMD.1** *Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.* **CC.G-GMD.2** – *unit supplement to be developed* Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. **CC.G-GMD.3** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **CC.G-GMD.4** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typographic grid systems based on ratios). **CC.G-MG.3**	***The Pythagorean Theorem*** • Discovering the Pythagorean theorem by comparing the areas of the squares constructed on the sides of a right triangle • Proving the Pythagorean theorem using an area argument • Applying the Pythagorean theorem in a variety of situations ***Surface Area and Volume*** • Understanding the role of units in measuring surface area and volume • Establishing standard units for surface area and volume, especially those based on a unit of length • Recognizing that a solid figure's surface area alone does not determine its volume • Developing principles relating the volume and surface area of a prism to the area and perimeter of its base • Discovering that the ratio of the surface areas of similar solids is equal to the square of the ratio of their corresponding linear dimensions, and that the ratio of the volumes of similar solids is equal to the cube of the ratio of their corresponding linear dimensions ***Trigonometry*** • Reviewing right-triangle trigonometry • Finding the ranges of the basic trigonometric functions (for acute angles) • Using the terminology and notation of inverse trigonometric functions ***Miscellaneous*** • Reviewing similarity • Reviewing the triangle inequality • Reviewing the angle sum property for triangles • Strengthening two- and three-dimensional spatial visualization skills • Examining the concept of tessellation and		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	discovering which regular polygons tessellate • Developing some properties of square-root radicals • Developing the general concept of an inverse function		
Unit Two: Cookies **Timeline: 18 days**			
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CC.A-CED.1** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **CC.A-CED.2** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.A-CED.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.A-REI.3** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.* ***CC.A-REI.5*** *– supplementary lesson is being developed by the publisher* Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.A-REI.6**	The central mathematical focus of *Cookies* is the formulation and solution of problems of optimization, or linear programming problems. In problems of this type, a linear function is to be optimized and a set of linear conditions constrains the possible solutions. Linearity is an important feature of these two-variable problems, in two ways: • The constraints are linear, so the feasible region is a polygon and its vertices can be found by solving pairs of linear equations. • The expression to be maximized or minimized is linear, so the points that give this expression a particular value lie on a straight line, and investigating a series of values produces a family of parallel lines. The linear programming problems that students encounter in this unit involve only two variables and a limited number of constraints. Their solutions are therefore easier to understand graphically, and the algebra needed to find their exact solutions is manageable. The main concepts and skills that students will encounter and practice during the unit are summarized here. ***Using Variables to Represent Problems*** • Expressing and interpreting constraints using inequalities • Expressing problem situations using systems of linear equations ***Working with Variables, Equations, and***	Can students express real-world situations in terms of equations and inequalities? Can students apply the distributive property? Can students use several methods for solving systems of linear equations in two variables? Can students define and recognize dependent, inconsistent, and independent pairs of linear equations? Can students solve non-routine equations using graphing calculators? Can students write and graph linear inequalities in two variables? Can students use principles of linear programming for two variables? Can students create linear programming problems with two variables?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.A-REI.7** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes. **CC.A-REI.12** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.F-IF.7a**	***Inequalities*** • Finding equivalent expressions and inequalities • Solving linear equations for one variable in terms of another • Developing and using a method for solving systems of two linear equations in two unknowns • Recognizing inconsistent systems and dependent systems ***Graphing*** • Graphing linear inequalities and systems of linear inequalities • Finding the equation of a straight line and the inequality for half plane • Using graphing calculators to draw feasible regions • Relating the intersection point of graphed lines to the common solution of the related equations • Using graphing calculators to estimate coordinates of points of intersection ***Reasoning Based on Graphs*** • Recognizing that setting a linear expression equal to a series of constants produces a family of parallel lines • Finding the maximum or minimum of a linear equation over a region • Examining how the parameters in a problem affect the solution • Developing methods of solving linear programming problems with two variables ***Creating Word Problems*** • Creating problems that can be solved using two equations in two unknowns • Creating problems that can be solved by linear programming methods		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit Three: Is There Really a Difference? **Timeline: 21 days**			
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data. **CC.S-ID.5** Distinguish between correlation and causation. **CC.S-ID.9** Understand that statistics is a process for making inferences about population parameters based on a random sample from that population. **CC.S-IC.1** Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.S-IC.2** Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **CC.S-IC.3** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.S-IC.4** Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment. **CC.S-IC.5** Evaluate reports based on data. **CC.S-IC.6** Construct and interpret two-way frequency tables of data when two categories are associated with each	The unit explores two categories of problems: • Problems that compare a single population to a theoretical model (the theoretical-model case) • Problems that compare two distinct populations (the two-population case) Students learn that statisticians often presume that a "neutral" hypothesis, called a null hypothesis, holds unless there is clear evidence to the contrary. In the context of the two categories of problems, the null hypothesis is that the single population *does* fit the model or that the two populations being studied *are* the same. Students learn that to evaluate the null hypothesis, they must examine whether the observed data could reasonably have occurred under that null hypothesis. In the course of studying such questions, students will • work with double-bar graphs to explore data • form hypotheses and corresponding null hypotheses • develop an intuitive sense for evaluating differences between sets of data • learn ways of organizing and presenting data • learn about designing and carrying out statistical studies This unit builds on students' prior experience with statistical ideas in the Year 1 unit *The Pit and the Pendulum.* In that unit, students worked with the normal distribution and used the standard deviation statistic as their primary tool. In this unit, students use the chi-square statistic, or x^2 statistic. In the main activities of the unit, students use the x^2 statistic only in the case of one degree of freedom. Supplemental activities explore more general use of the statistic.	Can students draw inferences from statistical data? Can students design, conduct, and interpret statistical experiments? Can students make and test statistical hypotheses? Can students formulate null hypotheses and understand its role in statistical reasoning? Can students use the χ^2 statistic? Do students understand that tests of statistical significance do not lead to definitive conclusions? Can students solve problems that involve conditional probability?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.* **CC.S-CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.* **CC.S-CP.5** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **CC.S-MD.4**	Although the unit makes intensive use of the x^2 statistic, the real emphasis is on broader statistical ideas, such as the null hypothesis, sampling fluctuation, and hypothesis testing. The main concepts and skills that students will encounter and practice during the course of this unit are summarized by category here. ***Setting Up Statistical Investigations*** • Distinguishing between data snooping and hypothesis testing • Describing the characteristics of a good sample • Making null hypotheses • Using proportional reasoning to analyze the consequences of a null hypothesis • Designing and conducting statistical experiments ***Interpreting Data*** • Making hypotheses about larger populations by analyzing sample data • Constructing and drawing inferences from charts, tables, and graphs, including frequency bar graphs and double-bar graphs • Determining whether to accept or reject a null hypothesis • Understanding the consequences of rejecting a null hypothesis • Interpreting statistical experiments and communicating the outcomes ***The x^2 Statistic*** • Developing intuition about the meaning of the x^2 statistic • Using simulations to estimate the x^2 distribution • Interpreting the x^2 distribution curve as a probability table • Calculating and interpreting the x^2 statistic in order to compare data from real-world situations to theoretical models		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Calculating and interpreting the x^2 statistic in order to compare two populations • Using the x^2 statistic to make decisions • Understanding some limitations in applying the x^2 statistic ***Related Concepts*** • Working with conditional probabilities • Using simulations to develop intuition and to obtain data about sampling fluctuation • Developing intuition about when differences in samples indicate that the larger populations are likely to be different • Understanding why neither numeric difference nor percentage difference is an adequate tool for measuring the "weirdness" of data • Reviewing the normal distribution and standard deviation and their applications to decision making		
Unit Four: Fireworks **Timeline: 13 days**			
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **CC.A-SSE.2** Factor a quadratic expression to reveal the zeros of the function it defines. **CC.A-SSE.3a** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **CC.A-SSE.3b** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.A-APR.1** *Know and apply the Remainder Theorem: For a*	*Fireworks* focuses on the use of quadratic functions to represent a variety of real-world situations and on the development of algebraic skills for working with those functions. Experiences with graphs play an important role in understanding the behavior of quadratic functions. The main concepts and skills students will encounter and practice during the unit are summarized here. ***Mathematical Modeling*** • Expressing real-world situations in terms of functions and equations • Applying mathematical tools to models of real-world problems • Interpreting mathematical results in terms of real-world situations	Can students solve quadratic equations by factoring? Can students relate the number of roots of a quadratic equation to the graph of the associated quadratic function? Can students use the method of completing the square to analyze the graphs of quadratic equations and to solve quadratic equations?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **CC.A-APR.2** - *unit supplement to be developed* Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.A-APR.3** Solve quadratic equations in one variable. **CC.A-REI.4** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.A-REI.4a** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.A-REI.4b** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.F-IF.7a** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.F-IF.7c** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.F-IF.8** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.F-IF.8a**	***Graphs of Quadratic Functions*** • Understanding the roles of the vertex and x-intercept in the graphs of quadratic functions • Recognizing the significance of the sign of the x^2 term in determining the orientation of the graph of a quadratic function • Using graphs to understand and solve problems involving quadratic functions ***Working with Algebraic Expressions*** • Using an area model to understand multiplication of binomials, factoring of quadratic expressions, and completing the square of quadratic expressions • Transforming quadratic expressions into vertex form • Simplifying expressions involving parentheses • Identifying certain quadratic expressions as perfect squares ***Solving Quadratic Equations*** • Interpreting quadratic equations in terms of graphs and vice versa • Estimating x-intercepts using a graph • Finding roots of an equation using the vertex form of the corresponding function • Using the zero product rule of multiplication to solve equations by factoring		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.F-IF.9** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **CC.F-BF.3**			
Unit Five: All About Alice **Timeline: 12 days**			
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **CC.N-RN.1** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **CC.N-RN.2** *Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.* ***CC.N-RN.3*** *– supplementary lesson is being developed by the publisher* Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate*	Unlike most other IMP units, All About Alice has no central problem to solve. Instead, there is a general context to the unit. In particular, the Alice story provides a metaphor for understanding exponents. When Alice eats an ounce of cake, her height is multiplied by a particular whole-number amount; when she drinks an ounce of beverage, her height is multiplied by a particular fractional amount. Using this metaphor, students reason about exponential growth and decay. Students use several approaches to extend exponentiation beyond positive integers: a contextual situation, algebraic laws, graphs, and number patterns. They then apply principles of exponents to study logarithms and scientific notation. The main concepts and skills students will encounter and practice during the course of this unit are summarized by category here.	Can students use exponential expressions, including zero, negative, and fractional exponents? Can students apply the laws of exponents? Can students use scientific notation? Can students use the concept of order of magnitude in estimation?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
mortgage payments. **CC.A-SSE.4** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.F-IF.7e** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.F-IF.8b** Find inverse functions. **CC.F-BF.4** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$. **CC.F-BF.4a** Verify by composition that one function is the inverse of another. **CC.F-BF.4b** Read values of an inverse function from a graph or a table, given that the function has an inverse. **CC.F-BF.4c** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **CC.F-BF.5** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.F-LE.1** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.F-LE.1a**	***Extending the Operation of Exponentiation*** • Defining the operation for an exponent of zero • Defining the operation for negative integer exponents • Defining the operation for fractional exponents ***Laws of Exponents*** • Developing the additive law of exponents • Developing the law of repeated exponentiation ***Graphing*** • Describing the graphs of exponential functions • Comparing graphs of exponential functions for different bases • Describing the graphs of logarithmic functions • Comparing graphs of logarithmic functions for different bases ***Logarithms*** • Understanding the meaning of logarithms • Making connections between exponential and logarithmic equations ***Scientific Notation*** • Converting numbers from ordinary notation to scientific notation, and vice versa • Developing principles for doing computations using scientific notation • Using the concept of order of magnitude in estimation		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **CC.F-LE.1c** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **CC.F-LE.3**			

Assessment Opportunities in this Unit:

End-of-Unit Assessments:
Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

Ongoing Assessment:
Assessment is a component in providing the best possible ongoing instructional program for students. Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills.

