Interactive Mathematics Program Curriculum Framework

School: Delaware STEM Academy **Curricular Tool:** IMP **Grade or Course** Year 1 (grade 9)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Unit One: Patterns **Timeline: 6 weeks**			
Interpret expressions that represent a quantity in terms of its context. **CC.A-SSE.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **CC.F-IF.1** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* **CC.F-IF.3** Write a function that describes a relationship between two quantities. **CC.F-BF.1** Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.F-BF.1a** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **CC.F-BF.2** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**CC.G-CO.1**	Patterns emphasizes extended, open-ended exploration and the search for patterns. Important mathematics introduced or reviewed in Patterns includes In-Out tables, functions, variables, positive and negative numbers, and basic geometry concepts related to polygons. Proof, another major theme, is developed as part of the larger theme of reasoning and explaining. Students' ability to create and understand proofs will develop over their four years in IMP; their work in this unit is an important start. This unit focuses on several mathematical ideas: • Finding, analyzing, and generalizing geometric and numeric patterns • Analyzing and creating In-Out tables • Using variables in a variety of ways, including to express generalizations • Developing and using general principles for working with variables, including the distributive property • Working with order-of-operations rules for arithmetic • Using a concrete model to understand and do arithmetic with positive and negative integers • Applying algebraic ideas, including In-Out tables, in geometric settings • Developing proofs concerning	Can students use variables and algebraic expressions to represent concrete situations, generalize results, and describe functions? Can students use different representations of functions—symbolic, graphical, situational, and numerical—and understanding the connections between these representations? Can students use function notation? Can students model, and computing with signed numbers? Can students solve equations using trial and error?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
	consecutive sums and other topics		
Unit Two: Game of Pig **Timeline: 7 weeks**			
Represent data with plots on the real number line (dot plots, histograms, and box plots). **CC.S-ID.1** Distinguish between correlation and causation. **CC.S-ID.9** Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.S-IC.2** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **CC.S-CP.1** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **CC.S-CP.2** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **CC.S-CP.3** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. **CC.S-CP.6** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.* **CC.S-CP.7** *– supplementary lesson is being developed by the publisher*	As an introduction to the probability concepts and skills needed to analyze the game of Pig, students work on a variety of problems involving chance occurrences. Through these experiences, they develop an understanding of the concept of expected value and learn to calculate expected value using an area model. They also encounter some real-life "games," such as buying insurance and playing the lottery, and discover that in such situations, expected value may not be the sole criterion for making a decision. In the unit activities, students explore these important mathematical ideas: • Learning what constitutes a "complete strategy" for a game and developing and analyzing strategies • Calculating probabilities as fractions, decimals, and percents by emphasizing equally likely outcomes and by constructing mathematical models, including area models and tree diagrams • Determining whether events are independent • Using the idea of "in the long run" to develop the concept of expected value and calculating and interpreting expected values • Solving problems involving conditional probability • Making and interpreting frequency bar graphs • Using simulations to estimate probabilities and compare strategies	Can students apply basic methods for calculating probabilities? Can students construct area models and tree diagrams? Can students distinguish between theoretical and experimental probabilities? Can students plan and carry out simulations? Can students collect and analyze data? Can students construct frequency bar graphs? Can students calculate, and interpret expected value? Can students apply the concept of expected value to real-world situations?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. **CC.S-CP.8** - *unit supplement to be developed* Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **CC.S-MD.1** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **CC.S-MD.2** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **CC.S-MD.3** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **CC.S-MD.5** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **CC.S-MD.5a** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* **CC.S-MD.5b** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **CC.S-MD.6**	• Comparing the theoretical analysis of a situation with experimental results • Examining how the number of trials in a simulation affects the results		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). **CC.S-MD.7**			
Unit Three: The Overland Trail **Timeline: 7 weeks**			
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **CC.N-Q.1** Define appropriate quantities for the purpose of descriptive modeling. **CC.N-Q.2** Interpret parts of an expression, such as terms, factors, and coefficients. **CC.A-SSE.1a** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.* **CC.A-SSE.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **CC.A-SSE.3** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CC.A-CED.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **CC.A-REI.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.A-REI.3**	Building on students' work in *Patterns*, this unit develops the central mathematical idea of functions and their representations. Students will move among the following four "faces" of functions: situations, graphs, tables and rules. The focus of this unit is on linear functions. Students will use starting values and rate of change to characterize linear functions, build In-Out tables, draw graphs, and write equations to represent specific contexts. They will use tables, graphs, and symbols to solve linear equations and systems of linear equations. They will fit lines to real data and use graphs and symbols representing these lines to solve problems in the context of the unit. The main concepts and skills that students will encounter and practice during the course of this unit can be summarized by category. **Constraints and Decision Making** • Creating examples that fit a set of constraints • Finding numbers that fit several conditions • Using tables of information and lines of best fit to make predictions and estimates • Working with mean and median **Algorithms, Variables, and Notation**	Can students interpret graphs and use graphs to represent situations? Can students relate graphs to their equations, with emphasis on linear relationships? Can students solve pairs of linear equations by graphing? Can students fit equations to data, both with and without graphing calculators? Can students develop and use principles for equivalent expressions, including the distributive property? Can students use the distributive property? Can students apply principles for equivalent equations to solve equations? Can students solve linear equations in one variable? Do students understand relationships between the algebraic expression defining a linear function and the graph of that function?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.A-REI.10** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **CC.A-REI.11** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.F-IF.2** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **CC.F-IF.6**	• Strengthening understanding of the distributive property • Developing numeric algorithms for problem situations • Expressing algorithms in words and symbols • Interpreting algebraic expressions in words using summary phrases • Developing meaningful algebraic expressions **Basics of Graphing** • Reviewing the coordinate system • Interpreting graphs intuitively and using graphs intuitively to represent situations • Making graphs from tabular information • Quantifying graphs with appropriate scales • Using graphs to represent two-variable equations and data sets • Using multiple representations—graphs, tables, and algebraic relationships—to describe situations **Linear Equations, Graphs, and Situations** • Finding and interpreting lines of best fit intuitively • Seeing the role of constant rate in linear situations • Using rates and starting values, or other data points, to create equations for straight lines • Laying the groundwork for the concept of slope • Using the point of intersection of two graphs to find values that satisfies two conditions		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.F-IF.7** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.F-IF.7b** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.F-LE.1a** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **CC.F-LE.1b** Interpret the parameters in a linear or exponential function in terms of a context. **CC.F-LE.5** Represent data on two quantitative variables on a scatter plot and describe how the variables are related. **CC.S-ID.6** Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.* **CC.S-ID.6a** *Informally assess the fit of a model function by plotting and analyzing residuals.* **CC.S-ID.6b** – *unit supplement to be developed* Fit a linear function for scatter plots that suggest a linear association. **CC.S-ID.6c** Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data. **CC.S-ID.7**	• Solving linear equations for one variable in terms of another • Solving problems involving two linear conditions • Solving linear equations in one variable **Graphs and Technology** • Making and interpreting graphs on a graphing calculator • Using the zoom and trace features to get information from a graphing calculator		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Compute (using technology) and interpret the correlation coefficient of a linear fit. ***CC.S-ID.8*** *– supplementary lesson is being developed by the publisher*			
Unit Four: The Pit and the Pendulum **Timeline: 7 weeks**			
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **CC.N-Q.3** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* **CC.A-SSE.1b** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.F-IF.7** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.F-IF.7b** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **CC.F-BF.3** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table). **CC.F-LE.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **CC.S-ID.2**	This unit draws on and extends students' work in the first three units. It blends scientific experiments with the statistical concepts of normal distribution and standard deviation and the algebra of functions and graphs. The main concepts and skills that students will encounter and practice during the course of this unit are summarized below. References to graphing calculators should be understood to include other technology that might be available. **Experiments and Data** • Planning and performing controlled scientific experiments • Working with the concept of period • Recognizing and accommodating for the phenomenon of measurement variation • Collecting and analyzing data • Expressing experimental results and other data using frequency bar graphs **Statistics** • Recognizing the normal distribution as a model for certain kinds of data • Making area estimates to understand the normal distribution • Developing concepts of data spread, especially standard deviation • Working with symmetry and concavity in connection with the normal distribution and standard deviation • Applying standard deviation and the normal distribution in problem contexts	• Can students describe normal distributions and their properties? • Can students use mean and standard deviation? • Can students use normal distribution, mean, and standard deviation?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **CC.S-ID.3** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. **CC.S-ID.4** Understand that statistics is a process for making inferences about population parameters based on a random sample from that population. **CC.S-IC.1** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **CC.S-MD.4**	• Distinguishing between population standard deviation and sample • Calculating the mean and standard deviation of data sets, both by hand and with calculators • Using standard deviation to decide whether a variation in experimental results is significant **Functions and Graphs** • Using function notation • Using graphing calculators to explore the graphs of various functions • Fitting a function to data using a graphing calculator • Making predictions based on curve-fitting		
Unit Five: Shadows **Timeline: 7 weeks**			
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* **CC.A-CED.4** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**CC.G-CO.1** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a rigid motion on a figure; given two figures, use the definition of congruence	The concept of similarity is the central theme of this unit. Through this concept, students explore the following important ideas from geometry and algebra. **Similarity and Congruence** • Developing intuitive ideas about the meaning of "same shape" and learning the formal definitions of similar and congruent • Discovering the special properties of triangles in connection with similarity, as well as other features of triangles as special polygons	Do students understand the meaning of angles and angle measurement? Can students apply the relationships among angles of polygons, including angle-sum formulas? Can students apply criteria for similarity and congruence? Can students use properties of similar polygons to solve real-	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
in terms of rigid motions to decide if they are congruent. **CC.G-CO.6** Explain using rigid motions the meaning of congruence for triangles as the equality of all corresponding pairs of sides and all corresponding pairs of angles. **CC.G-CO.7** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence. **CC.G-CO.8** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **CC.G-CO.9** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **CC.G-CO.10** – *supplementary lessons are being developed by the publisher to cover theorems not already included in the curriculum.* Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangle are parallelograms with congruent diagonals.* **CC.G-CO.11** – *supplementary lessons are being developed by the publisher to cover theorems not already included in the curriculum.* Verify experimentally the properties of dilations: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **CC.G-SRT.1a**	• Understanding the role of similarity in defining the trigonometric functions of sine, cosine and tangent **Proportional Reasoning and the Algebra of Proportions** • Understanding the meaning of proportionality in connection with similarity • Developing equations of proportionality from situations involving similar figures • Understanding the role of proportionality in nongeometric situations • Developing techniques for solving equations involving fractional expressions **Polygons and Angles** • Developing angle sum formulas for triangles and other polygons • Discovering the properties of angles formed by a transversal across parallel lines • Discovering the triangle inequality and investigating its extension to polygons **Logical Reasoning and Proof** • Working with the concept of counterexample in understanding the criteria for similarity • Proving conjectures about vertical and polygon angle sums • Understanding the role of the parallel postulate in proofs **Right Triangles and Trigonometry** • Learning standard terminology for triangles, including hypotenuse, leg, opposite side, and adjacent side • Learning the right triangle definitions of *sine*, *cosine*, and *tangent*	world problems? Can students use similarity to define right-triangle trigonometric functions? Can students apply right-triangle trigonometry to real-world problems? Do students understand the meaning of angles and their measurement? Do students recognize relationships among angles of polygons, including angle-sum formulas? Can students define and apply properties of similarity and congruence? Can students use properties of similar polygons to solve real-world problems? Can students use similarity to define right-triangle trigonometric functions? Can students apply right-triangle trigonometry to real-world problems?	

