Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Counting to 20 and to 100**
Designed by: **Michelle Hawley, Innovative Schools**
Based on Pearson *enVisionMATH*
Content Area: **Mathematics**
Grade Level(s): **Kindergarten**

Summary of Unit

In this unit, students expand their counting skills from 10 up to 100. Students spend time representing the quantities in words, numerals, and pictures, and using these representations interchangeably. This work begins with counting items in neat arrays, and progresses into items that are randomly placed. Then, students begin work with estimation. Students are given opportunities to look at various representations of quantities, and develop a greater understanding of what various quantities look like. Time is spent looking at important patterns on the hundreds chart in order to build connections between numbers. These skills lay a critical foundation for concepts of addition and subtraction (and future operation work).

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Counting and Cardinality:

Know number names and the count sequence.

K.CC.1. Count to 100 by ones and by tens.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.
K.CC.4a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
K.CC.4b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
K.CC.4c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Big Idea(s)

- Counting is useful.
- The base ten numeration system is used to recognize, identify, and group numbers.
- Numbers can be put together and taken apart into ten ones and remaining ones.
- Counting tells us how many are in a set no matter which order the objects are counted.
- The base ten numeration system is a way to record numbers using the numbers 0-9, groups of ten, and place value.
- Relationships can be described and generalizations can be made for numbers that repeat in predictable ways.
- Decade numbers are built on groups of ten.
- Mathematical practices can be applied to solve problems.

Unit Enduring Understanding(s)

Students will understand…

- Different strategies can be used to count a group of objects.
- Place value is built upon making groups of ten.
- Numbers work in predictable ways.
- Strategies are useful for problem solving.

Unit Essential Questions(s)

- How do I use my reasoning skills to solve problems?
- How can we group objects to see how many we have?
- Why does changing the arrangement of the objects not affect the quantity?
- Why is it important to represent numbers in different ways?
- How are problems solved by using objects to act out the actions in the problem?
- How are numbers counted and written using a hundreds chart?
- How are patterns used for counting on a hundreds chart?
- What is a numeric pattern?
- How does a hundreds chart organize numbers?
- How are problems solved by using objects to act out the actions in the problem?

Knowledge and Skills

Students will know…

- Pattern for counting
- Strategies for counting groups of objects
- Vocabulary:
  - Eleven
  - Twelve
  - Thirteen

Fourteen
Fifteen
Sixteen
Seventeen
Eighteen
Nineteen
Twenty
Thirty
Estimate
About how many
Hundred chart
Row
Column
Count by

Students will be able to…

- Count to 120 out loud
- Read and write numerals to 120
- Count groups of objects (up to 10 if scattered; up to 20 if arranged in arrays)
- Count on from a given number

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

---

Suggested Performance/Transfer Task(s)

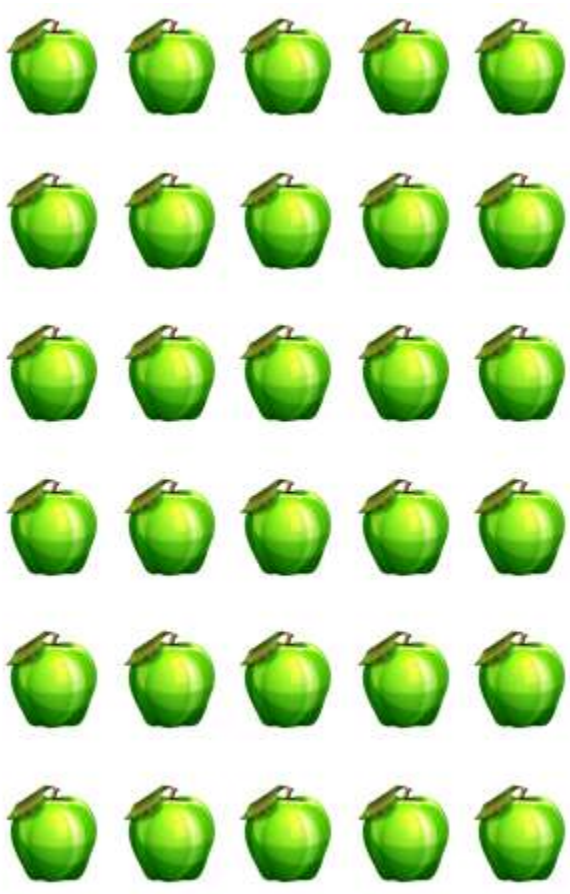
Adapted from:
https://gradekcommoncoremath.wikispaces.hcpss.org/Assessing+KCC1

CC Task 1	
Domain	Counting and Cardinality
Cluster	Know number names and the count sequence.
Standard(s)	**K.CC.1** Count to 100 by ones and tens. **K.CC.2** Count forward beginning from a given number within the known sequence (instead of beginning at 1).
Materials	Teacher recording sheets, bag of 10 small items to count, picture of apples (attached - one for each student) *Note: quantities in the bag and picture may be increased as appropriate for children who have demonstrated greater mastery during formative assessments (continue with the same rubric)*
Task	1. Start at 81 and count as far as you can (stop student after 100). 2. Count by 10's as far as you can (stop student after 100). 3. Begin counting with the number 12. I'll tell you when to stop. (Stop student at 21.) Repeat: • Begin at 38. Stop student at 51. • Begin at 62. Stop student at 72.

	4. Give student the bag of objects. How many _____ are in this bag? 5. Give student a copy of the picture of objects. How many ___ are in this picture? (Allow student to mark picture if necessary)

Rubric(s)

Continuum of Understanding		
Needs Improvement	• Struggles to count by ones past ten • May not show understanding of "count by tens" • Demonstrates difficulty counting on from the given numbers • May cross one of the decades correctly • Counts objects in the bag incorrectly • Counts apples in the picture incorrectly	***SEE ATTACHED CLASS DATA RECORDING SHEET***
Developing Understanding	• Starts counting on by ones, but does not make it to 100 (may omit or repeat numbers in the sequence) • Demonstrates some understanding of counting by tens, but does so incorrectly • Counts from one or more of the given numbers incorrectly ***watch for students who may still be struggling with "teen" number words* • Incorrectly counts over one of the decade(s) • Counts the objects in the bag correctly • Counts more than ten – but not all – of the apples in the picture correctly	
Complete Understanding	• Counts to 100 by ones from 81 • Counts to 100 by tens • Counts on from all given numbers correctly • Counts correctly, crossing over all the decades correctly • Counts objects in the bag correctly • Counts apples in the picture correctly to 20	
Advanced Understanding	• Counts beyond 100 by ones • Counts beyond 100 by tens • Counts on from all given numbers correctly • Counts correctly, crossing across decades and hundreds correctly • Counts objects in bag correctly • Counts apples in the picture correctly beyond 20	



5

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Name: Date:

Student…

1. Correctly counts from 81 to _____ by ones
2. Correctly counts from 0 to _____ by tens
3. Counts on from:

   ____12 ____38 ____62
4. Counts bag of objects correctly to ______
5. Counts apples correctly to ______

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments
- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1:

5-1: Students work on counting, reading and writing 11 and 12 using items presented in neat arrays. Students learn to use these representations interchangeably as appropriate.

5-2: Students work on counting, reading and writing 13, 14 and 15 using items presented in neat arrays. Students learn to use these representations interchangeably as appropriate.

5-3: Students work on counting, reading and writing 16 and 17 using items presented in neat arrays. Students learn to use these representations interchangeably as appropriate.

5-4: Students work on counting, reading and writing 18, 19 and 20 using items presented in neat arrays. Students learn to use these representations interchangeably as appropriate.

5-5: Students use logical reasoning as a strategy for solving problems involving missing number sequences for numbers 0-20. This reinforces all three representations students have used for the numbers 0-20.

Lesson 2:

6-1: Students work on counting, reading and writing numbers up to 30. Students learn to use these representations interchangeably as appropriate.

6-2: Students use visual benchmarks as a tool for estimation. Students begin working with more visual representations that are not in neat arrays.

6-3: Students find missing values on a hundreds chart, using the counting patterns they have been practicing.

6-4: Students count by tens up to 100.

6-5: Students use a hundreds chard to uncover both visual and numerical patterns. This lesson also lays groundwork for even and odd patterns, and reinforces patterns of counting by tens.

6-6: Students look for patterns as a strategy for solving problems involving numbers to 100.

Resources and Teaching Tips

- Pearson enVisionMATH: https://www.pearsonsuccessnet.com

Differentiation

Extension:

- Allow students to practice counting objects that are scattered.
- Each textbook section includes suggested small group games to play for students who have mastered the skill and can extend the concepts.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.

