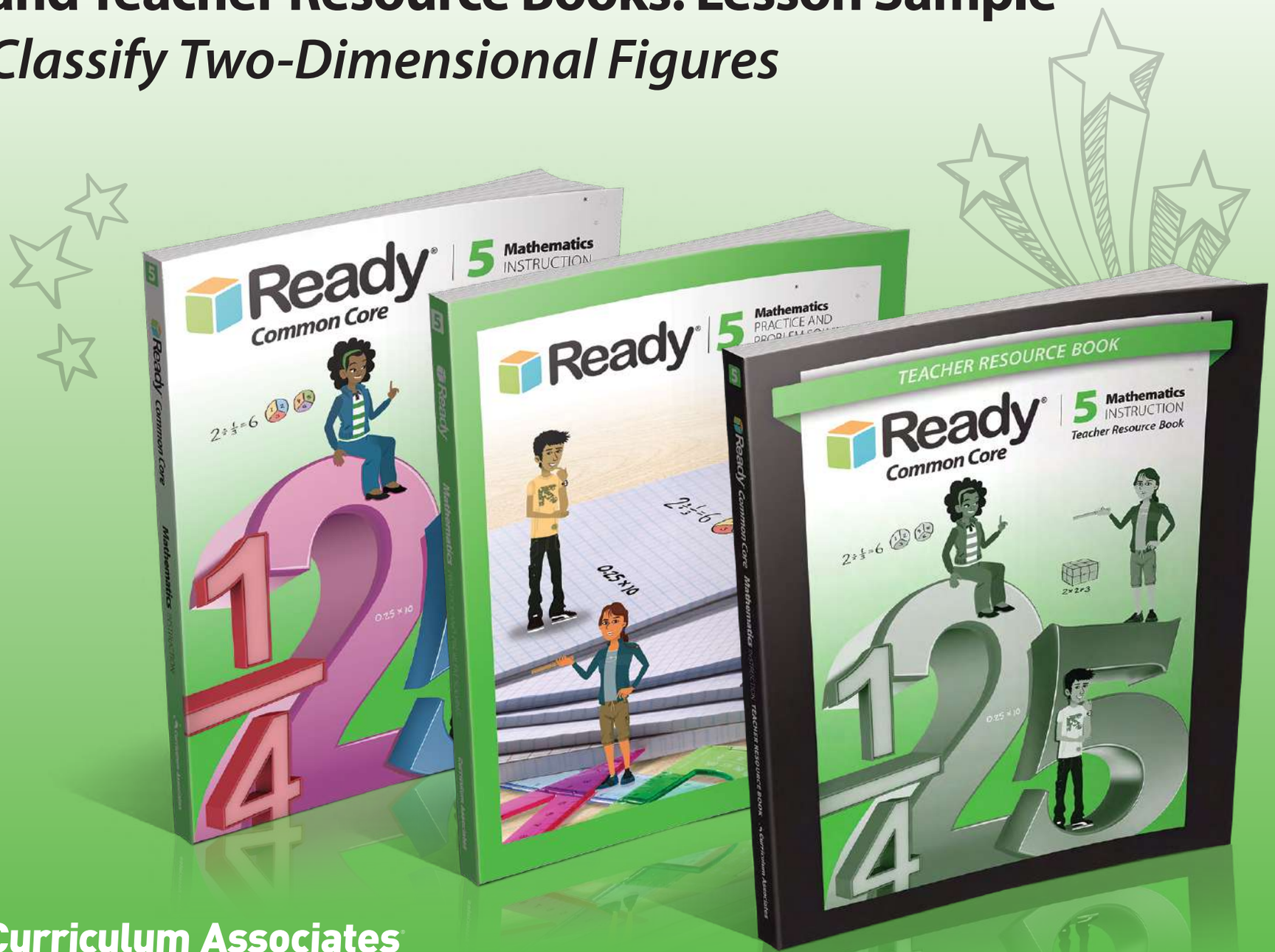
Ready® Common Core | 5 Mathematics INSTRUCTION

Student Instruction, Practice and Problem Solving, and Teacher Resource Books: Lesson Sample

Classify Two-Dimensional Figures



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5 Mathematics INSTRUCTION

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Standards in boldface are the focus standards that address major lesson content.

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Standards

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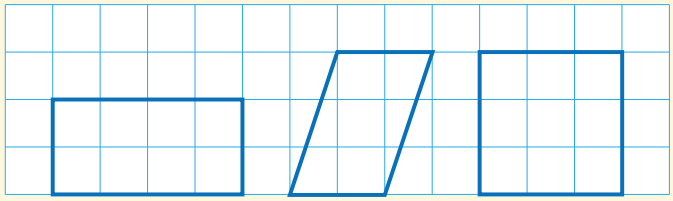
Classify Two-Dimensional Figures

Use What You Know

In this lesson, you will classify polygons based on their properties. Take a look at this problem.

Arrange the polygons below so that a polygon can also be called by the name of the polygon before it. Order them from left to right.



A *B* *C*

a. Complete the table below. Put a check in each box if the polygon has the property listed.

Property	Polygon *A*	Polygon *B*	Polygon *C*
4 sides			
2 pairs of parallel sides			
2 pairs of sides of equal length			
4 right angles			
4 sides of equal length			

b. Write the most specific name for each polygon from the list below.

quadrilateral parallelogram rectangle square

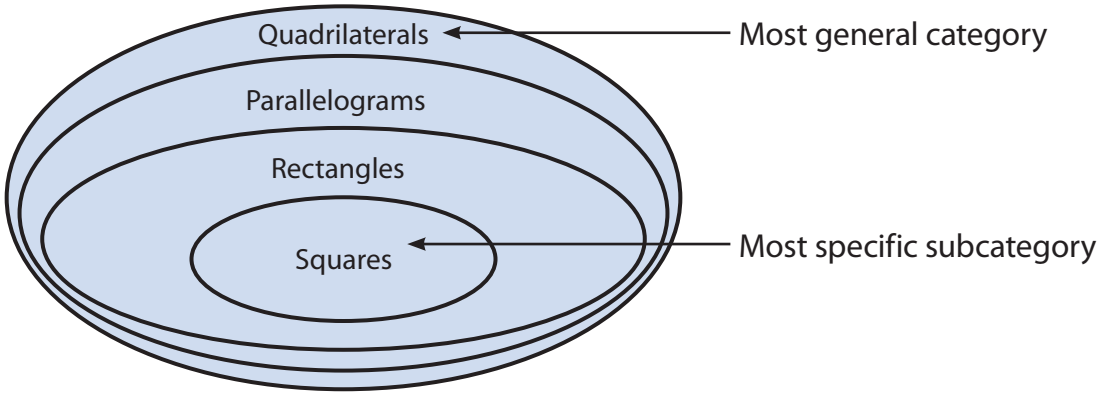
A: _____ *B*: _____ *C*: _____

c. How would you arrange the polygons so each shape has all the properties of the shape(s) before it? _____

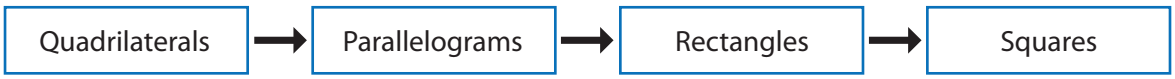
Find Out More

Shapes can be classified according to their properties. When you order categories of polygons by their properties, you put them in a **hierarchy**. A hierarchy organizes categories from the most to least general. One model you can use to show a hierarchy is a Venn diagram.

A Venn diagram can show categories and subcategories. This Venn diagram shows that squares have all the properties that rectangles have, plus more. This means all squares are also rectangles. A square is also a parallelogram and a quadrilateral.



You can also use a flow chart to show the hierarchy of quadrilaterals. The most general category is at the left, while the most specific is at the right. This means that a figure that belongs in one category also belongs in all categories to the left.



Reflect

1 How are the flow chart and the Venn diagram alike? How are they different?

Learn About

Ordering Shapes in a Hierarchy

Read the problem below. Then explore different ways to classify figures in a hierarchy.

Classify the following triangles from the most general to the most specific: scalene triangle, isosceles triangle, and equilateral triangle. Use a tree diagram to classify them as types of triangles.

Model It

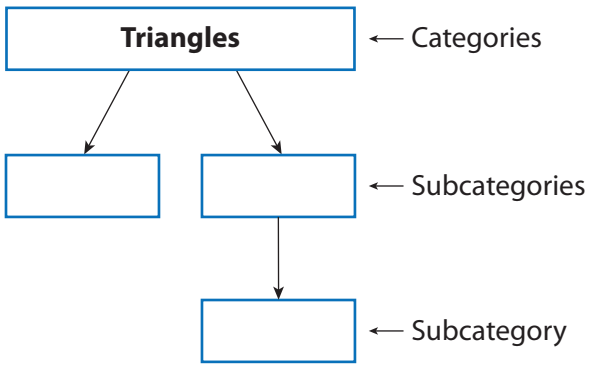
You can understand the problem by listing the properties of the triangles in a table before arranging them in a tree diagram.

Types of Triangles	Properties of Sides
Isosceles	2 or 3 sides of equal length
Scalene	no sides of equal length
Equilateral	3 sides of equal length

Model It

You can represent the problem with a tree diagram.

A tree diagram can also be used to show a hierarchy. Put the most general category as the top branch. Then put the more specific subcategories as the branches.



Connect It Now you will solve the problem from the previous page by using the table to complete a tree diagram.

2 Why is "Triangles" in the top row of the tree diagram? _____

_____

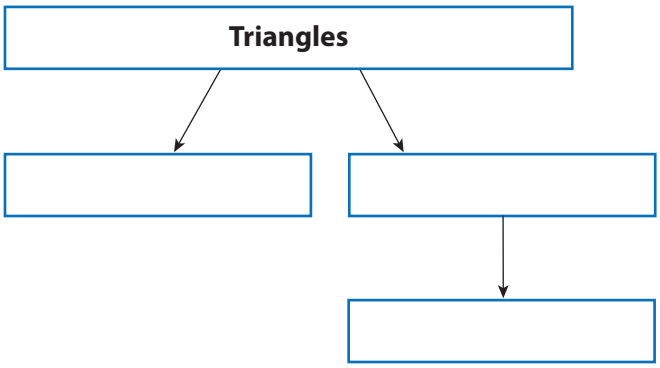
3 Write "Scalene" and "Isosceles" in the second row of the tree diagram at the right.

Why are those categories separate?

_____

_____

_____



4 Write "Equilateral" beneath "Isosceles."

Why can all equilateral triangles be classified as isosceles triangles?

_____

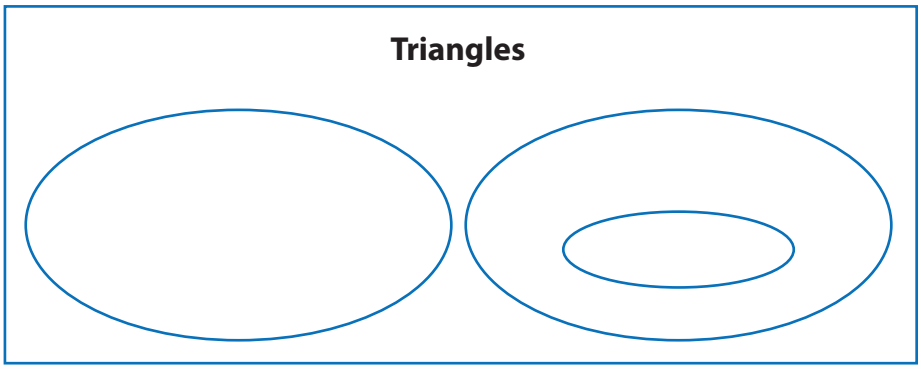
5 How can you use a tree diagram to order figures? _____

_____

_____

Try It Use what you learned about ordering figures in a hierarchy to solve this problem.