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessments
Verify experimentally the properties of dilations: The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **CC.G-SRT.1b** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides. **CC.G-SRT.2** Use the properties of similarity transformations to establish the AA criterion for similarity of triangles. **CC.G-SRT.3** Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.* **CC.G-SRT.4** Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures. **CC.G-SRT.5** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.G-SRT.6** Explain and use the relationship between the sine and cosine of complementary angles. **CC.G-SRT.7** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CC.G-SRT.8** Find the point on a directed line segment between two given points that divide the segment in a given ratio. **CC.G-GPE.6**	• Using sine, cosine, and tangent to solve real-world problems **Experiments and Data Analysis** • Planning and carrying out controlled experiments • Collecting and analyzing data • Identifying key features in graphs of data **Mathematical Modeling** • Using a geometric diagram to represent a real-world situation • Using scale drawings to solve problems • Applying properties of similar triangles to real-world situations • Exploring how models provide insight in a variety of situations		

Assessment Opportunities in this Unit:

End-of-Unit Assessments:

Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

On-Going Assessments:

Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills. Students' written and oral work provides many opportunities for teachers to gather this information.

- *Presentations on Calculator Exploration:* These presentations will give you information on how comfortable students are with calculators and open-ended investigation.
- *Pulling Out Rules:* This activity will help you gauge how well students understand the basic ideas of In-Out tables and evaluate their ability in writing rules to describe tables.
- *You're the Chef:* This summary activity will tell you how well students understand the arithmetic of positive and negative integers.
- *Presentations on Consecutive Sums:* These presentations will indicate how students are developing in their ability to conduct independent mathematical investigations.
- *An Angular Summary:* This activity will help you gauge students' understanding of the sum of the angles in a polygon and related formulas.
- *Border Varieties:* This activity will reflect students' understanding of the use of variables.
- *Pig Strategies:* This activity will help you gauge how well students understand the rules of Pig and assess their comfort level with the idea of strategy.
- *0 to 1, or Never to Always:* This activity will illustrate students' grasp of the 0-to-1 scale for probability.
- *Two-Dice Sums and Products:* This activity will show how well students understand and can work with two-dimensional area models.
- *Spinner Give and Take:* This activity can provide a baseline of students' initial understanding of the meaning of "the long run," in preparation for work with expected value.
- *Spins and Draws:* This activity will tell you how well students understand and can work with expected value.
- *A Fair Deal for the Carrier?:* This activity will inform you about students' ability to find probabilities in two-stage situations.
- *Little Pig Strategies:* This activity will tell you how well prepared students are for the detailed analysis of Little Pig.
- *The Best Little Pig:* This activity will inform you of students' grasp of the big picture in the analysis of Little Pig.
- *Creating Families:* This assignment will give you information on how well students can deal with verbal constraints.
- *Laced Travelers:* This activity will tell you whether students can put arithmetic processes into words.
- *Ox Expressions at Home:* This assignment will help you assess how well students understand meaningful algebraic expressions
- *Graph Sketches:* This activity will give you a sense of how well students understand graphs.
- *Who Will Make It?* This activity can help you gauge students' ability to make meaningful inferences from graphs.
- *All Four, One--Linear Functions:* This assignment will give you information about students' understanding of the connections among different ways to represent a situation.
- *Straight Line Reflections:* This activity will give you a sense of how well students understand concepts related to straight-line graphs.
- *More Fair Share for Hired Hands:* This assignment can provide information on student understanding of the connection between graphs and equations.
- *Family Comparisons by Algebra:* This activity will help you evaluate students' ability to represent situations using equations and their facility with solving linear equations.
- *Initial Experiments:* This activity will tell you how well students understand the idea of isolating a single variable.
- *Pulse Analysis:* This assignment will tell you about students' understanding of mean and frequency bar graphs.
- *Kai and Mai Spread Data:* This activity will give you a baseline of information about students' understanding of data spread.
- *Penny Weight Revisited:* This activity will guide you in determining students' intuitive understanding of standard deviation.
- *Pendulum Conclusions:* This assignment will tell you how well students can reason using the concept of standard deviation.