Intervention:

- Encourage use of manipulatives as a support for counting and skip counting practice.
- Each textbook section includes suggested small group games to play with students who struggle with the concept.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.
- Each textbook topic contains a "reteaching" section for use with students who are struggling with the concepts.
- Each textbook topic assessment has an item analysis that aligns it with the coordinating Intervention System materials as well.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3)

Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and completing problem-based activities. Students will relate their learning to the world around them by counting objects from the environment.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- *Splat Square* hundreds chart game: http://www.oswego.org/ocsd-web/games/SplatSquares/splatsq99.html
- Various counting games to reinforce concepts: http://www.kidsmathgamesonline.com/counting.html

Content Connections

Content Standards integrated within instructional strategies

Counting concepts can be implemented in any of the content areas. Students can practice counting items during learning centers and science data collections, count jumping jacks during physical education, and even whisper count in the hallway. Morning meeting time and the count up to the 100th day of school also provide additional connections to counting skills.

Many books are available that address counting concepts.

Curious George Learns to Count from 1 to 100, by H.A. Rey

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Place Value**
Designed by: **Michelle Hawley, Innovative Schools**
Based on *Pearson enVisionMATH*
Content Area: **Mathematics**
Grade Level(s): **1**

Summary of Unit

In this unit, counting concepts are expanded to include numbers up to 120. Students also develop more advanced counting strategies, such as counting by tens. When counting up to 120, concepts of place value are reinforced during lessons to help students understand the importance of groups of ten in our number system. Students continue building the understanding that all two digit numbers are composed of groups of tens plus some ones, and can be decomposed as well. This understanding is later applied to discuss hundreds, tens and some ones. Students spend time learning about concepts of "more" and "less." Finally, students are expected to use these place value concepts to compare and order numbers – both verbally and in written form. The place value concepts developed in this unit are integral to the later concepts of addition and subtraction with two-digit numbers and building an understanding of multiplication.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Number and Operations in Base Ten:

Extend the counting sequence.

CC.1.NBT.1: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

CC.1.NBT.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

CC.1.NBT.2a: 10 can be thought of as a bundle of ten ones — called a "ten."

CC.1.NBT.2b: The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

CC.1.NBT.2c: The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

CC.1.NBT.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

CC.1.NBT.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Big Idea(s)

- Numbers are used to tell how many.
- Looking for patterns in numbers can help us understand strategies for more efficient counting.
- Counting by 10's can help us count large quantities quickly.
- Skip counting can help us count items faster.
- Using standard numerals is important for communicating mathematically.
- Place value tells us the value of each digit in a number.
- Our number system is based upon sets of 10.
- Numbers can be given in different forms, but still maintain equivalence
- There are strategies that help us compare and order numbers.
- Comparison symbols are used to record relationships between quantities.
- Numbers can be made larger or smaller by the order in which you place the digits given.

Unit Enduring Understanding(s)

Students will understand…

- Counting strategies help us count efficiently
- All numbers are based on sets of tens
- Studying number patterns help us develop efficient methods for working with them
- Place value is critical to comparing and ordering numbers

Unit Essential Questions(s)

- How are patterns recognized on the hundreds charts?
- Why are place value patterns useful when counting?
- How can we use objects, images, or other representations to show addition?
- How can we use objects and drawings to solve word problems?
- How is skip counting used to find the total number of objects in a collection of equal groups?
- How can identifying patterns help solve problems?
- What number patterns can be identified in a group of numbers?
- How is counting groups of tens and ones used to tell how many there are in all?
- How can numbers be named in more than one way but still have the same value?
- How can numbers 10 and higher be shown?
- How can numbers 10 and higher be written?
- How can numbers 10 and higher be read?
- How can numbers 10 and higher be counted?
- How can symbols be used to show relationships between two numbers?
- How is place value used to compare numbers?
- How are place value and comparison strategies used to order numbers?

- What does a comparison symbol tell you?
- Why is making an organized list of outcomes helpful when solving problems?

Knowledge and Skills

Students will know…

- Strategies for skip counting
- The digits of a number have different values, based upon the placement within that number
- Comparison symbols are used to capture relationships between numbers
- Numbers can be ordered by using place value concepts
- Vocabulary:
  - Place Value
  - Hundreds
  - Tens
  - Ones
  - Digits
  - Row
  - Column
  - Skip Counting
  - More
  - Less
  - Fewer
  - Greater than >
  - Less than <
  - Equal to =

Students will be able to…

- Count to 120
- Write, read, name and represent numbers up to 120
- Use strategies to find 1, 2 or 10 more/less than a number
- Skip count
- Compose numbers from groups of tens and ones
- Decompose numbers into groups tens and ones
- Compare two numbers
- Order three numbers
- Use problem solving strategies

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

---

Suggested Performance/Transfer Task(s)

(The item questions should be read to the students.)

Use these cards to make a true comparison statement. Use each card one time.

7	2	6	9

_____ _____ < _____ _____

How do you know your comparison statement is true? Use pictures, numbers and/or words to explain your thinking.

Here is Sam's answer:

69 < 27

Is Sam correct? Use pictures, numbers and/or words to explain your thinking.

Rubric(s)

Each line is worth 1 point:

Part I:

- Student creates an accurate comparison statement, using each number one time
- Student explains thinking using pictures, numbers, and/or words to support the response
- Student demonstrates accurate understanding of place value concepts and uses these as supporting evidence for a correct comparison statement
- Student thinking is clearly represented

Part II:

- Student correctly responds that Sam's answer is incorrect
- Student explains reasons for Sam's incorrect answer using pictures, numbers and/or words appropriately
- Student demonstrates accurate understanding of place value concepts and uses these as supporting evidence for determining whether Sam's response is correct
- Student demonstrates understanding of the use of comparison symbols (this may include using a trick for remembering which symbol is "greater" vs. "less" than)
- Student thinking is clearly represented

Total: ___

Exceeds: 9 points

Proficient: 7-8 points

Nearing: 5-6 points

Below: 0-4 points

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments
- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1: Building counting and skip-counting concepts

7-1: Students review the numbers eleven through nineteen. Students work with ten frames to reinforce the concept of "a group of ten and some ones."

7-2: Students continue the work with ten frames and counters, developing concepts of "more" and "less" by finding two more or two less on the ten frames.

7-3: Students move into skip counting by tens, as a means of learning the names for each group of ten.

7-4: Students use patterns on the hundreds chart to develop counting skills for ones and tens.

7-5: Students use skip counting by 2's, 5's and 10's as a strategy for counting groups of objects.

7-6: Formative assessment opportunity: The lesson culminates in an activity that involves students skip counting using tables as problem-solving strategy.

Lesson 2: Flexibility with Place Value and Groups of Tens

8-1: Students continue working with groups of tens and ones to deepen understanding of the relationships between groups of objects, the quantity name, and the associated numeral(s) written.

8-2: Students begin working with numbers over 100, built on the concept of counting by tens.

8-3: Students build an understanding of the efficiency in counting groups of tens first and adding on the ones to tell how many items there are in large groups.

8-4: Students continue the work with tens and ones by using expanded form, cementing the concept that a number's value is the same as adding its tens and ones values together

8-5: Students develop flexibility in composing and decomposing numbers greater than ten.

8-6: Formative assessment opportunity: The lesson culminates in an activity that involves students decomposing numbers using an organized list as a problem-solving strategy.

Lesson 3: Comparing and Ordering Numbers

9-1: Students begin working with concept of one more/less and ten more/less

9-2: Students use patterns on a hundreds chart to reinforce concepts of one more/less and ten more/less.

9-3: Students use place value to compare numbers. Comparison statements are written using the comparison symbols >, <, and =.

9-4: Students work to put three numbers into numerical order using place value strategies.

9-5: Formative assessment opportunity: The lesson culminates in an activity that involves students comparing and ordering numbers using an organized list as a problem-solving strategy.

Resources and Teaching Tips

- Pearson enVisionMATH *(*https://www.pearsonsuccessnet.com)
- Math concept games and activities as provided in each section in the text

Differentiation

Extension:

- Encourage students to count by addition numbers, beyond 2's, 5's and 10's.
- Have students continue to find for multiple ways to represent numbers (mentally).
- Provide students opportunities to expand understanding to other manipulatives, such as number lines.
- Begin building foundation for multiplication when skip counting (one group of two, two groups of two, three groups of two, etc.).
- Each textbook section includes suggested small group games to play for students who have mastered the skill and can extend the concepts.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.