Students' written and oral work provides many opportunities for teachers to gather this information. Here are some recommendations of written assignments and oral presentations to monitor especially carefully that will offer insight into student progress.

- How Many Can You Find?: This assignment will inform you about how well students have understood the basics about the meaning of area.
- That's All There Is!: This activity will tell you how comfortable students are with a more open-ended approach to area.
- More Gallery Measurements: This activity will provide information on students' grasp of the fundamentals of right-triangle trigonometry.
- Any Two Sides Work, Make the Lines Count, and The Power of Pythagoras: These assignments will tell you about students' comfort with using the Pythagorean theorem.
- Leslie's Fertile Flowers: In this activity, students need to combine ideas about area with use of the Pythagorean theorem, so it will give you a sense of their facility with these concepts.
- More Fencing, Bigger Corrals: This activity, which involves how changes in linear dimensions affect area, will help you decide how much work students need on this topic.
- Not a Sound: This assignment will give you feedback on students' grasp of the concept of surface area.
- Inequality Stories, Part I: This assignment will give you information about students' understanding of how real-life contexts can be expressed in algebraic terms using inequalities.
- Profitable Pictures: This activity will tell you how well students understand how profit lines can be used to determine an optimal value.
- Changing What You Eat: In this assignment, students will demonstrate their understanding of how changing specific parameters in a problem affects the solution.
- Get the Point: This investigation will give you insight into students' abilities to think about systems of linear equations in flexible ways.
- A Reflection on Money: This assignment will give you information about students' comfort levels with solving systems of linear equations.
- "How Many of Each Kind?" Revisited: This activity will tell you how well students have synthesized the ideas of the unit.

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- Changing the Difference, Part I: This work will give you information on students' sense of how probabilities behave with large samples.
- Loaded or Not?: This activity will tell you how well students can interpret experimental data.
- Decisions with Deviation: This assignment will provide information about students' understanding of how to use the normal distribution.
- Measuring Weirdness with χ^2: This activity will give you information about students' understanding of how to calculate and use the χ^2 statistic.
- Late in the Day: This assignment will give you feedback on how well students can set up and analyze a situation using the χ^2 statistic.
- "Two Different Differences" Revisited: This activity will give you information on students' abilities to do a complete analysis of a situation using the χ^2 statistic.
- Using Vertex Form will illustrate students' ability to pull together and use the various components of the vertex form of a quadratic.
- Squares and Expansions will demonstrate students' developing understanding of the technique of completing the square.
- How Much Can They Drink? will provide information on students' developing understanding of how to find the maximum value of a quadratic function to find the solution to a problem in context.
- Another Rocket will show how well students are prepared to address the unit problem.
- A Fireworks Summary is a reflective piece in which students summarize their work on the unit problem.
- A Quadratic Summary is a reflective piece in which students summarize their understanding of the big ideas of the unit.
- Graphing Alice: This assignment will give you information about how well students understand the basic Alice metaphor and about their comfort with nonlinear graphs.
- Having Your Cake and Drinking Too: This activity will reveal students' ability to work with the Alice metaphor in a complex situation.
- Negative Reflections: This assignment will tell you how well students understand the extension of exponentiation to negative exponents.
- All Roads Lead to Rome: This activity will give you information on students' ability to synthesize a variety of approaches to understanding a mathematical concept.
- Alice on a Log: This assignment will give you information on students' understanding of the basics about logarithms.

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

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Interactive Mathematics Program Curriculum Framework

School: First State Military Academy **Curricular Tool:** IMP **Grade or Course:** Year 3 (grade 11)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit One: Orchard Hideout **Timeline: 17 days**			
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**CC.G-CO.1** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **CC.G-CO.9** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **CC.G-CO.12** *Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.* ***CC.G-CO.13*** *– supplementary lesson is being developed by the publisher* *Derive the formula A = ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.* ***CC.G-SRT.9*** *– supplementary lesson is being developed by the publisher* *Prove that all circles are similar.* ***CC.G-C.1*** *– supplementary lesson is being developed by the publisher*	The central unit problem concerns a couple who have planted an orchard of trees in careful rows and columns on a circular lot. The couple realizes that, after a while, the trunks of their trees will become so thick that they will no longer be able to see out from the center of the orchard. In other words, the orchard will become a "hideout." The main unit question is this: **How soon after the couple plant the orchard will the center of the lot become a true "orchard hideout"?** Students' search for the answer to this question leads them to the study of several aspects of geometry. Students use the Pythagorean Theorem to measure distances within the orchard, leading to development of the distance formula. As a sidelight to their work with the distance formula, students construct the general equation of a circle. Giving the initial size of the trees in terms of circumference and the growth rate in terms of cross-sectional area motivates development of the area and circumference formulas for a circle. While solving the unit problem, students encounter a variety of tangents (both	Can students explain the relationship of the area and circumference of a circle to its radius? Do students understand the significance of using regular polygons to approximate the area and circumference of a circle? Can students justify locus descriptions of various geometric entities, such as perpendicular bisectors and angle bisectors? Can students apply properties of parallel lines? Can students identify possible intersections of lines and planes? Can students use "if and only if" in describing sets of points fitting given criteria? Can students define and use the concept of the converse of a statement?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **CC.G-C.2** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **CC.G-C.3** *Construct a tangent line from a point outside a given circle to the circle.* ***CC.G-C.4*** *- supplementary lesson is being developed by the publisher* Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **CC.G-GPE.1** Derive the equation of a parabola given a focus and directrix. **CC.G-GPE.2** Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola. **CC.G-GPE.3** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **CC.G-GPE.4** *Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).* ***CC.G-GPE.5*** *– supplementary lesson is being developed by the publisher*	figuratively and literally). One result is a proof that a tangent to a circle is perpendicular to the radius at the point of tangency. They use the technique of completing the square to put certain quadratic equations into standard form to find the radius and center of the circles they represent. Other ideas arise through the unit's POWs. For example, students prove basic facts about perpendicular bisectors and angle bisectors, thereby establishing the existence of both circumscribed and inscribed circles for triangles. The main concepts and skills students will encounter and practice during the unit are summarized below. ***Coordinate geometry*** • Using the Cartesian coordinate system to organize a complex problem • Developing and applying the distance formula • Developing the standard form for the equation of a circle with a given center and radius • Finding the distance from a point to a line in a coordinate setting • Developing and applying the midpoint formula ***Circles*** • Using similarity to see that the circumference of a circle should be a constant times its radius, and that the area of a circle should be a constant times the square of its radius • Finding formulas for the perimeter and area of regular polygons circumscribed about a circle		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula. **CC.G-GPE.7** Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. **CC.G-GMD.3** Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **CC.G-MG.1**	• Using circumscribed polygons to see that the "circumference coefficient" for the circle is twice the "area coefficient" for the circle • Defining π and understanding why it appears in the formulas for both the circumference and the area of a circle • Developing and applying formulas for the circumference and area of a circle ***Synthetic Geometry*** • Identifying and describing a set of points satisfying a geometric condition • Discovering and proving that the set of points equidistant from two given points is the perpendicular bisector of the segment connecting the given points • Defining the distance from a point to a line and proving that the perpendicular distance is the shortest • Discovering and proving that any line through the midpoint of a segment is equidistant from the endpoints of the segment • Discovering and proving that the set of points equidistant from two intersecting lines consists of the bisectors of the angles formed by the lines ***Algebra*** • Using the technique of completing the square to transform equations of circles into standard form • Using algebra in a variety of proofs involving coordinates and angles ***Logic*** • Understanding and using the phrases "if-then" and "if and only if" in		

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	definitions and proofs • Working with converses ***Miscellaneous*** • Using symmetry to help analyze a problem • Learning about Pythagorean triples		
Unit Two: Meadows or Malls? **Timeline: 27 days**			
Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.N-VM.6** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **CC.N-VM.7** Add, subtract, and multiply matrices of appropriate dimensions. **CC.N-VM.8** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **CC.N-VM.9** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **CC.N-VM.10** *Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.* ***CC.N-VM.11*** *– supplementary unit being is developed by the publisher* *Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.* ***CC.N-VM.12*** *- supplementary unit is being developed by the publisher*	The main concepts and skills that students will encounter and practice during this unit are: ***General Linear Programming*** • Seeing that for two-variable problems, the optimal value always occurs at a corner point of the feasible region • Generalizing the corner-point principle to more than two variables • Recognizing that for two-variable problems, corner points can be found as the intersections of lines corresponding to constraint equations or inequalities • Generalizing the method of finding corner points to more than two variables ***Solving Linear Equations*** • Using substitution, graphing, and guess-and-check methods to solve systems of linear equations in two variables • Developing and using the elimination method to solve systems of linear equations in two or more variables • Using the concepts of **inconsistent**, **dependent,** and **independent** systems of equations ***Geometry in the Plane and in 3-Space***	Can students use the elimination method for solving systems of linear equations in up to four variables? Can students extend the concepts of dependent, inconsistent, and independent systems of linear equations to more than two variables? Can students use matrices? Can students use the operations of matrix addition and multiplication in the context of applied problems? Can students use of matrices to represent systems of linear equations? Can students use the identity element and inverse in the context of matrices? Can students use matrices and matrix inverses to solve systems of linear equations? Can students relate the existence of matrix inverses to the uniqueness of the solution of corresponding systems of	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.A-CED.3** Represent a system of linear equations as a single matrix equation in a vector variable. **CC.A-REI.8** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.A-REI.9**	• Extending the concept of coordinates to three variables by introducing a third axis perpendicular to the first two • Graphing linear equations in three variables and recognizing that these graphs are planes in 3-space • Seeing that two distinct points always determine a unique line and that two distinct lines in the plane determine a unique point unless the lines are parallel • Examining the possible intersections of planes in 3-space • Relating the possible intersections of lines and planes to the algebra of solving linear systems in two or three variables ***Matrix Algebra*** • Using matrices to represent information • Using problem situations to motivate and develop the definitions of matrix addition and multiplication • Examining whether matrix operations have certain properties, such as associativity and commutativity ***Matrices and Systems of Linear Equations*** • Seeing that systems of linear equations are equivalent to certain types of matrix equations • Recognizing the role of identity and inverse elements in solving certain types of matrix equations • Finding matrix inverses by hand by solving systems of linear equations • Understanding the relationship between a system of linear equations	linear equations? Can students use calculators to multiply and invert matrices and to solve systems of linear equations? Can students apply the concepts of linear programming to problems with several variables? Can students use equations of planes in three-dimensional coordinate geometry? Can students define polar coordinates? Do students recognize graphs of polar equations?	