- *Graphing Summary:* This activity will give you information on what students know about the shape of graphs of various functions.
- *Mathematics and Science:* This assignment will give you insight into what students see as the key ideas of the unit.
- *Shadow Data Gathering* and *Working with Shadow Data:* These activities, which ask students to set up and conduct controlled experiments (as in the unit *The Pit and the Pendulum*), will provide evidence of their understanding of the unit problems.
- *Similar Problems:* This assignment will provide evidence of students' ability to write and solve proportions derived from similar figures.
- *Angles and Counterexamples:* This activity will help you assess students' ability to create and solve linear equations derived from a geometric context and their developing understanding of similarity.
- *Angles, Angles, Angles:* This assignment will give you information on students' knowledge of facts about angles created by intersecting lines (including transversals of parallel lines) and interior angles of polygons.
- *Mirror Madness:* This activity will tell you whether students can use the reflective property of mirrors along with the concept of similarity to do indirect measurement.
- *A Shadow of a Doubt:* This activity will provide evidence about whether students understand the general solution to the lamp shadow problem.
- *The Tree and the Pendulum:* This assignment will illustrate students' ability to use trigonometry to do indirect measurement.
- *A Bright, Sunny Day:* This activity will provide evidence of students' understanding of the general solution to the sun shadow problem.

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

Interactive Mathematics Program Curriculum Framework

School: Delaware STEM Academy **Curricular Tool:** IMP **Grade or Course:** Year 2 (grade 10)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit One: Do Bees Build it Best? **Timeline: 8 weeks**			
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **CC.N-Q.3** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.A-REI.2** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.A-REI.4b** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.* **CC.G-SRT.4** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CC.G-SRT.8** Prove the Laws of Sines and Cosines and use them to solve problems. **CC.G-SRT.10**	The regular form of a honeycomb is striking. Viewed end on, honeycomb cells resemble the hexagonal tiles on a bathroom floor. But a honeycomb is a three-dimensional object, a collection of right hexagonal prisms. Why do bees build their honeycombs this way? Concepts of measurement—especially area, surface area, and volume—are the mathematical focus of this unit. The main concepts and skills that students will encounter and practice during the unit are summarized by category here. ***Area*** • Understanding the role of units in measuring area • Establishing standard units for area, especially those based on units of length • Recognizing that a figure's perimeter alone does not determine its area • Discovering formulas for the areas of rectangles, triangles, parallelograms, and trapezoids • Establishing that a square has the greatest area of all rectangles with a fixed perimeter • Developing a formula for the area of a regular polygon with a given perimeter in terms of the number of sides • Discovering that for a fixed perimeter, the more sides a regular polygon has, the greater its area • Discovering that the ratio of the areas of similar figures is equal to the square of the ratio of their corresponding linear dimensions	Can students measure area using both standard and nonstandard units? Can students use several methods for finding areas of polygons, including development of formulas for area of triangles, rectangles, parallelograms, trapezoids, and regular polygons? Can students find surface area and volume for three-dimensional solids, including prisms and cylinders? Can students apply the Pythagorean theorem? Can students prove the Pythagorean theorem? Can students maximize area for a given perimeter? Do students understand the relationship between the areas and volumes of similar figures? Can students create successful tessellations? Can students apply right triangle trigonometry to area and perimeter problems?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **CC.G-SRT.11** Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **CC.G-GMD.1** *Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.* **CC.G-GMD.2** *– unit supplement to be developed* Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. **CC.G-GMD.3** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **CC.G-GMD.4** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typographic grid systems based on ratios). **CC.G-MG.3**	***The Pythagorean Theorem*** • Discovering the Pythagorean theorem by comparing the areas of the squares constructed on the sides of a right triangle • Proving the Pythagorean theorem using an area argument • Applying the Pythagorean theorem in a variety of situations ***Surface Area and Volume*** • Understanding the role of units in measuring surface area and volume • Establishing standard units for surface area and volume, especially those based on a unit of length • Recognizing that a solid figure's surface area alone does not determine its volume • Developing principles relating the volume and surface area of a prism to the area and perimeter of its base • Discovering that the ratio of the surface areas of similar solids is equal to the square of the ratio of their corresponding linear dimensions, and that the ratio of the volumes of similar solids is equal to the cube of the ratio of their corresponding linear dimensions ***Trigonometry*** • Reviewing right-triangle trigonometry • Finding the ranges of the basic trigonometric functions (for acute angles) • Using the terminology and notation of inverse trigonometric functions ***Miscellaneous*** • Reviewing similarity • Reviewing the triangle inequality • Reviewing the angle sum property for triangles • Strengthening two- and three-dimensional spatial visualization skills • Examining the concept of tessellation and		

<table>
<tr><th>Standards Alignment</th><th>Unit Concepts /
Big Ideas from IMP</th><th>Essential Questions</th><th>Assessment</th></tr>
<tr><td></td><td>discovering which regular polygons tessellate
• Developing some properties of square-root radicals
• Developing the general concept of an inverse function</td><td></td><td></td></tr>
<tr><td colspan="4">Unit Two: Cookies
Timeline: 7 weeks</td></tr>
<tr><td>Create equations and inequalities in one variable and use them to solve problems.
Include equations arising from linear and quadratic functions, and simple rational and exponential functions. CC.A-CED.1

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. CC.A-CED.2

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. CC.A-CED.3

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.A-REI.3

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. CC.A-REI.5 – supplementary lesson is being developed by the publisher

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. CC.A-REI.6</td><td>The central mathematical focus of Cookies is the formulation and solution of problems of optimization, or linear programming problems. In problems of this type, a linear function is to be optimized and a set of linear conditions constrains the possible solutions. Linearity is an important feature of these two-variable problems, in two ways:
• The constraints are linear, so the feasible region is a polygon and its vertices can be found by solving pairs of linear equations.
• The expression to be maximized or minimized is linear, so the points that give this expression a particular value lie on a straight line, and investigating a series of values produces a family of parallel lines.
The linear programming problems that students encounter in this unit involve only two variables and a limited number of constraints. Their solutions are therefore easier to understand graphically, and the algebra needed to find their exact solutions is manageable.
The main concepts and skills that students will encounter and practice during the unit are summarized here.
Using Variables to Represent Problems
• Expressing and interpreting constraints using inequalities
• Expressing problem situations using systems of linear equations
Working with Variables, Equations, and</td><td>Can students express real-world situations in terms of equations and inequalities?

Can students apply the distributive property?

Can students use several methods for solving systems of linear equations in two variables?

Can students define and recognize dependent, inconsistent, and independent pairs of linear equations?

Can students solve non-routine equations using graphing calculators?

Can students write and graph linear inequalities in two variables?

Can students use principles of linear programming for two variables?

Can students create linear programming problems with two variables?</td><td>All assessments are listed at the end of the curriculum map.</td></tr>
</table>