6 Complete the Venn diagram below to show the hierarchy of isosceles, scalene, and equilateral triangles.



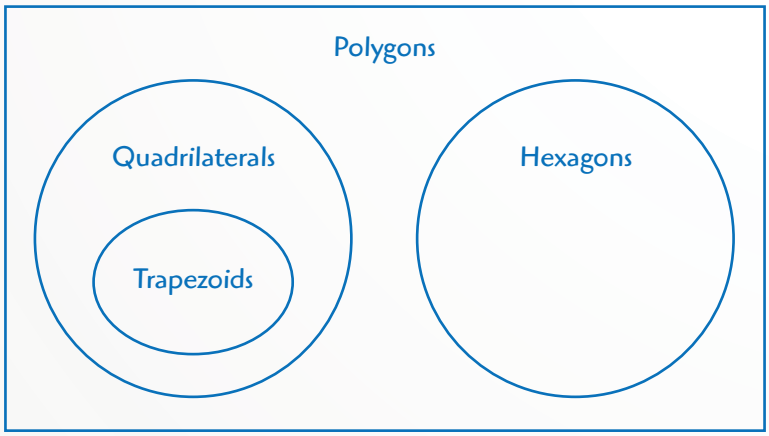
Practice Classifying Two-Dimensional Figures

Study the example below. Then solve problems 7–9.

Example

Create a Venn diagram to show the hierarchy of quadrilaterals, polygons, trapezoids, and hexagons.

Look at how you could show your work using a Venn diagram.



A shape can never be both a hexagon and a quadrilateral. So these regions do not overlap.

Pair/Share
Recreate the hierarchy with a tree diagram.

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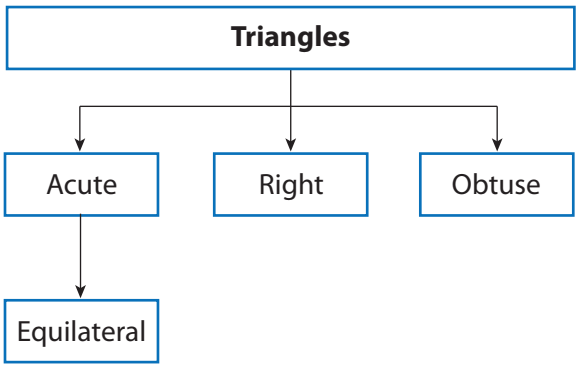
7 Look at the tree diagram below. Write a statement about the relationship between acute triangles and equilateral triangles.



Which type of triangle is the most specific?

Pair/Share
Write a statement about the relationship between acute triangles and obtuse triangles.

Solution _____

_____

8 Create a Venn diagram to show the hierarchy of the polygons described in the chart.

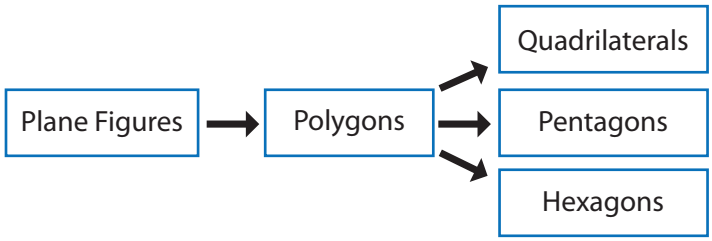
Polygon	Description
Trapezoid	quadrilateral with at least 1 pair of parallel sides
Isosceles Trapezoid	trapezoid with at least 2 sides of equal length
Parallelogram	quadrilateral with 2 pairs of parallel sides

"At least 2" means 2 or more.

Pair/Share

Draw one example of a polygon in each separate category of your Venn diagram.

9 Look at the flow chart below.



The flow chart is like a tree diagram. But the arrows show that the hierarchy moves from left to right instead of top to bottom.

Which statement is true? Circle the letter of the correct answer.

A A plane figure is always a polygon.

B All polygons are plane figures.

C All hexagons are also pentagons and quadrilaterals.

D A hexagon is not a plane figure.

Brad chose **C** as the correct answer. How did he get that answer?

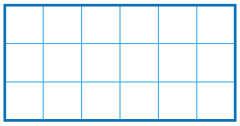
Pair/Share

Does Brad's answer make sense?

Practice Classifying Two-Dimensional Figures

Solve the problems.

1 Look at the shape below.



Which is a correct classification for this shape from LEAST specific to MOST specific?

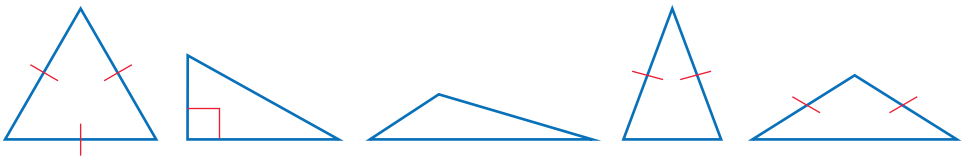
A polygon, quadrilateral, rectangle

B quadrilateral, parallelogram, square

C polygon, quadrilateral, square

D quadrilateral, rectangle, square

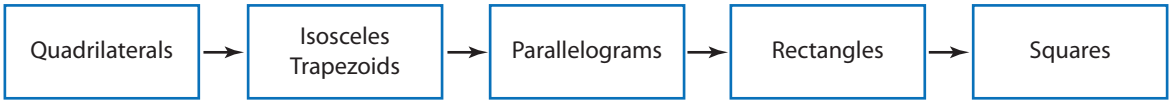
2 Classify the triangles shown below as "scalene," "isosceles," or "obtuse." Sides that are the same length are marked with a slash. Draw the triangles in the correct column of the table. If a triangle fits more than one classification, draw it in all the columns that apply.



Scalene	Isosceles	Obtuse

3 The word "isosceles" can be used to describe any polygon with at least 2 sides of equal length.

Look at the flow chart below.



Part A Draw an example of an isosceles trapezoid.

Part B Explain how isosceles trapezoids relate to parallelograms.

Part C Can you use the term "isosceles" to describe a rectangle? Explain your reasoning.

Self Check **Go back and see what you can check off on the Self Check on page 283.**

5.G.B.3

Understand Properties of Two-Dimensional Figures

Think It Through

How do we group polygons into categories?

Polygons are grouped into categories by their **attributes**, or properties, such as the number of sides or angles, the side lengths, and the angle measures. All polygons in the same category share certain properties. Some properties of polygons are described in the table below.

Property	Description	Example
Scalene	no sides of equal length	
Isosceles	at least 2 sides of equal length	
Equilateral	all sides of equal length	
Regular	all sides of equal length and all angles of equal measure	
Irregular	at least 1 side and 1 interior angle are not equal in measure to the other sides and angles	
Right	at least 1 pair of perpendicular sides	
Parallel sides	at least 1 pair of opposite sides that will never intersect, no matter how far they are extended	

Think **Can a polygon be categorized in more than one way?**

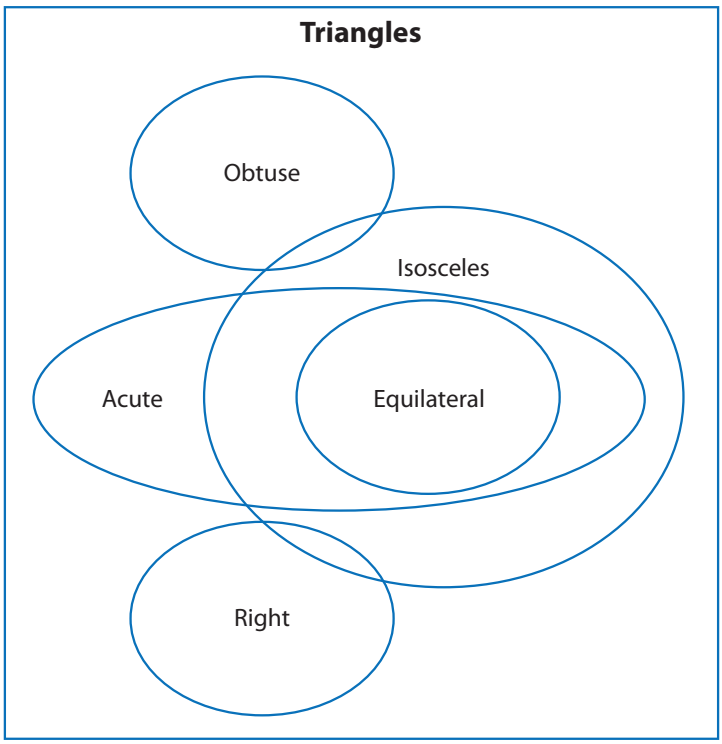
Think about how a quadrilateral is defined. It is a polygon with 4 sides. So any shape with 4 sides can be called both a polygon and a quadrilateral. If the quadrilateral has two pairs of parallel sides, then it can also be called a parallelogram.

Every parallelogram is a quadrilateral because every parallelogram has 4 sides. But not all quadrilaterals are parallelograms because not all quadrilaterals have two pairs of parallel sides.

Shade a polygon above that can be named both a quadrilateral and parallelogram.

Think How can you show the relationships among polygons with a diagram?

A Venn diagram is a useful tool for organizing categories of polygons that share properties.



The Venn diagram shows a triangle can never be both right and obtuse.

Notice the "Right" category partly overlaps the "Isosceles" category. This means a right triangle may also have all the properties of an isosceles triangle. Also notice that the "Right" category does not overlap the "Obtuse" category. That means a right triangle can never have all the properties of an obtuse triangle.

The "Equilateral" category is nested completely inside the "Isosceles" category. This shows that equilateral triangles are a subcategory of isosceles triangles. So all equilateral triangles share all the properties of isosceles triangles.

Reflect

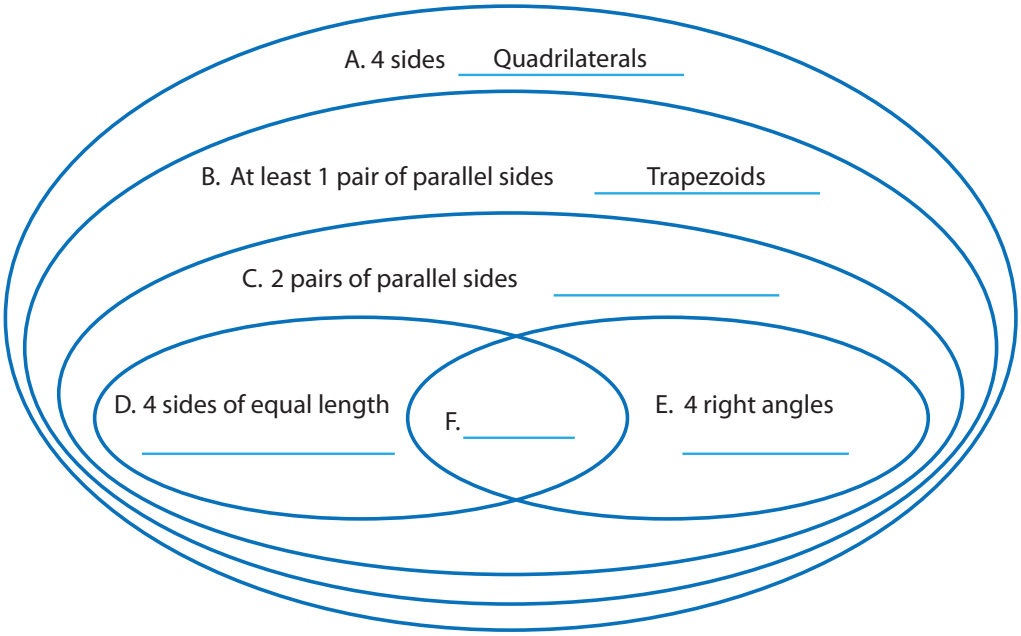
1 What does it mean that the Venn diagram shows "Obtuse" partially overlapping "Isosceles?"

Think About Properties Shared by Polygons

Let's Explore the Idea

A Venn diagram can help you understand what properties are shared by categories of polygons.

2 The Venn diagram shows categories of quadrilaterals with different properties. Write the name of each category that fits the description.



3 Use the Venn diagram to fill in the table below.

Category	Properties	Name
A	4 sides	Quadrilaterals
B	4 sides, at least 1 pair of parallel sides	Trapezoids
C	4 sides, 2 pairs of parallel sides	
D	4 sides, 2 pairs of parallel sides, 4 sides of equal length	
E		
F		

Let's Talk About It

Use the Venn diagram to help you understand how properties are shared by categories of quadrilaterals.