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	having a unique solution and the coefficient matrix being invertible ***Technology*** • Entering matrices and doing matrix operations on a graphing calculator • Using matrix inversion on a graphing calculator to solve systems of linear equations		
Unit Three: Small World, Isn't It? **Timeline: 15 days**			
Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **CC.A-SSE.3c** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* **CC.A-SSE.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.F-IF.8b** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **CC.F-BF.2** Recognize situations in which a quantity grows or decays by a	The main concepts and skills that students will encounter and practice during this unit are: ***Rate of Change*** • Evaluating average rate of change in terms of the coordinates of points on a graph • Understanding the relationship between the rate of change of a function and the appearance of its graph • Realizing that in many contexts, the rate of growth or decline with respect to time in a population is proportional to the population ***Slope and Linear Functions*** • Developing an algebraic definition of slope • Proving, using similarity, that a line has a constant slope • Understanding the significance of a negative slope for a graph and an applied context • Seeing that the slope of a line is equal to the coefficient of x in the $y = a + bx$ representation of the line • Using slope to develop equations for lines	Can students use exponential and logarithmic functions and describe their graphs? Do students understand the relationship between logarithms and exponents? Do students understand that the derivative of an exponential function is proportional to the value of the function? Can students use the general laws of exponents? Do students understand the meaning and significance of *e*? Can students approximate data with an exponential function? Can students define slope and understand its relationship to rate of change and to equations for straight lines? Can students develop equations for straight lines from two points and from point-slope information? Can students apply various	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
constant percent rate per unit interval relative to another. **CC.F-LE.1c** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table). **CC.F-LE.2** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **CC.F-LE.4** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **CC.G-MG.2** Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.* **CC.S-ID.6a**	***Derivatives*** • Developing the concept of the derivative of a function at a point • Seeing that the derivative of a function at a point is the slope of the tangent line at that point • Finding numerical estimates for the derivatives of functions at specific points • Working with the derivative of a function as a function in itself • Realizing that for functions of the form $y = b^x$, the derivative at each point of the graph is proportional to the y-value at that point ***Exponential and Logarithmic Functions*** • Using exponential functions to model real-life situations • Strengthening understanding of logarithms • Reviewing and applying the principles that $a^b \bullet a^c = a^{b+c}$ and $(a^b)^c = a^{bc}$ • Understanding and using the fact that $a^{log}a^{\ b} = b$ • Discovering that any exponential function can be expressed using any positive number other than 1 as a base • Learning the meaning of the terms *natural logarithm* and *common logarithm* • Using an exponential function to fit a curve to numerical data ***The Number* e *and Compound Interest*** • Estimating the value of b for which the function $y = b^x$ has a derivative at each point on its graph equal to the y-value at that point • Developing and using a formula for compound interest	formulas from coordinate geometry, including: • Distance formula? • Midpoint formula? • Equation of a circle with arbitrary center and radius? Can students find the distance from a point to a line? Do students understand the meaning of the derivative of a function at a point and its relationship to instantaneous rate of change? Can students approximate the value of a derivative at a given point?	

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Seeing that expressions of the form $(1+1/n)^n$ have a limiting value, called *e*, as *n* increases without bound • Learning that the limiting value *e* is the same number as the special base for exponential functions		
Unit Four: Pennant Fever **Timeline: 10 days**			
Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **CC.A-APR.4** – *unit supplement to be developed* Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. **CC.A-APR.5** Use permutations and combinations to compute probabilities of compound events and solve problems. **CC.S-CP.9**	The main concepts and skills that students will encounter and practice during this unit are: ***Probability and statistics*** • Developing a mathematical model for a complex probability situation • Using area diagrams and tree diagrams to find and explain probabilities • Using a simulation to understand a situation, to help analyze probabilities, and to support a theoretical analysis • Finding expected value • Finding and using probabilities for sequences of events • Using specific problem contexts to develop the binomial distribution and finding a formula for the associated probabilities • Using probability to evaluate null hypotheses ***Counting principles*** • Developing systematic lists for complex situations • Using the multiplication principle for choosing one element from each of several sets • Defining and using the concepts of permutation and combination	Can students apply principles for finding the probability for a sequence of events? Can students systematically list possibilities for complex problems? Can students use combinatorial and permutation coefficients in the context of real-world situations, and understanding the distinction between combinations and permutations? Can student use Pascal's triangle? Can students use the binomial distribution? Can students express the physical laws of falling bodies in terms of quadratic functions?	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Understanding and using standard notation for counting permutations and combinations • Developing formulas for the permutation and combinatorial coefficients ***Pascal's triangle and combinatorial coefficients*** • Finding patterns and properties within Pascal's triangle • Recognizing that Pascal's triangle consists of combinatorial coefficients • Explaining the defining pattern and other properties of Pascal's triangle using the meaning of combinatorial coefficients • Developing and explaining the binomial theorem		
Unit Five: High Dive **Timeline: 16 days**			
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **CC.A-SSE.1b** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.* **CC.A-APR.6** – *unit supplement to be developed* *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.* **CC.A-APR.7** – *supplementary unit is being developed by publisher*	The main concepts and skills that students will encounter and practice during this unit are: ***Trigonometry*** • Extending the trigonometric functions to all angles • Reinforcing the importance of similarity in the definitions of the trigonometric functions • Graphing the trigonometric functions and variations on those functions • Defining the inverse trigonometric functions and principal values • Discovering and explaining the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, and other trigonometric identities • Defining polar coordinates and finding rectangular coordinates from	Can students apply right-triangle trigonometry to real-world situations? Can students extend the right-triangle trigonometric functions to circular functions? Can students use trigonometric functions to work with polar coordinates? Can students define radian measure? Can students graph the sine and cosine functions and variations of these functions? Can students use inverse	All assessments are listed at the end of the curriculum map.

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Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.F-IF.7e** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.F-TF.2** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **CC.F-TF.5** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **CC.F-TF.7** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. **CC.F-TF.8**	polar coordinates and vice versa ***Physics*** • Developing quadratic expressions for the height of free-falling objects, based on the principle of constant acceleration • Recognizing that a person falling from a moving object will follow a different path than someone falling from a stationary object ***Quadratic Equations*** • Developing simple quadratic equations to describe the behavior of falling objects	trigonometric functions? Can students apply various trigonometric formulas, including: • The Pythagorean identity? • Formulas for the sine and cosine of a sum of angles? • The law of sines and the law of cosines?	

Assessment Opportunities in this Unit:

End-of-Unit Assessments: Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

On-Going Assessments:
Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills. Students' written and oral work provides many opportunities for teachers to gather this information.

- *Sprinkler in the Orchard*
- *Proving with Distance—Part I or Proving with Distance—Part II*

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- *Polygoning the Circle*
- *Orchard Growth Revisited*
- *Cable Ready*
- *Hiding in the Orchard*
- Presentations of *Programming Puzzles*
- Presentations or write-ups of *Just the Plane Facts*
- *Three Variables, Continued*
- *Matrices in the Oven*
- *Inverses and Equations*
- Presentations of *Meadows or Malls? Revisited*
- *How Many More People?*
- *Points, Slopes, and Equations*
- *Photo Finish*
- *What's It All About?*
- *Slippery Slopes*
- *Return to "A Crowded Place"*
- *Baseball Probabilities*
- *How Likely Is All Wins?*
- *Monthly Matches*
- *Cones from Bowls, Bowls from Cones*
- *Who's on First?*
- *About Bias*
- *Race for the Pennant Revisited*
- *As the Ferris Wheel Turns*
- *Testing the Definition*
- *More Beach Adventures*
- *A Practice Jump*
- *Moving Cart, Turning Ferris Wheel*

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

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Curriculum Framework for Pre-Calculus

School: First State Military Academy **Tool: *Calculus I with Pre-Calculus* (Larson & Edwards)** **Teacher: _____**

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit P: Prerequisites **Timeline : 15 days**			
Interpret expressions that represent a quantity in terms of its context.* **CC.9-12.A.SSE.1** Interpret parts of an expression, such as terms, factors, and coefficients. **CC.9-12.A.SSE.1a** Interpret complicated expressions by viewing one or more of their parts as a single entity. **CC.9-12.A.SSE.1b** Use the structure of an expression to identify ways to rewrite it. **CC.9-12.A.SSE.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*[1] **CC.9-12.A.SSE.3** Factor a quadratic expression to reveal the zeros of the function it defines. **CC.9-12.A.SSE.3a** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **CC.9-12.A.SSE.3b** Use the properties of exponents to transform expressions for exponential functions. **CC.9-12.A.SSE.3c**	Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of	**Essential Questions:** How can solutions to algebraic equations and inequalities be found? How are linear equations and inequalities graphically represented? **Learning Targets:** Students will identify different types of equations. Students will solve linear equations in one variable and equations that lead to linear equations. Students will solve quadratic equations by factoring, extracting square roots, completing the square, and using the quadratic formula. Students will solve polynomial equations of degree three or greater. Students will solve equations using radicals. Students will solve equations with absolute value. Students will represent solutions of linear	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

[1] Specific modeling standards are indicated by a star symbol (*).

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.9-12.A.APR.3** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC.9-12.A.APR.6** Create equations and inequalities in one variable and use them to solve problems. **CC.9-12.A.CED.1** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **CC.9-12.A.CED.2** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context.. **CC.9-12.A.CED.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **CC.9-12.A.CED.4** Explain each step in solving a simple equation as	operations and exponents, and the conventions of algebraic notation. An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	inequalities in one variable. Students will represent use properties of inequalities to create equivalent inequalities and solve inequalities in one variable. Students will solve inequalities involving absolute value. Students will solve polynomial inequalities and rational inequalities. Students will plot points in the Cartesian plane. Students will use the distance formula to find the distance between two points and use the midpoint formula to find the midpoint of a line segment. Students will use a coordinate plane to model and solve real-life problems. Students will sketch graphs of equations. Students will find x- and y- intercepts of graphs of equations. Students will use symmetry to sketch graphs of equations. Students will find equations of and sketch graphs of circles. Students will use graphs of equations in solving real-life problems. Students will use slope to graph linear	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **CC.9-12.A.REI.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.9-12.A.REI.3** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables		equations in two variables. Students will find the slope of a line given two points on the line. Students will write linear equations in two variables. Students will use slope to identify parallel and perpendicular lines. Students will use slope and linear equations in two variables to model and solve real-life problems.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
algebraically and graphically. **CC.9-12.A.REI.7** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.9-12.A.REI.10** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **CC.9-12.G.CO.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **CC.9-12.G.GPE.1** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **CC.9-12.G.GPE.5** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **CC.9-12.G.GPE.6**			
Unit One: Functions and their Graphs **Timeline : 15 days**			
Use the structure of an **CC.9-12.A.SSE.2** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1**	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are	**Essential Questions:** How are functions recognized, represented, and evaluated? How are graphs of functions sketched and analyze? How can graphs of functions be	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems

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Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **CC.9-12.F.IF.1** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.9-12.F.IF.2** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship..* **CC.9-12.F.IF.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* **CC.9-12.F.IF.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **CC.9-12.F.IF.6** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** Write a function defined by an expression in different but equivalent forms to reveal and explain different	purely mathematical. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.	transformed? How can functions be combined? How to find inverse functions? How can functions be used to model mathematical data? **Learning Targets**: Students will determine whether relations between two variables are functions. Students will use function notation and evaluate functions. Students will find the domains of functions. Students will use functions to model and solve real-life problems Students will use the Vertical Line Test for functions. Students will find the zeros of functions. Students will determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions. Students will identify and graph linear functions. Students will identify and graph step and other piecewise-defined functions. Students will identify even and odd functions.	**Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
properties of the function. **CC.9-12.F.IF.8** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.9-12.F.IF.8a** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).**CC.9-12.F.IF.9** Write a function that describes a relationship between two quantities.* **CC.9-12.F.BF.1** Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.9-12.F.BF.1a** Combine standard function types using arithmetic operations. **CC.9-12.F.BF.1b** (+)[2] Compose functions. **CC.9-12.F.BF.1c** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f*(*x*), *f*(*kx*), and *f*(*x + k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **CC.9-12.F.BF.3** Find inverse functions. **CC.9-12.F.BF.4** Solve an equation of the form f(x) = c for a simple		Students will recognize graphs of common functions. Students will use vertical and horizontal shifts to sketch graphs of functions. Students will use reflections to sketch graphs of functions. Students will use non-rigid transformations to sketch graphs of functions. Students will add, subtract, multiply and divide functions. Students will find the composition of one function with another function. Students will use combinations and compositions of functions to model and solve real-life problems. Students will find inverse functions informally and verify that two functions are inverse functions of each other. Students will use graphs of functions to determine whether functions have inverse functions. Students will use graphs of functions to determine whether functions have inverse functions.	