Intervention:

- Allow students continued access to manipulatives as they work to develop number sense, counting, and comparison strategies.
- Use visuals and manipulatives to help students name numbers in different ways.
- Students can play games with a partner, matching pictures of objects, ten frames, hundreds charts, etc., with the appropriate numerals saying the number name out loud.
- Each textbook section includes suggested small group games to play with students who struggle with the concept.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.
- Each textbook topic contains a "reteaching" section for use with students who are struggling with the concepts.
- Each textbook topic assessment has an item analysis that aligns it with the coordinating Intervention System materials as well.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and

completing problem-based activities. Students are encouraged to use manipulatives and multiple forms of modeling in order to gain an understanding of the content. Students will relate their learning to the world around them by counting and comparing real-world objects.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- Oregon City School District: Link to various online resources and interactive games to support Place Value and Number Sense development: http://www.orecity.k12.or.us/staff/curriculum_resources/mathematics/place_value_activities
- Online base ten manipulatives: base ten blocks, ten bears in a boat, etc. http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Content Connections

Content Standards integrated within instructional strategies

Suggested Literacy Connections:

- *The King's Commissioners*, by Marilyn Burns
- *Sir Cumference and All the King's Tens: A Math Adventure* by Cindy Neuschwander
- *Best Counting Book Ever*, by Richard Scarry

Counting strategies are critical to work in all content areas.

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Addition and Subtraction Strategies**
Designed by: **Michelle Hawley, Innovative Schools**
Based on Pearson *enVisionMATH*
Content Area: **Mathematics**
Grade Level(s): **2**

Summary of Unit

In this unit, students work to increase their understanding of the inverse relationship between addition and subtraction. They also work to develop fluency with these operations by making use of common strategies. These strategies include adding 0/subtracting 0, adding 1/subtracting 1, adding 2/subtracting 2, doubles, near doubles and making ten. Students also begin laying the foundation for the Commutative and Associative Properties of Addition, as they begin to manipulate quantities to make problems easier to solve. Of course, using these properties is also a highly effective strategy for problems with more than two addends.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Operations and Algebraic Thinking:

CC.2.OA.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1

CC.2.OA.2: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Numbers in Base Ten:

CC.2.NBT.9: Explain why addition and subtraction strategies work, using place value and the properties of operations.

Big Idea(s)

- Adding 0, 1, and 2
- Doubles
- Adding three numbers
- Making 10
- Subtraction facts
- Subtraction is related to addition
- Subtracting doubles

- Inverse Operations

Unit Enduring Understanding(s)

Students will understand…

- Strategies help us answer addition and subtraction problems.
- Addition and subtraction are inverse operations to each other.
- There are a set of problem solving strategies that can be used in many situations.

Unit Essential Questions(s)

- What are strategies for adding numbers?
- How are addition facts derived from 0-, 1-, and 2-more than a number?
- What are related doubles facts and how are they used to solve addition problems?
- Why can numbers be grouped and added in any order?
- How can using 10 help you solve addition problems?
- What are strategies for subtracting numbers?
- How are subtraction facts derived from 0-, 1-, and 2-less than a number?
- What is an inverse relationship?
- How can the relationship between addition and subtraction be used to answer subtraction problems?
- How is 10 used to help solve subtraction problems?

Knowledge and Skills

Students will know…

- Addition facts to 20 (fluent)
- Subtraction facts to 20 (fluent)
- Problem solving strategies
- Vocabulary:
  - Doubles
  - Near doubles
  - Number sentence

Students will be able to…

- Add and subtract numbers to 20
- Add three numbers
- Use appropriate strategies to solve problems

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

---

Suggested Performance/Transfer Task(s)

Adapted from http://www.illustrativemathematics.org/illustrations/1396

Preparation: copy and cut apart a set of cards for each student, numbered 1 through 10. (sheet follows)

Directions: Demonstrate the process for students. Using a set of cards, randomly select FIVE of the cards. Using a target number of 11, use *at least* two of the cards to create a number sentence that equals the target number of 11. For example, if the cards 2, 5, 3, 4, and 9 and are drawn, one possible number sentence using at least two of these would be: 5+4+2=11. Another possible sentence would be 2+3+5+1 = 11 (student used at least two cards, and figured out the necessary remaining addend).

Model writing out:

Cards: 2, 5, 3, 4, 9

5 + 4 + 2 = 11
2 + 3 + 5 + 1 = 11

Make sure students are clear that they can use addition and subtraction, but must use at least TWO of their cards in EACH of their sentences. They may choose to reuse some of their cards in the second number sentence.

If necessary, repeat this process with some additional target numbers 10 or less (to avoid jeopardizing the assessed target numbers).

Have students record their work on the sheet that follows.

Since addition and subtraction within 20 is a required fluency for grade 2, the goal is to have students complete this task without any support. However, if necessary, manipulatives may be provided.

Number Cards (cut apart):

1	2	3
4	5	6
7	8	9
10		

Name: _____ Date: _____

Pick 5 Cards from your set. Write the numbers here:

_____ _____ _____ _____ _____

Use two or more of your cards to make a number sentence that equals 13:

What strategies did you used to make your number sentence? Use pictures, numbers and words to explain why your answer is correct:

Use two or more of your cards to make another number sentence that equals 13:

What strategies did you used to make your second number sentence? Use pictures, numbers and words to explain why your answer is correct:

Mix up all of your cards and pick 5 new ones. Write the numbers here:

_____ _____ _____ _____ _____

Use two or more of your cards to make a number sentence that equals 20:

What strategies did you used to make your number sentence? Use pictures, numbers and words to explain why your answer is correct:

Use two or more of your cards to make another number sentence that equals 20:

What strategy did you use to make your second number sentence? Use pictures, numbers and words to explain why your answer is correct:

Rubric(s)

Characteristic	Point value	Total Possible
Student creates four correct – and distinct number sentences (adding zeroes does not create a distinct sentence). For each: 2 – student creates a full number sentence, with equal sign and total quantity shown 1 – Student correctly identifies two or more addends, but may forget the equal sign and total quantity. 0 –Student does not demonstrate any concept of writing addition or subtraction sentences (may just write down two or three numbers)	2 points each	/8
Each number sentence uses at least two of the five numbers drawn.	1 point each	/4
Explanations: 2 – Student provides strong and appropriate explanation of thinking. Explanation may include student-drawn used of base ten blocks, hundreds chart, ten frames, manipulatives, etc. 1 – Student explanation appears to be on the right track, but explanation lacks clarity. 0 – Student explanation is either absent or completely off topic.	2 points each	/8
	TOTAL	/20

Strategies used:

While strategies used are not part of the rubric score, it may be useful to note which strategy the student uses to explain work. Some are more sophisticated than others (counting on fingers versus using near doubles). This information can be used to help guide future work with addition and subtraction concepts.

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments
- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1:

2-1: This lesson focuses on adding 0, 1 or 2 to numbers. Students look for patterns that will help them solve addition problems with 0, 1, and 2. In addition to manipulatives, several representations are used to help build conceptual knowledge for students. At this stage, it is critical that students understand adding zero does not change the value.

2-2: The strategy introduced in this lesson is doubles. Again, students are exposed to the use of different representations and manipulatives to build their knowledge.

2-3: Students build on their doubles knowledge by using near doubles (doubles plus one/ doubles minus one). Students are taught to anchor these facts on the doubles facts using manipulatives and visuals.

2-4: This lesson introduces students to the Commutative Property of Addition. Students work with manipulatives to build and understanding of this property.

2-5: In this lesson, students develop strategies for adding three numbers together. While reinforcing the Commutative Property of Addition, this lesson also shows students the Associative Property of Addition. Students are encouraged to look for tens and doubles when choosing the first two numbers to add.

2-6: This lesson teaches students to redistribute quantities as a strategy for adding numbers. Students are encouraged create tens. For example, 9+5 is equivalent to 10+4.

2-7: This lesson teaches students to use pictures and number sentences to solve addition problems involving three addends.

Lesson 2:

3-1: Students are reintroduced to subtraction as the invers of addition. They begin with subtraction of 0, 1 and 2 from a number. This work will be useful later when students are working again with near doubles.

3-2: Students use the doubles facts they learned in Lesson 1 to help them solve subtraction problems. These are used in the context of unknown addend problems (e.g., 6 + ? = 12)

3-3: Again building on their previous addition strategies, students how answer subtraction problems by creating the equivalent unknown added problems. These problems include facts with numbers up to ten.

3-4: Students build their missing addend knowledge by working on problems with a total quantity up to 18.

3-5: In this lesson, students focus on subtracting in two parts. First, they subtract back to make ten and then subtracting the remaining portion.

3-6: This lesson teaches students to think critically about the problem they are trying to solve. In this case, students are given two questions, and must decide in which order to answer the problems. These problems include both addition and subtraction facts.

