Area of a sector and segment of a circle

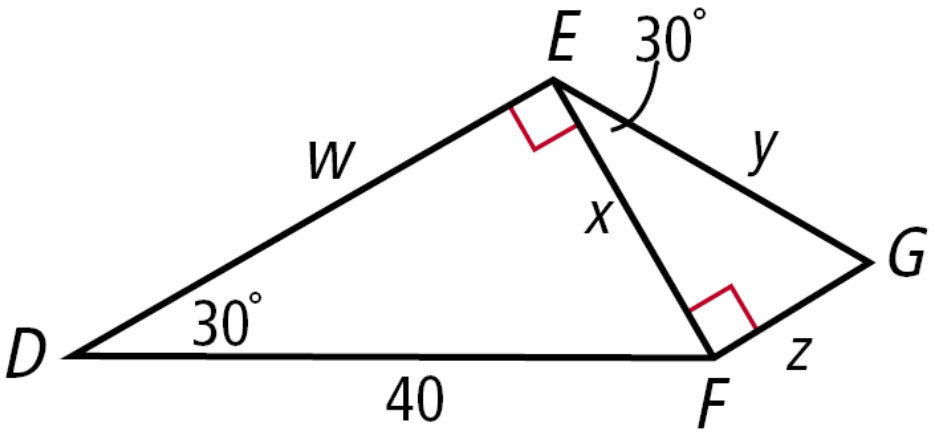
Warm Up

1. Find *w*, *y*, and *z*. Give the answers in simplest radical form.

$w = 20\sqrt{3}$

$y = \dfrac{40\sqrt{3}}{3}$

$z = \dfrac{20\sqrt{3}}{3}$

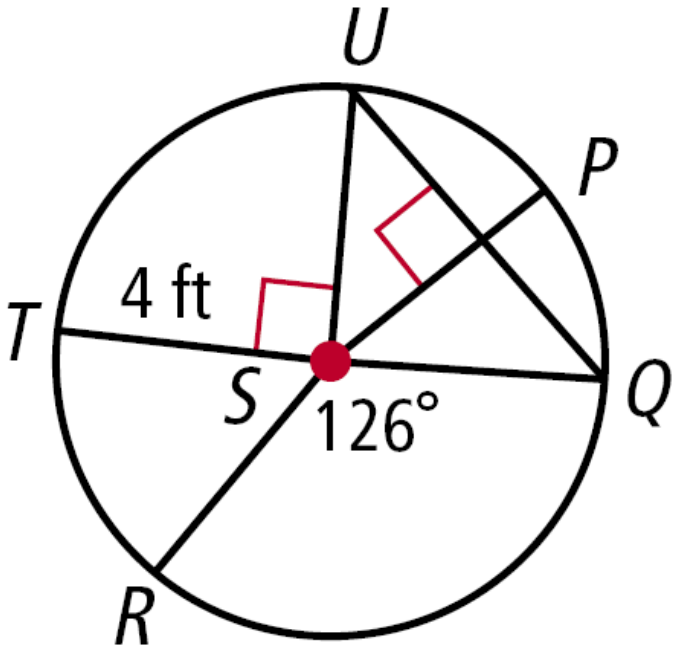


Warm Up Continued

Find each measure.

2. $\overset{\frown}{RT}$ 54°

3. UQ $4\sqrt{2}$ ft



Objectives

Find the area of sectors.

Find arc lengths.

Vocabulary

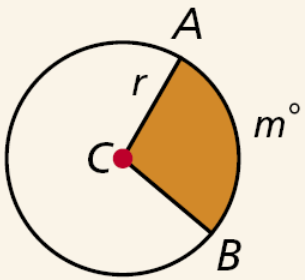
sector of a circle

segment of a circle

arc length

The area of a sector is a fraction of the circle containing the sector. To find the area of a sector whose central angle measures $m°$, multiply the area of the circle by $\frac{m°}{360°}$.