[2] Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by an addition sign (+).

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
function f that has an inverse and write an expression for the inverse. **CC.9-12.F.BF.4a** (+) Verify by composition that one function is the inverse of another. **CC.9-12.F.BF.4b** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. **CC.9-12.F.BF.4c** (+) Produce an invertible function from a non-invertible function by restricting the domain. **CC.9-12.F.BF.4d**		Students will use the Horizontal Line Test to determine if functions are one-on-one. Students will find inverse functions analytically. Students will use mathematical models to approximate sets of data points. Students will use the *regression* feature of a graphing utility to find the equation of a least squares regression line. Students will write mathematical models for direct variation. Students will write mathematical models for direct variations as an n^{th} power. Students will write mathematical models for inverse variation. Students will write mathematical models for joint variation.	
Unit Two: Polynomial and Rational Functions **Timeline: 15 days**			
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **CC.9-12.N.CN.1** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **CC.9-12.N.CN.2** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **CC.9-12.N.CN.3**	Knowledge of number properties in the Real Number System can be use to develop and apply properties of the Complex Number System. How to extend and apply the conceptual understanding of arithmetic structures	**Essential Questions:** How are equations of parabolas and other polynomial functions written and graphed? How are polynomial functions divided to find roots? How are operations performed on complex numbers? How is the domain and asymptotes of a function found?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **CC.9-12.N.CN.4** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. **CC.9-12.N.CN.5** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **CC.9-12.N.CN.6** Solve quadratic equations with real coefficients that have complex solutions. **CC.9-12.N.CN.7** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **CC.9-12.N.CN.8** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **CC.9-12.N.CN.9** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **CC.9-12.A.APR.2**	and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. The concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	**<u>Learning Targets:</u>** Students will analyze graphs of quadratic functions. Students will write quadratic functions in standard form and use the results to sketch graphs of quadratic functions. Students will find minimum and maximum values of quadratic functions in real-life applications. Students will use transformations to sketch graphs of polynomial functions. Students will use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions. Students will find and use zeros of polynomial functions as sketching aids. Students will divide polynomials using long division. Students will use synthetic division to divide polynomials by binomials of the form (x-k/). Students will use the Remainder Theorem and the Factor Theorem. Students will use polynomial division to answer questions about real-life problems. Students will use the imaginary unit i to write complex numbers. Students will add, subtract and multiply	Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.9-12.A.APR.3** Prove polynomial identities and use them to describe numerical relationships. **CC.9-12.A.APR.4** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC.9-12.A.APR.6** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **CC.9-12.A.APR.7** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.9-12.A.REI.2** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** Solve quadratic equations by inspection (e.g., for $x^2 =$		complex numbers. Students will use complex conjugates to write the quotient of two complex numbers in standard form. Students will find complex solutions of quadratic equations. Students will understand and use the Fundamental Theorem of Algebra. Students will find all the zeros of a polynomial function. Students will write a polynomial function with real coefficients, given its zeros. Students will find the domains of rational functions. Students will find the vertical and horizontal asymptotes of graphs of rational functions. Students will analyze and sketch graphs of rational functions. Students sketch graphs of rational functions that have start asymptote. Students will use rational functions to model and solve real-life problems.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.9-12.F.IF.7b** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7c** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7d** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret			