4 Is every property of parallelograms also a property of all rectangles? _____

Is every property of rectangles also a property of all parallelograms? _____

Explain what the Venn diagram shows about the relationship between rectangles and parallelograms. _____

_____

_____

Classify each inference statement as *true* or *false*. If false, explain.

5 The opposite angles of any parallelogram have the same measure. Therefore, the opposite angles of any rhombus have the same measure. _____

_____

6 The diagonals of any square are the same length. Therefore, the diagonals of any rhombus are the same length. _____

_____

Try It Another Way

The flow chart below shows another way to think about how quadrilaterals are categorized.

Use the flow chart to describe the statements as *true* or *false*.

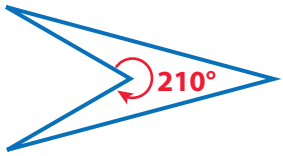
7 In every rectangle the two diagonals have the same length. Therefore, in every parallelogram the two diagonals must have the same length. _____

8 Every rhombus has at least 2 lines of symmetry. Therefore, every square has at least 2 lines of symmetry. _____

Connect Ideas About Properties of Polygons

Talk through these problems as a class. Then write your answers below.

9 Categorize All polygons are either **convex** or **concave**. A convex polygon has all interior angles less than 180°. A triangle is an example of a convex polygon. A concave polygon has at least 1 interior angle greater than 180°. The quadrilateral below is an example of a concave polygon.

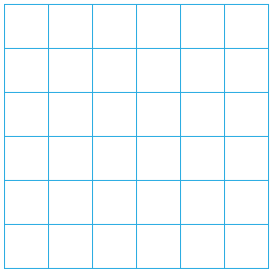


Categorize concave polygons, convex polygons, triangles, quadrilaterals, and rectangles in a Venn diagram. Draw an example of each polygon in the diagram.

10 Explain Nadriette said that a rectangle can never be called a trapezoid. Explain why Nadriettte's statement is incorrect. _____

_____

11 Create Describe the properties of a shape that is both a rectangle and a rhombus. Name the shape and use the grid below to draw an example.



_____

_____

_____

Apply Ideas About Properties of Polygons

12 Put It Together Use what you have learned about classifying polygons to complete this task.

Part A Create a tree diagram to show the following types of triangles: acute, obtuse, right, isosceles, and equilateral. Make sure to include the category "Triangle." Use information in the table to help you.

Triangle	Types of Angles
Acute	all acute angles
Right	2 acute angles and 1 90° angle
Obtuse	2 acute angles and 1 obtuse angle
Scalene	acute, right, or obtuse
Isosceles	acute, right, or obtuse
Equilateral	all acute angles

Part B Write a statement that is always true about the relationship between obtuse triangles and equilateral triangles.

Part C Write a statement that is sometimes true about the relationship between acute triangles and isosceles triangles.

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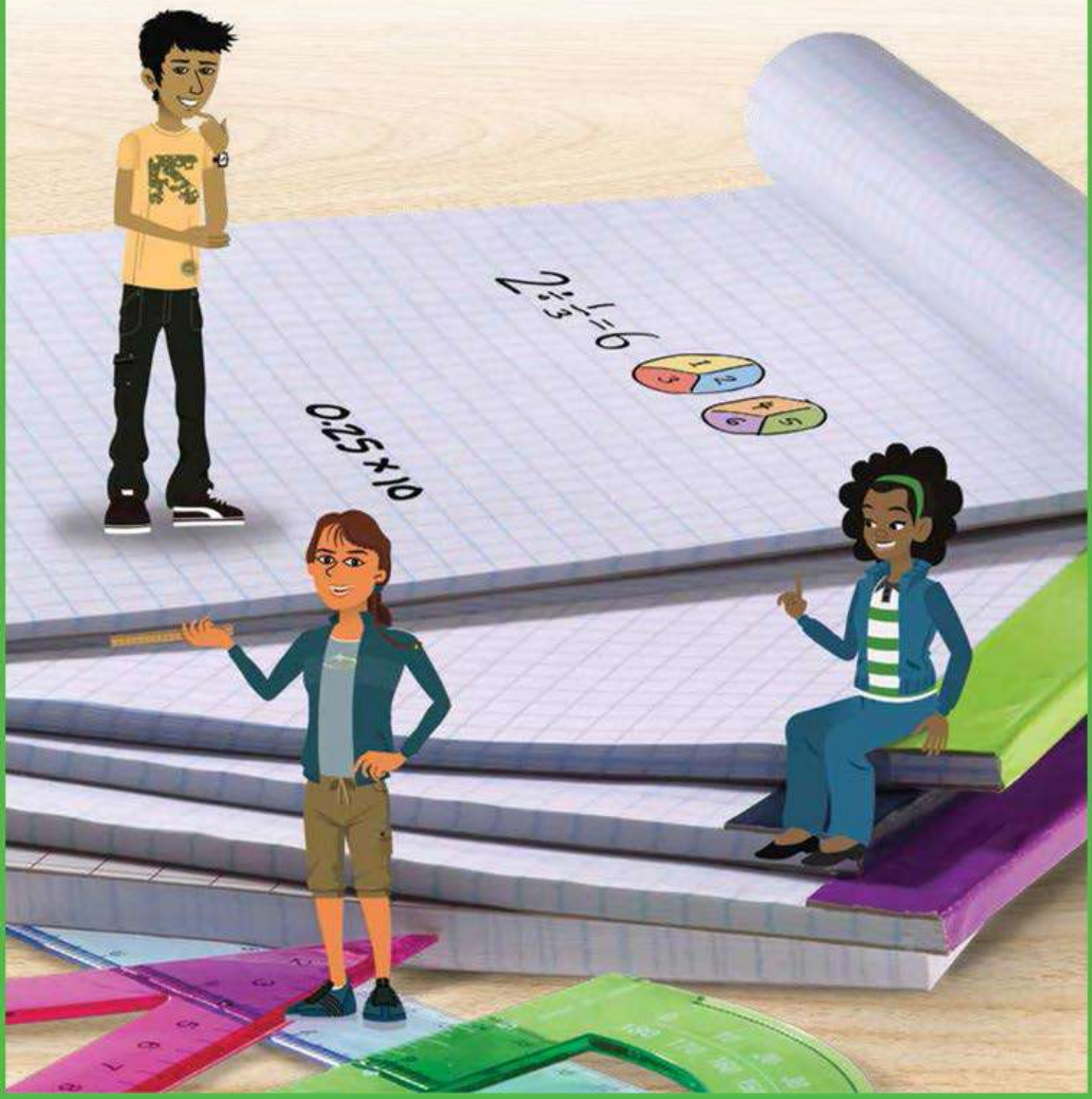

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Unit 2: Number and Operations—Fractions

Family Letter available with every lesson.

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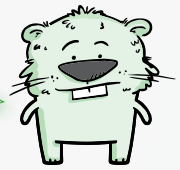
Family Letter available with every lesson.

Dear Family,

This week your child is learning to classify two-dimensional figures.

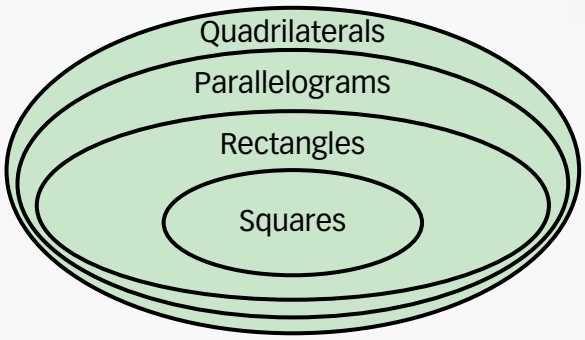


You can classify all polygons, or special two-dimensional figures, by their properties. Some properties of figures are the number of sides they have, whether the sides are perpendicular or parallel, and what kinds of angles they have.

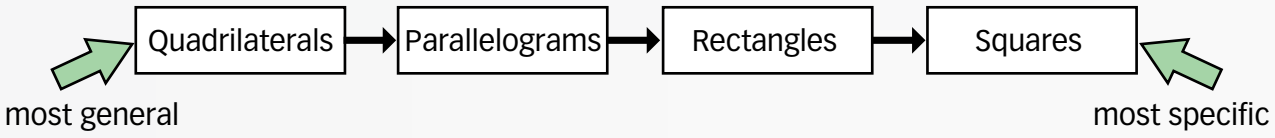
You can use a hierarchy to rank categories of figures. At the top of the hierarchy is the category for the most general group. As you go down a hierarchy, you can see how more specific groups are related.

You can use a hierarchy to show how figures such as squares, rectangles, parallelograms, and other quadrilaterals (four-sided figures) are related. A useful way to show categories in a hierarchy is with a Venn diagram.

The Venn diagram at the right shows that quadrilaterals are the most general category. All figures that have four sides are quadrilaterals. Parallelograms, rectangles, and squares are kinds of quadrilaterals.



Another way that your child is learning to classify figures is with a flow chart. The flow chart below shows the hierarchy of quadrilaterals from left to right.



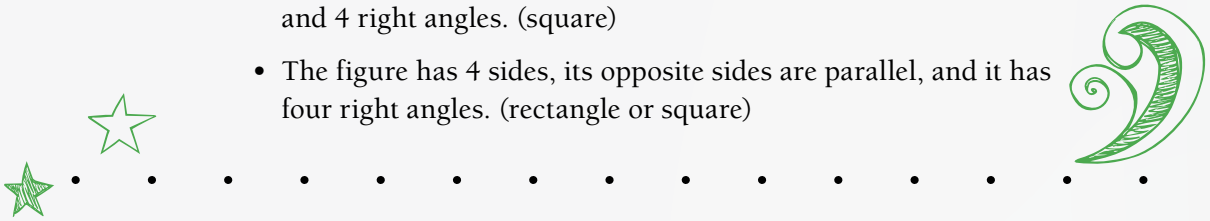
Invite your child to share what he or she knows about classifying two-dimensional figures by doing the following activity together.

NEXT

Classifying Two-Dimensional Figures Activity

Work together with your child to draw a figure based on a description of the figure's properties.

- Use the dot paper below. One person describes properties of a figure and the other person draws and names the figure based on the description of its properties.
- Here are some examples:
  - The figure is a quadrilateral that has at least 1 pair of parallel sides (trapezoid, parallelogram, rectangle, square).
  - The figure has 4 sides of equal length, 2 pairs of parallel sides, and 4 right angles. (square)
  - The figure has 4 sides, its opposite sides are parallel, and it has four right angles. (rectangle or square)



Classify Two-Dimensional Figures

Name: _____

Prerequisite: Identify Parallel and Perpendicular Lines

Study the example problem that shows how to sort shapes based on parallel and perpendicular sides. Then solve problems 1–6.

Example

Mark each shape that appears to have at least one pair of parallel sides with the symbol ||. Mark each shape that appears to have at least one pair of perpendicular sides with the symbol ⊥.

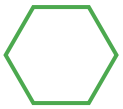
Parallel sides are always the same distance apart and will never cross. Perpendicular sides form a right angle (90°).

triangle rectangle parallelogram trapezoid right triangle

1 Look at the shapes in the example. Write the name of the shapes that belong in each group shown in the table below.

parallel sides only	perpendicular sides only	parallel and perpendicular sides

2 Which group from problem 1 does each shape shown below belong in?



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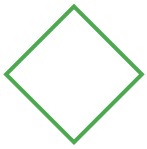


_____

_____

3 Draw a shape that does not belong to any of the groups in problem 1.

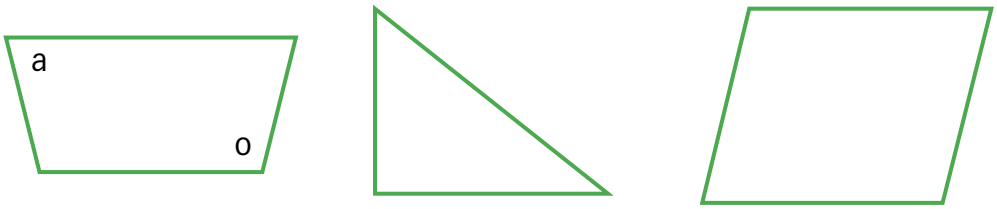
Solve.