Sector of a Circle

TERM	NAME	DIAGRAM	AREA
A **sector of a circle** is a region bounded by two radii of the circle and their intercepted arc.	sector *ACB*		$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$

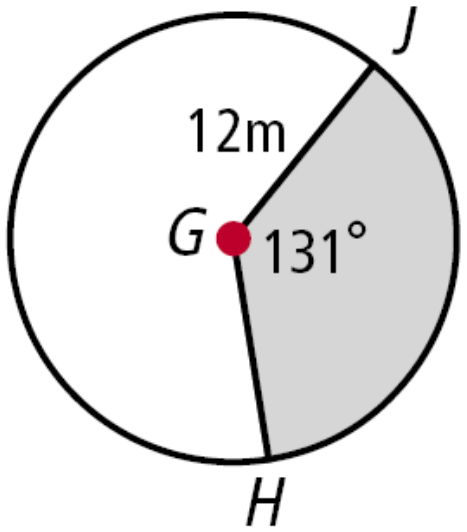
Helpful Hint

Write the degree symbol after m in the formula to help you remember to use degree measure not arc length.

Example 1A: Finding the Area of a Sector

Find the area of each sector. Give answers in terms of π and rounded to the nearest hundredth.

sector *HGJ*



$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$

Use formula for area of sector.

$$= \pi (12)^2 \left(\frac{131^\circ}{360^\circ}\right)$$

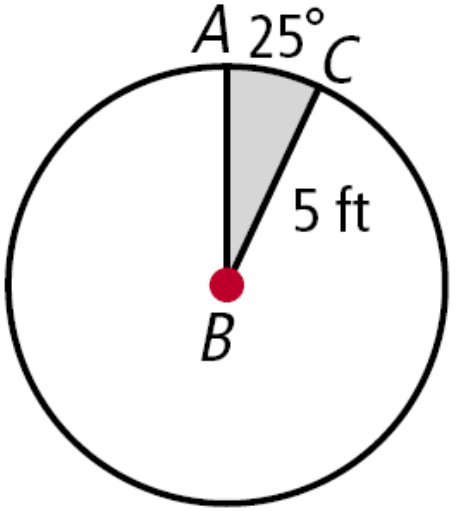
Substitute 12 for r and 131 for m.

$= 52.4\pi \text{ m}^2 \approx 164.62 \text{ m}^2$ *Simplify.*

Example 1B: Finding the Area of a Sector

Find the area of each sector. Give answers in terms of π and rounded to the nearest hundredth.

sector *ABC*



$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$

Use formula for area of sector.

$$= \pi (5)^2 \left(\frac{25^\circ}{360^\circ}\right)$$

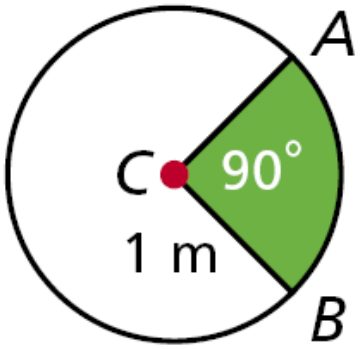
Substitute 5 for r and 25 for m.

$$\approx 1.74\pi \text{ ft}^2 \approx 5.45 \text{ ft}^2$$

Simplify.

Check It Out! Example 1a

Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.



sector *ACB*

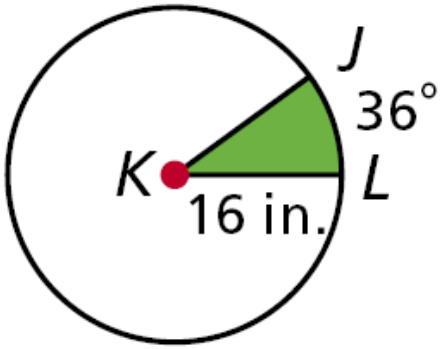
$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= \pi (1)^2 \left(\frac{90^\circ}{360^\circ}\right)$$ *Substitute 1 for r and 90 for m.*

$= 0.25\pi \text{ m}^2 \approx 0.79 \text{ m}^2$ *Simplify.*

Check It Out! Example 1b

Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.



sector *JKL*

$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= \pi(16)^2 \left(\frac{36^\circ}{360^\circ}\right)$$ *Substitute 16 for r and 36 for m.*

$= 25.6\pi \text{ in}^2 \approx 80.42 \text{ in}^2$ *Simplify.*

Example 2: Automobile Application

A windshield wiper blade is 18 inches long. To the nearest square inch, what is the area covered by the blade as it rotates through an angle of 122°?

$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= \pi (18)^2 \left(\frac{122^\circ}{360^\circ}\right)$$ *r = 18 in.*

$$\approx 345 \text{ in}^2$$ *Simplify.*

Check It Out! Example 2

To the nearest square foot, what is the area watered in Example 2 (p. 765) as the sprinkler rotates through a semicircle?