Resources and Teaching Tips

- Pearson enVisionMAth *(*https://www.pearsonsuccessnet.com)
- Math concept games and activities

Differentiation

Extension:

- Encourage students who already memorized addition and subtraction facts to add and subtract with multiples of tens and hundreds.
- Each textbook section includes suggested small group games to play for students who have mastered the skill and can extend the concepts.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.

Intervention:

- Provide students the opportunity to play math games that reinforce addition and/or subtraction concepts to 12.
- Each textbook section includes suggested small group games to play with students who struggle with the concept.
- Each textbook section contains three different leveled homework assignments that address the content of the section in different ways.
- Each textbook topic contains a "reteaching" section for use with students who are struggling with the concepts.
- Each textbook topic assessment has an item analysis that aligns it with the coordinating Intervention System materials as well.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and completing problem-based activities. Students will relate their learning to the world around them by using addition and subtraction strategies in the environment in which they live.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- Link to various online games for addition and subtraction: http://resources.woodlands-junior.kent.sch.uk/maths/interactive/

Content Connections

Content Standards integrated within instructional strategies

Students should be encouraged to add and subtract numbers to 20 (and beyond) whenever possible across the curriculum.

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title:	**Patterns in Multiplication**
Designed by:	**Natalie Sadorf, Innovative Schools**
	Based on *enVisionMATH* and *NYC Department of Education*
Content Area:	**Mathematics**
Grade Level(s):	**3**

Summary of Unit

In this unit patterns are explored and analyzed. The understanding of and ability to use multiplication and division is introduced as the basis for all further mathematics work in the unit. As students move through *Patterns in Multiplication*, the foundations to work with fractions, decimals, and percent are introduced. Students learn that area is a measure of the space inside a region or how many squares it takes to cover a region. As with other attributes, students must first understand the attribute of area before measuring it. Students realize that the concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Using this model, students should be able to create arrays to solve real-life problems involving multiplication and apply this concept with addition, subtraction, and division to solve equations involving two steps or more to find the solution.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **CC.3.OA.3**

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **CC.3.OA.7**

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **CC.3.OA.8**

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. **CC.3.OA.9**

Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **CC.3.NBT.3**

Big Idea(s)

- Patterns in numbers and shapes can be used to solve problems and determine area.
- Addition and subtraction are related operations.
- Multiplication and division are related operations.
- Skip counting can be used to identify patterns in numbers.
- The Identity Property of Multiplication can be used to find factors and solve multiplication problems.
- The Zero Property of Multiplication can be used to find factors and solve multiplication problems.

Unit Enduring Understanding(s)

Students will understand…

- Area models are related to addition and multiplication.
- Area covers a certain amount of space using square units.
- Multiplication is repeated addition.
- Multiplication can be used to find the area of rectangles.
- Area models of rectangles and squares help us understand the commutative property of multiplication.
- A product can have more than two factors.
- Area in measurement is equivalent to the product in multiplication.

Unit Essential Questions(s)

- How can multiple math operations be used to solve real world problems?
- How can multiplication and addition be used to determine a rectangle's area?
- How can multiplication products be displayed on a 1-100 chart?
- How can multiplication used when reading a pictograph?
- How can the knowledge of area be used to solve real world problems?
- How can the same area measure produce rectangles with different dimensions?
- How can we use patterns to solve problems?
- How can you describe various patterns

Knowledge and Skills

Students will know…

- Multiplication and division notation
- Methods of recording multiplication strategies using equations and arrays
- Skip-counting strategies
- Mental strategies for multiplying single-digit numbers
- Partial products for multiplication (partial products can be notated using equations and/or arrays and area)
- Doubling and halving strategies
- All products of two one-digit numbers

Students will be able to…

- Understand concepts of area and relate area to multiplication and addition.
- Find the area of a rectangle with whole- number side lengths by tiing it.
- Multiply side lengths to find areas of rectangles with whole-number side lengths in context of solving real world and mathematical problems.
- Construct and analyze area models with the same product.
- Describe and extend numeric patterns.
- Determine addition and multiplication patterns.
- Understand the commutative property's relationship to area.
- Create arrays and area models to find different ways to decompose a product.
- Use arrays and area models to develop understanding of the distributive property.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

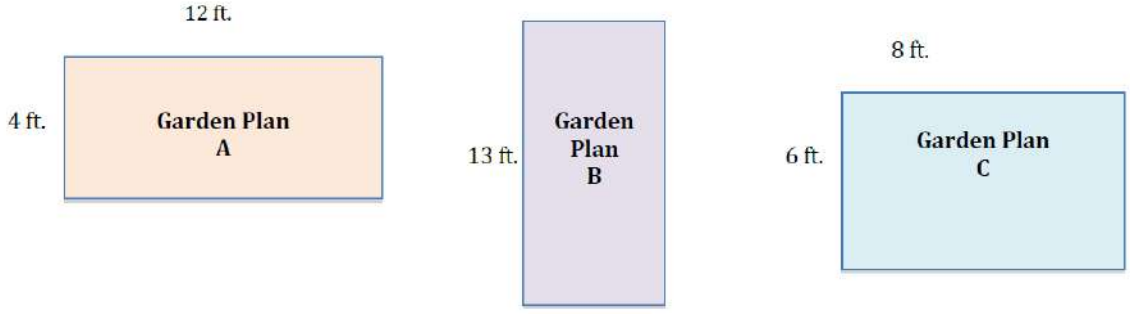
Suggested Performance/Transfer Task(s)

Designing Our Garden

During this task, students extend their knowledge of area defined as counting the same size squares in a rectilinear array partitioned into rows and columns. Students explore the concept of area using operations of both multiplication and addition in this real-world problem. Within the three parts of this task, students will complete a series of questions and then design a garden for their school using specific given measurements.

Part 1

Our school principal wants to make the entrance of the school look beautiful by adding a fenced garden of roses. She has drafted three different garden plans:



1a) If Garden Plan A and Garden Plan B have the same perimeter, what is the unknown side length of Garden Plan B?

1b) The principal wants to plant the largest number of flowers while using the least amount of fencing. Which plan should he choose? Justify your response.

1c) Which two garden plans have the same area?

1d) What is the total area of all three garden plans?

Part 2

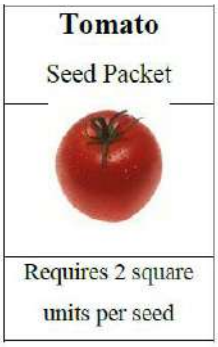
Your school has decided to build a garden on the roof to be used for growing vegetables for school lunches.

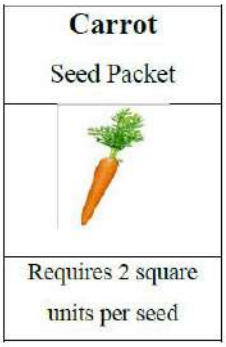
2a) The school cafeteria worker would like to plant 5 rows of vegetables, in a sequence of 4 heads of cabbage followed by 6 potatoes in each row. Showing your work, explain how many total vegetables will grow this winter.

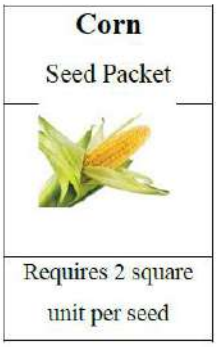
2b) Draw the garden and label the cabbage patch and the potato patch.

Part 3

A new garden layout is designed, covering 48 square feet. Joseph and Michael made posters explaining how many vegetable seeds can be planted.







Seed Posters:

Joseph

Tomatoes	6
Carrots	9
Lettuce	3
Corn	8

Michael

Tomatoes	12
Carrots	3
Lettuce	3
Corn	6

3a) Using the seed packet information above, determine which student is correct.

3b) Show and explain how the correct class poster garden plot can be partitioned into exactly *four rectangular* vegetable patches. Draw a rectangle with an area of 48 square units. Label the length and width. Use the rectangle to represent the garden and label each vegetable patch.