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.A-REI.7** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes. **CC.A-REI.12** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.F-IF.7a**	***Inequalities*** • Finding equivalent expressions and inequalities • Solving linear equations for one variable in terms of another • Developing and using a method for solving systems of two linear equations in two unknowns • Recognizing inconsistent systems and dependent systems ***Graphing*** • Graphing linear inequalities and systems of linear inequalities • Finding the equation of a straight line and the inequality for half plane • Using graphing calculators to draw feasible regions • Relating the intersection point of graphed lines to the common solution of the related equations • Using graphing calculators to estimate coordinates of points of intersection ***Reasoning Based on Graphs*** • Recognizing that setting a linear expression equal to a series of constants produces a family of parallel lines • Finding the maximum or minimum of a linear equation over a region • Examining how the parameters in a problem affect the solution • Developing methods of solving linear programming problems with two variables ***Creating Word Problems*** • Creating problems that can be solved using two equations in two unknowns • Creating problems that can be solved by linear programming methods		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit Three: Is There Really a Difference? **Timeline: 8 weeks**			
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data. **CC.S-ID.5** Distinguish between correlation and causation. **CC.S-ID.9** Understand that statistics is a process for making inferences about population parameters based on a random sample from that population. **CC.S-IC.1** Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.S-IC.2** Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **CC.S-IC.3** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.S-IC.4** Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment. **CC.S-IC.5** Evaluate reports based on data. **CC.S-IC.6** Construct and interpret two-way frequency tables of data when two categories are associated with each	The unit explores two categories of problems: • Problems that compare a single population to a theoretical model (the theoretical-model case) • Problems that compare two distinct populations (the two-population case) Students learn that statisticians often presume that a "neutral" hypothesis, called a null hypothesis, holds unless there is clear evidence to the contrary. In the context of the two categories of problems, the null hypothesis is that the single population *does* fit the model or that the two populations being studied *are* the same. Students learn that to evaluate the null hypothesis, they must examine whether the observed data could reasonably have occurred under that null hypothesis. In the course of studying such questions, students will • work with double-bar graphs to explore data • form hypotheses and corresponding null hypotheses • develop an intuitive sense for evaluating differences between sets of data • learn ways of organizing and presenting data • learn about designing and carrying out statistical studies This unit builds on students' prior experience with statistical ideas in the Year 1 unit *The Pit and the Pendulum.* In that unit, students worked with the normal distribution and used the standard deviation statistic as their primary tool. In this unit, students use the chi-square statistic, or x^2 statistic. In the main activities of the unit, students use the x^2 statistic only in the case of one degree of freedom. Supplemental activities explore more general use of the statistic.	Can students draw inferences from statistical data? Can students design, conduct, and interpret statistical experiments? Can students make and test statistical hypotheses? Can students formulate null hypotheses and understand its role in statistical reasoning? Can students use the χ^2 statistic? Do students understand that tests of statistical significance do not lead to definitive conclusions? Can students solve problems that involve conditional probability?	All assessments are listed at the end of the curriculum map.

Standards Alignment	**Unit Concepts / Big Ideas from *IMP***	**Essential Questions**	**Assessment**
object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.* **CC.S-CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.* **CC.S-CP.5** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **CC.S-MD.4**	Although the unit makes intensive use of the x^2 statistic, the real emphasis is on broader statistical ideas, such as the null hypothesis, sampling fluctuation, and hypothesis testing. The main concepts and skills that students will encounter and practice during the course of this unit are summarized by category here. ***Setting Up Statistical Investigations*** • Distinguishing between data snooping and hypothesis testing • Describing the characteristics of a good sample • Making null hypotheses • Using proportional reasoning to analyze the consequences of a null hypothesis • Designing and conducting statistical experiments ***Interpreting Data*** • Making hypotheses about larger populations by analyzing sample data • Constructing and drawing inferences from charts, tables, and graphs, including frequency bar graphs and double-bar graphs • Determining whether to accept or reject a null hypothesis • Understanding the consequences of rejecting a null hypothesis • Interpreting statistical experiments and communicating the outcomes ***The x^2 Statistic*** • Developing intuition about the meaning of the x^2 statistic • Using simulations to estimate the x^2 distribution • Interpreting the x^2 distribution curve as a probability table • Calculating and interpreting the x^2 statistic in order to compare data from real-world situations to theoretical models		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Calculating and interpreting the x^2 statistic in order to compare two populations • Using the x^2 statistic to make decisions • Understanding some limitations in applying the x^2 statistic ***Related Concepts*** • Working with conditional probabilities • Using simulations to develop intuition and to obtain data about sampling fluctuation • Developing intuition about when differences in samples indicate that the larger populations are likely to be different • Understanding why neither numeric difference nor percentage difference is an adequate tool for measuring the "weirdness" of data • Reviewing the normal distribution and standard deviation and their applications to decision making		
Unit Four: Fireworks **Timeline: 6 weeks**			
Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. **CC.A-SSE.2** Factor a quadratic expression to reveal the zeros of the function it defines. **CC.A-SSE.3a** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **CC.A-SSE.3b** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.A-APR.1** *Know and apply the Remainder Theorem: For a*	*Fireworks* focuses on the use of quadratic functions to represent a variety of real-world situations and on the development of algebraic skills for working with those functions. Experiences with graphs play an important role in understanding the behavior of quadratic functions. The main concepts and skills students will encounter and practice during the unit are summarized here. ***Mathematical Modeling*** • Expressing real-world situations in terms of functions and equations • Applying mathematical tools to models of real-world problems • Interpreting mathematical results in terms of real-world situations	Can students solve quadratic equations by factoring? Can students relate the number of roots of a quadratic equation to the graph of the associated quadratic function? Can students use the method of completing the square to analyze the graphs of quadratic equations and to solve quadratic equations?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **CC.A-APR.2** - *unit supplement to be developed* Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.A-APR.3** Solve quadratic equations in one variable. **CC.A-REI.4** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.A-REI.4a** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.A-REI.4b** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.F-IF.7a** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.F-IF.7c** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.F-IF.8** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.F-IF.8a**	***Graphs of Quadratic Functions*** • Understanding the roles of the vertex and x-intercept in the graphs of quadratic functions • Recognizing the significance of the sign of the x^2 term in determining the orientation of the graph of a quadratic function • Using graphs to understand and solve problems involving quadratic functions ***Working with Algebraic Expressions*** • Using an area model to understand multiplication of binomials, factoring of quadratic expressions, and completing the square of quadratic expressions • Transforming quadratic expressions into vertex form • Simplifying expressions involving parentheses • Identifying certain quadratic expressions as perfect squares ***Solving Quadratic Equations*** • Interpreting quadratic equations in terms of graphs and vice versa • Estimating x-intercepts using a graph • Finding roots of an equation using the vertex form of the corresponding function • Using the zero product rule of multiplication to solve equations by factoring		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.F-IF.9** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\, f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **CC.F-BF.3**			
Unit Five: All About Alice **Timeline: 5 weeks**			
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **CC.N-RN.1** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **CC.N-RN.2** *Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.* ***CC.N-RN.3*** *– supplementary lesson is being developed by the publisher* Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate*	Unlike most other IMP units, All About Alice has no central problem to solve. Instead, there is a general context to the unit. In particular, the Alice story provides a metaphor for understanding exponents. When Alice eats an ounce of cake, her height is multiplied by a particular whole-number amount; when she drinks an ounce of beverage, her height is multiplied by a particular fractional amount. Using this metaphor, students reason about exponential growth and decay. Students use several approaches to extend exponentiation beyond positive integers: a contextual situation, algebraic laws, graphs, and number patterns. They then apply principles of exponents to study logarithms and scientific notation. The main concepts and skills students will encounter and practice during the course of this unit are summarized by category here.	Can students use exponential expressions, including zero, negative, and fractional exponents? Can students apply the laws of exponents? Can students use scientific notation? Can students use the concept of order of magnitude in estimation?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
mortgage payments. **CC.A-SSE.4** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.F-IF.7e** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.F-IF.8b** Find inverse functions. **CC.F-BF.4** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$. **CC.F-BF.4a** Verify by composition that one function is the inverse of another. **CC.F-BF.4b** Read values of an inverse function from a graph or a table, given that the function has an inverse. **CC.F-BF.4c** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **CC.F-BF.5** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.F-LE.1** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.F-LE.1a**	***Extending the Operation of Exponentiation*** • Defining the operation for an exponent of zero • Defining the operation for negative integer exponents • Defining the operation for fractional exponents ***Laws of Exponents*** • Developing the additive law of exponents • Developing the law of repeated exponentiation ***Graphing*** • Describing the graphs of exponential functions • Comparing graphs of exponential functions for different bases • Describing the graphs of logarithmic functions • Comparing graphs of logarithmic functions for different bases ***Logarithms*** • Understanding the meaning of logarithms • Making connections between exponential and logarithmic equations ***Scientific Notation*** • Converting numbers from ordinary notation to scientific notation, and vice versa • Developing principles for doing computations using scientific notation • Using the concept of order of magnitude in estimation		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **CC.F-LE.1c** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **CC.F-LE.3**			

Assessment Opportunities in this Unit:

End-of-Unit Assessments:
Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

Ongoing Assessment:
Assessment is a component in providing the best possible ongoing instructional program for students. Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills.