A right angle is an angle that looks like a square corner and measures 90°.

An acute angle has a smaller opening than a right angle.

An obtuse angle has a wider opening than a right angle but is not a straight line.

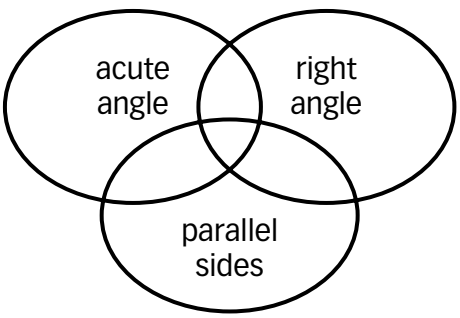
4 Finish marking each angle in these shapes: "a" for acute, "r" for right, and "o" for obtuse.

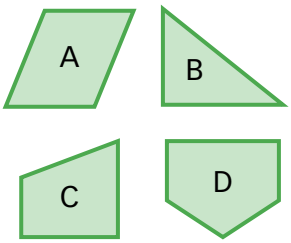


5 Write the name of each shape from problem 4 that belongs in each group shown in the table below.

acute and right angles	acute and obtuse angles

6 Where does each shape belong in the Venn diagram below? Write the letter of the shape in the section that it belongs in.





Name: _____

Order Shapes in a Hierarchy

Study the example showing how to order shapes in a hierarchy. Then solve problems 1–6.

Example

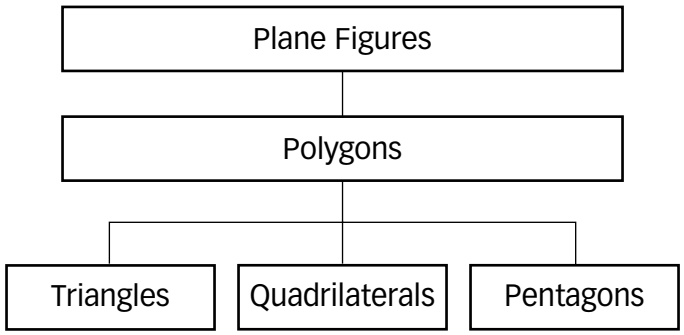
A hierarchy starts with the most general category and then shows how more specific groups are related. Draw a tree diagram relating the shapes in the table.

Shape	Description
plane figure	a two-dimensional shape
polygon	a closed plane figure with straight sides
triangle	a polygon with 3 sides
quadrilateral	a polygon with 4 sides
pentagon	a polygon with 5 sides

Triangles, quadrilaterals, and pentagons have all the properties that polygons have. They have other properties, too. Because triangles, quadrilaterals, and pentagons have different properties from each other, they appear side-by-side.

Polygons have all the properties that plane figures have. Polygons also have properties that plane figures don't have. Polygons appear right below plane figures in the hierarchy.

Tree Diagram



1 Fill in the blanks.
Triangles are both _____ and _____.

2 A circle is a plane figure. It does not have straight sides, so it is not a polygon. Where in the hierarchy should "Circles" go? Explain.

_____

_____

_____

_____

_____

Vocabulary

hierarchy a ranking of categories based on properties.

Solve.

3 Mark an X in the column if the shape always has that property.

Shape	4 sides	2 pairs of parallel sides	4 right angles
parallelogram			
rectangle			
quadrilateral			

4 Use the table in problem 3 to make a flow chart that shows the relationship between the three shapes. Order the shapes from general to specific going from left to right.

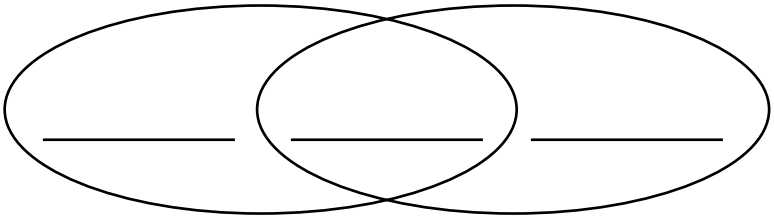
5 Where would you include squares in the flow chart in problem 4? Explain.

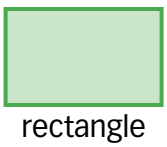
_____

_____

_____

6 Fill in the Venn diagram that shows the relationship between rectangles, squares, and rhombuses. Explain what the diagram shows about squares.





rectangle

square

rhombus

_____

_____

_____

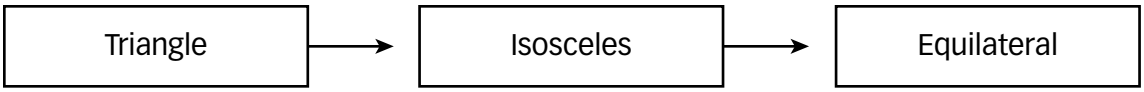
_____

Name: _____

Classify Two-Dimensional Figures

Solve the problems.

1 Look at the flow chart below.



Which statement is true? Circle the letter of all that apply.

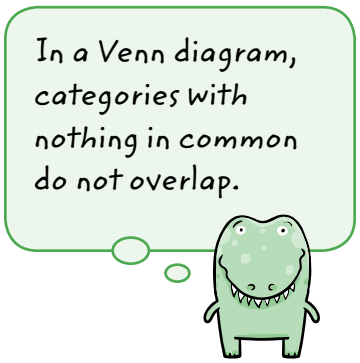
A Equilateral triangles can be classified as isosceles triangles.

B Isosceles triangles have all the properties that equilateral triangles have.

C Isosceles triangles can be classified as equilateral triangles.

D Equilateral triangles have all the properties that isosceles triangles have.

2 Create a Venn diagram to show the hierarchy of triangles, quadrilaterals, isosceles triangles, and polygons.



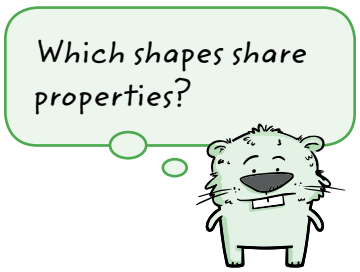
3 Use the diagram in problem 2. Write two different statements that describe the relationships between the shapes.

Solution: _____

_____

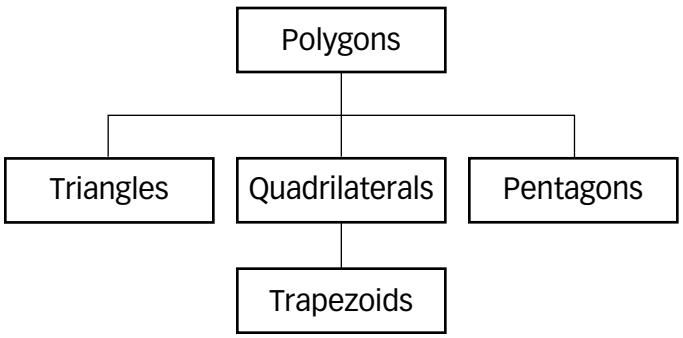
_____

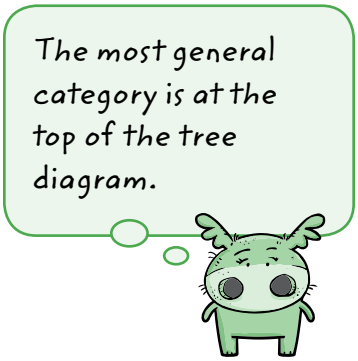
_____



Solve.

4 Look at the tree diagram below.





Which statement is true? Circle the letter of the correct answer.

A All polygons are triangles, quadrilaterals, and pentagons.

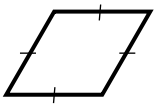
B All quadrilaterals are trapezoids.

C All triangles and quadrilaterals are polygons.

D Triangles, quadrilaterals, and pentagons all have the same properties.

Dina chose **B** as the correct answer. How did she get that answer?

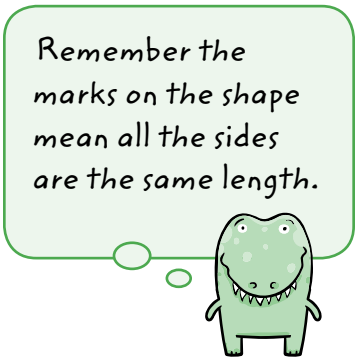
5 Chen wrote some names that can be used to classify this shape in order from LEAST specific to MOST specific.



quadrilateral, parallelogram, square, rhombus

Do you agree with what he did? Explain.

Solution: _____



TEACHER RESOURCE BOOK

Ready® Common Core

5 Mathematics INSTRUCTION

Teacher Resource Book

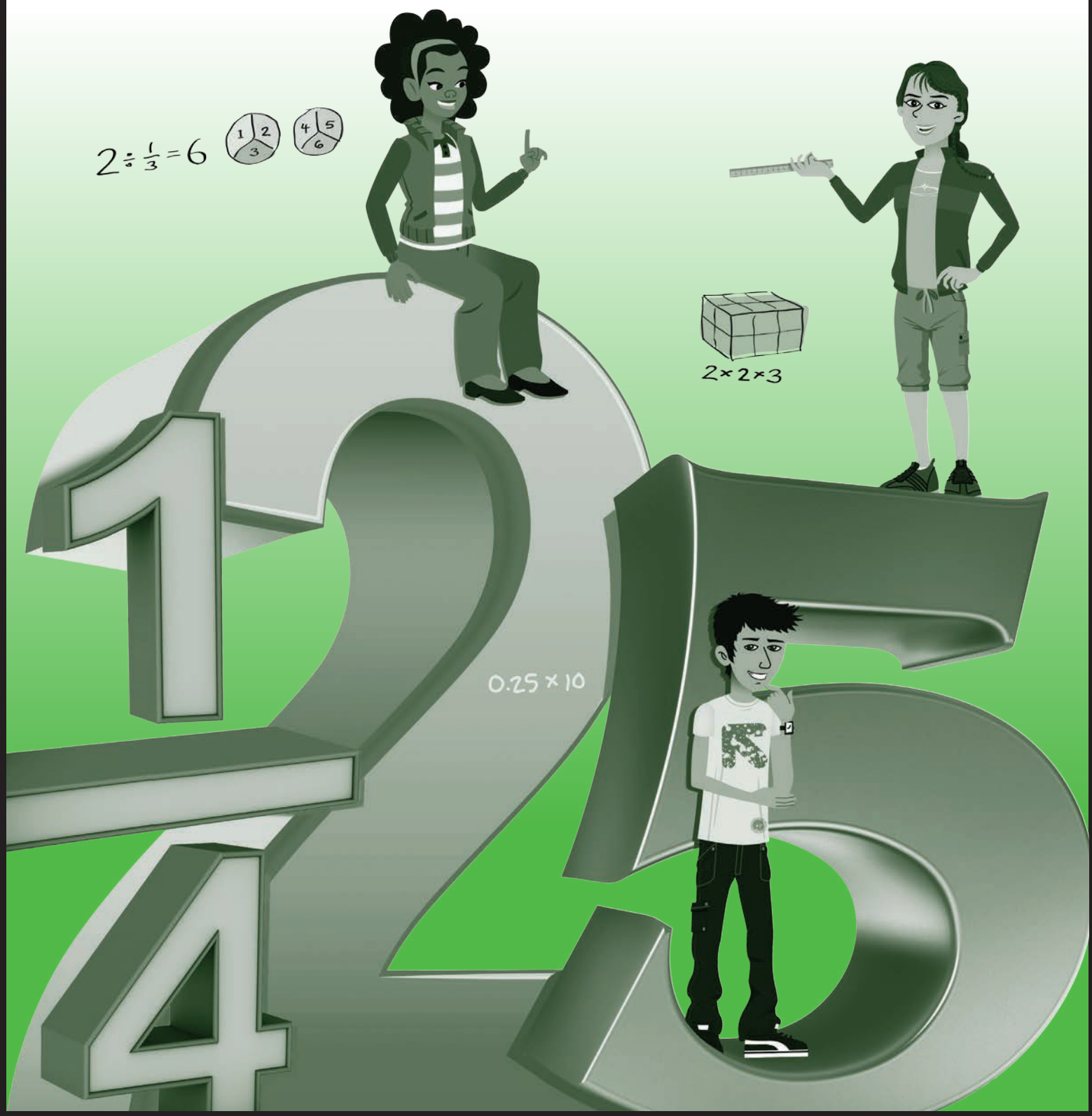


Table of Contents

Mathematics Lessons

M = Lessons that have a major emphasis in the Common Core Standards
S/A = Lessons that have supporting/additional emphasis in the Common Core Standards
Standards in boldface are the focus standards that address major lesson content.