Rubric(s)

Problem	Characteristics	Points	Total Possible Points
1	a) Student calculates unknown side of the garden plot in Plan B as 3 feet.	1 point	8 Points
	b) Student determines that the design which maximizes area while minimizing perimeter is Plan C. Student provides strong explanation for their choice.	1 point 2 points	
	c) Student determines that Plan A and Plan C have the same area.	1 point	
	d) Student determines a total garden area of 135 square feet.	1 point	
	Student uses appropriate units (linear or square as needed) on *every* written measurement in the response.	2 points (total)	
2	a) Student determines the total number of vegetables to be 100 (5 rows of 10 vegetables each or 5 rows of 4 cabbage adjoining 5 rows of 6 potatoes). Student provides strong explanation for their response.	1 point 2 points	4 points

	Student draws a rectangle showing 5 x (4 + 6)	1 point	
3	a) Student determines that Michael's total vegetable area is 48 square feet (Joseph's is 52 square feet). Student provides strong explanation to support their selection of Michael's poster.	1 point 2 points	9 points
	b) Student draws a rectangle with an area of 48 square feet. Student labels the rectangle appropriately. Student subdivides the rectangle into four acceptable rectangular patches (Tomatoes = 24 sq. ft; Carrots = 6 sq. ft.; Lettuce = 6 sq. ft.; Corn = 12 sq. ft.) . Student labels the vegetable patches with names. Student uses appropriate units (linear or square as needed) on *every* written measurement in the response	1 point 1 point 4 points 1 point 2 points (total)	
Total Points			**21 Points**

Meets Proficiency: 18-21 points
Below Proficiency: 15-18 points
Well Below Proficiency: Less than 15 points

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments

- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1
Teacher will introduce the Identify Property of Multiplication and the Zero Property of Multiplication.

Teacher:
The Identity Property of Multiplication is defined as a product of 1 and any number is that number. The Zero Property of Multiplication is defined as the product of any number and 0 is 0.

Ask students:
How can you use counters and equal groups to show multiplication by 1 or 0?

Maya has 8 plates. She puts one mango on each plate. How many mangos are there?

Explain how to solve the problem:
Draw 8 circles to show the 8 plates. Put 1 counter in each circle to show the mangos. Write a multiplication sentence to show the total.

$8 \times 1 = 8$
$1 \times 8 = 8$

Solution:
There are 8 mangos on the plates.

Students will solve the following problem:

Maya has 5 baskets. There are no mangos in them. How many mangos are in the 5 baskets all together?

1) Draw circles to show the baskets.

2) Use counters to show the mangos. How many counters do you need?

3) Write a multiplication sentence to show the total.

Lesson 2

Teacher will present a chart of factor and products of 9 to the class.

Teacher:

This chart shows the 9s facts. Look at each row in the chart. Notice that the tens digit of the product is always 1 less than the underlined factor.

7 × 9 = 63

Look at each product in the chart. Notice that the sum of the digits in the product is always 9.

7 × 9 = 63 -> 6 + 3 = 9

Ask students:

Find the product of 8 × 9 =

Explain how to solve the problem:

What number is 1 less than 8? Write 7 in the tens place of the product.

8 × 9 = 7 ?

Seven plus what number is 9? Write 2 in the ones place of the product.

8 × 9 = 7 2

Solution:

8 x 9 = 72 [7 + 2 = 9]

Students will solve several multiplication problems, multiplying by 9.

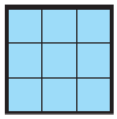
Lesson 3

Teacher will introduce square arrays, factors, and area.

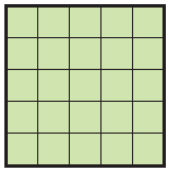
Teacher:

Introduce square arrays to the class.

3 x 3 = 3 x 3 = 9; 9 is a square number.



5 x 5 = 5 x 5 = 25; 25 is a square number.



Ask students:

Draw arrays to find the products.

1) 9 x 9
2) 8 x 8
3) 1 x 1

Does the array show a square number? If not, how many squares could be added to make it a square number?

4)  *5)*  *6)* 

Review student answers.
Introduce application problems and have students solve:
7) Chris is hanging pictures in a gallery, in a square array. There are 4 rows of 4 pictures. How many pictures are there in all?

a. What do you know? What do you want to find out?
b. Plan You can draw an array. Will the array be a square?
c. Solve Draw the array. Use the array to solve the problem. There are ____ pictures in all.
d. Look Back Use another multiplication strategy to solve the problem. Did you get the same answer?

Lesson 4

Teacher:

The nature club is going on a hike. Each person brings 4 bottles of water. How many bottles in all will 3 people bring? You can use doubles facts to help learn the 3s facts. Since multiplying by 3 is like adding a number 3 times, you can double the number and add one more group.

Ask students:

How many bottles in all will 3 people bring? Find the product of 4 × 3 =

Explain how to solve the problem:

Multiply 4 by 2; 4 × 2 = 8

Then add a third group of 4; 8 + 4 =12

Write the multiplication equation; 4 × 3 = 12

Solution:

Three people will bring 12 bottles of water.

Students will solve several multiplication problems, multiplying by 3.

Lesson 5

Teacher:

Rosa's uncle sends her a postcard every week. Rosa has 3 postcards of fish. She has 9 times as many postcards of birds. How many postcards of fish and birds does she have?

Discuss with students:

The problem asks how many postcards of fish and birds Rosa has. Rosa has 3 fish postcards. She has 9 times as many bird postcards.

It takes more than one step to solve this problem.
Break the problem into parts.
- *First, find the number of bird postcards.*
- *Then find the sum of the bird and fish postcards.*

Explain how to solve the problem:
Find the number of bird postcards. 9 x 3 = 27
Add to find the total. 27 + 3 = 30

Solution:
Rosa has 30 postcards of fish and birds.

Resources and Teaching Tips

- http://www.murrieta.k12.ca.us/Page/4056
- Pearson enVisionMAth *(*https://www.pearsonsuccessnet.com)
- https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
- http://mnpsnc.wikispaces.com/file/view/Math_Perform_Task_Grade_3%2520(08-08).pdf
- http://www.nsa.gov/academia/_files/collected_learning/elementary/geometry/school_garden.pdf
- Number lines
- Base Ten blocks
- Math concept games and activities

Differentiation

Extension

- Give students the area of the garden. Have them determine how much rope to purchase and how to plant the garden (this would yield multiple possible solutions, depending upon the factors of the area).
- Increase the size of the garden so students can work with manipulatives to craft the garden out of pop cubes, then draw it to scale on graph paper.

Intervention

- Students use manipulatives and grid paper to design their garden and then draw the design on the same grid paper.
- Reduce "special needs" column for the plants.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and completing problem-based activities. Students will relate their learning to the world around them by identifying shapes in the environment in which they live and creating shape patterns when designing their gardens.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- Unit Webiner: https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
- http://illuminations.nctm.org/activitydetail.aspx?id=27
- http://www.korthalsaltes.com/
- Virtual geoboards http://www.k-5mathteachingresources.com/2nd-grade-geometry.html

Content Connections

Content Standards integrated within instructional strategies

Science

- In science class, students may work in groups to build their gardens using seeds provided by the teacher. The gardens may be in a space outside or planted in bins in the classroom. Students will link their understanding of area to plants, accurately determining how many of each plant can be planted within the given space.

Literature

- Students may keep an observation journal documenting and predicting growing patterns they observe in their gardens.

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title:	**Place Value**
Designed by:	**Natalie Sadorf, Innovative Schools**
	Based on *enVisionMATH* and *nsa.gov* (4th grade mathematics)
Content Area:	**Mathematics**
Grade Level(s):	**4**

Summary of Unit

This unit helps pre-assess students' previous knowledge and misconceptions about place value and number sense. The strategies they use to solve the problem demonstrate students' understanding about a number of concepts including place value, grouping, computation, number sense, patterns, and mathematical communication. This unit develops and reviews place value concepts through millions. It involves the utilization of manipulatives. The lessons focus on a students' basic understanding of place value and the reinforcement of the concept. It is ideal for a classroom setting where differentiation is implemented and can be used in a large or small group setting.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. **CC.4.NBT.1**

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **CC.4.NBT.2**

Use place value understanding to round multi-digit whole numbers to any place. **CC.4.NBT.3**

Big Idea(s)

- Identifying whole numbers through millions
- Place value can be determined using the Base 10 Model.
- Numbers can be represented in different forms: Standard Form, Expanded Form, and Word Form.

- Numbers can be ordered through millions using a Place Value Chart.
- Numbers can be compared using symbols and words.

Unit Enduring Understanding(s)

Students will understand…

- Our number system is based on groups of ten. When we make a "10" in one place value, it is recorded in the next greater place value.
- In a multi-digit whole number, a digit in one place represents ten times what it would represent in the place immediately to its right.
- Place value can be used to compare and order numbers.
- Rounding whole numbers is a process for finding the multiple of 10, 100, and so on closest to a given number.

Unit Essential Questions(s)

- In what ways can you represent multi-digit numbers?
- How do the digits in a multi-digit number relate to each other?
- How can you use place value to compare numbers?
- How can you use place value and comparing to order numbers?
- How can place value help you to round multi-digit numbers to any place?