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
these in terms of a context. **CC.9-12.F.IF.8a** Use the properties of exponents to interpret expressions for exponential functions. **CC.9-12.F.IF.8b**			
Unit Three: Limits and Their Properties **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Calculus, along with geometric and analytic information, can explain the observed local and global behavior of a function. Limits can be determined using algebra, graphs and/or tables of data. The concept of a limit is one of the foundations of Calculus. The limit of a function is the value approached by f(x) as x is approaching a given value or infinity.	**Essential Questions:** What is the limit of a function? How is it found and interpreted? How are limits evaluated when they cannot be solved through direct substitution? What are the limits of functions at infinity and limits of sequences? How can limits be used in finding areas as defined by a function? **Learning Targets:** Students will understand what calculus is and how it compares with pre-calculus. Students will understand that the tangent line problem is basic to calculus. Students will understand that the area problem is also basic to calculus. Students will estimate a limit using a numerical or graphical approach. Students will learn different ways that a limit can fail to exist.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will study and use a formal definition of limit. Students will evaluate a limit using properties of limits. Students will develop and use a strategy for finding limits. Students will evaluate a limit using dividing out and rationalizing techniques. Students will evaluate a limit using the Squeeze Theorem. Students will determine continuity at a point and continuity on an open interval. Students will determine one-sided limits and continuity on a closed interval. Students will use properties of continuity. Students will understand and use the Intermediate Value Theorem. Students will determine infinite limits from the left and from the right. Students will find and sketch the vertical asymptotes of the graph of a function.	
Unit Four: Differentiation **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving	The derivative is the instantaneous rate of change at a given point. Derivatives can be	**Essential Questions:** What is the derivative of a given function? What are related rates and how are they found?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	used to analyze curves. Derivatives can be used to model rates of change. Derivatives can be used in optimization problems.	**Learning Targets:** Students will find the slope of the tangent line to a curve at a point. Students will use the limit definition to find the derivative of a function. Students will understand the relationship between differentiability and continuity. Students will find the derivative of a function using the Constant Rule. Students will find the derivative of a function using the Power Rule. Students will find the derivative of a function using the Constant Multiple Rule. Students will find the derivative of a function using the Sum and Difference Rules. Students will use derivatives to find rates of change. Students will find the derivative of a function using the Product Rule. Students will find the derivative of a function using the Quotient Rule. Students will find the higher-order derivative of a function. Students will find the derivative of a composite function using the Chain Rule.	problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will find the derivative of a function using the General Power Rule. Students will simplify the derivative of a function using algebra. Students will distinguish between functions written in implicit form and explicit form. Students will use implicit differentiation to find the derivative of a function. Students will find a related rate. Students will use related rates to solve real-life problems.	
Unit Five: Applications of Differentiation **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The limit of a function is the value approached by f(x) as x is approaching a given value or infinity. The first derivative determines increasing or decreasing and the second derivative determines concavity.	**Essential Questions:** In what ways can derivatives be used to determine characteristics of a function? **Learning Targets:** Students will understand the definition of extrema of a function on an interval. Students will understand the definition of relative extrema of a function on an open interval. Students will find extrema on a closed interval. Students will understand and use Rolle's Theorem. Students will understand and use the Mean Value Theorem.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will determine intervals on which a function is increasing or decreasing. Students will apply the First Derivative Test to find relative extrema of a function. Students will determine intervals on which a function is concave upward or concave downward. Students will find any points of inflection of the graph of a function. Students will apply the Second Derivative Test to find relative extrema of a function. Students will determine (finite) limits at infinity. Students will determine the horizontal asymptotes, if any, of the graph of a function. Students will determine infinite limits at infinity. Students will analyze and sketch the graph of a function. Students will use calculus to solve applied minimum and maximum problems. Students will understand the concept of a tangent line approximation. Students will compare the value of the differential, *dy*, with the actual change in *y*, Δy.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will estimate a propagated error using a differential. Students will find the differential of a function using differentiation formulas.	
Unit Six: Integration **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Integration is a summation process. The Fundamental Theorems of Calculus relate differentiation and integration as inverse functions. Antiderivatives follow directly from derivatives. Antiderivatives can be used to solve initial condition problems, including separable differential equations. There are several numerical techniques to approximate the definite integral.	**Essential Questions:** In what ways can you evaluate integrals? **Learning Targets:** Students will write the general solution of a differential equation. Students will use indefinite integral notation for antiderivatives. Students will use basic integration rules to find antiderivatives. Students will find a particular solution of a differential equation. Students will use sigma notation to write and evaluate a sum. Students will understand the concept of area. Students will use rectangles to approximate the area of a plane region. Students will find the area of a plane region using limits. Students will understand the definition of a Riemann sum.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will evaluate a definite integral using limits. Students will evaluate a definite integral using properties of definite integrals. Students will evaluate a definite integral using the Fundamental Theorem of Calculus. Students will understand and use the Mean Value Theorem for Integrals. Students will find the average value of a function over a closed interval. Students will understand and use the Second Fundamental Theorem of Calculus. Students will understand and use the Net Change Theorem. Students will use pattern recognition to find an indefinite integral. Students will use a change of variables to find an indefinite integral. Students will use the General Power Rule for Integration to find an indefinite integral. Students will use a change of variables to evaluate a definite integral. Students will evaluate a definite integral involving an even or odd function. Students will approximate a definite integral	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		using the Trapezoidal Rule. Students will approximate a definite integral using Simpson's Rule. Students will analyze the approximate errors in the Trapezoidal Rule and Simpson's Rule.	
Unit Seven: Systems of Equations and Matrices (WEB) **Timeline: 15 days**			
(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.9-12.N.VM.6** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **CC.9-12.N.VM.7** (+) Add, subtract, and multiply matrices of appropriate dimensions. **CC.9-12.N.VM.8** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **CC.9-12.N.VM.9** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **CC.9-12.N.VM.10** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **CC.9-12.N.VM.11**	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	**Essential Questions:** How is the solution to a system of equations calculated? How is the solution to a system of inequalities calculated? **Learning Targets:** Students will write, graph, and solve systems of linear equations in two variables. Students will write, graph, and solve multivariable linear systems. Students will write, graph, and solve systems of inequalities. Students will use matrices to model and solve systems of equations. Students will operate on matrices. Students will find and use the inverse of a square matrix. Students will find and use the determinant of a square matrix.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **CC.9-12.N.VM.12** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **CC.9-12.A.REI.7** (+) Represent a system of linear equations as a single matrix equation in a vector variable. **CC.9-12.A.REI.8** (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.9-12.A.REI.9** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.9-12.A.REI.10** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find		Students will use Crammer's rule to solve problems.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* **CC.9-12.A.REI.11** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **CC.9-12.A.REI.12**			
Unit Eight: Exponential and Logarithmic Functions **Timeline: 15 days**			
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.* **CC.9-12.N.RN.1** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **CC.9-12.N.RN.2** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **CC.9-12.N.RN.3** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** Graph exponential and logarithmic functions, showing intercepts and end behavior, and	Writing, graphing, and, recognizing characteristics of exponential and logarithmic functions. The properties of exponential and logarithmic functions are used to solve equations, and model and solve real-world problems.	**Essential Questions:** How are exponential functions written, evaluated, and graphed? How are logarithmic functions written, evaluated, and graphed? How are exponential and logarithmic functions used to solve real-world problems? **Learning Targets:** Students will recognize and evaluate exponential functions with base *a.* Students will graph exponential functions. Students will recognize, evaluate and graph exponential functions with base *e.* Students will use exponential functions to model and solve real-life problems. Students will recognize and evaluate logarithmic functions with base *a.*	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. **CC.9-12.F.IF.8b** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.9-12.F.IF.9** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **CC.9-12.F.BF.5** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.9-12.F.LE.1** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.9-12.F.LE.1a** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.		Students will graph logarithmic functions. Students will recognize, evaluate and graph natural logarithmic functions. Students will use logarithmic functions to model and solve real-life problems. Students will use the change-of-base formula to rewrite and evaluate logarithmic expressions. Students will use properties of logarithms to evaluate or rewrite logarithmic expressions. Students will use properties of logarithms to expand or condense logarithmic expressions. Students will use logarithmic functions to model and solve real-life problems. Students will solve simple exponential and logarithmic equations. Students will solve more complicated exponential equations. Students will solve more complicated logarithmic equations. Students will use exponential and logarithmic equations to model and solve real-life problems. Students will recognize the five most common types of models involving exponential and logarithmic functions.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
CC.9-12.F.LE.1b Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **CC.9-12.F.LE.1c** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.LE.2** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **CC.9-12.F.LE.3** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **CC.9-12.F.LE.4** Interpret the parameters in a linear or exponential function in terms of a context. **CC.9-12.F.LE.5**		Students will use exponential growth and decay functions to model and solve real-life problems. Students will use Gaussian functions to model and solve real-life functions. Students will use logistic growth functions to model and solve real-life problems. Students will use logarithmic functions to model and solve real-life problems.	
Unit Nine: Trigonometric Functions **Timeline: 15 days**			
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.9-12.F.TF.1** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.9-12.F.TF.2** (+) Use special triangles to determine geometrically	Real-life quantities and triangle relationships can be modeled and solved using Trigonometric functions. The basic characteristics of trigonometric	**Essential Questions:** How can angles be described, radian measures be used, and degree measures be used? How can trigonometric functions be evaluated using the unit circle? How are trigonometric functions evaluated for acute angles?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
the values of sine, cosine, tangent for □π/3,□ π/4 and□ π/6, and use the unit circle to express the values of sine, cosine, and tangent for □π-*x*,□ π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. **CC.9-12.F.TF.3** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **CC.9-12.F.TF.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* **CC.9-12.F.TF.5** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **CC.9-12.F.TF.6** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.7** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.9-12.G.SRT.6** Explain and use the relationship between the sine and cosine of complementary angles. **CC.9-12.G.SRT.7** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* **CC.9-12.G.SRT.8**	functions, their inverses, and their reciprocals are evaluated and graphed. Introduction to radian measures and the unit circle. Trigonometric functions can be used to solve real-world problems	How are fundamental trigonometric identities used? How can reference angles be used to evaluate trigonometric functions of any angle? How can the graphs of sine and cosine be sketched? How can the graphs of tangent, cotangent, secant, and cosecant be sketched? How can inverse trigonometric functions be evaluated? How are real-life problems involving right triangles, directional bearings, and harmonic motion solved? **<u>Learning Targets:</u>** Students will describe angles. Students will use radian measure. Students will use degree measure. Students will use angles to model and solve real-life problems. Students will identify a unit circle and describe its relationship to real numbers. Students will evaluate trigonometric functions using the unit circle. Students will use the domain and period to evaluate sine and cosine functions.	Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will use a calculator to evaluate trigonometric functions. Students will evaluate trigonometric functions of acute angles. Students will use fundamental trigonometric identities. Students will use trigonometric functions to model and solve real-life problems. Students will evaluate trigonometric functions of any angle. Students will use reference angles to evaluate trigonometric functions. Students will sketch the graphs of basic sine and cosine functions. Students will use amplitude and period to help sketch the graphs of sine and cosine functions. Students will sketch translations of the graphs of sine and cosine functions. Students will use sine and cosine functions to model real-life data. Students will sketch the graphs of tangent functions. Students will sketch the graphs of cotangent functions.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will sketch the graphs of secant and cosecant functions. Students will sketch the graphs of damped trigonometric functions. Students will evaluate and graph the inverse sine function. Students will evaluate and graph the other inverse trigonometric functions. Students will evaluate and graph the compositions of trigonometric functions. Students will solve real-life problems involving right triangles. Students will solve real-life problems involving directional bearings. Students will solve real-life problems involving harmonic motion.	
Unit Ten: Topics in Analytic Geometry **Timeline: 15 days**			
(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **CC.9-12.G.GPE.3**	Classifying conic sections by their equations is used when solving problems. Conic sections are rotated to simplify their equation. Parametric and polar equations are both written and graphed.	**Essential Questions:** How is each conic section recognized? How are problems involving parabolas, ellipses, and hyperbolas solved? How is a conic section classified on the basis of its general equation? How can equations describe the motion of a point on a plane?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
	Represent and solve problems involving conic sections by using polar coordinates.	Why is the position of a point on a plane described using distance and angle rather than x- and y-coordinates? What do graphs of polar equations look like? How are conic sections on polar coordinates represented? **Learning Targets:** Students will recognize a conic as the intersection of a plane and double-napped cone. Students will write equations of parabolas in standard from and graph parabolas. Students will use the reflective property of parabolas to solve real-life problems. Students will write equations of ellipses in standard form and graph ellipses. Students will use implicit differentiation to find the slope of a line tangent to an ellipse. Students will use properties of ellipses to model and solve real-life problems. Students will find eccentricities of ellipses. Students will write equations of hyperbolas in standard form. Students will find asymptotes of and graph hyperbolas.	Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will use implicit differentiation to find the slope of a line tangent to a hyperbola. Students will use properties of hyperbolas to solve real-life problems. Students will classify conics from their general equations. Students will evaluate sets of parametric equations for given values of the parameter. Students will sketch curves that are represented by sets of parametric equations. Students will rewrite sets of parametric equations as single rectangular equations by eliminating the parameter. Students will find sets of parametric equations for graphs. Students will find the slope of a tangent line to a curve given by a set of parametric equations. Students will understand the polar coordinate system. Students will rewrite rectangular coordinates and equations in polar form and vice versa. Students will find the slope of a tangent line to a polar graph. Students will graph polar equations by point plotting.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will use symmetry, zeros and maximum *r*-values to sketch graphs of polar equations. Students will recognize special polar graphs. Students will define conic in terms of eccentricity. Students will write and graph equations of conic in polar form. Students will use equations of conic in polar form to model real-life problems.	
Unit Eleven: Additional Topics in Trigonometry **Timeline: 15 days**			
(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, $\|v\|$, $\|\|v\|\|$, *v*). **CC.9-12.N.VM.1** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **CC.9-12.N.VM.2** (+) Solve problems involving velocity and other quantities that can be represented by vectors. **CC.9-12.N.VM.3** (+) Add and subtract vectors. **CC.9-12.N.VM.4** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **CC.9-12.N.VM.4a**	Trigonometry is applicable to oblique triangles, vectors, and complex numbers. By applying vectors and vector notation, trigonometry is used to solve real-world problems. Writing and operating using complex numbers in trigonometry.	**Essential Questions:** What trigonometry skills are used to solve and find areas of oblique triangles? What are vector quantities and two vector components? How are they represented and evaluated? How is trigonometry used to represent and perform operations on complex numbers? **Learning Targets:** Students will use the Law of Sines to solve oblique triangles (AAS, ASA, or SSA). Students will find the areas of oblique triangles. Students will use the Law of Sines to model and solve real-life problems.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **CC.9-12.N.VM.4b** Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **CC.9-12.N.VM.4c** (+) Multiply a vector by a scalar. **CC.9-12.N.VM.5** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **CC.9-12.N.VM.5a** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). **CC.9-12.N.VM.5b** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **CC.9-12.G.SRT.10** (+) Derive the formula A=1/2absin(c)) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **CC.9-12.G.SRT.9** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **CC.9-12.G.SRT.11**		Students will use the Law of Cosines to solve oblique triangles (SSS or SAS). Students will use Heron's Area Formula to find the area of a triangle. Students will use the Law of Cosines to model and solve real-life problems. Students will represent vectors as directed line segments. Students will write the component forms of vectors. Students will perform basic vector operations and represent them graphically. Students will write vectors as linear combinations of unit vectors. Students will find the direction angles of vectors. Students will use vectors to model and solve real-life problems. Students will find the dot product of two vectors and use the properties of the dot product. Students will find the angle between two vectors and determine whether two vectors are orthogonal. Students will write a vector as the sum of two vector components.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Students will use vectors to find the work done by a force. Students will plot complex numbers in the complex plane and find absolute values or complex numbers. Students will write the trigonometric forms of complex numbers. Students will multiply and divide complex numbers written in trigonometric form. Students will use DeMoivre's Theorem to find powers of complex numbers. Students will find *n*th roots of complex numbers.	

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Curriculum Framework for Statistics

School: **First State Military Academy** **Curricular Tool:** ***Elementary Statistics*** **Teacher:** _____

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Statistics **Timeline : 2 weeks**			
Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **CC.9-12.S.IC.1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **CC.9-12.S.IC.3** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.9-12.S.IC.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.4**	Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation. In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of	Understand and be able to describe the difference between descriptive and inferential statistics Understand and be able to identify and interpret the relationships between sample and population. Know and be able to identify and describe the different types of variables Understand how conveniences and volunteer samples result in biased samples Understand the differences among and be able to identify experiments, observational studies, and judgment samples. Understand and be able to describe the single-stage sampling methods of “simple random sample” and “systematic sampling’ Understand and be able to describe the multistage sampling methods of “stratified sampling” and “cluster sampling” Understand that variability is inherent in everything and in the sampling process	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	independence and conditional probability, which can be approached through the analysis of two-way tables.		
Unit Two: Descriptive Analysis and Presentation of Single-Variable Data **Timeline: 1 week**			
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **CC.9-12.S.CP.1** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **CC.9-12.S.CP.2** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **CC.9-12.S.CP.3** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly*	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments	Create and interpret graphical displays, including pie charts, bar graphs, Pareto diagrams, dotplots, and stem-and-leaf plots displays Understand and be able to describe the difference between grouped and ungrouped frequency distributions, frequency and relative frequency and cumulative relative frequency Identify and describe the parts of a frequency distribution; class boundaries, class width, and class midpoint Create and interpret frequency histograms, and relative frequency histograms Identify the shapes of distributions Compute, describe, and compare the four measures of central tendency: mean, median, mode and midrange. Understand the effect of outliers on each of the four measures of central tendency Compute, describe, compare, and interpret the two measures of position: quartiles, percentiles, and z-scores. Create and interpret boxplots Understand the empirical rule and Chebyshev's	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. **CC.9-12.S.CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **CC.9-12.S.CP.5**		theorem and be able to assess a set of data's compliance to these rules Know when and when not to use certain statistics-graphic and numeric	
Unit Three: Linear Systems and Matrices **Timeline: 1 week**			
(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.9-12.N.VM.6** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.9-12.A.REI.7** (+) Represent a system of linear equations as a single matrix equation in a vector variable.	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing	Understand and be able to present and describe data in the form of two qualitative variables, both in contingency table format and appropriate graphs Understand and be able to present and describe data in the form of one qualitative variable and one quantitative variable, in both table format and appropriate graphs. Understand and be able to present and describe the relationship between two quantitative variables using a scatter diagram Understand and be able to explain a linear relationship Compute, describe, and interpret a correlation coefficient Compute, describe, and interpret a line of best fit Define and understand the difference between correlation and causation	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
CC.9-12.A.REI.8 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.9-12.A.REI.9** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **CC.9-12.S.ID.7**	predictions with data. Causation implies correlation yet correlation does not imply causation.	Determine and explain possible luring variables and their effects on a linear relationship Understand and be able to explain the slope of the line of best fit with respect to the context it is presented in Understand and be able to explain the y-intercept of the line of best fit with respect to the context it is presented in Create a scatter diagram with the line of best fit drawn on it Compute prediction values based on the line of best fit Understand and be able to explain what predication values are. Understand that predictions should be made lonely for values within the sample domain and that caution must be exercised for values outside that domain.	
Unit Four: Probability **Timeline: 1 week**			
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.9-12.S.IC.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.	Understand and be able to describe the basic concept of probability Understand and describe a simple event Understand and be able to describe the differences between empirical, theoretical, and subjective probabilities Compute and interpret relative frequencies Identify and describe a sample space for an	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
determine if they are independent. **CC.9-12.S.CP.2** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **CC.9-12.S.CP.3** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **CC.9-12.S.CP.5**	Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. .	experiment Construct tables, tree diagrams, and/or Venn diagrams to aid in computing and interpreting probabilities Understand the properties of probability numbers: 1. $0 \leq$ each $P(A) \leq 1$ 2. $\sum P(A) = 1$ Understand , describe, and use the law of large numbers to determine probabilities Understand, compute, and interpret odds of an event Understand that compound events involve the occurrence of more than one event Construct, describe, compute, and interpret a conditional probability Understand and be able to utilize the complement rule Compute probabilities of compound events using the addition rule. Compute probabilities of compound events using the multiplication rule Understand, describe, and determine mutually exclusive events Compute probabilities of compound events using the addition rule for mutually exclusive events Understand , describe, and determine independent events	