$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= \pi (360)^2 \left(\frac{180^\circ}{360^\circ}\right)$$ *d = 720 ft, r = 360 ft*

$$\approx 203{,}575 \text{ ft}^2$$ *Simplify.*

A **segment of a circle** is a region bounded by an arc and its chord.

Area of a Segment

area of segment = area of sector − area of triangle

Example 3: Finding the Area of a Segment

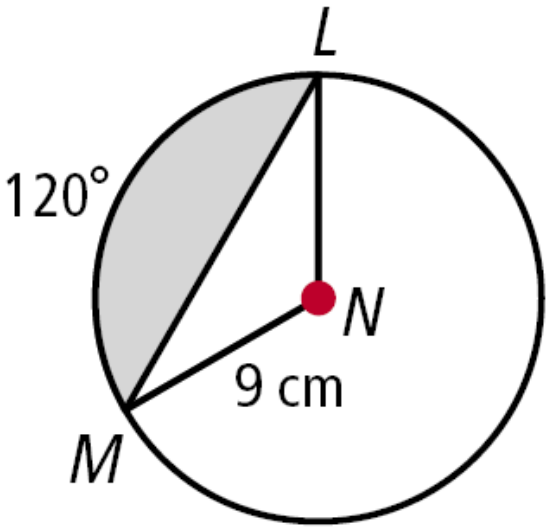
Find the area of segment *LNM* to the nearest hundredth.

Step 1 Find the area of sector *LNM*.

$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= \pi (9)^2 \left(\frac{120^\circ}{360^\circ}\right)$$ *Substitute 9 for r and 120 for m.*

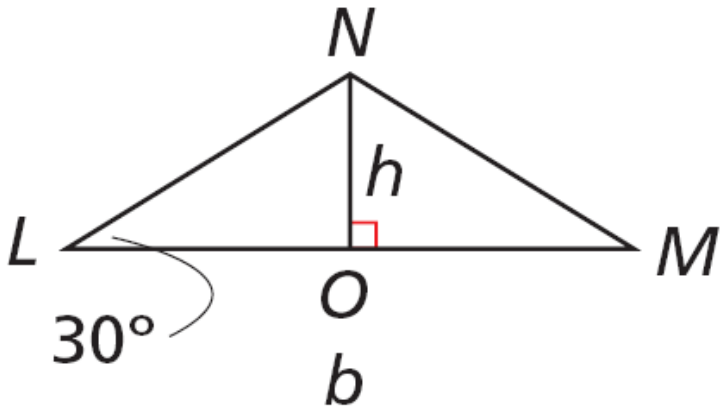
$$= 27\pi \text{ cm}^2$$ *Simplify.*



Example 3 Continued

Find the area of segment *LNM* to the nearest hundredth.

Step 2 Find the area of ΔLNM. Draw altitude $\overline{NO}$.



$$A = \frac{1}{2}bh = \frac{1}{2}\left(9\sqrt{3}\right)(4.5) \qquad LM = 9\sqrt{3} \text{ cm, and } h = 4.5 \text{ cm}$$

$$= 20.25\sqrt{3} \text{ cm}^2 \qquad \textit{Simplify.}$$

Remember!

In a 30°-60°-90° triangle, the length of the leg opposite the 60° angle is $\sqrt{3}$ times the length of the shorter leg.

Example 3 Continued

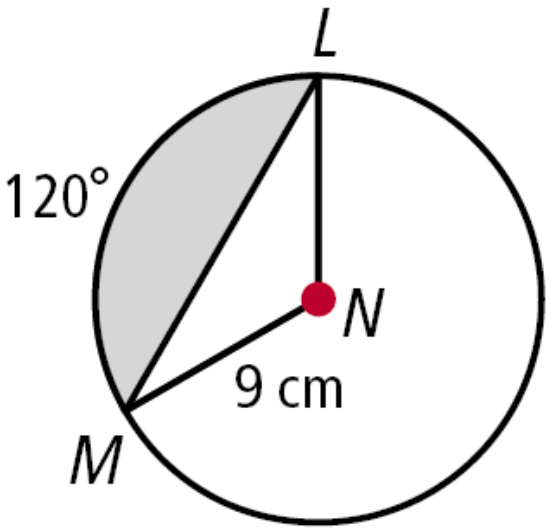
Find the area of segment *LNM* to the nearest hundredth.