Students' written and oral work provides many opportunities for teachers to gather this information. Here are some recommendations of written assignments and oral presentations to monitor especially carefully that will offer insight into student progress.

- How Many Can You Find?: This assignment will inform you about how well students have understood the basics about the meaning of area.
- That's All There Is!: This activity will tell you how comfortable students are with a more open-ended approach to area.
- More Gallery Measurements: This activity will provide information on students' grasp of the fundamentals of right-triangle trigonometry.
- Any Two Sides Work, Make the Lines Count, and The Power of Pythagoras: These assignments will tell you about students' comfort with using the Pythagorean theorem.
- Leslie's Fertile Flowers: In this activity, students need to combine ideas about area with use of the Pythagorean theorem, so it will give you a sense of their facility with these concepts.
- More Fencing, Bigger Corrals: This activity, which involves how changes in linear dimensions affect area, will help you decide how much work students need on this topic.
- Not a Sound: This assignment will give you feedback on students' grasp of the concept of surface area.
- Inequality Stories, Part I: This assignment will give you information about students' understanding of how real-life contexts can be expressed in algebraic terms using inequalities.
- Profitable Pictures: This activity will tell you how well students understand how profit lines can be used to determine an optimal value.
- Changing What You Eat: In this assignment, students will demonstrate their understanding of how changing specific parameters in a problem affects the solution.
- Get the Point: This investigation will give you insight into students' abilities to think about systems of linear equations in flexible ways.
- A Reflection on Money: This assignment will give you information about students' comfort levels with solving systems of linear equations.
- "How Many of Each Kind?" Revisited: This activity will tell you how well students have synthesized the ideas of the unit.

- Changing the Difference, Part I: This work will give you information on students' sense of how probabilities behave with large samples.
- Loaded or Not?: This activity will tell you how well students can interpret experimental data.
- Decisions with Deviation: This assignment will provide information about students' understanding of how to use the normal distribution.
- Measuring Weirdness with χ^2: This activity will give you information about students' understanding of how to calculate and use the χ^2 statistic.
- Late in the Day: This assignment will give you feedback on how well students can set up and analyze a situation using the χ^2 statistic.
- "Two Different Differences" Revisited: This activity will give you information on students' abilities to do a complete analysis of a situation using the χ^2 statistic.
- Using Vertex Form will illustrate students' ability to pull together and use the various components of the vertex form of a quadratic.
- Squares and Expansions will demonstrate students' developing understanding of the technique of completing the square.
- How Much Can They Drink? will provide information on students' developing understanding of how to find the maximum value of a quadratic function to find the solution to a problem in context.
- Another Rocket will show how well students are prepared to address the unit problem.
- A Fireworks Summary is a reflective piece in which students summarize their work on the unit problem.
- A Quadratic Summary is a reflective piece in which students summarize their understanding of the big ideas of the unit.
- Graphing Alice: This assignment will give you information about how well students understand the basic Alice metaphor and about their comfort with nonlinear graphs.
- Having Your Cake and Drinking Too: This activity will reveal students' ability to work with the Alice metaphor in a complex situation.
- Negative Reflections: This assignment will tell you how well students understand the extension of exponentiation to negative exponents.
- All Roads Lead to Rome: This activity will give you information on students' ability to synthesize a variety of approaches to understanding a mathematical concept.
- Alice on a Log: This assignment will give you information on students' understanding of the basics about logarithms.

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

Interactive Mathematics Program Curriculum Framework

School: Delaware STEM Academy **Curricular Tool: IMP** **Grade or Course: Year 3 (grade 11)**

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit One: Orchard Hideout **Timeline: 7 weeks**			
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **CC.G-CO.1** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **CC.G-CO.9** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **CC.G-CO.12** *Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.* ***CC.G-CO.13*** *– supplementary lesson is being developed by the publisher* *Derive the formula A = ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.* ***CC.G-SRT.9*** *– supplementary lesson is being developed by the publisher* *Prove that all circles are similar.* ***CC.G-C.1*** *– supplementary lesson is being developed by the publisher*	The central unit problem concerns a couple who have planted an orchard of trees in careful rows and columns on a circular lot. The couple realizes that, after a while, the trunks of their trees will become so thick that they will no longer be able to see out from the center of the orchard. In other words, the orchard will become a "hideout." The main unit question is this: **How soon after the couple plant the orchard will the center of the lot become a true "orchard hideout"?** Students' search for the answer to this question leads them to the study of several aspects of geometry. Students use the Pythagorean Theorem to measure distances within the orchard, leading to development of the distance formula. As a sidelight to their work with the distance formula, students construct the general equation of a circle. Giving the initial size of the trees in terms of circumference and the growth rate in terms of cross-sectional area motivates development of the area and circumference formulas for a circle. While solving the unit problem, students encounter a variety of tangents (both	Can students explain the relationship of the area and circumference of a circle to its radius? Do students understand the significance of using regular polygons to approximate the area and circumference of a circle? Can students justify locus descriptions of various geometric entities, such as perpendicular bisectors and angle bisectors? Can students apply properties of parallel lines? Can students identify possible intersections of lines and planes? Can students use "if and only if" in describing sets of points fitting given criteria? Can students define and use the concept of the converse of a statement?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **CC.G-C.2** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **CC.G-C.3** *Construct a tangent line from a point outside a given circle to the circle.* ***CC.G-C.4*** *- supplementary lesson is being developed by the publisher* Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **CC.G-GPE.1** Derive the equation of a parabola given a focus and directrix. **CC.G-GPE.2** Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola. **CC.G-GPE.3** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **CC.G-GPE.4** *Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).* ***CC.G-GPE.5*** *– supplementary lesson is being developed by the publisher*	figuratively and literally). One result is a proof that a tangent to a circle is perpendicular to the radius at the point of tangency. They use the technique of completing the square to put certain quadratic equations into standard form to find the radius and center of the circles they represent. Other ideas arise through the unit's POWs. For example, students prove basic facts about perpendicular bisectors and angle bisectors, thereby establishing the existence of both circumscribed and inscribed circles for triangles. The main concepts and skills students will encounter and practice during the unit are summarized below. ***Coordinate geometry*** • Using the Cartesian coordinate system to organize a complex problem • Developing and applying the distance formula • Developing the standard form for the equation of a circle with a given center and radius • Finding the distance from a point to a line in a coordinate setting • Developing and applying the midpoint formula ***Circles*** • Using similarity to see that the circumference of a circle should be a constant times its radius, and that the area of a circle should be a constant times the square of its radius • Finding formulas for the perimeter and area of regular polygons circumscribed about a circle		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula. **CC.G-GPE.7** Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. **CC.G-GMD.3** Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **CC.G-MG.1**	• Using circumscribed polygons to see that the "circumference coefficient" for the circle is twice the "area coefficient" for the circle • Defining π and understanding why it appears in the formulas for both the circumference and the area of a circle • Developing and applying formulas for the circumference and area of a circle ***Synthetic Geometry*** • Identifying and describing a set of points satisfying a geometric condition • Discovering and proving that the set of points equidistant from two given points is the perpendicular bisector of the segment connecting the given points • Defining the distance from a point to a line and proving that the perpendicular distance is the shortest • Discovering and proving that any line through the midpoint of a segment is equidistant from the endpoints of the segment • Discovering and proving that the set of points equidistant from two intersecting lines consists of the bisectors of the angles formed by the lines ***Algebra*** • Using the technique of completing the square to transform equations of circles into standard form • Using algebra in a variety of proofs involving coordinates and angles ***Logic*** • Understanding and using the phrases "if-then" and "if and only if" in		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	definitions and proofs • Working with converses ***Miscellaneous*** • Using symmetry to help analyze a problem • Learning about Pythagorean triples		
Unit Two: Meadows or Malls? **Timeline: 11 weeks**			
Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.N-VM.6** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **CC.N-VM.7** Add, subtract, and multiply matrices of appropriate dimensions. **CC.N-VM.8** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **CC.N-VM.9** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **CC.N-VM.10** *Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.* ***CC.N-VM.11*** *– supplementary unit being is developed by the publisher* *Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.* ***CC.N-VM.12*** *- supplementary unit is being developed by the publisher*	The main concepts and skills that students will encounter and practice during this unit are: ***General Linear Programming*** • Seeing that for two-variable problems, the optimal value always occurs at a corner point of the feasible region • Generalizing the corner-point principle to more than two variables • Recognizing that for two-variable problems, corner points can be found as the intersections of lines corresponding to constraint equations or inequalities • Generalizing the method of finding corner points to more than two variables ***Solving Linear Equations*** • Using substitution, graphing, and guess-and-check methods to solve systems of linear equations in two variables • Developing and using the elimination method to solve systems of linear equations in two or more variables • Using the concepts of **inconsistent**, **dependent,** and **independent** systems of equations ***Geometry in the Plane and in 3-Space***	Can students use the elimination method for solving systems of linear equations in up to four variables? Can students extend the concepts of dependent, inconsistent, and independent systems of linear equations to more than two variables? Can students use matrices? Can students use the operations of matrix addition and multiplication in the context of applied problems? Can students use of matrices to represent systems of linear equations? Can students use the identity element and inverse in the context of matrices? Can students use matrices and matrix inverses to solve systems of linear equations? Can students relate the existence of matrix inverses to the uniqueness of the solution of corresponding systems of	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.A-CED.3** Represent a system of linear equations as a single matrix equation in a vector variable. **CC.A-REI.8** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.A-REI.9**	• Extending the concept of coordinates to three variables by introducing a third axis perpendicular to the first two • Graphing linear equations in three variables and recognizing that these graphs are planes in 3-space • Seeing that two distinct points always determine a unique line and that two distinct lines in the plane determine a unique point unless the lines are parallel • Examining the possible intersections of planes in 3-space • Relating the possible intersections of lines and planes to the algebra of solving linear systems in two or three variables ***Matrix Algebra*** • Using matrices to represent information • Using problem situations to motivate and develop the definitions of matrix addition and multiplication • Examining whether matrix operations have certain properties, such as associativity and commutativity ***Matrices and Systems of Linear Equations*** • Seeing that systems of linear equations are equivalent to certain types of matrix equations • Recognizing the role of identity and inverse elements in solving certain types of matrix equations • Finding matrix inverses by hand by solving systems of linear equations • Understanding the relationship between a system of linear equations	linear equations? Can students use calculators to multiply and invert matrices and to solve systems of linear equations? Can students apply the concepts of linear programming to problems with several variables? Can students use equations of planes in three-dimensional coordinate geometry? Can students define polar coordinates? Do students recognize graphs of polar equations?	