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Compute probabilities of compound events using the multiplication rule for independent events Recognize and compare the differences between mutually exclusive events and independent events	
Unit Five: Probability Distributions (Discrete Variables) **Timeline: 1 week**			
(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **CC.9-12.S.MD.3**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments	Understand that random variables is a numerical quantity whose value depends on the conditions and probabilities associated with an experiment Understand the difference between a discrete and a continuous random variable Be able to construct a discrete probability distribution based on an experiment or given function Understand the terms *mutually exclusive* and *all-inclusive* as they apply to the variable for probability distributions Understand the similarities and differences between frequency distributions and probability distributions Understand and be able to utilize the two main properties of probability distribution to verify compliance Understand that a probability distribution in a theoretical probability distribution and that the mean and standard deviation (μ and σ, respectively) are parameters. Compute, describe, and interpret the mean and standard deviation of a probability distribution	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Understand the key elements of a binomial experiment and be able to define *x* ,*n* ,*p* and *q*. Know and be able to calculate binomial probabilities using the binomial probability function Understand and be able to use Table 2 in Appendix B, Binomial Probabilities, to determine binomial probabilities. Compute, describe, and interpret the mean and standard deviation of a binomial probability distribution	
Unit Six: Normal Probability Distributions **Timeline: 1 week**			
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **CC.9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **CC.9-12.S.ID.4**	Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation.	Understand the difference between a discrete and continuous random variable Understand the relationship between the empirical rule and the normal cure Understand that a normal curve is a bell-shaped curve, with total area under the curve equal to 1 Understand that the normal curve is symmetrical about the mean , with an area of 0.5000 on each side of the mean Be able to draw a normal curve, labeling the mean and various *z*-scores Understand and be able to use Table 3, Areas of the Standard Normal Distributions, in Appendix B Calculate probabilities for intervals defined on the standard normal distribution	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Determine *z*-values for corresponding intervals on the standard normal distribution. Compute, describe, and interpret a *z*-value for a data value from a normal distribution Compute *z*-scores and probabilities for applications of the normal distribution Draw, compute, and interpret *z* of alpha notation, $z(\alpha)$ Understand the key elements of a binomial experiment: *x, n, p, q.* Know its mean and standard deviation formulas Understand that the normal distribution can be used to calculate binomial probabilities, provided certain conditions are met Understand and be able to use the continuity correction factor when calculating *z*-scores. Compute *z*-scores and probabilities for normal approximation to the binomial	
Unit Seven: Sample Variability **Timeline: 1 week**			
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.9-12.S.IC.4**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.	Understand what a sampling distribution of a sample statistic is and that the distribution is obtained from repeated samples, all of the same size Be able to form a sampling distribution for a mean, median, or range based on a small, finite population Understand that a sampling distribution is a probability distribution for a sample statistic.	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.	Understand and be able to present and describe the sampling distribution of sample means and the central limit theorem Understand and be able to explain the relationship between the sampling distribution of sample means and the central limit theorem Determine and be able to explain the effect of sample size on the standard error of the mean Understand when and how the normal distribution can be used to find probabilities corresponding to sample means Compute, describe, and interpret *z*-scores corresponding to known values of $\bar{x}$ Compute *z*-scores and probabilities for applications of the sampling distribution of sample means	
Unit Eight: Introduction to Statistical Inferences **Timeline: 2 weeks**			
Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **CC.9-12.S.IC.1**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random	Understand the difference between descriptive statistics and inferential statistics. Understand that an unbiased statistic has a sampling distribution with a mean that is equal to the population parameter being estimated. **With respect to confidence intervals:** Understand that a confidence interval is an interval estimate of a population parameter, with a degree of certainty, used when the population parameter is unknown	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.	Understand that a point estimate for a population parameter is the value of the corresponding sample statistic Understand that the level of confidence is the long-run proportion of the intervals, which will contain the true population parameters, based on repeated sampling Understand and be able to describe the key components for a confidence interval: point estimate, level of confidence, confidence coefficient, maximum error of estimate, lower confidence limit, and upper confidence limit	
Unit Nine: Applications of Chi-Square **Timeline : 1 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Chi-Square Statistic Inferences Concerning Multinomial Experiments Inferences Concerning Contingency Tables	Understand that enumerative data are data that can be counted and placed into categories Understand that the chi-square distributions will be used to test hypotheses involving enumerative data Understand the properties of the chi-square distribution and how series of distribution based on sample size (using degrees of freedom as the index) Understand the key elements of multinomial experiment and be able to define n, k, O_i, and P_i Know and be able to calculate $E = np$ Know and be able to calculate a chi-square statistic: $x^2 = \sum_{(\text{all cells})} (o - E)^2/E$ Know and be able to calculate the degrees of freedom for a multinomial experiment (df = k – 1)	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Perform, describe, and interpret a hypothesis test for a multinomial experiment using the chi-square distribution with the *p*-value approach and/or the classical approach Understand and know the definition of independence of two events Know and be able to calculate expected values using $E_{ij} = R_1 \bullet C_j/n$ Know and be able to calculate the degrees of freedom for a test of independence or homogeneity [df = (r – 1)(c – 1) Perform , describe, and interpret a hypothesis test for a test of independence or homogeneity using the chi-square distribution with the *p*-value approach and/or the classical approach Understand the differences and similarities between tests of independence and tests of homogeneity	
Unit Ten: Analysis of Variance **Timeline: 1 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	Introduction to the Analysis of Variance Technique The Logic behind ANOVA Applications of Single-Factor ANOVA	Understand that analysis of variance technique (ANOVA) are use to test differences among more than two means Understand that ANOVA uses variances to complete the testing of several means Understand that the *F*-distribution is used to test the ratio of the variation between the means being tested to the variation within the samples being tested Understand that if the variation between the means	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		is significantly more than the variation within the samples ,then the means are considered unequal Compute, describe, and interpret a hypothesis test for the differences among several means, using the *F*-distribution with the *p*-value approach and/or the classical approach	
Unit Eleven: Linear Correlation and Regression Analysis **Timeline: 1 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Linea Correlation Analysis Inferences about the Linear Correlation Coefficient Linear Regression Analysis Inferences Concerning the Slope of the Regression Line Confidence Intervals for Regression Understanding the Relationship between Correlation and Regression	Understand what bivariate data, independent variable, and dependent variable are Understand that the linear correlation coefficient, *r*, measures the strength of the linear relationship between two variables Understand that the centroid for bivariate data is $(\bar{x}, \bar{y})$. Understand that the centroid is used in the calculation of the correlation coefficient Understand that covariance is a measure of linear dependency but that it is affected by the spread of the data Understand that the correlation coefficient , *r*, standardizes covariance so that relative strengths can be compared Understand that the assumptions for inferences about the linear correlation coefficient are that the ordered pairs form a random sample and that the *y* values at each *x* have a normal distribution. Inferences will utilize the *t*-distribution using $(n - 2)$ degrees of freedom Compute, describe, and interpret a confidence	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		interval for the population correlation coefficient, *p*, using Table 10 in Appendix B Perform, describe, and interpret a hypothesis test for the population correlation coefficient, *p*, using the *t*-distribution with the *p*-value approach and classical approach Understand that the significance of *r* does not imply a cause-and-effect relationship Understand that the estimate of the experimental error ,*e*, is the difference between the observed *y* and the predicted *y*, $(y - y)$, at a given value of *x* Understand that the variance about the line of best fit is the same as the variance of the error, *e* Understand that the line of best fit passes through the centroid Compute, describe, and interpret a confidence interval for population slope of the regression line, β_1, using the *t*-distribution Perform, describe, and interpret a confidence interval for population slope of the regression line, β_1, using the *t*-distribution with the *p*-value approach and classical approach Compute, describe, and interpret a confidence interval for the mean value of *y* for a particular *x*, $(\mu_{y/x0})$, using the *t*-distribution. Compute, describe and interpret a prediction interval for an individual value of *y* for a particular *x*, (y_{x0}), using the *t*-distribution Understand the difference between a confidence interval and a prediction interval for a *y* value at a particular *x* value	

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit Twelve: Elements of Nonparametric Statistics **Timeline: 1 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Nonparametric Statistics The Sign Test The Mann-Whitney *U* Test The Runs Test Rank Correlation	Understand that parametric methods are statistical methods that assume that the parent population is approximately normal or that the central limit theorem gives (at least approximately) a normal distribution of a test statistic Understand that nonparametric methods (distribution –free methods) do not depend on the distribution of the population being sampled Understand that the power of a test (1- β) is its ability to reject a false null hypothesis Understand that efficiency of a nonparametric test takes into account the power of a test and the required sample size. Understand that the sign test is the nonparametric alternative to the *t*-test for one mean and the difference between two dependent means Compute, describe, and interpret a confidence interval for a population median using the sign test Perform, describe, and interpret a hypothesis test for a single median using the sign test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the median of paired differences using the sign test with the *p*-value approach and classical approach Understand that the Mann-Whitney *U* test is the nonparametric alternative to the *t*-test for the difference between two independent means	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Perform, describe, and interpret a hypothesis test for the difference between two means using the Mann- Whitney *U* test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the difference between two means using the normal approximations to the Mann-Whitney *U* test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the randomness of data using the runs test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the randomness of data using normal approximation to the runs test with the *p*-value approach and classical approach Understand that the Spearman rank correlation coefficient is the nonparametric alternative to the Pearson linear correlation coefficient, *r*. Perform, describe, and interpret a hypothesis test for the significance of correlation between two variable s using the Spearman rank correlation coefficient with the *p*-value approach and classical approach	

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Curriculum Framework for Calculus

School: First State Military Academy Curricular Tool: *Calculus* 9th Ed. (Larson & Edwards) Teacher: _____