Knowledge and Skills

Students will know…

- Expanded notation can be used to show order, values of each digit, and the powers of 10.
- The Distributive Property of Multiplication can be modeled in an array as well as with expanded notation.
- Rounding a number to the largest place value can be accomplished by answering: "Is this number closest to N-thousand or N+1 thousand?"

Students will be able to…

- Read and write three-digit and four-digit numbers.
- Understand how digits within a multi-digit whole number relate to each other by their place value.
- Compare whole numbers through hundred thousands.
- Use knowledge of place value to compare and order numbers.
- Demonstrate how to use place value to round whole numbers.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

Population Search

Materials:

- Student instructions sheet
- Student recording sheet
- Student report sheet

Teachers will review with students what a census is and how it is taken. Use a census report from other states (see example on page 12; Georgia cities) as an example of this take for students.

Students should display proficiency during this task in the following areas covered in this unit and apply learning from previous units:

- read numbers to one million
- write numbers in the millions using commas to indicate periods
- round numbers to the nearest 10s, 100s, and 1000s
- compare rounded numbers and express their relationship using >, <, or =
- estimate sum and/or difference of numbers
- apply estimation to solve problems; and determine when it is necessary to apply
- estimation strategies

Task:

You are writing an article for next year's fourth grade class to read. This article should be a "how-to" article that describes the process for comparing and rounding any set of large numbers.

1. Before writing your article, look at the table of populations in ten different cities in Delaware.
2. On the recording sheet, list the cities in order from largest population to smallest population.
3. Answer the questions at the bottom of the recording sheet.

Write your article based on your work you completed.

Rubric(s)

Delaware Population Rubric

Exceeds Standard	Meets Standard	Partially Meets Standard	Does Not Meet Standard
___ Orders all population data from largest to smallest ___ Rounds all population values to the nearest 1,000 ___ Writes all population data in word form ___ Writes all population data in expanded form ___ Records project in article format, with strong, descriptive explanations of how to round and compare large numbers (student makes generalizations to all place values)	___ Orders 8 or more of the population data values correctly from largest to smallest ___ Rounds 8 or more of the population data values to the nearest 1,000 ___ Writes 8 or more of the population data values correctly in word form ___ Writes 8 or more population data values correctly in expanded form ___Writes 2 correct comparison statements using symbols ___ Records project in article format, with good description of how to round and compare large numbers (student may explain well, but does not generalize to all place values)	___ Orders 2-7 of the population data values correctly from largest to smallest ___ Rounds 2-7 of the population data values to the nearest 1,000 ___ Writes 2-7 of the population data values correctly in word form ___ Writes 2-7 population data values correctly in expanded form ___Writes 1 correct comparison statement using symbols OR 2 correct statements using words ___ Writes an article that provides a vague description of how to round and compare large numbers (may only use samples from the given table without making generalizations)	___ Does not put population data in order from largest to smallest ___ Rounds fewer than 2 of the population values to the nearest 1,000 ___ Writes fewer than two of the population data values in word form ___ Does fewer than two of the population data values correctly in expanded form ___Does not write any comparison statements using symbols OR writes 1 correct statement using words ___ Does not include an article or article is off-topic

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments
- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Throughout the unit, students will be able to compose, write, and identify large numbers through millions. This unit will aid the students in their conceptual understanding of composing numbers which will later lead to them decomposing numbers when doing expanded notation and calculating mentally during the computation of addition, subtraction, multiplication, and division.

Lesson 1

Teacher will provide each student (or pair of students) with a set of ten number cards with various numbers on each card. Teacher will instruct the students to place the cards on their desk. Explain that a clue will be given and they must identify the number that matches the clue. Instruct the students to hold up the card with the number on it that matches each clue. Give the class one clue at a time. Once the students identify the numbers by holding up their cards, call on one student to read the number aloud.

Clues:

1. Show me the two-digit number.
2. Show me a number with the two in the tens place.
3. Show me a number with nine in the ones place.
4. Show me a three-digit number.
5. Show me a number with eight in the hundreds place.
6. Show me a four-digit number.
7. Show me a number with four in the thousands' place.
8. Show me the largest number.
9. Show me the smallest number.

Teacher will instruct the students to put their cards in order from least to greatest. Explain to the student that they are going to learn about place value through millions. Distribute sets of base ten blocks to each pair of students. Review the values of each type of place value (ones, tens, hundreds, and thousands blocks).

While holding up a unit: *This is called a unit it represents ones*. While holding up a long strip: *This is called a strip. It represents tens.*

Instruct the students to construct the four four-digit numbers that are written on the board using the base ten blocks. *Look at the number 6,482 on the board. Show me how you would represent this number using the place value blocks.*

Ask students:
How many unit blocks it takes to make a ten-rod?
How many ten rods it takes to make a hundred flat?
How many hundred flats it takes to make a thousand cubes?

Call students up to the front of the class to demonstrate making one thousand with the cubes. Discuss with the students how many cubes it would take to make ten thousand (ten), hundred thousand (100), and a million (1,000). Emphasize the pattern of how it takes ten of each place value representation to make the value of the next placeholder.

Write the number 3,425,689 on the board. Ask a volunteer to read the number. Discuss the place value of each number. *What is the value of 6? (600) What is the value of 5? (5,000)*

Introduce to the ten thousands, hundred thousands, and millions place. Label each on the board. Explain to the students that they are going to make a millions place value chart.

Provide each student with a place value chart, and demonstrate how to label the place values and make the charts. Instruct students to cut out their number cards. Ask students to place the cards in the correct placeholder as the teacher reads a number aloud. *I am going to tell you a number. The number is 9,324. Use your number cards to represent the number on your place value chart.* Repeat with different numbers.

Lesson 2
Write various numbers on the board. Have the students read and identify various place values to the millions. Example: Write 2,365,476 on the board. Ask the students: *What is the value of the 5? What is the value of the 2?*

Challenge the students to create a number that has a seven in the ten thousands place. The students can use any number they choose to complete the chart. Ask each student to share his/her numbers with a partner by first showing and then reading the number.

Explain to the students that they are going to learn how to write large numbers in expanded form. Demonstrate the meaning of expand by writing 2,365 on the board. Discuss with the students how expanded form means that you break down a number indicating the place value of each digit.

Model the expanded form 2000 + 300 + 60 + 5

Instruct the students to place a 3 in the thousands place and 0's in the hundreds, tens, and ones place. *What number did you create? (3,000)*
Help students make the connection between 3,000 and writing numbers in expanded form.

Explain to the students that in expanded form they put a plus next to each of the numbers. *Write the number 7,582 in expanded form. (7,000 + 500 + 80 + 2)*
What is 7,000 plus 500 plus 80 plus 2? (7,582)

Work through the tens and ones in the same manner with the students. Model with one additional six- or seven-digit number.

Lesson 3

Building 1,000 (activity)

This activity helps students begin to make some connections about place value, as well as gives them a visual perspective and sense of the number 1,000. This task helps assess students' previous knowledge and misconceptions about place value and number sense. The strategies they use to solve the problem demonstrate students' understanding about a number of concepts including place value, grouping, computation, number sense, patterns, and mathematical communication.

It is important that students understand that they are actually building the number 1,000. Have students brainstorm inexpensive and appropriate building materials. You will need to provide ample space for students to build and store their designs.

Materials:

- Building materials such as straws, toothpicks, noodles, string, pennies, paper clips, etc.
- Tape measures, rulers, yardsticks
- Poster board, markers, overheads for presenting, tape, glue, etc.

Students will answer the following questions on the student recording form:

- What does 1,000 look like? How long is it? How tall is it? How big is it? How much space will it take up?
- To answer these questions, decide what type of material you would like to use to show 1,000. Next, make a prediction about the size you think your 1,000 will be.
- Next, using words and pictures, explain what you did to make your prediction.

Once students have answered these questions, they will use materials they've chosen to create their model of 1,000. Then they will answer the following questions, and prepare their presentation for their classmates.

- What strategies did you use to create 1,000?
- What did you learn from this investigation of 1,000? Did you notice any patterns or connections?
- When you have completed this task, plan a presentation of your investigation for the class.

Lesson 4

Explain to the students that they will learn to write numbers in word form. Discuss reasons why they need to write the word form of numbers. Point to numbers around the classroom or write them on the board. *Write the number I point to in word form.*

Distribute activity sheets that have numbers written as words to the class. Write numbers on the board and have students identity the number in word form on their sheets.

Explain to the students the rules for writing number word forms:

Start with the highest digit and do not place the word "and" in between two numbers.

Write numbers on the board in word form and instruct the students to write the standard form on in their notebooks. Repeat as needed with different numbers.

Draw a sample of a check on the board and as students if they know what you drew, or show students a check. Describe a real life scenario and demonstrate how to fill in the amount on the check. Demonstrate how to add the amount on the check.