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	having a unique solution and the coefficient matrix being invertible ***Technology*** • Entering matrices and doing matrix operations on a graphing calculator • Using matrix inversion on a graphing calculator to solve systems of linear equations		
Unit Three: Small World, Isn't It? **Timeline: 6 weeks**			
Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **CC.A-SSE.3c** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* **CC.A-SSE.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **CC.F-IF.5** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.F-IF.8b** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **CC.F-BF.2** Recognize situations in which a quantity grows or decays by a	The main concepts and skills that students will encounter and practice during this unit are: ***Rate of Change*** • Evaluating average rate of change in terms of the coordinates of points on a graph • Understanding the relationship between the rate of change of a function and the appearance of its graph • Realizing that in many contexts, the rate of growth or decline with respect to time in a population is proportional to the population ***Slope and Linear Functions*** • Developing an algebraic definition of slope • Proving, using similarity, that a line has a constant slope • Understanding the significance of a negative slope for a graph and an applied context • Seeing that the slope of a line is equal to the coefficient of x in the $y = a + bx$ representation of the line • Using slope to develop equations for lines	Can students use exponential and logarithmic functions and describe their graphs? Do students understand the relationship between logarithms and exponents? Do students understand that the derivative of an exponential function is proportional to the value of the function? Can students use the general laws of exponents? Do students understand the meaning and significance of *e*? Can students approximate data with an exponential function? Can students define slope and understand its relationship to rate of change and to equations for straight lines? Can students develop equations for straight lines from two points and from point-slope information? Can students apply various	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
constant percent rate per unit interval relative to another. **CC.F-LE.1c** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table). **CC.F-LE.2** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **CC.F-LE.4** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **CC.G-MG.2** Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.* **CC.S-ID.6a**	***Derivatives*** • Developing the concept of the derivative of a function at a point • Seeing that the derivative of a function at a point is the slope of the tangent line at that point • Finding numerical estimates for the derivatives of functions at specific points • Working with the derivative of a function as a function in itself • Realizing that for functions of the form $y = b^x$, the derivative at each point of the graph is proportional to the y-value at that point ***Exponential and Logarithmic Functions*** • Using exponential functions to model real-life situations • Strengthening understanding of logarithms • Reviewing and applying the principles that $a^b \cdot a^c = a^{b+c}$ and $(a^b)^c = a^{bc}$ • Understanding and using the fact that $a^{\log_a b} = b$ • Discovering that any exponential function can be expressed using any positive number other than 1 as a base • Learning the meaning of the terms *natural logarithm* and *common logarithm* • Using an exponential function to fit a curve to numerical data ***The Number* e *and Compound Interest*** • Estimating the value of b for which the function $y = b^x$ has a derivative at each point on its graph equal to the y-value at that point • Developing and using a formula for compound interest	formulas from coordinate geometry, including: • Distance formula? • Midpoint formula? • Equation of a circle with arbitrary center and radius? Can students find the distance from a point to a line? Do students understand the meaning of the derivative of a function at a point and its relationship to instantaneous rate of change? Can students approximate the value of a derivative at a given point?	

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Seeing that expressions of the form $(1+1/n)^n$ have a limiting value, called *e*, as *n* increases without bound • Learning that the limiting value *e* is the same number as the special base for exponential functions		
Unit Four: Pennant Fever **Timeline: 4 weeks**			
Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. **CC.A-APR.4** – *unit supplement to be developed* Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. **CC.A-APR.5** Use permutations and combinations to compute probabilities of compound events and solve problems. **CC.S-CP.9**	The main concepts and skills that students will encounter and practice during this unit are: ***Probability and statistics*** • Developing a mathematical model for a complex probability situation • Using area diagrams and tree diagrams to find and explain probabilities • Using a simulation to understand a situation, to help analyze probabilities, and to support a theoretical analysis • Finding expected value • Finding and using probabilities for sequences of events • Using specific problem contexts to develop the binomial distribution and finding a formula for the associated probabilities • Using probability to evaluate null hypotheses ***Counting principles*** • Developing systematic lists for complex situations • Using the multiplication principle for choosing one element from each of several sets • Defining and using the concepts of permutation and combination	Can students apply principles for finding the probability for a sequence of events? Can students systematically list possibilities for complex problems? Can students use combinatorial and permutation coefficients in the context of real-world situations, and understanding the distinction between combinations and permutations? Can student use Pascal's triangle? Can students use the binomial distribution? Can students express the physical laws of falling bodies in terms of quadratic functions?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	• Understanding and using standard notation for counting permutations and combinations • Developing formulas for the permutation and combinatorial coefficients ***Pascal's triangle and combinatorial coefficients*** • Finding patterns and properties within Pascal's triangle • Recognizing that Pascal's triangle consists of combinatorial coefficients • Explaining the defining pattern and other properties of Pascal's triangle using the meaning of combinatorial coefficients • Developing and explaining the binomial theorem		
Unit Five: High Dive **Timeline: 6 weeks**			
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* **CC.A-SSE.1b** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.* **CC.A-APR.6** – *unit supplement to be developed* *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.* **CC.A-APR.7** – *supplementary unit is being developed by publisher*	The main concepts and skills that students will encounter and practice during this unit are: ***Trigonometry*** • Extending the trigonometric functions to all angles • Reinforcing the importance of similarity in the definitions of the trigonometric functions • Graphing the trigonometric functions and variations on those functions • Defining the inverse trigonometric functions and principal values • Discovering and explaining the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$, and other trigonometric identities • Defining polar coordinates and finding rectangular coordinates from	Can students apply right-triangle trigonometry to real-world situations? Can students extend the right-triangle trigonometric functions to circular functions? Can students use trigonometric functions to work with polar coordinates? Can students define radian measure? Can students graph the sine and cosine functions and variations of these functions? Can students use inverse	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.F-IF.4** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.F-IF.7e** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.F-TF.2** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **CC.F-TF.5** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **CC.F-TF.7** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. **CC.F-TF.8**	polar coordinates and vice versa ***Physics*** • Developing quadratic expressions for the height of free-falling objects, based on the principle of constant acceleration • Recognizing that a person falling from a moving object will follow a different path than someone falling from a stationary object ***Quadratic Equations*** • Developing simple quadratic equations to describe the behavior of falling objects	trigonometric functions? Can students apply various trigonometric formulas, including: • The Pythagorean identity? • Formulas for the sine and cosine of a sum of angles? • The law of sines and the law of cosines?	