Step 3

area of segment = area of sector *LNM* – area of ΔLNM

$= 27\pi - 20.25\sqrt{3}$

$\approx 49.75 \text{ cm}^2$



Check It Out! Example 3

Find the area of segment *RST* to the nearest hundredth.

Step 1 Find the area of sector *RST*.

$$A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$$

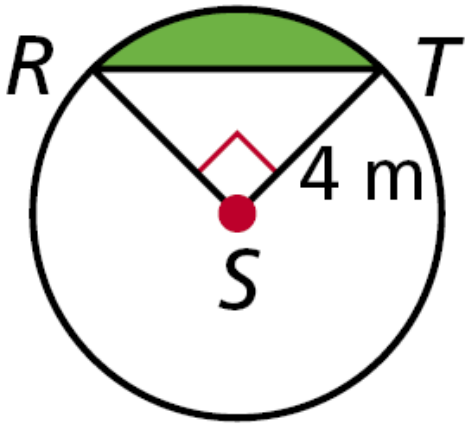
Use formula for area of sector.

$$= \pi (4)^2 \left(\frac{90^\circ}{360^\circ}\right)$$

Substitute 4 for r and 90 for m.

$$= 4\pi \text{ m}^2$$

Simplify.



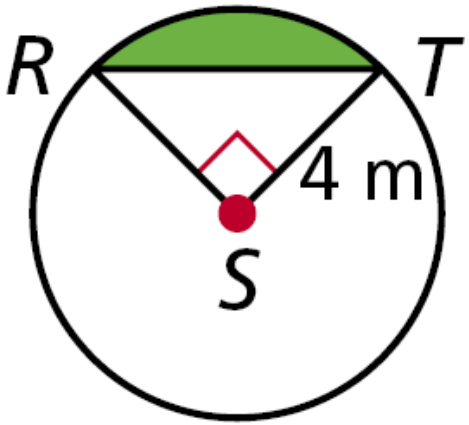
Check It Out! Example 3 Continued

Find the area of segment *RST* to the nearest hundredth.

Step 2 Find the area of ΔRST.

$$A = \frac{1}{2}bh = \frac{1}{2}(4\text{ m})(4\text{ m})$$ *ST = 4 m, and RS = 4m.*

$$= 8\text{ m}^2$$ *Simplify.*



Check It Out! Example 3 Continued

Find the area of segment *RST* to the nearest hundredth.

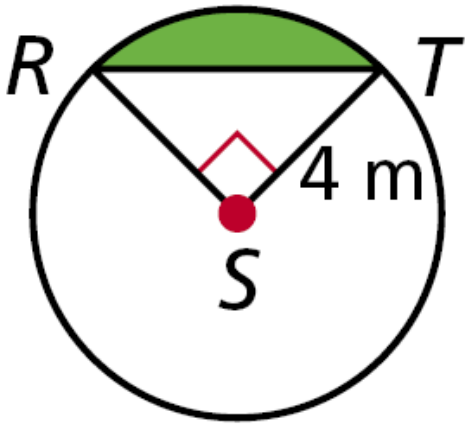
Step 3

area of segment = area of sector *RST* – area of ΔRST

$$= 27\pi - 20.25\sqrt{3}$$

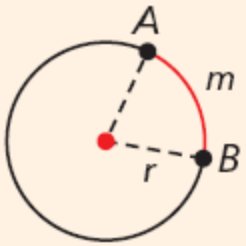
$$= 4\pi - 8$$

$$\approx 4.57 \text{ m}^2$$



In the same way that the area of a sector is a fraction of the area of the circle, the length of an arc is a fraction of the circumference of the circle.

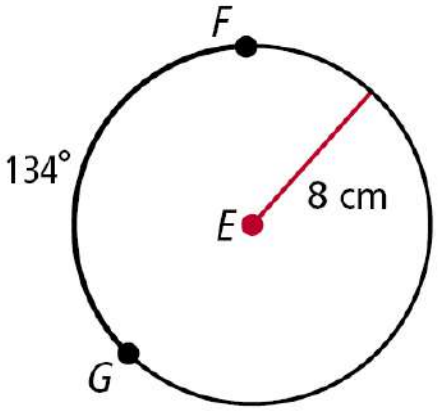
Arc Length

TERM	DIAGRAM	LENGTH
Arc length is the distance along an arc measured in linear units.		$L = 2\pi r\left(\frac{m°}{360°}\right)$

Example 4A: Finding Arc Length

Find each arc length. Give answers in terms of π and rounded to the nearest hundredth.