Mathematics Lessons, *continued*

Mathematics Lessons, *continued*

Mathematics Lessons, *continued*

LESSON OVERVIEW

Lesson 30
Classify Two-Dimensional Figures

CCSS Focus

Domain
Geometry

Cluster
B. Classify two-dimensional figures into categories based on their properties.

Standards
5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.

Standards for Mathematical Practices (SMP)
2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

Lesson Objectives

Content Objectives
- Classify two-dimensional figures in a hierarchy based on properties of the figures.
- Draw and use flow charts, Venn diagrams, and tree diagrams to show the hierarchical relationship of two-dimensional figures.

Language Objectives
- Define the key term *hierarchy* and discuss its meaning with a partner.
- List relationships among two-dimensional figures shown by flow charts, Venn diagrams, and tree diagrams.

Prerequisite Skills
- Recognize parallel and perpendicular lines.
- Recognize right, acute, and obtuse angles.
- Sort two-dimensional figures based on the kinds of sides they have and on the kinds of angles they have.

Lesson Vocabulary
- **hierarchy** a ranking of categories based on properties

Review the following key terms.
- **polygon** a closed two-dimensional shape made with three or more line segments
- **Venn diagram** a drawing that shows relationships among groups

Learning Progression

In Grade 4 students classified two-dimensional figures that included quadrilaterals, hexagons, trapezoids, and triangles. Students classified figures based on properties of sides and angles, such as parallel or perpendicular sides and right, acute, or obtuse angles. Students classified triangles based on lengths of sides and kinds of angles and named triangles as equilateral, isosceles, or scalene, as well as right, acute, or obtuse.

In this lesson students analyze categories of polygons based on their properties and relate the categories in a hierarchy. Students use the properties of figures to show categories of polygons in a hierarchical relationship from most general to most specific. They organize properties of figures in a table and classify figures in a hierarchy by using visual models such as Venn diagrams, flow charts, and tree diagrams. In this lesson, the hierarchical relationships between categories of figures have no overlap or are entirely contained within another category.

In the next lesson students will classify polygons in hierarchies with categories that have some overlap. Students will use more complex Venn diagrams to help them visualize properties that are shared by categories of polygons. For example, students will classify triangles based on both side lengths and angle measures. Students will continue to use visual models such as tables, Venn diagrams, flow charts, and tree diagrams to organize properties of polygons and to show more complex hierarchical relationships between categories of polygons.

Lesson Pacing Guide

Whole Class Instruction

Day 1
45–60 minutes

Toolbox: Interactive Tutorial
Classify Two-Dimensional Figures

Introduction
- Use What You Know *10 min*
- Find Out More *10 min*
- Reflect *5 min*

Modeled and Guided Instruction

Learn About Ordering Shapes in a Hierarchy
- Model It/Model It *10 min*
- Connect It *10 min*
- Try It *5 min*

Practice and Problem Solving
Assign pages 323–326.

Day 2
45–60 minutes

Guided Practice

Practice Classifying Two-Dimensional Figures
- Example *5 min*
- Problems 7–9 *15 min*
- Pair/Share *15 min*
- Solutions *10 min*

Practice and Problem Solving
Assign pages 327–328.

Day 3
45–60 minutes

Independent Practice

Practice Classifying Two-Dimensional Figures
- Problems 1–3 *20 min*
- Quick Check and Remediation *10 min*
- Hands-On or Challenge Activity *15 min*

Toolbox: Lesson Quiz
Lesson 30 Quiz

Small Group Differentiation

Teacher-Toolbox.com

Reteach
Ready Prerequisite Lessons *45–90 min*

Grade 4
- Lesson 32 Classify Two-Dimensional Figures

Teacher-led Activities
Tools for Instruction *15–20 min*

Grade 5 *(Lesson 30)*
- Subcategories of Plane Figures

Student-led Activities
Math Center Activities *30–40 min*

Grade 4 *(Lesson 32)*
- 4.57 Triangle Vocabulary Match
- 4.58 Classifying Shapes

Grade 5 *(Lesson 30)*
- 5.51 Organize Polygons on a Venn Diagram
- 5.52 Organize Triangles on a Venn Diagram

Personalized Learning

i-Ready.com

Independent
i-Ready Lessons *10–20 min*

Grade 4 *(Lesson 32)*
- Quadrilaterals
- Classifying Triangles

Introduction

At a Glance

Students identify the properties of three polygons and use the properties to determine the most specific name for each polygon. Then students arrange the polygons from most general to least general. Then students explore the hierarchy of the polygons from the previous page in a Venn diagram and a flow chart.

Step By Step

- Work through **Use What You Know** as a class.
- Tell students that this page models using properties of polygons to determine the most specific name for a polygon.
- Have students read the problem at the top of the page.
- Review the meaning of parallel lines. [Lines that never intersect and always remain the same distance apart.] Draw examples of parallel lines on the board.
- Ask students to describe a right angle. [An angle that looks like a square corner and measures 90°.] Draw an example of a right angle.
- Discuss the properties that each polygon has as students complete the table.
- Ask students what properties all three polygons have in common. [4 sides, 2 pairs of parallel sides, 2 pairs of sides of equal length] Ask: *Which polygon has only those properties?* [Polygon *B*] Ask: *Which polygon has more properties than Polygon B but does not have all the properties listed in the table?* [Polygon *A*] *Which polygon has all the properties listed in the table?* [Polygon *C*] Discuss the most specific name for each polygon and point out that students can arrange the polygons from most general to least general based on the polygons' properties.
- Ask students to explain their answers for the remaining problems.

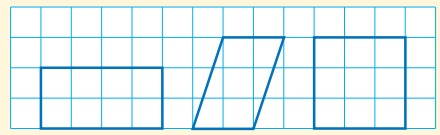
▶ Mathematical Discourse 1 and 2

Classify Two-Dimensional Figures

Use What You Know

In this lesson, you will classify polygons based on their properties. Take a look at this problem.

Arrange the polygons below so that a polygon can also be called by the name of the polygon before it. Order them from left to right.



a. Complete the table below. Put a check in each box if the polygon has the property listed.

Property	Polygon *A*	Polygon *B*	Polygon *C*
4 sides	X	X	X
2 pairs of parallel sides	X	X	X
2 pairs of sides of equal length	X	X	X
4 right angles	X		X
4 sides of equal length			X

b. Write the most specific name for each polygon from the list below.

quadrilateral parallelogram rectangle square

A: rectangle *B*: parallelogram *C*: square

c. How would you arrange the polygons so each shape has all the properties of the shape(s) before it? parallelogram, rectangle, square

▶ Mathematical Discourse

1 *Which is the most specific description of a breakfast menu item: a fried egg, an egg, or a fried egg over easy? Describe how this compares to determining the most specific name for polygons A, B, and C.*

Responses should indicate an understanding that the description that provides the most information is the most specific description.

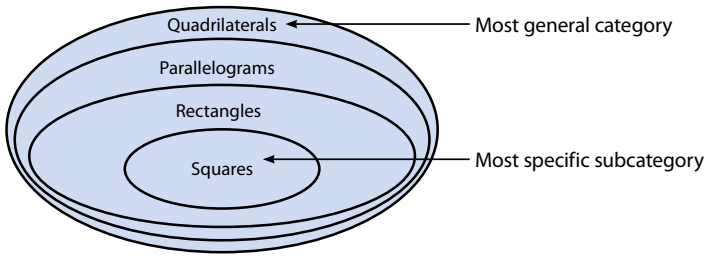
2 *Why might it make more sense to call polygon C a square when you could also call it a rectangle, a parallelogram, or a quadrilateral?*

Calling polygon *C* a square gives the most information possible about the figure.

Find Out More

Shapes can be classified according to their properties. When you order categories of polygons by their properties, you put them in a **hierarchy**. A hierarchy organizes categories from the most to least general. One model you can use to show a hierarchy is a Venn diagram.

A Venn diagram can show categories and subcategories. This Venn diagram shows that squares have all the properties that rectangles have, plus more. This means all squares are also rectangles. A square is also a parallelogram and a quadrilateral.



You can also use a flow chart to show the hierarchy of quadrilaterals. The most general category is at the left, while the most specific is at the right. This means that a figure that belongs in one category also belongs in all categories to the left.

Quadrilaterals → Parallelograms → Rectangles → Squares

Reflect

1 How are the flow chart and the Venn diagram alike? How are they different?

Possible answer: They are alike because they order quadrilaterals first and squares last. They are different because the Venn diagram has a larger area for the largest category, and the flow chart shows each category in same-sized boxes.

Step By Step

- You may wish to review Venn diagrams with your class before discussing this page.
- Read **Find Out More** as a class.
- Have students look at the Venn diagram. Work from the outermost category to the innermost. Guide students to understand that a parallelogram is a quadrilateral because it has all of the properties of a quadrilateral plus some additional properties. Ask students to use the table on the previous page to name those additional properties. [2 pairs of parallel sides, 2 pairs of sides of equal length]
- Similarly, a rectangle is a parallelogram because it has all of the properties of a parallelogram plus an additional property. Ask students to use the table on the previous page to name the additional property. [4 right angles]
- Finally, have students explain why a square is a rectangle. [A square has all of the properties of a rectangle plus the property that it has 4 sides of equal length.]
- Point out that the flow chart shows the same relationships between the categories of quadrilaterals and that the hierarchy in the flow chart goes from the left, with the most general category, to the right, with the most specific category.

▶ Real-World Connection

▶ Concept Extension

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 323–324** after students have completed this section.

▶ Concept Extension

Explore ordering three-dimensional figures.

Point out that just as two-dimensional figures can be ordered from general to most specific, so can three-dimensional figures. Show students an example of a generic prism [faces are parallelograms], a rectangular prism, and a cube. Point out that the faces of a generic prism are parallelograms, the faces of a rectangular prism are rectangles, and the faces of a cube are squares. Ask students to use what they learned about parallelograms, rectangles, and squares to order the three-dimensional figures from most general to most specific. [generic prism, rectangular prism, cube]

▶ Real-World Connection

Encourage students to think about everyday places or situations in which people might see or talk about a hierarchy.

Example: A hierarchy can be applied to answer the question "Where do you live?" For example, you could answer based on your continent, country, state, city, town, neighborhood, or street name. All answers would be accurate, but the name of your street would be the most specific answer for where you live.

Modeled and Guided Instruction

At a Glance

Students use a table to organize the properties of triangles. Then students use a tree diagram to arrange the triangles in a hierarchy. Then students revisit this problem to complete a tree diagram to understand the relationships among isosceles, scalene, and equilateral triangles and to order the triangles in a hierarchy.

Step By Step

- Read the problem at the top of the page as a class.

Model It

- Read **Model It**. Be sure that students understand that a triangle can be called isosceles if it has 2 or 3 sides of equal length. Point out that you could also say that an isosceles triangle has at least 2 sides of equal length.

▶ **Mathematical Discourse 1**

Model It

- Read **Model It**. Ask students to explain why Triangles is the more general category. [All of the other categories are specific types of triangles.]

▶ **English Language Learners**

SMP TIP Attend to Precision
Students attend to precision when they use clear and precise language to describe the properties of isosceles, scalene, and equilateral triangles. *(SMP 6)*

▶ **Mathematical Discourse 2**

Learn About **Ordering Shapes in a Hierarchy**

Read the problem below. Then explore different ways to classify figures in a hierarchy.