Assessment Opportunities in this Unit:

End-of-Unit Assessments: Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

On-Going Assessments:
Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills. Students' written and oral work provides many opportunities for teachers to gather this information.

- *Sprinkler in the Orchard*
- *Proving with Distance—Part I or Proving with Distance—Part II*

- *Polygoning the Circle*
- *Orchard Growth Revisited*
- *Cable Ready*
- *Hiding in the Orchard*
- Presentations of *Programming Puzzles*
- Presentations or write-ups of *Just the Plane Facts*
- *Three Variables, Continued*
- *Matrices in the Oven*
- *Inverses and Equations*
- Presentations of *Meadows or Malls? Revisited*
- *How Many More People?*
- *Points, Slopes, and Equations*
- *Photo Finish*
- *What's It All About?*
- *Slippery Slopes*
- *Return to "A Crowded Place"*
- *Baseball Probabilities*
- *How Likely Is All Wins?*
- *Monthly Matches*
- *Cones from Bowls, Bowls from Cones*
- *Who's on First?*
- *About Bias*
- *Race for the Pennant Revisited*
- *As the Ferris Wheel Turns*
- *Testing the Definition*
- *More Beach Adventures*
- *A Practice Jump*
- *Moving Cart, Turning Ferris Wheel*

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.

Interactive Mathematics Program Curriculum Framework

School: Delaware STEM Academy **Curricular Tool:** IMP **Grade or Course** Year 4 (grade 12)

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Unit One: The Diver Returns **Timeline: 6 weeks**			
Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form *a* + *bi* with *a* and *b* real. **CC.N-CN.1** Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **CC.N-CN.2** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **CC.N-CN.3** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **CC.N-CN.4** *Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (1 – √3i)3 = 8 because (1 – √3i) has modulus 2 and argument 120°. **CC.N-CN.5** - supplementary lesson is being developed by publisher* *Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **CC.N-CN.6** – supplementary lesson is being developed by publisher*	This unit uses key ideas from *High Dive*, including the extension of the trigonometric functions and the physics of objects falling from rest. The unit builds on those ideas, especially extending the physics principles to include motion with both horizontal and vertical initial components, which students learn to express as vectors. This leads to a study of quadratic equations and the need to express a solution in terms of the coefficients. That work culminates in the development of the quadratic formula and an introduction of complex numbers. The main concepts and skills that students will encounter and practice during the unit are summarized below. ***Trigonometry and Geometry*** • Using the extended trigonometric functions • Applying the principle that the tangent to a circle is perpendicular to the radius at the point of tangency ***Physics*** • Reinforcing the idea that a person falling from a moving object will follow a different path than someone falling from a stationary object • Expressing velocity in terms of vertical and horizontal components • Representing the motion of falling objects when the vertical and horizontal components of the initial velocity are both nonzero ***Quadratic Equations*** • Recognizing the importance of quadratic equations in the analysis of falling objects • Developing the quadratic formula	Can students apply the quadratic formula? Can students express the physical laws of falling bodies in terms of quadratic functions? Can students use complex numbers to solve certain quadratic equations? Can students extend right-triangle trigonometric functions to circular functions?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
Solve quadratic equations with real coefficients that have complex solutions. **CC.N-CN.7** *Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).* ***CC.N-CN.8*** *– supplementary lesson is being developed by publisher* *Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.* ***CC.N-CN.9*** *– supplementary lesson is being developed by publisher* Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, ***v***). **CC.N.VM.1** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **CC.N-VM.2** Solve problems involving velocity and other quantities that can be represented by vectors. **CC.N-VM.3** Add vectors end-to-end, componentwise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **CC.N-VM.4a** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **CC.N-VM.4b** Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the	• Using the quadratic formula to solve quadratic equations • Finding a general solution for the falling time of objects with an initial vertical velocity ***Complex Numbers*** • Seeing the need to extend the number system to solve certain quadratic equations • Establishing basic ideas about complex number arithmetic • Representing complex numbers in the plane and seeing addition of complex numbers as a vector sum		

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **CC.N-VM.4c** *Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.* ***CC.N-VM.5a*** *– supplementary lesson is being developed by publishers* *Compute the magnitude of a scalar multiple c**v** using $\|c\mathbf{v}\| = \|c\|\mathbf{v}$. Compute the direction of c**v** knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$).* ***CC.N-VM.5b****- supplementary lesson is being developed by publisher*			
Unit Two: The World of Functions **Timeline: 6 weeks**			
Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.F-IF.7c** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **CC.F-IF.7d** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **CC.F-BF.1b** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at*	Over the course of this unit, students develop a wide range of ideas about functions. The main concepts and skills that students will encounter and practice during the unit are summarized below. ***General Notions Regarding Functions*** • Recognizing four ways of representing a function—tabular, graphical, algebraic, and situational—and moving from one representation to another • Formally defining functions as sets of ordered pairs • Reviewing some basic families of functions ***Properties of Specific Families of Functions*** • Finding, describing, and proving patterns in the tables of linear, quadratic, cubic, and exponential functions based on the algebraic form of the functions	Can students apply families of functions from several perspectives: • Through their algebraic representations? • In relationship to their graphs? • As tables of values? • In terms of real-world situations that they describe? Can students describe the effect of changing parameters on functions in a given family? Can students describe end behavior and asymptotes of	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
the location of the weather balloon as a function of time. **CC.F-BF.1c** Find inverse functions. **CC.F-BF.4** *Produce an invertible function from a non-invertible function by restricting the domain.* **CC.F-BF.4d** – *unit supplement to be developed* *Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.* **CC.F-TF.6** – *supplementary lesson is being developed by the publisher*	• Seeing the sets of linear and exponential functions as two-parameter families and comparing the two types of growth • Applying the concepts of direct and inverse proportionality and constants of proportionality • Using absolute value functions and step functions to model problem situations • Using rational functions to model problem situations ***End Behavior and Asymptotes of Functions*** • Finding vertical and horizontal asymptotes for specific functions and finding functions with given asymptotes • Relating asymptotic behavior to situations • Characterizing end behavior of functions and finding the behavior of particular functions ***Fitting Functions to Data*** • Finding the specific function in a given family to fit a given situation or set of data • Developing a measure of "quality of fit" of a function to a set of data • Applying the least-squares criterion for quality of fit • Using a calculator's regression feature to find a function that fits a given set of data ***Combining and Modifying Functions*** • Arithmetic operations on functions —Describing situations using arithmetic combinations of functions —Relating arithmetic operations on functions to graphs —Formally defining arithmetic operations on functions • Composite Functions —Developing the concept of composition of functions based on situations —Defining composition notation —Establishing that composition is not commutative	rational functions? Can students apply the algebra of functions, including composition and inverse functions? Can students explain the least-squares approximation and use a calculator's regression capability to do curve-fitting?	