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Limits and Their Properties **Timeline : 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Introduction/Preview of Calculus Finding limits Evaluating limits analytically Continuity and one-sided limits Infinite limits	**Essential Questions:** How does calculus compare with pre-calculus? What are the two basics of calculus? How are limits found graphically and numerically upon existence? How are limits evaluated analytically: using properties of limits, dividing out, rationalizing techniques, and the Squeeze Theorem? How can continuity be determined at a point and on an open interval, How are one-sided limits determined? What is the Intermediate Value Theorem and how is it applied? How are infinite limits determined and vertical asymptotes found? **Learning Targets:** Understand what calculus is and how it compares to pre-calculus. Understand that the tangent line problem is basic to calculus. Understand that the area problem is basic to calculus. Estimate a limit using numerical or graphical approaches. Learn different ways that a limit can fail to exist. Use the formal definition of a limit. Evaluate and find a limit.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Determine continuity at a point and continuity on an open interval. Determine one-sided limits and continuity on a closed interval. Use properties of continuity. Understand and use the Intermediate Value Theorem. Determine infinite limits from the left and from the right. Find and sketch the vertical asymptotes of the graph of a function.	
Unit Two: Differentiation **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated	The derivative and the tangent line Basic differentiation rules and rates of change Product and quotient rules and high-order derivatives The chain rule Implicit differentiation Related rates	**Essential Questions:** Why is understanding the definition of limit important when finding the derivation of a function? What is the relationship between differentiability and continuity? How is the derivative of a function found using: ▪ basic differentiation rules? ▪ the Product Rule? ▪ the Quotient Rule? ▪ the Chain Rule? ▪ the General Power Rule? ▪ implicit differentiation? What is related rate? **Learning Targets:** Find the slope of the tangent line to a curve at a point. Use the limit definition to find the derivative of a function. Understand the relationship between differentiability and continuity.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
reasoning.		Find the derivative of a function using the Constant Rule. Find the derivative of a function using the Power Rule. Find the derivative of a function using the Constant Multiple Rule. Find the derivative of a function using the Sum and Difference Rules. Find the derivative of a sine and cosine function. Use derivatives to find rates of change. Find the derivative of a function using the Product Rule. Find the derivative of a function using the Quotient Rule. Find the derivative of a trigonometric function. Find a higher-order derivative of a function. Find the derivative of a function using the Chain Rule. Find the derivative of a function using the General Power Rule. Simplify the derivative of a function using algebra. Find the derivative of a trigonometric function using the Chain Rule. Distinguish between functions written in implicit from and explicit form. Use implicit differentiation to find the derivative of a function. Find a related rate.	
Unit Three: Applications of Differentiation **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common	Extrema on an Interval	**Essential Questions:** How is a derivative used to locate the minimum and maximum values	**Suggested Formative Assessment:**

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Rolle's Theorem and the Mean Value Theorem Increasing and Decreasing Functions and the First Derivative test Concavity and the Second Derivative test Limits an Infinity A summary of the curve Sketching Optimization problems Newton's Method Differentials	of a function on a closed interval? How do numerous results in this chapter depend on two important theorems called Rolle's Theorem and the Mean Value Theorem? How is the first derivative used to determine whether a function is increasing or decreasing? How is the second derivative used to determine whether the graph of a function is concave upward or concave downward? What is the horizontal asymptotes of the graph of a function? How are optimization problems solved? How are approximation techniques used to solve problems? **Learning Targets:** Understand the definition of extrema of a function on an interval. Understand the definition of relative extrema of a function on an open interval. Find extrema on a closed interval. Understand and use the Rolle's Theorem and the Mean Value Theorem. Determine intervals on which a function is increasing or decreasing. Apply the First Derivative Test to find relative extrema of a function. Determine intervals on which a function is concave upward or concave downward. Find any points of inflection of the graph of a function. Apply the Second Derivative Test to find extrema of a function.	Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Determine finite and infinite limits at infinity. Determine the horizontal asymptotes, if any, of a graph of function. Analyze and sketch the graph of a function. Solve and apply minimum and maximum problems. Approximate a zero of a function using Newton's Method. Understand the concept of a tangent line approximation. Compare the value of a differential, *y*. Estimate a propagated error using a differential. Find the differential of a function using differentiation formulas.	
Unit Four: Integration **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically.	Antiderivatives and Indefinite Integration Area Riemann Sums and Definite Integrals The Fundamental Theorem of Calculus Integration by Substitution Numerical Integration	**Essential Questions:** What basic rules are used to evaluate indefinite integrals? How is the sum of a plane region evaluated and approximated? How is a limit used to evaluate a definite integral? How is the Fundamental Theorem of Calculus used to evaluate a definite integral? What methods can be used to evaluate different types of definite and indefinite integrals? How are the Trapezoidal Rule and Simpson's Rule used to approximate a definite integral? **Learning Targets:** Write a general solution of a differential equation.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		Use indefinite integral notation for antiderivatives. Use basic integration rules to find antiderivatives. Find a particular solution of a differential equation. Use sigma notation to write and evaluate a sum. Understand the concept of area and approximate the area of a plane region. Find the area of a plane region using limits. Understand the definition of a Riemann sum. Evaluate a definite integral using limits and properties of definite integrals. Evaluate a definite integral using the Fundamental Theorem of Calculus and the Second Fundamental Theorem of Calculus. Understand and use the Mean Value Theorem for Integrals. Find the average value of a function over a closed interval Use patterns recognition to evaluate an indefinite integral. Use a change of variables to evaluate an indefinite integral. Use the General Power Rule for Integration to evaluate an indefinite integral. Use a change of variables to evaluate a definite integral. Evaluate a definite integral involving an even or odd function.	
Unit Five: Logarithmic, Exponential, and Other Transcendental Functions **Timeline: 12 days**			

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts: evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.7** This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The Natural Logarithmic Function: Differentiation and Integration Inverse Functions Exponential Functions: Differentiation and Integration Bases Other than e and Applications Inverse Trigonometric Functions: Differentiation and Integration Hyperbolic Functions	**Essential Questions:** What are the properties of natural logarithmic functions? How is the derivative and antiderivative of a natural logarithmic function found? Do all functions have an inverse function? What are the properties of the natural exponential function? How are the derivative and antiderivative of the natural exponential function found? What are the properties, derivatives, and antiderivatives of logarithmic and exponential functions that have bases other than *e*? What are the properties of inverse trigonometric functions? How are the derivatives and antiderivatives of inverse trigonometric functions found? What are the properties of hyperbolic functions? How are the derivative and antiderivative of hyperbolic functions found? **Learning Targets:** Develop and use properties of the natural logarithmic function. Understand the definition of the number *e.* Find derivatives of functions involving the natural logarithmic function. Use the Log Rule for Integration to integrate a rational function. Integrate trigonometric functions. Verify that one function is the inverse function of another function. Determine whether a function has an inverse function. Find the derivative of an inverse function. Develop properties of the natural exponential function.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Differentiate natural exponential functions. Integrate natural exponential functions. Define exponential functions that have bases other than *e*. Differentiate and integrate exponential functions that have bases other than *e*. Use exponential functions to model compound interest and exponential growth. Develop properties of the six inverse trigonometric functions. Differentiate an inverse trigonometric function. Review the basic differentiations rules for elementary functions. Integrate functions whose antiderivatives involve inverse trigonometric functions. Use the method of completing the square to integrate a function. Review the basic integration rules involving elementary functions. Develop properties of hyperbolic functions. Differentiate and integrate hyperbolic functions. Develop properties of inverse hyperbolic functions. Differentiate and integrate functions involving inverse hyperbolic functions.	
Unit Six: Differential Equations **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for	Slope fields and Euler's method	**Essential Questions:** How are the slope filed of differential equations and a particular solution found?	**Suggested Formative Assessment:** Lesson Exercises