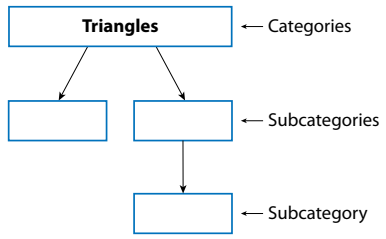
Classify the following triangles from the most general to the most specific: scalene triangle, isosceles triangle, and equilateral triangle. Use a tree diagram to classify them as types of triangles.

Model It **You can understand the problem by listing the properties of the triangles in a table before arranging them in a tree diagram.**

Types of Triangles	Properties of Sides
Isosceles	2 or 3 sides of equal length
Scalene	no sides of equal length
Equilateral	3 sides of equal length

Model It **You can represent the problem with a tree diagram.**

A tree diagram can also be used to show a hierarchy. Put the most general category as the top branch. Then put the more specific subcategories as the branches.



▶ **Mathematical Discourse**

1 *The table shows the properties of sides of isosceles, scalene, and equilateral triangles. What kinds of triangles could you put in the table if you wanted to show the properties of angles of triangles?*

You could put right triangles, acute triangles, and obtuse triangles in the table.

2 *How is a tree diagram similar to a Venn diagram?*

Responses may vary but should indicate an understanding that both show a hierarchy of items. Both would show the relationships between isosceles, scalene, and equilateral triangles.

▶ **English Language Learners**

Point out that a *tree diagram* gets its name because its shape resembles a tree with many branches.

Connect It **Now you will solve the problem from the previous page by using the table to complete a tree diagram.**

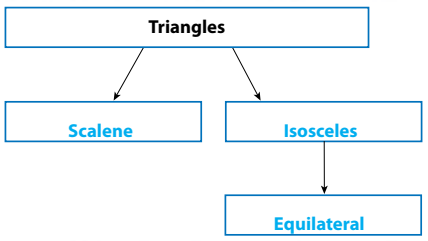
2 Why is "Triangles" in the top row of the tree diagram? _____

Possible answer: That's the most general category.

3 Write "Scalene" and "Isosceles" in the second row of the tree diagram at the right.

Why are those categories separate?

A scalene triangle cannot have sides of equal length.



4 Write "Equilateral" beneath "Isosceles."

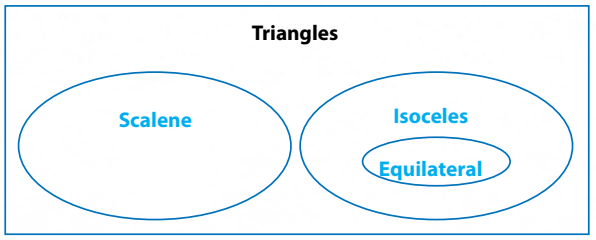
Why can all equilateral triangles be classified as isosceles triangles?

They have at least 2 sides of equal length.

5 How can you use a tree diagram to order figures? Possible answer: Place the most general category of shape at the top and more specific subcategories of shapes beneath.

Try It **Use what you learned about ordering figures in a hierarchy to solve this problem.**

6 Complete the Venn diagram below to show the hierarchy of isosceles, scalene, and equilateral triangles.



Step By Step

Connect It

- Read **Connect It** as a class. Be sure to point out that the questions refer to the problem on the previous page.
- Guide students to understand that all triangles can be classified as either scalene or isosceles because scalene triangles have no sides of equal length and isosceles triangles have at least 2 sides of equal length. Since "at least 2" means 2 or more, equilateral triangles are also isosceles triangles.

SMP TIP **Model with Mathematics**
Students model the hierarchical order of triangles using a tree diagram. Point out that they could also use a flow chart or Venn diagram to model the order. *(SMP 4)*

▶ Concept Extension

Explore Venn diagrams.

Point out that the Venn diagram for Quadrilaterals, Parallelograms, Rectangles, and Squares shown on the Introduction page uses an oval for the category Parallelograms. The category Parallelograms is set within an oval because there are figures that are quadrilaterals but not parallelograms. Ask students to name or draw such a shape. [kite]

Explain that when drawing a Venn diagram for triangles based on the properties of sides of triangles, you do need to set the category Triangles within an oval because Scalene and Isosceles triangles make up the entire category of Triangles. There are no triangles that are not Scalene or Isosceles. All triangles fit in one of these two categories.

Try It

- Ask students how they would show that all equilateral triangles are isosceles triangles using a Venn diagram. [The oval for Equilateral would be nested inside the oval for Isosceles.]

6 Solution
See completed Venn diagram on the Student Book page. Students may draw a Venn diagram with an outside category of triangles, two non-overlapping subcategories of scalene and isosceles, and a category of equilateral that is nested inside isosceles.

Error Alert Students who draw *equilateral* such that it overlaps both *scalene* and *isosceles* may not understand what overlapping categories of a Venn diagram represent.

Ready **Mathematics** PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 325–326** after students have completed this section.

Guided Practice

At a Glance

Students use tables, Venn diagrams, tree diagrams, and flow charts to classify plane figures in a hierarchy.

Step By Step

- Ask students to solve the problems individually and label categories in their diagrams.
- **Pair/Share** When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

Solutions

Example A Venn diagram illustrating the hierarchy is shown. Students may begin by creating a table or list of properties.

7 Solution

Students may say that all equilateral triangles are acute triangles. They may also say that some acute triangles are equilateral triangles.

DOK 3

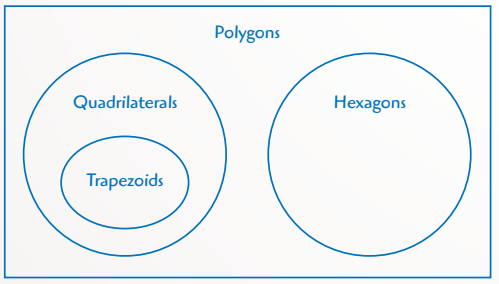
Practice Classifying Two-Dimensional Figures

Study the example below. Then solve problems 7–9.

Example

Create a Venn diagram to show the hierarchy of quadrilaterals, polygons, trapezoids, and hexagons.

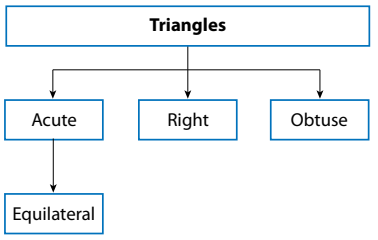
Look at how you could show your work using a Venn diagram.



A shape can never be both a hexagon and a quadrilateral. So these regions do not overlap.

Pair/Share
Recreate the hierarchy with a tree diagram.

7 Look at the tree diagram below. Write a statement about the relationship between acute triangles and equilateral triangles.



Which type of triangle is the most specific?

Pair/Share
Write a statement about the relationship between acute triangles and obtuse triangles.

Solution All equilateral triangles are acute triangles.

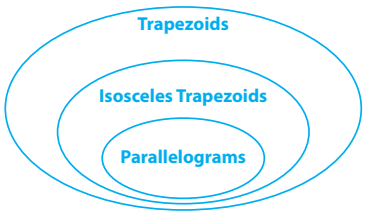
Teacher Notes

8 Create a Venn diagram to show the hierarchy of the polygons described in the chart.

Polygon	Description
Trapezoid	quadrilateral with at least 1 pair of parallel sides
Isosceles Trapezoid	trapezoid with at least 2 sides of equal length
Parallelogram	quadrilateral with 2 pairs of parallel sides

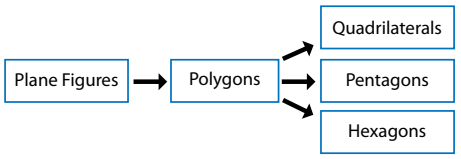
"At least 2" means 2 or more.

Possible Venn diagram:



Pair/Share
Draw one example of a polygon in each separate category of your Venn diagram.

9 Look at the flow chart below.



The flow chart is like a tree diagram. But the arrows show that the hierarchy moves from left to right instead of top to bottom.

Which statement is true? Circle the letter of the correct answer.

A A plane figure is always a polygon.

B All polygons are plane figures.

C All hexagons are also pentagons and quadrilaterals.

D A hexagon is not a plane figure.

Brad chose **C** as the correct answer. How did he get that answer?

Possible answer: Brad confused the flow chart with a tree diagram.

Pair/Share
Does Brad's answer make sense?

Teacher Notes

Solutions

8 Solution

See possible student work on the Student Book page; Students use the descriptions in the table to create a Venn diagram.

DOK 3

9 Solution

B: "Polygons" belong to the category "Plane Figures" because a figure that belongs in one category also belongs in all categories to the left.

Explain to students why the other two answer choices are not correct:

A is not correct because a figure that belongs in one category also belongs in all categories to the left (so a polygon is always a plane figure), but a figure that belongs in one category does not necessarily belong in all categories to the right (so a plane figure is not always a polygon).

D is not correct because a figure that is a hexagon also belongs in all categories to the left, so a hexagon is also a plane figure.

DOK 3

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 327–328** after students have completed this section.

Independent Practice

At a Glance

Students classify plane figures in a hierarchy based on their properties to answer questions that might appear on a mathematics test.

Solutions

1 Solution

A; The most general, or least specific, name for the shape shown is polygon. The shape has 4 sides, so it is also a quadrilateral. The shape has 2 pairs of sides of equal length and 4 right angles, so it is also a rectangle.

DOK 2

2 Solution

See the Student Book page for the completed table; A scalene triangle has 3 sides of different lengths. An isosceles triangle has at least 2 sides of equal length. An obtuse triangle has an angle greater than 90°.

DOK 1

Practice Classifying Two-Dimensional Figures

Solve the problems.

1 Look at the shape below.

Which is a correct classification for this shape from LEAST specific to MOST specific?

A polygon, quadrilateral, rectangle

B quadrilateral, parallelogram, square

C polygon, quadrilateral, square

D quadrilateral, rectangle, square

2 Classify the triangles shown below as "scalene," "isosceles," or "obtuse." Sides that are the same length are marked with a slash. Draw the triangles in the correct column of the table. If a triangle fits more than one classification, draw it in all the columns that apply.

Scalene	Isosceles	Obtuse

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Quick Check and Remediation

- Ask students to draw a Venn diagram to classify the following shapes in order from most general to most specific: parallelogram, polygon, rhombus, quadrilateral. [polygon, quadrilateral, parallelogram, rhombus] Remind students that a rhombus is a parallelogram with four sides of equal length.
- For students who are struggling, use the chart to guide remediation.
- After providing remediation, check students' understanding. Ask students to draw a Venn diagram to classify the following shapes in order from most general to most specific: parallelogram, square, rhombus, quadrilateral [quadrilateral, parallelogram, rhombus, square]
- If a student is still having difficulty, use *Ready Instruction*, Grade 4, Lesson 32.

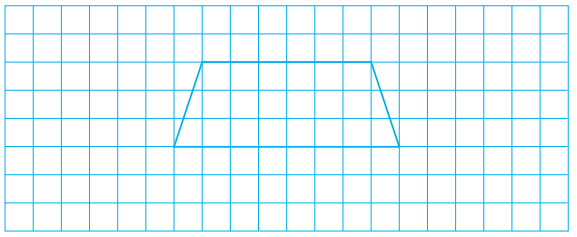
If the error is . . .	Students may . . .	To remediate . . .
any order other than the correct order	be able to identify all of the properties of the given shapes	Have students make a table with the names of the shapes as the headings of four columns. Work with students to list the properties of each of the shapes so that students can see, for example, that a rhombus has all the properties that parallelograms have plus one more.

3 The word "isosceles" can be used to describe any polygon with at least 2 sides of equal length.

Look at the flow chart below.

Part A Draw an example of an isosceles trapezoid.

Possible isosceles trapezoid:



Part B Explain how isosceles trapezoids relate to parallelograms.

Possible answer: Isosceles trapezoids have at least 2 sides of equal length and at least 1 pair of parallel sides. Parallelograms have 2 pairs of sides of equal length and 2 pairs of parallel sides, so all parallelograms are isosceles trapezoids.