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	—Composing and decomposing functions • Inverse functions —Formally defining the concept of inverse function —Finding a general algebraic equation for the inverse of a linear function —Relating the concept of inverse function to graphs, tables, and situations —Seeing that the graph of an inverse function is a reflection of the graph of the original function • Transformations of functions —Finding the graphs and tables of transformations of functions —Using functional notation and understanding its use in characterizing the transformations of functions		
Unit Three: The Pollster's Dilemma **Timeline: 7 weeks**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The main concepts and skills that students will encounter and practice during the unit are summarized below. ***General Sampling Concepts*** • Establishing methods of good polling, including random sampling • Using sampling from a known population to analyze the reliability of samples • Distinguishing between sampling with replacement and sampling without replacement, and comparing the two methods • Using the terminology true proportion and sample proportion • Identifying simplifying assumptions in analyzing sampling ***Specific Results on Sampling with Replacement*** • Making probability bar graphs for various distributions • Developing the concept of a theoretical distribution for sampling results from a	Can students use a binomial distribution to model a polling situation? Can students distinguish between sampling with replacement and sampling without replacement? Do students understand that the central limit theorem is a statement about approximating a binomial distribution by a normal distribution? Can students use area estimates to understand and use a normal distribution table? Can students extend concepts of mean and standard deviation from sets of data to probability distributions?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	given population • Using combinatorial coefficients to find the theoretical distribution of poll results for polls of various sizes • Generalizing that sampling results fit a binomial distribution ***The Central Limit Theorem and the Normal Distribution*** • Seeing intuitively that as poll size increases, the distribution of sample proportions becomes approximately normal • Reviewing the concept of normal distribution • Using estimates of areas to understand the normal distribution table • Applying the central limit theorem for the case of binomial distributions ***Mean and Standard Deviation*** • Reviewing the steps for computation of standard deviation • Seeing that the "large number of trials" method for computing mean and standard deviation is independent of the number of trials • Extending the concepts of mean and standard deviation from sets of data to probability distributions • Defining the concept of variance • Finding formulas for the mean and standard deviation of the distribution of poll results in terms of the poll size and the true proportion • Deciding what to use for σ if the true proportion is unknown, and finding the maximum value of σ for polling problems ***Confidence Levels and Margin of Error*** • Using the terminology confidence level, confidence interval, and margin of error • Seeing how poll size affects the standard	Can students create formulas for mean and standard deviation for binomial sampling situations? Can students use the normal approximation for binomial sampling to assess the significance of poll results? Can students apply the concepts of confidence interval, confidence level, and margin of error? Do students understand the relationship between poll size and margin of error?	

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	deviation of poll results • Establishing confidence intervals in terms of sample proportions and standard deviation • Seeing how the term margin of error is commonly used in news reporting • Estimating the size of a poll based on the reported margin of error		
Unit Four: How Much? How Fast? **Timeline: 6 weeks**			
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.F-TF.1** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. **CC.F-TF.3** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **CC.F-TF.4** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.* ***CC.G-C.5*** *– supplementary lesson is being developed by the publisher*	This unit focuses on key ideas and techniques from calculus and their applications in various settings. The main concepts and skills that students will encounter and practice during the unit are summarized below. ***Accumulation*** • Recognizing that the area under a rate curve represents an accumulation • Estimating amount of total accumulation based on linear approximations of a situation • Creating and analyzing graphs for accumulation as a function of time ***Derivatives*** • Reviewing the concept of a derivative as an instantaneous rate of change • Estimating derivatives from graphs • Developing formulas for derivatives of simple polynomial functions • Developing formulas for derivatives of the sine and cosine functions • Establishing principles for the derivative of a sum or constant multiple ***The Fundamental Theorem of Calculus*** • Seeing that an accumulation function is an antiderivative of the corresponding rate function • Finding areas and volumes using	Can students estimate derivatives from graphs, and develop formulas for derivatives of some basic functions? Do students understand accumulation as the antiderivative of a corresponding rate function? Can students define radian measure?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
	antiderivatives ***Trigonometry*** • Defining radian measure • Using radians in sine and cosine functions ***Geometry*** • Developing formulas for the volumes of pyramids and cones		
Unit Five: As the Cube Turns **Timeline: 8 weeks**			
Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **CC.F-TF.9** Model transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus stretch in a specific direction). **CC.G-CO.2** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **CC.G-CO.3** Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. **CC.G-CO.4** Given a specified rotation, reflection or translation and a geometric figure, construct the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Construct a sequence of transformations that will carry a given figure onto another. **CC.G-CO.5** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of	The main concepts and skills that students will encounter and practice during the unit are summarized below. ***Coordinate Geometry*** • Expressing geometric transformations—translations, rotations, and reflections—in terms of coordinates in two and three dimensions • Finding coordinates a fractional distance along a line segment in two and three dimensions • Reviewing graphing in three dimensions • Finding the projection of a point onto a plane from the perspective of a fixed point and developing an algebraic description of the projection process • Studying the effect of change of viewpoint on projections • Reviewing polar coordinates ***Matrices*** • Reviewing the algebra of matrices • Using matrices to express geometric transformations in two and three dimensions ***Programming*** • Learning to use a technical manual • Using loops in programming • Understanding programs from their code • Designing and programming animations ***Synthetic Geometry and Trigonometry*** • Reviewing formulas relating the sine of an	Can students express geometric transformations—translations, rotations, and reflections—in analytic terms? Can students use matrices to represent geometric transformations? Can students develop an analytic expression for projection onto a plane from a point perspective? Can students represent a line in 3-dimensional space algebraically? Can students create programming loops? Can students write and interpret programs? Can students use a graphing calculator to create programs involving animation?	All assessments are listed at the end of the curriculum map.

Standards Alignment	Unit Concepts / Big Ideas from *IMP*	Essential Questions	Assessment
two-dimensional objects. **CC.G-GMD.4**	angle to the cosine of a related angle • Deriving the formula for the area of a triangle in terms of the lengths of two sides and the sine of the included angle • Deriving formulas for the sine and cosine of the negative of an angle • Deriving formulas for the sine and cosine of the sum of two angles and related variations		

Assessment Opportunities in this Unit:

End-of-Unit Assessments: Each unit concludes with in-class and take-home assessments. The in-class assessment is intentionally short so that time pressures will not affect student performance. Students may use graphing calculators and their notes from previous work when they take the assessments.

On-Going Assessments:
Ongoing assessment includes the daily work of determining how well students understand key ideas and what level of achievement they have attained in acquiring key skills. Students' written and oral work provides many opportunities for teachers to gather this information.

- *As the Ferris Wheel Turns*
- *Free Fall*
- *The Simplified Dive, Revisited*
- *Big Push*
- *Complex Numbers and Quadratic Equations*
- *Three O'clock Drop*
- *Vector Velocities*
- *The Diver's Success*
- *What Good Are Functions?*
- *Exponential Tables*
- *Families Have Many Different Members*
- *Name That Family!*
- *The Cost of Pollution*
- *Better Braking*
- *Graphs of the Theory*
- *Gifts Aren't Always Free*
- *A Normal Poll*
- *The Search Is On!*
- *What Does It Mean?*
- *"The Pollster's Dilemma" Revisited*
- *Leaky Faucet*

- *A Distance Graph*
- *Zero to Sixty*
- *A Pyramid of Bright Ideas*
- *A Solar Summary*
- *Learning the Loops*
- *Move That Line!*
- *Oh, Say What You Can See*
- *Swing That Line!*
- *And Fred Brings the Lunch*
- *Find Those Corners!*
- *Work on POW 9: An Animated POW (The outline is turned in for An Animated Outline, the write-up is turned in for An Animated POW Write-up, and presentations are made following that.)*

NOTE: When developed in Phase II, individual units will better define the assessment tools and demonstrate how they will be used formatively and summative.