$\overset{\frown}{FG}$



$$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= 2\pi(8)\left(\frac{134^\circ}{360^\circ}\right)$$ *Substitute 8 for r and 134 for m.*

$\approx 5.96\pi$ cm ≈ 18.71 cm *Simplify.*

Example 4B: Finding Arc Length

Find each arc length. Give answers in terms of π and rounded to the nearest hundredth.

an arc with measure 62° in a circle with radius 2 m

$$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= 2\pi(2)\left(\frac{62^\circ}{360^\circ}\right)$$ *Substitute 2 for r and 62 for m.*

$$\approx 0.69\pi \text{ m} \approx 2.16 \text{ m}$$ *Simplify.*

Check It Out! Example 4a

Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.

$\overset{\frown}{GH}$

$$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$$

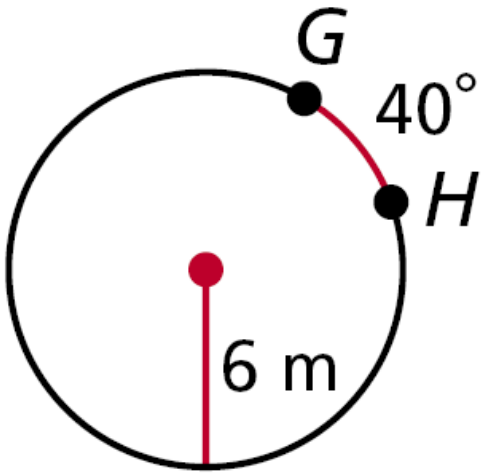
Use formula for area of sector.

$$= 2\pi(6)\left(\frac{40^\circ}{360^\circ}\right)$$

Substitute 6 for r and 40 for m.

$$= \frac{4}{3}\pi \text{ m} \approx 4.19 \text{ m}$$

Simplify.



Check It Out! Example 4b

Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.

an arc with measure 135° in a circle with radius 4 cm

$$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$$ *Use formula for area of sector.*

$$= 2\pi(4)\left(\frac{135^\circ}{360^\circ}\right)$$ *Substitute 4 for r and 135 for m.*

$= 3\pi$ cm ≈ 9.42 cm *Simplify.*

Lesson Quiz: Part I

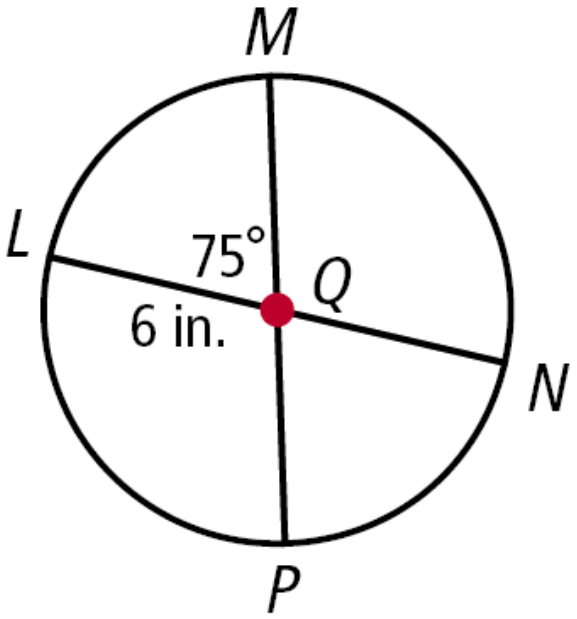
Find each measure. Give answers in terms of π and rounded to the nearest hundredth.

1. area of sector *LQM*

7.5π in^2 $\approx$ 23.56 in^2

2. length of $\overset{\frown}{NP}$

$2.5\ \pi$ in. $\approx$ 7.85 in.



Lesson Quiz: Part II

3. The gear of a grandfather clock has a radius of 3 in. To the nearest tenth of an inch, what distance does the gear cover when it rotates through an angle of 88°?

≈ 4.6 in.

4. Find the area of segment *GHJ* to the nearest hundredth.

≈ 55.94 m^2