Part C Can you use the term "isosceles" to describe a rectangle? Explain your reasoning.

Yes. Possible explanation: A rectangle has at least two sides of equal length.

Self Check **Go back and see what you can check off on the Self Check on page 283.**

Solutions

3 **Part A Solution**

Students draw a trapezoid that has two sides of equal length; see possible drawing on the Student Book page.

Part B Solution

See possible student explanation on the Student Book page.

Part C Solution

Yes; see possible student explanation on the Student Book page.

DOK 3

▶ Hands-On Activity

Build quadrilaterals that fit the given conditions.

Materials: geoboards and geobands

Have students make a shape that fits conditions you supply. Ask students to name the shape they made.

- four sides, opposite sides are parallel, no right angles [parallelogram (or rhombus)]
- four sides, opposite sides are parallel, four right angles [rectangle (or square)]
- four sides, opposite sides are parallel, four right angles, all sides are of equal length [square]

If time permits, provide conditions for students to build different triangles and have students build and name the triangles you have described.

▶ Challenge Activity

Name the figure in different ways.

Challenge students to provide as many different names as they can for figures that you draw.

Label important features such as sides of equal length and right angles in your drawings.

Include figures such as an equilateral triangle, a parallelogram, a square, a rectangle, and a rhombus. Have students justify why each of the names they use applies to the figure.

LESSON OVERVIEW

Lesson 31
Understand Properties of Two-Dimensional Figures

CCSS Focus

Domain
Geometry

Cluster
B. Classify two-dimensional figures into categories based on their properties.

Standards
5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

Standards for Mathematical Practices (SMP)
2 Reason abstractly and quantitatively.

4 Model with mathematics.

6 Attend to precision.

7 Look for and make use of structure.

Lesson Objectives

Content Objectives
- Recognize that two-dimensional figures can be categorized based on shared attributes and properties.
- Use Venn diagrams, flow charts, and tree diagrams to model how attributes are shared by categories of polygons.

Language Objectives
- Discuss the definitions of key terms *attribute, property, category,* and *subcategory* with a partner and use the terms in conversation.
- Draw Venn diagrams, flow charts, and tree diagrams to show properties that are shared by categories of polygons.
- List inferences about attributes of sub-categories of quadrilaterals and triangles shown in hierarchy diagrams.

Prerequisite Skills
- Recognize parallel and perpendicular lines.
- Recognize right, acute, and obtuse angles.
- Classify two-dimensional figures in a hierarchy based on properties of the figures.

Lesson Vocabulary
- **convex polygon** a polygon with interior angles all measuring less than 180°
- **concave polygon** a polygon with at least one interior angle measuring greater than 180°

Review the following key term.

- **attribute** any characteristic of an object or shape, like number of sides, color, angle measure, etc.

Learning Progression

In the previous Grade 5 lesson students analyzed categories of polygons based on their properties and related the categories in a hierarchy. They classified figures in a hierarchy by using visual models such as Venn diagrams, flow charts, and tree diagrams. The hierarchical relationships between categories did not overlap or one category was entirely encompassed within another category.

In this lesson students classify two-dimensional figures based on attributes that are shared and not shared. Students show relationships among categories of figures that have overlap among the categories. For example, students may classify a triangle as acute but not isosceles, or as isosceles but not acute, or as both acute and isosceles. Students continue to use visual models such as tables, Venn diagrams, flow charts, and tree diagrams to organize attributes of figures and to show more complex hierarchical relationships between categories of figures.

In Grade 6 students will work with figures in the coordinate plane and will find the area of two-dimensional figures.

Lesson Pacing Guide

Whole Class Instruction

Day 1
45–60 minutes

Toolbox: Interactive Tutorial
Understand Properties of Two-Dimensional Figures

Introduction
- Think It Through Question *10 min*
- Think *10 min*
- Think *15 min*
- Reflect *10 min*

Practice and Problem Solving
Assign pages 331–332.

Day 2
45–60 minutes

Guided Instruction

Think About Properties Shared by Polygons
- Let's Explore the Idea *20 min*
- Let's Talk About It *15 min*
- Try It Another Way *10 min*

Practice and Problem Solving
Assign pages 333–334.

Day 3
45–60 minutes

Guided Practice

Connect Ideas About Properties of Polygons
- Categorize *10 min*
- Explain *5 min*
- Create *5 min*

Independent Practice

Apply Ideas About Properties of Polygons
- Put It Together *20 min*
- Pair/Share *10 min*

Practice and Problem Solving
Assign pages 335–336.

Day 4
45–60 minutes

- On-Level, Intervention, or Challenge Activity *20 min*

Toolbox: Lesson Quiz
Lesson 31 Quiz

Small Group Differentiation

Teacher-Toolbox.com

Reteach
Ready Prerequisite Lessons *45–90 min*

Grade 4
- Lesson 32 Classify Plane Figures

Teacher-led Activities
Tools for Instruction *15–20 min*

Grade 5 *(Lesson 31)*
- Classify Plane Figures

Student-led Activities
Math Center Activities *30–40 min*

Grade 4 *(Lesson 32)*
- 4.57 Triangle Vocabulary Match
- 4.58 Classifying Shapes

Grade 5 *(Lesson 31)*
- 5.49 Classify Quadrilaterals
- 5.5 Classify Triangles

Personalized Learning

i-Ready.com

Independent
i-Ready Lessons *10–20 min*

Grade 4 *(Lesson 32)*
- Quadrilaterals
- Classifying Triangles

Introduction

At a Glance

Students explore how to describe polygons by their attributes, or properties. Students learn that a polygon can be categorized in more than one way. Then students use a Venn diagram to explore the relationship between isosceles, equilateral, right, acute, and obtuse triangles.

Step By Step

- Introduce the question at the top of the page.

▶ English Language Learners

- Review the meanings of parallel [lines that never intersect and the same distance apart] and perpendicular [lines that intersect at a 90° angle].
- Explain the meaning of the marks on the polygons in the Example column that indicate sides that are the same length. Review the meaning of the marks that indicate right angles.
- Introduce the meaning of a regular polygon [all sides are of equal length and all angles are of equal measure]. Ask students to name a regular quadrilateral. [square]
- Read the **Think** question with students.
- Point out that since an isosceles triangle has at least two sides of equal length and an equilateral triangle has three sides of equal length, an equilateral triangle can also be categorized as an isosceles triangle.

▶ Mathematical Discourse 1 and 2

Understand Properties of Two-Dimensional Figures

Think It Through

How do we group polygons into categories?

Polygons are grouped into categories by their **attributes**, or properties, such as the number of sides or angles, the side lengths, and the angle measures. All polygons in the same category share certain properties. Some properties of polygons are described in the table below.

Property	Description	Example
Scalene	no sides of equal length	
Isosceles	at least 2 sides of equal length	
Equilateral	all sides of equal length	
Regular	all sides of equal length and all angles of equal measure	
Irregular	at least 1 side and 1 interior angle are not equal in measure to the other sides and angles	
Right	at least 1 pair of perpendicular sides	
Parallel sides	at least 1 pair of opposite sides that will never intersect, no matter how far they are extended	

Think **Can a polygon be categorized in more than one way?**

Think about how a quadrilateral is defined. It is a polygon with 4 sides. So any shape with 4 sides can be called both a polygon and a quadrilateral. If the quadrilateral has two pairs of parallel sides, then it can also be called a parallelogram.

Every parallelogram is a quadrilateral because every parallelogram has 4 sides. But not all quadrilaterals are parallelograms because not all quadrilaterals have two pairs of parallel sides.

Shade a polygon above that can be named both a quadrilateral and parallelogram.

▶ Mathematical Discourse

1 *Triangles can be categorized by their angle measures, as well as their side lengths. Which way of categorizing triangles do you think is more useful? Explain your reasoning.*

Responses should indicate an understanding that both categories are useful depending on the context of the situation.

2 *In how many different ways can a polygon be categorized? Explain your reasoning.*

Responses may vary but should indicate an understanding that it depends on the polygon. For example, a quadrilateral can be categorized two ways, as a polygon and a quadrilateral. A parallelogram can be categorized three ways, as a polygon, quadrilateral, and parallelogram.

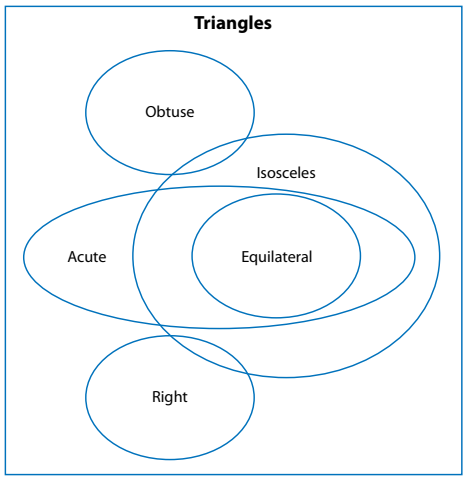
▶ English Language Learners

It is important that students understand the phrase "at least" when discussing properties of polygons. Use the phrase "at least" in sentences to help students better understand the meaning of the phrase. For example, to say that a cell phone costs at least $75 means it costs $75 or more. If you say that you need to study for at least 2 hours tonight, you mean 2 hours or more.

Have students tell you what "at least two sides of equal length" means. [two or more sides of equal length] Ask: *If a triangle has three sides of equal length, can you say that it has at least two sides of equal length?* [yes]

Think **How can you show the relationships among polygons with a diagram?**

A Venn diagram is a useful tool for organizing categories of polygons that share properties.



The Venn diagram shows a triangle can never be both right and obtuse.

Notice the "Right" category partly overlaps the "Isosceles" category. This means a right triangle may also have all the properties of an isosceles triangle. Also notice that the "Right" category does not overlap the "Obtuse" category. That means a right triangle can never have all the properties of an obtuse triangle.

The "Equilateral" category is nested completely inside the "Isosceles" category. This shows that equilateral triangles are a subcategory of isosceles triangles. So all equilateral triangles share all the properties of isosceles triangles.

Reflect

1 What does it mean that the Venn diagram shows "Obtuse" partially overlapping "Isosceles?"

Possible answer: An obtuse triangle can also be an isosceles triangle.

Step By Step

- Read the **Think** question with students.
- Tell students that categories that do not overlap in a Venn diagram do not share any properties. Ask students why an equilateral triangle does not share any properties with an obtuse triangle. [Obtuse triangles have an angle that measures greater than 90°, but all the angles of an equilateral triangle have the same measure and the measure is less than 90°.]
- Point out that categories that overlap can share properties. For example, a triangle can be both right and isosceles.
- Review with students that a subcategory that is nested completely inside another category shares all the properties of the category in which it is nested.
- Have students read and reply to the **Reflect** question.

▶ Mathematical Discourse 3 and 4

▶ Visual Model

▶ Visual Model

Draw triangles in the Venn diagram.

Work with students to draw an example of a triangle for each category and subcategory in the Venn diagram. Include markings to indicate right angles and sides of the same length as appropriate.

▶ Mathematical Discourse

3 *Can you think of another kind of mathematical relationship that could be modeled with a Venn diagram?*

Responses may vary. Possible answer: whole numbers, prime numbers, and composite numbers.

4 *How could you explain to a friend why a Venn diagram is a useful tool?*

Responses should indicate an understanding that a Venn diagram visually shows categories so that you can easily determine what properties items do and do not share.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 331–332** after students have completed this section.

Guided Instruction

At a Glance

Students complete a Venn diagram and table of properties for quadrilaterals. Then students use a Venn diagram to understand how properties are shared by categories of quadrilaterals. Students use a flow chart to answer questions about the properties of quadrilaterals.