Distribute blank checks (these can be printed as templates online) and instruct the students to complete the checks.

What are the 3 ways that we have learned how to write numbers?

Discuss what each term means and some examples of each.

Draw a chart like this on the board leaving out certain cells (indicated in red).

STANDARD FORM	EXPANDED FORM	WORD FORM
2,678	2000 + 600 + 70 + 8	two thousand, six hundred seventy eight
56,071	50,000 +6000 +70 +1	fifty six thousand, seventy one
87	80 + 7	eighty seven
399	300 + 90 + 9	three hundred ninety nine

Instruct the students to come to the board and complete the chart.

Resources and Teaching Tips

- http://www.nsa.gov/academia/_files/collected_learning/elementary/arithmetic/place_value_whole_numbers.pdf
- Performance Task adapted from Richmond County School System - http://www.rcboe.org
- Pearson enVisionMAth (https://www.pearsonsuccessnet.com)
- https://www.Georgia standards.org
- http://www.prometheanplanet.com/en-us/Resources/Item/109644/place-value-through-100-000
- Place Value chart
- Number lines
- Base Ten blocks
- Math concept games and activities

Differentiation

Extension

- Collections- As a class or grade-level project, collect some type of object with the objective of reaching some specific quantity-for

example, 1,000 or 10,000 buttons, walnuts, old pencils, jar lids, pieces of junk mail, soup labels, or cereal box tops.

- Illustrations- Sometimes it is easier to create large amounts. For example, start a project where students draw 100 or 200 or even 500 dots on a sheet of paper.

Intervention

- Using the base ten blocks, the students could build a number and have a peer determine what the number was created and orally give the value of each place contingent upon the blocks.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and completing problem-based activities. Students will relate their learning to the world around them by recognizing 1,000 and building a structure of 1,000.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- http://real.doe.k12.ga.us/content/math/destination_math/msc3/msc3/msc3/MSC3/MSC3/Module1/Unit1/Session1/Tutorial.html
  This resource provides videos which can be used as an introduction to the concept or as a form as remediation.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L367
  Use this resource as a follow-up lesson to extend place value understanding.
- http://www.prometheanplanet.com/en-us/Resources/Item/109644/place-value-through-100-000
  A lesson for your ActivSlate or SmartBoard to reinforce basic place value ideas through 100,000

Content Connections

Content Standards integrated within instructional strategies

Social Studies:

- Students will be introduced to census reports. Students will understand, investigate, and analyze census data.

Literature:

- *A Place for Zero* by Angeline Sparagna Lopresti
- *A Millions Dots* by Andrew Clements
- *How Much Is a Million?* by David M. Schwartz
- *Sir Cumference and All the King's Tens* by Cindy Neuschwander
- *Two of Everything* by Lily Toy Hong
- *One Grain of Rice* by Demi

Name _____ Date _____

Building 1,000
(Lesson 3)

What does 1,000 look like? How long is it? How tall is it? How big is it? How much space will it take up? Describe the model for 1,000 that you will create:

_____
_____
_

What type of material you would like to use to show 1,000? Make a prediction about the size of your 1,000
model._____
_____
_____

Next, using words and pictures, explain what you did to make a prediction about the size of your model.
_____
_____
_____

After you have created your model, answer the following questions:

What strategies did you use to create 1,000?
_____
_____
_____

What did you learn from this investigation of 1,000? Did you notice any patterns or connections?
_____
_____
_____

When you have completed this task, plan a presentation of your investigation for the class.

Populations of Delaware Cities

(Source: U.S. Census Bureau, 2010)

Lewes	2,747
Georgetown	6,422
Wilmington	70,851
Dagsboro	805
Rehoboth Beach	1,327
Milford	9,559
Dover	36,047
Laurel	3,708
Newark	31,454
Middletown	18,871

Population Recording Sheet

City	Population Rounded to the Nearest 1,000	Population in Word Form	Population in Expanded Form

1. Which city has the largest population? Smallest population?

2. Write two statements comparing city populations using numbers and symbols.

3. Using the information on your recording sheet, write a “how to” article detailing the process for comparing and rounding any set of large numbers. Be sure to use complete sentences and math vocabulary.

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title:	**Volume of Solids**
Designed by:	**Natalie Sadorf, Innovative Schools**
	Based on *enVisionMATH* and *Mails Elementary School (5th grade mathematics)*
Content Area:	**Mathematics**
Grade Level(s):	**5**

Summary of Unit

In this unit, students will explore measuring using customary capacity units and metric units. Students will determine that a liter is about a quart. Students locate items in the classroom that weigh about a pound and about a kilogram. Students create a table to represent the relationship between a kilogram and a pound. Students investigate the difference between a centimeter and an inch. They identify the 2.5: 1 relationship between the two by measuring a variety of items. Students investigate the relationship among feet, yards, and meters. They determine that a meter is a bit longer than a yard. Students review customary and standard measurements. Through this unit students realize volume of three-dimensional figures can be related to multiplication and addition. Volume of three-dimensional figures is measured in cubic units. Two-dimensional figures can be classified according to their properties.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **CC.5.MD.3**

A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **CC.5.MD.3a**

A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. **CC.5.MD.3b**

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **CC.5.MD.4**

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **CC.5.MD.5**

Big Idea(s)

- Measurement units are used to express equations
- Customary units are used to express measurement.
- Metric units are used to express measurement.
- Solid figures are also called three-dimensional figures.
- Solid figures look similar to plane shapes but take up space.

Unit Enduring Understanding(s)

Students will understand…

- Relationships between measurement units of the same length can be expressed as an equation.
- Relationships exist that enable you to convert between units of length by multiplying and dividing.
- Relationships between measurement units of the same capacity can be expressed as a ratio.
- Relationships between measurement units of weights/mass can be expressed as a ratio.

Unit Essential Questions(s)

- What are customary measurement units?
- How are customary units related?
- What are metric measurements units?
- How are metric units related?
- How do I know which unit size is reasonable for my task?

Knowledge and Skills

Students will know…

- Standard measurement units and metric units can be used interchangeably.
- Volume is an attribute of solid figures relating length, width, and height (depth).
- Volume is "filling" the inside space if a 3D shape with unit cubes.
- Volume is additive: The volumes of two non-overlapping rectangular prisms (or solids) can be added to find a total volume.
- The area of a rectangular base can be utilized when calculating the volume.
- Volume can be thought of as creating layers of unit cubes in a solid

Students will be able to…

- Identify three-dimensional shapes according to faces, edges, and vertices.
- Identify different views of a solid figure.
- Determine the volume of rectangular solids.
- Use formulas to find the volume of rectangular prisms.
- Find the volume of irregular solids.
- Use the properties of solid figures to compare them to one another.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

Breakfast Boxes

In this task, students will be designing three different sizes of cereal boxes. They will need to determine the dimensions for the original box and then use the appropriate operations to enlarge or reduce the size of the original box to meet the specifications of the manufacturer.

Students should have had practice determining the volume of rectangular prisms. In addition, they should be familiar with the terminology "half the size of" and "three times the size of" and be able to determine relative dimensions. They should also be able to determine the correct unit of measure for given item (centimeters/inches or meters/feet/yards)

Students may believe that in order to make the boxes "half the size" or "three times the size" they need to adjust each dimension (length, width, height) by half or three times. They need to investigate how the total volume is affected by changing the dimensions and determine "half" and "three time" by calculating total volume.

Teacher should begin this task by showing several cereal boxes and asking them to estimate the dimensions of the box. They could even measure a cereal box to find out what the appropriate dimensions could be.

Materials

- Ruler
- Grid or graph paper
- Centimeter cubes / Base Ten blocks

Grouping

Individual/ pairs

Students will be able to justify why they chose which unit of measure to use, and explain if their answer is reasonable. In this task student will demonstrate their understanding of volume by determining how much cereal can be packaged in each cereal box.