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Differential Equations: Growth and Decay Separation of Variables and the Logistics Equation First-Order Linear Differential Equations	How can you use an exponential function to model growth and decay? How is the separation of variables used to solve a differential equation? What is a first-order linear differential equation, and a Bernoulli differential equation? **Learning Targets:** Use initial conditions to find particular solutions of differential equations. Use slope fields to approximate solutions of differential equations. Use Euler's Method to approximate solutions of differential equations. Use separation of variables to solve a simple differential equation. Use exponential functions to model growth and decay in applied problems. Recognize and solve differential equations that can be solved by separations of variables. Recognize and solve homogeneous differential equations. Use differential equations to model and solve applied problems. Solve and analyze logistic differential equations. Solve a first-order linear differential equation. Use linear differential equations to solve applied problems. Solve a Bernoulli differential equation.	Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit Seven: Applications of Integration **Timeline: 12 days**			
(+) Give an informal argument using Cavalier's Principle for the formulas for the volume of a sphere and other solid figures. **CC.9-12.G.GMD.2** This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Area of region Between Two Curves Volume: The Disk Method and Shell Method Arc Length and Surfaces of Revolution Work Moments, Centers of Mass, and Centroids Fluid pressure and force	**Essential Questions:** In what ways is the definite integral used to find the area of a region bounded by two curves? How is the volume of a solid of revolution by the disk and shell method calculated? How is the length of a curve and the surface area of a surface of revolution? How is the work done by a constant force and by variable force calculated? What are centers of mass and centriods? What is fluid pressure and fluid force? **Learning Targets:** Find the area of a region between two curves using integration. Find the area of a region between intersecting curves using integration. Describe integration as an accumulation process. Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross sections. Find the volume of a solid of revolution using the shell method. Compare the uses of the disk method and the shell method.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Find the arc length of a smooth curve. Find the area of a surface of revolution. Find the work done by a constant force. Find the work done by a variable force. Understand the definition of mass. Find the center of mass in a one-dimensional system. Find the center of amass in a two-dimensional system. Find the center of mass of a planar lamina. Use Theorem of Pappus to find the volume of a solid of revolution. Find fluid pressure and fluid force.	
Unit Eight Integration Techniques, L'Hopital Rule, and Improper Integrals **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools	Basic Integration Rules Integration by Parts Trigonometric Integrals Trigonometric Substitution Partial Fractions Integration by Tables and other Integration Techniques Indeterminate Forms and L'Hopital Rule Improper Integrals	**Essential Questions:** How does an integrand fit to one of the basic integration rules? How is an antiderivative found using integration by parts? How are trigonometric integrals found? How is trigonometric substitution used to evaluate an integral? How is partial fraction decomposition used to integrate rational functions? Using a table of integrals and using reduction formulas, how can you evaluate an indefinite integral? What is L'Hopital Rule?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		How is an improper integral evaluated? **<u>Learning Targets:</u>** Review procedure for fitting an integrand to one of the basic integration rules. Find an antiderivative using integration by parts. Use a tabular method to perform integration by parts. Solve trigonometric integrals involving powers of sine and cosine. Solve trigonometric integrals involving powers of secant and tangent. Solve trigonometric integrals involving sine-cosine products with different angles. Use trigonometric substitution to solve an integral. Use integrals to model and solve real-life applications. Understand the concept of partial fraction decomposition. Use partial fraction decomposition with linear factors to integrate rational functions. Use partial fraction decomposition with quadratic factors to integrate rational functions. Evaluate an indefinite integral using a table of integrals. Evaluate an indefinite integral using reduction formulas. Evaluate an indefinite integral involving rational functions of sine and cosine. Recognize limits that produce indeterminate forms. Apply L'Hopitals's Rule to evaluate a limit.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Evaluate an improper integral that has an infinite limit of integration. Evaluate an improper integral that has an infinite discontinuity.	
Unit Nine: Infinite Series **Timeline: 12 days**			
(+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example Pascal's Triangle. **CC.9-12.A.APR.5** This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated	Sequences Series and Convergence The Integral Test and p-Series Comparison of Series Alternating series The Ratio and Root tests Taylor Polynomials and Approximations Power Series Representation of Functions by Power Series Taylor and Maclaurin Series	**Essential Questions:** How is it determined whether a sequence converges or diverges? How is it determined whether an infinite series converges or diverges? How is Taylor and Maclaurin polynomial approximation of elementary functions found? What is the radius and interval of convergence of a power series? How are functions by power series represented? What is the Taylor and Maclaurin series for a function? **Learning Targets:** List the terms of a sequence. Determine whether a sequence converges or diverges. Write a formula for the *nth* term of a sequence. Use properties of monotonic sequences and bounded sequences. Understand the definition of a convergent infinite series. Use properties of infinite geometric series. Use the *n*th-Term Test for Divergence of an infinite series. Use the Integral Test to determine whether an infinite series	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
reasoning. .		converges or diverges. Use properties of *p*-series and harmonic series. Use the Direct Comparison Test to determine whether a series converges or diverges. Use the Limit Comparison Test to determine whether a series converges or diverges. Use the Alternating Series Test to determine whether an infinite series converges. Use the Alternating Series Remainder to approximate the sum of an alternating series. Classify a convergent series as absolutely or conditionally convergent. Rearrange an infinite series to obtain a different one. Use the Ratio Test to determine whether a series converges or diverges. Use the Root Test to determine whether a series converges or diverges. Review the test for convergence and divergence of an infinite series. Find polynomial approximations of elementary functions and compare them with the elementary functions. Find Taylor and Maclaurin polynomial approximations of elementary functions. Use the remainder of a Taylor polynomial. Understand the definition of a power series.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Find the radius and interval of convergence of a power series. Determine the endpoint convergence of a power series. Differentiate and integrate a power series. Find a geometric power series that represents a function. Construct a power series using series operations. Find a Taylor or Maclaurin series for a function. Find a binomial series. Use a basic list of Taylor series to find other Taylor series.	
Unit Ten: Conics, Parametric Equations, and Polar Coordinates **Timeline: 12 days**			
(+) Derive the equations of ellipses and Hyperbolaas given the foci, using the fact that the sum or difference of distances from the foci is constant. **CC.9-12.G.GPE.3** This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	Conics and Calculus Plane Curves and Parametric Equations Parametric Equations and Calculus Polar Coordinates and Polar Graphs Area and Arc Length in Polar Coordinates Polar Equations of Conics and Kepler's Laws	**Essential Questions:** In what ways are equations of a parabola, an ellipse, and a hyperbola written and analyzed? What does a sketch of a curve represented by parametric equations look like? How can a set of parametric equations be used to find the slope of a tangent line to a curve and the arc length of a curve? What is the graph of an equation in polar form, the slope of a tangent line to a polar graph, and how are special polar graphs identified? How is the area of a region bounded by a polar graph and the arc length of a polar graph found? What is the polar equation of a conic? **Learning Targets:**	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		Understand the definition of a conic section. Analyze and write equations of parabolas using properties of parabolas. Analyze and write equations of ellipses using properties of ellipses. Analyze and write equations of hyperbolas using properties of hyperbolas. Sketch the graph of a curve given by a set of parametric equations. Eliminate the parameter in a set of parametric equations. Find a set of parametric equations to represent a curve. Understand two classic calculus problems, the tautochrone and brachistochrone problems. Find the slope of a tangent line to a curve given by a set parametric equations. Find the arc length of a curve given by a set of parametric equations. Find the area of a surface of revolution (parametric form). Understand the polar coordinate system. Rewrite rectangular coordinates and equations in polar form and vice versa. Sketch the graph of an equation given in polar form. Find the slope of a tangent line to a polar graph. Identify several types of special polar graphs. Find the area of a region bounded by a polar graph.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Find the points of intersection of two polar graphs. Find the arc length of a polar graph. Find the area of a surface of revolution (polar form). Analyze and write polar equations of conics. Understand and use Kepler's Laws of planetary motion.	
Unit Eleven Vectors and the Geometry of Space **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Vectors in the Plane Space Coordinates and Vectors in Space The Dot Product of Two Vectors The Cross Product of Two vectors in Space Lines and Planes in Space Surfaces in Space Cylindrical and Spherical Coordinates	**Essential Questions:** How are vectors written? What are the basic vector operations? How are vectors represented graphically? Why are points plotted in a three-dimensional coordinate system, and how are vectors analyzed in space? How is the dot product of two vectors (in plane or in space) found? How is the cross product of two vectors (in space) found? How are the equations of lines and planes in space found and graphed? How are equations of cylindrical and quadric surfaces and of surfaces of revolution written and graphed? How can cylindrical and spherical coordinates be used to represent surfaces in space? **Learning Targets:** Write the component form of a vector.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Perform vector operations ad interpret the results geometrically. Write a vector as a linear combination of standard unit vectors. Use vectors to solve problems involving force or velocity. Understand the three-dimensional rectangular coordinate system. Analyze vectors in space. Use three-dimensional vectors to solve real-life problems. Use properties of the dot product of two vectors. Find the angle between two vectors using the dot product. Find the direction cosines of a vector in space. Find the projection of a vector onto another vector . Use vectors to find the work done by a constant force. Find the cross product of two vectors in space. Use the triple scalar product of three vectors in space. Write a set of parametric equations for a line space. Write a linear equation to represent a plane in space. Sketch the plane given by a linear equation. Find the distances between points, planes, and lines in space. Recognize and write equations of cylindrical surfaces. Recognize and write equations of quadric surfaces.	

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Recognize and write equations of surfaces of revolution. Use cylindrical coordinates to represent surfaces in space. Use spherical coordinates to represent surfaces in space.	
Unit Twelve: Vector-Valued Functions **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Vector-Valued Functions Differentiation and Integration of Vector-Valued Functions Velocity and Acceleration Tangent Vectors and Normal Vectors Arc Length and Curvature	**Essential Questions** How is a space curve represented by a vector-valued function analyzed? How are the concepts of limits and continuity applied to vector-valued functions? In what ways can vector-valued functions be differentiated and integrated? What is the velocity and acceleration associated with a vector-value function? How can a vector-valued function be used to analyze projectile motion? What are tangent vectors and normal vectors? What is the arc length and the curvature of a curve? **Learning Targets:** Analyze and sketch a space curve given by a vector-valued function. Extend the concepts of limits and continuity to vector-valued functions. Apply differentiate and integrate to vector-valued functions. Describe the velocity and acceleration associated with a vector-valued function. Use vector-valued function to analyze projectile motion.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Find a unit tangent vector and a principal unit normal vector at appoint on a space curve. Find the tangential and normal components of acceleration. Find the arc length of a space curve. Use the arc length parametric to describe a plane curve or space curve. Find the curvature of a curve at a point on the curve. Use vector valued function to find frictional force.	
Unit Thirteen: Functions of Several Variables **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure.	Introduction to Functions of Several Variables Limits and Continuity Partial Derivatives Differentials Chain Rules for Functions of Several Variables Directional Derivatives and Gradients Tangent Planes and Normal Lines Extrema of Functions of Two Variable And their Applications	**Essential Questions:** What do level curves and level surfaces look like when graphed? How are limit continuity determined? What is a partial derivative? How can you find and use a total differential and determine differentiability? What is the Chain Rule? What is the directional derivative and the gradient? What is the equation of a tangent plane and the equation of a normal line to a surface What is the angle of inclination of a plane? How are absolute and relative extrema identified?	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
8. Look for and express regularity in repeated reasoning.	Langrange Multipliers	How is an optimization problem including constrained optimization using Lagrange multiplier solved? What is the method of least squares? **Learning Targets:** Determine the domain of a several variable function. Understand the graph of a several variable function, contour maps, and level fields. Understand the Cobb-Douglas Production Function. Understand how to find the limit of several variable functions. Determine the continuity of a several variable function. Understand how to find and use a partial derivative of a several variable function. Find and use total differentials and determine differentiability of a several variable function. Use the Chain Rules for Functions of Several Variables to find a partial derivative implicitly. Understand how to find the Directional Derivatives and Gradients. Find an equation of tangent planes and normal line to a surface. Determine the angle of inclination of a plane. Determine the extrema of Functions of Two Variable and absolute. Solve an optimization problem using Langrange Multipliers. Use the method of least squares.	
Unit Fourteen: Multiple Integrations **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common	Iterated Integrals and	**Essential Questions:** How is an iterated integral and find the area of a plane region	**Suggested Formative Assessment:**

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Area in the Plane Double Integrals and Volume Change of Variables: Polar Coordinates Center of Mass and Moments of Inertia Surface Area Triple Integrals and Applications Triple Integrals in Cylindrical and Spherical Coordinates Change of Variables: Jacobians	evaluated? How is a double integral used to find the volume of a solid region? How are double integrals in polar coordinates found and written? What is the mass of a planar lamina, the center of mass of a planar lamina, and moments of inertia using double integrals? How can a double integral used to find the area of a surface? How can a triple integral be used to find the volume, center of mass, and moments of inertia of a solid region? How are triple integrals in cylindrical and spherical coordinates written and evaluated? How does a Jacobian change variables in double integral? **Learning Targets:** Evaluate iterated integrals. Find the area in the Plane. Find double integrals. Use a double integral to find the volume of a solid region. Write a double integral in polar coordinates. Evaluate a double integral in polar coordinates. Determine the mass of a planar lamina and the center of mass of a planar lamina. Determine moments of inertia using double integrals. Use a double integral to find the area of a surface. Write a triple integral. Use a triple integral to find the volume, center of mass, and moments of inertia of a solid region.	Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		How can you write and evaluate triple integrals in cylindrical and spherical coordinates? Triple Integrals in Cylindrical and Spherical Coordinates. Use a Jacobian to change variables in double integral.	
Unit Fifteen: Vector Analysis **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Vector Fields Line Integrals Conservative Vector Fields and Independence of Path Greens' Theorem Parametric Surfaces Divergence Theorem Stokes's Theorem	**Essential Questions:** What does the sketch of a vector field look like? What is a conservative vector field? How is a piecewise smooth parametrization found, written, evaluated on a line integral, and how does Green's Theorem apply? What is the Fundamental Theorem of Line Integrals? What is a parametric surface? What set of parametric equations represent a surface? How is a normal vector calculated? What is the tangent plane of a surface? What is the area of a surface? How is the Divergence Theorem used? What is Stokes's Theorem and how is it applied? **Learning Targets:** Sketch the graph of a vector field. Determine whether a vector field is conservative. Find a potential function, curl, and divergence.	**Suggested Formative Assessment:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Suggested Summative Assessment** Capstones Lesson quizzes Unit Test

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Understand and use the concept of a piecewise smooth curve. Write and evaluate a line integral. Write and evaluate a line integral of a vector field. Write and evaluate a line integral in differential form. Use Greens' Theorem. Use the Fundamental Theorem of Line Integrals, independence of path, and conservation of energy of vector fields. Determine a set of parametric equations to represent a surface. Find the normal vector, tangent plane, and area of a parametric surface. Evaluate a surface integral. Determine the orientation of a surface. Evaluate a flux integral. Use the Divergence Theorem. Use Stokes's Theorem to evaluate a lien integral or a surface integral. Analyze rotation of liquid using curl.	

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