Step By Step

Let's Explore the Idea

- Tell students that they will have time to work individually on the problems on this page and then share their responses in groups. You may choose to work through problem 2 together as a class.
- Note that a trapezoid is defined as a quadrilateral with at least one pair of parallel sides.
- Take note of students who are still having difficulty and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

Student Misconception Alert

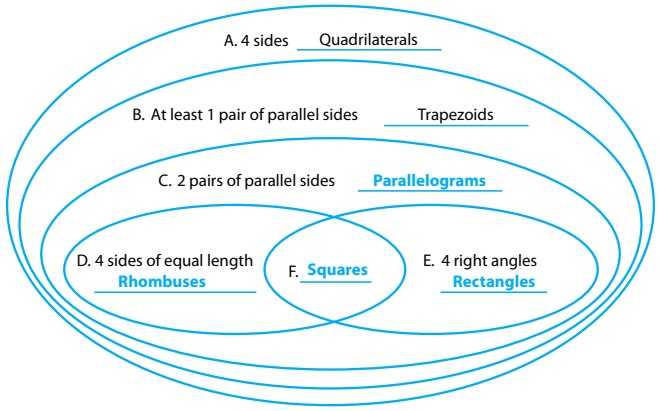
When completing the table, students may notice that categories A through D repeat the properties of the preceding category. Students may mistakenly extend this pattern to category E. Point out that category E is not nested inside category D, so it does not share all the properties of category D. Emphasize that it is nested inside categories A, B, and C, so it does share all the properties of those categories.

▶ Mathematical Discourse 1 and 2

Think About **Properties Shared by Polygons**

Let's Explore the Idea **A Venn diagram can help you understand what properties are shared by categories of polygons.**

2 The Venn diagram shows categories of quadrilaterals with different properties. Write the name of each category that fits the description.



3 Use the Venn diagram to fill in the table below.

Category	Properties	Name
A	4 sides	Quadrilaterals
B	4 sides, at least 1 pair of parallel sides	Trapezoids
C	4 sides, 2 pairs of parallel sides	Parallelograms
D	4 sides, 2 pairs of parallel sides, 4 sides of equal length	Rhombuses
E	4 sides, 2 pairs of sides that are parallel and of equal length, 4 right angles	Rectangles
F	4 sides, 2 pairs of parallel sides, 4 sides of equal length, 4 right angles	Squares

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▶ Mathematical Discourse

1 *What pattern do you see in the table?*

Responses should indicate an understanding that a nested category repeats all the properties of the category in which it is nested.

2 *Explain why a square is also a rectangle, a rhombus, a parallelogram, a trapezoid, and a quadrilateral.*

Responses may vary but should indicate an understanding that a square shares the properties of each of the other figures.

Let's Talk About It

Use the Venn diagram to help you understand how properties are shared by categories of quadrilaterals.

4 Is every property of parallelograms also a property of all rectangles? yes

Is every property of rectangles also a property of all parallelograms? no

Explain what the Venn diagram shows about the relationship between rectangles and parallelograms. Possible answer: All rectangles are parallelograms, but only some parallelograms are rectangles.

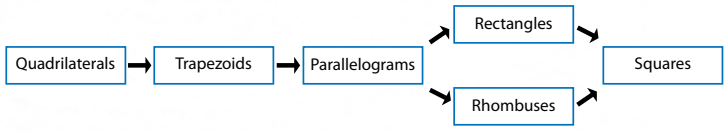
Classify each inference statement as *true* or *false*. If false, explain.

5 The opposite angles of any parallelogram have the same measure. Therefore, the opposite angles of any rhombus have the same measure. true

6 The diagonals of any square are the same length. Therefore, the diagonals of any rhombus are the same length. false; Possible explanation: Not all rhombuses are squares.

Try It Another Way

The flow chart below shows another way to think about how quadrilaterals are categorized.



Use the flow chart to describe the statements as *true* or *false*.

7 In every rectangle the two diagonals have the same length. Therefore, in every parallelogram the two diagonals must have the same length. false

8 Every rhombus has at least 2 lines of symmetry. Therefore, every square has at least 2 lines of symmetry. true

Mathematical Discourse

3 *Luca says a four-sided plane figure with two pairs of parallel sides is a quadrilateral. Federico says it is a parallelogram. Who is correct? Explain your reasoning.*

Responses should indicate an understanding that both are correct, but *parallelogram* is a more precise name.

4 *How could you use the properties of rectangles to explain that not all rectangles are squares?*

Responses should include the fact that not all rectangles have four sides that are the same length.

Step By Step

Let's Talk About It

- Organize students into pairs or groups. You may also choose to work through problems 4–6 together as a class.
- If you choose to have students work in pairs or groups, walk around to each group, listen to, and join in on discussions at different points.
- Ask students to draw examples of figures in each category. Discuss whether or not the figure drawn could also belong to another category.

▶ Mathematical Discourse 3 and 4

SMP TIP Reason Abstractly
Students use reasoning skills to classify statements about properties of quadrilaterals as true or false. Have students share their reasoning with a partner. *(SMP 2)*

Try It Another Way

- Direct each group's attention to **Try It Another Way**. Have a volunteer from each group come to the board to explain the group's solutions to problems 7 and 8.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 333–334** after students have completed this section.

Guided Practice

At a Glance

Students demonstrate their understanding of categorizing polygons based on properties that are shared or not shared. Then students show the relationship between types of triangles with a tree diagram and words.

Step By Step

- Discuss each problem as a class using the discussion points outlined below.

Categorize

- You may choose to have students work in pairs to encourage sharing ideas.

SMP TIP Model with Mathematics
Point out to students that, in addition to a Venn diagram, they could also draw a flow chart or tree diagram to model the relationships. *(SMP 4)*

- Ask students how they chose the most general categories. [Triangles, quadrilaterals, and rectangles are all types of polygons; polygons are the most general category.]
- Discuss with students why triangles can never be concave polygons. [Each of the angles of a triangle has a measure less than 180 degrees. A concave polygon has at least one interior angle greater than 180 degrees.]

Explain

- This problem focuses on using words to explain how two shapes are related.
- Ask students to describe how rectangles are related to trapezoids. [All parallelograms are trapezoids. A rectangle is a parallelogram.] What other figures could be classified as trapezoids? [rectangles, rhombuses, and squares]

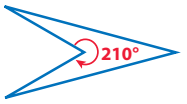
Create

- This problem gives students an opportunity to describe the properties of and draw a shape given certain criteria. Have students describe the methods they can use.

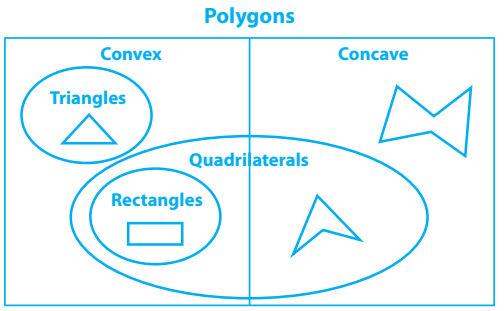
Connect Ideas About Properties of Polygons

Talk through these problems as a class. Then write your answers below.

9 Categorize All polygons are either **convex** or **concave**. A convex polygon has all interior angles less than 180°. A triangle is an example of a convex polygon. A concave polygon has at least 1 interior angle greater than 180°. The quadrilateral below is an example of a concave polygon.

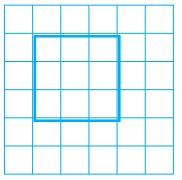


Categorize concave polygons, convex polygons, triangles, quadrilaterals, and rectangles in a Venn diagram. Draw an example of each polygon in the diagram.
Possible Venn diagram:



10 Explain Nadriette said that a rectangle can never be called a trapezoid. Explain why Nadriettte's statement is incorrect. **Possible answer: Since a rectangle has at least 1 pair of parallel sides, it can be called a trapezoid.**

11 Create Describe the properties of a shape that is both a rectangle and a rhombus. Name the shape and use the grid below to draw an example.



a parallelogram with 4 sides of equal length and 4 right angles: a square

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Scoring Rubrics

Part A

Points	Expectations
2	The tree diagram indicates a student's understanding of the relationship between the properties of triangles. The tree diagram correctly connects categories.
1	An effort was made to accomplish the task. The tree diagram demonstrates some understanding of the relationship between the properties of triangles, but the student's diagram is missing categories, misplaces categories, or incorrectly connects categories.
0	There is no tree diagram or the diagram shows little or no understanding of the relationship between the properties of triangles.

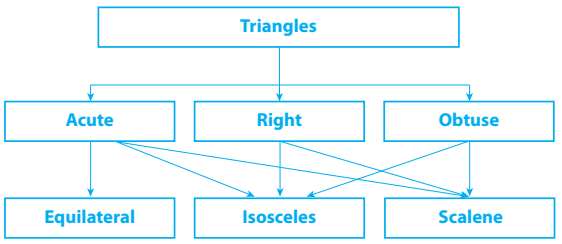
Apply Ideas About Properties of Polygons

12 Put It Together Use what you have learned about classifying polygons to complete this task.

Part A Create a tree diagram to show the following types of triangles: acute, obtuse, right, isosceles, and equilateral. Make sure to include the category "Triangle." Use information in the table to help you.

Triangle	Types of Angles
Acute	all acute angles
Right	2 acute angles and 1 90° angle
Obtuse	2 acute angles and 1 obtuse angle
Scalene	acute, right, or obtuse
Isosceles	acute, right, or obtuse
Equilateral	all acute angles

Possible tree diagram:



Part B Write a statement that is always true about the relationship between obtuse triangles and equilateral triangles.

Possible answer: An equilateral triangle can never be classified as an obtuse triangle.

Part C Write a statement that is sometimes true about the relationship between acute triangles and isosceles triangles.

Possible answer: An isosceles triangle can be an acute triangle.

Independent Practice

Step By Step

Put It Together

- Direct students to complete the **Put It Together** task on their own.
- Explain that the categories in each row of the tree diagram can be placed in any order as long as they are correctly connected to the categories in the row beneath them.
- As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.
- If time permits, have students share their tree diagrams with a partner.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 335–336** after students have completed Guided Practice.

Part B

Points	Expectations
2	The student wrote a statement about the relationship between obtuse triangles and equilateral triangles that is always true.
1	An effort was made to accomplish the task, but the statement the student wrote is not always true.
0	There is no response or the response shows no evidence of understanding the relationship between obtuse and equilateral triangles.

Part C

Points	Expectations
2	The student wrote a statement about the relationship between acute triangles and isosceles triangles that is sometimes true.
1	An effort was made to accomplish the task, but the statement the student wrote is not true.
0	There is no response or the response shows no evidence of understanding the relationship between acute and isosceles triangles.

Differentiated Instruction

▶ Intervention Activity

Model and categorize triangles.

Materials: geoboards, strips of paper

Group students into pairs. Tell students to write each of the following categories on its own strip of paper, and include a description of each: triangle, right triangle, acute triangle, obtuse triangle, scalene triangle, isosceles triangle, and equilateral triangle. Have one student display a triangle on the geoboard. Have the other student choose as many strips of paper as possible that describe the triangle shown. Then have students switch roles.

If time allows, repeat for quadrilaterals, parallelograms, trapezoids, rectangles, rhombuses, and squares.

▶ On-Level Activity

Create a model of a Venn diagram.

Materials: string, paper, scissors

Distribute string and scissors to each student. Have students draw and cut out a quadrilateral, trapezoid, parallelogram, rhombus, rectangle, and square. Tell students to label and write the properties of each figure on their cutouts. Then have students use string to make a Venn diagram for the figures, using their figures to label each section. Have students draw and cut out other quadrilaterals, write all the categories that describe them on the cutouts, and place them appropriately in their Venn diagram.

▶ Challenge Activity

Justify classifications of polygons.

Ask students to answer each of the following questions and to provide an explanation for each answer.

- Are all rectangles squares? [No, students may draw a rectangle that does not have 4 sides of equal length.]
- Are all squares rectangles? [Yes, students may list all of the properties of a rectangle and note that a square has all of these properties.]
- Is every quadrilateral a parallelogram or a trapezoid? [No, students may draw a quadrilateral that has no pairs of parallel sides.]
- Are all parallelograms rectangles? [No. Not all parallelograms have 90° angles.]
- Is there a quadrilateral that can be classified as a quadrilateral, parallelogram, rectangle, and rhombus? [Yes. A square shares all of the properties of these figures.]

Teacher Notes