Rubric(s)

Requirements	Points Earned	Comments
Cereal box is measured using a standard unit (can be rounded to the nearest units since the focus is volume)	/2	
Mini sized box drawn is half as tall as original cereal box as measured	/2	
Mini sized box drawn is half as wide as original cereal box as measured	/2	
Mini sized box drawn is half as deep as original cereal box as measured	/2	
Super-sized box drawn is three times as tall as original cereal box as measured	/2	
Super-sized box drawn is three times as wide as original cereal box as measured	/2	
Super-sized box drawn is three times as deep as original cereal box as measured	/2	
1) Student labels: length, width and height of each box (1 point per dimension per box) Student calculates correct volumes for each of the boxes drawn (2 points per box)	/15	
2) Student selects a box as the best seller (1 point) Student gives reasonable explanation why the selected box is better for consumers (2 points) Student gives reasonable explanation for the selected box to be a better seller (2	/5	

points)		
Student includes appropriate unit labels on written measurements on all components of the task: **6 = All labels correct and present** **3 = Mislabeled or missed 2-5 labels measurements** **0 = did not include any correct labels or mislabeled more than 5 labels**	/6	
Total Points	**/40**	

Well Below Proficiency: below 25 points
Below Proficiency: 25 - 34 points
Meets Proficiency: 35 - 40 points

Other Evidence

- Student Exercises
- Peer questioning
- Classroom Discussions
- Quick Check sheets
- Vocabulary checks
- Visual Learning Bridges
- Problem Solving Challenges
- Exit Tickets
- Centers
- Topic Tests
- Text-based Performance Assessments
- Benchmark Tests
- Problem-Based Interactive Learning Activities

Student Self-Assessment and Reflection

- Students will explain and reflect on mathematical problems by completing journal prompts.
- Students interact and participate in classroom discussion and reflection/feedback conversation.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

As students develop their understanding of volume, they learn that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic

inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit.

Lesson 1

In geometry we have learned about figures that are two-dimensional. Two-dimensional figures are plane figures and we often think of them as "flat" figures. A plane figure does not have other dimensions of the figure shown. A plane figure is just that, it is a figure that is flat and does not have depth to it. Solid figures are known as polyhedrons, solids with flat surfaces that are polygons.

Introduce students to three-dimensional (solid) figures: Prism, Cylinders, Pyramids, Cones, Spheres. Have students provide examples of these figures that they recognize in everyday life.

Students will engage in a scavenger hunt around the classroom, located and identifying solid figures.

Lesson 2

Teacher will introduce volume to students.

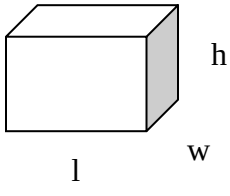
In this unit you will learn how to calculate the volume of common solids to include definition of volume, prisms, cylinders, pyramids, cones, and spheres. For the volume of a solid, there is a similar definition to area. The definition for volume is the unit cubed. A unit cube can be 1" x 1" x 1" or 1 yd x 1 yd x 1 yd or 1 ft x 1 ft x 1 ft or a cube by some other unit. Therefore, volume is the sum of all unit cubes.

Using Base Ten Blocks, students will fill bins, boxes, and hollow containers with unit cubes to understand volume. Students will compare the number of units needed to fill each container with a partner. Conduct a class discussion to talk about the results and comparisions.

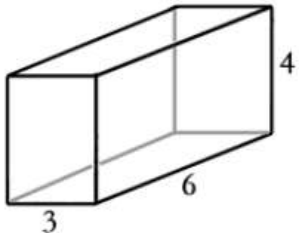
Lesson 3

Prisms: V = Bh = lwh

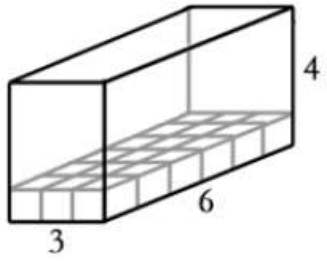
Let's take a look at a diagram of an actual prism. A diagram will help us understand what volume actually means and how volume is calculated.



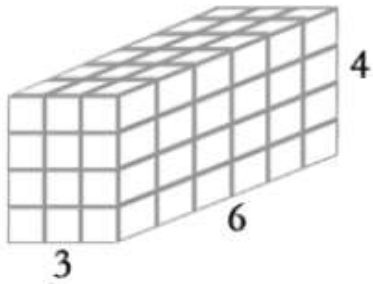
Look at the prism below. It has a length of 3 units, a width of 6 units, and a height of 4 units.



To understand what volume means, let's start by filling the bottom of the prism with unit cubes. This means the bottom of the prism will act as a box and will hold as many cubes as possible without stacking them on top of each other.



We need to count these cubes to arrive at the total volume. We could count them one at a time or use the formula for finding volume of a prims. Since the length is 3 units and the width is 6 units, the bottom layer of cubes measure rows and columns that are 3 units by 6 units. So, a quick way of counting the blocks would be to multiply 3 by 6. In general, we would multiply the length of the prism by its width. Multiplying the numbers, we arrive at 18 cubes resting on the prism's base. If we imagine the prism like a building, we could stack cubes on top of each other until the prism is completely filled. It would be filled so that all cubes are touching each other such that no space existed between cubes.



We already know there are 18 cubes on the bottom level and all levels contain the exact number of cubes. Therefore, we need only take that bottom total of 18 and multiply it by 4 because there are four levels to the prism. 18 x 4 = 72 total cubes to our original prism.

Students will complete the following problems:

1) If l = 7 ft and w = 5 ft and h = 2 ft, then what is the volume?

   V = Bh
   V = lwh
   V = (7 ft)(5 ft)(2 ft)
   V = 70 ft^3

2) If l = 12 yds and w = 10 yds and h = 1/2 yd, then what is the volume?

   V = Bh
   V = lwh
   V = (12 yds)(10 yds)(1/2 yd)
   V = 60 yds^3

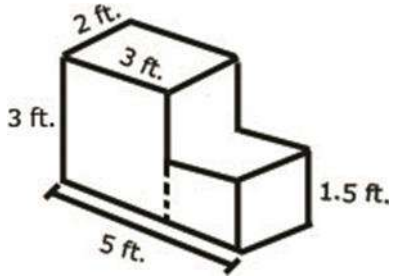
Lesson 4

Students will explore volume through the activities below. The teacher will circulate around the classroom and work with small groups to solidify understanding.

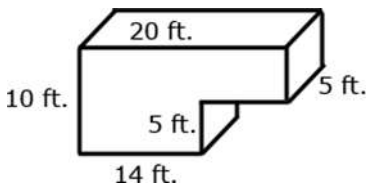
- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions. Students then discuss the relationships between length, width, height, and volume.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below. Allow multiple students to share out strategies for calculating the volume.



- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below. Allow multiple students to share out strategies for calculating the volume.



Resources and Teaching Tips

- Math Guide - http://www.mathguide.com/lessons/Volume.html
- Common Core Georgia Performance Standards – Frameworks (https://www.georgiastandards.org/CommonCore/Common%20Core%20Frameworks/CCGPS_Math_K_Unit1FrameworkSE.pdf)
- Pearson enVisionMAth (https://www.pearsonsuccessnet.com)
- Inch Grid Paper

- Centimeter Grid Paper
- A variety of Smart Board lessons and activities.
- Base Ten blocks
- Math concept games and activities

Differentiation

Extension

- Have students find the answer to the following question: How big would a box have to be to hold enough cereal for your entire school?
- Have students design a box to hold 30 1-inch cube blocks

Intervention

- Students may work in partners.
- Students may use calculators.
- Using the base ten blocks, the students could build a number and have a peer determine what the number was created and orally give the value of each place contingent upon the blocks.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have the opportunity to further discover and understand the main topics of this unit through real-world application, reflecting in journal prompts, answering and discussing mathematical tasks, and completing problem-based activities. Students will relate their learning to the world around them by identifying measure of units used in everyday life, as well as measurements given in metric and customary units.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6

This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240
  In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Content Connections

Content Standards integrated within instructional strategies

Literature:

- *How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000?* by Helen Nolan and Nancy Walker - This fantastic book will help students visualize one 1,000 in many different situation. This book addresses length, weight, height, and estimation.
- *Measuring Penny* by Loreen Leedy - This book is a fantastic introduction to a measurement project. In this story, Lisa decides to use her dog Penny for a measurement assignment. This book addresses length, weight, height, cup, pint, quart, gallon, pound, inches, feet, centimeters, etc.
- *Millions to Measure* by David M. Schwartz - This book explores length, weight, and volume in both customary and metric measurements. This book addresses customary and metric units of measure, cup, pint, quart, gallon, pound, inches, feet, centimeters, etc.

Name _____ Date _____

Breakfast Boxes

You have been asked to create the packaging for a new kind of cereal. The manufacturer wants three different sized boxes:

- ✓ A standard sized cereal box
- ✓ A mini sized box that is half as tall, half as wide, and half as deep as the standard size
- ✓ A super-sized box that is three times as tall, three times as wide and three times as deep as the standard size.

1) Using grid paper, draw a possible design for each box. (17 points)

2) Label the dimensions and calculate the volume. (18 points)

3) Which box do you think would be the best seller? Explain how the volume of the box you chose as "best seller" will benefit consumers. Why is it a better choice than the other boxes? (5 points)

_____
_____
_____
_____
_____
_____
_____
_____