Big Ideas 3: Oscillations 1

1. Which of the following statements about a spring-block oscillator in simple harmonic motion about its equilibrium point is false?

- (A) The displacement is directly related to the acceleration.
- (B) The acceleration and velocity vectors always point in the same direction.
- (C) The acceleration vector is always toward the equilibrium point.
- (D) The acceleration and displacement vectors always point in opposite directions.

2. Which of the following are most likely to result in simple harmonic motion? Select two answers.

(A) A hole is drilled through one end of a meter stick, which is hung vertically from a frictionless axle. The bottom of the meter stick is displaced 12 degrees and released.(B) A hole is drilled through one end of a meter stick, which is hung vertically from a rough axle. The bottom of the meter stick is displaced 12 degrees and released. Every time the meter stick swings back and forth the axle squeaks.

(C) A block is hung vertically from a linear spring. The opposite end of the spring is attached to a stationary point. The entire apparatus is placed in deep space. The block is displaced 4 cm from equilibrium and released.

(D) A block is placed on a frictionless surface and attached to a non-linear spring. The opposite end of the spring is attached to a wall. The block is displaced 2 cm from equilibrium and released.

3. A spring of spring constant k is hung vertically from a fixed surface, and a block of mass M is attached to the bottom of the spring. The mass is released and the system is allowed to come to equilibrium as shown in the diagram at right.

(a) Derive an expression for the equilibrium position of the mass.

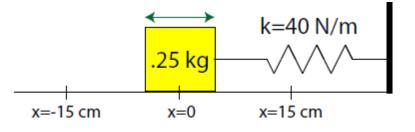
(b) The spring is now pulled downward and displaced an amount A. Derive an expression for the potential energy stored in the spring.

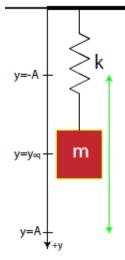
(c) At time t=0, the spring is released. Derive an expression for the period of the spring-block oscillator.

(d) Describe an experimental procedure you could use to verify your derivation of the period. Include all equipment required.

(e) How would you analyze the data to determine whether the experimental data verifies your derivation? What evidence from the analysis would be used to make the determination?

4. A spring of spring constant 40 N/m is attached to a fixed surface, and a block of mass 0.25 kg is attached to the end of the spring, sitting on a frictionless surface. The block is now displaced 15 centimeters and released at time t=0.

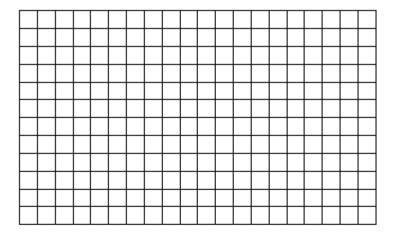




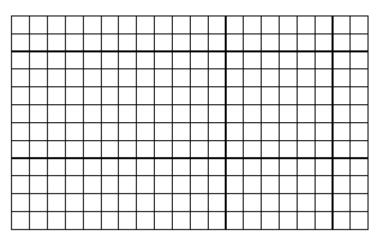
- (a) Draw a free body diagram for the block at t=0.
- (b) Determine the period of the spring-block oscillator.
- (c) Determine the speed of the block at position x=0.

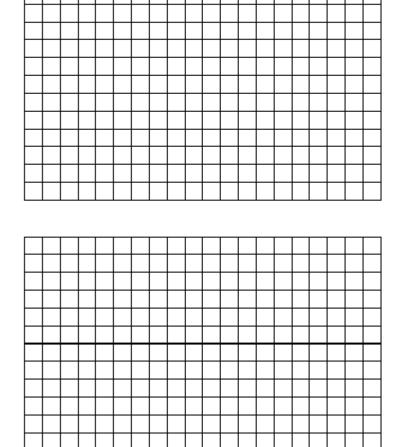
(d) Write an expression for the displacement of the block as a function of time.

(e) On the graphs below, plot the displacement, speed, and acceleration of the mass as a function of time. Explicitly label axes with units as well as any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(f) On the axes below, plot the kinetic energy of the mass and the elastic potential energy of the spring as functions of time. Label your plots K and U_s . Explicitly label axes with units as well as any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

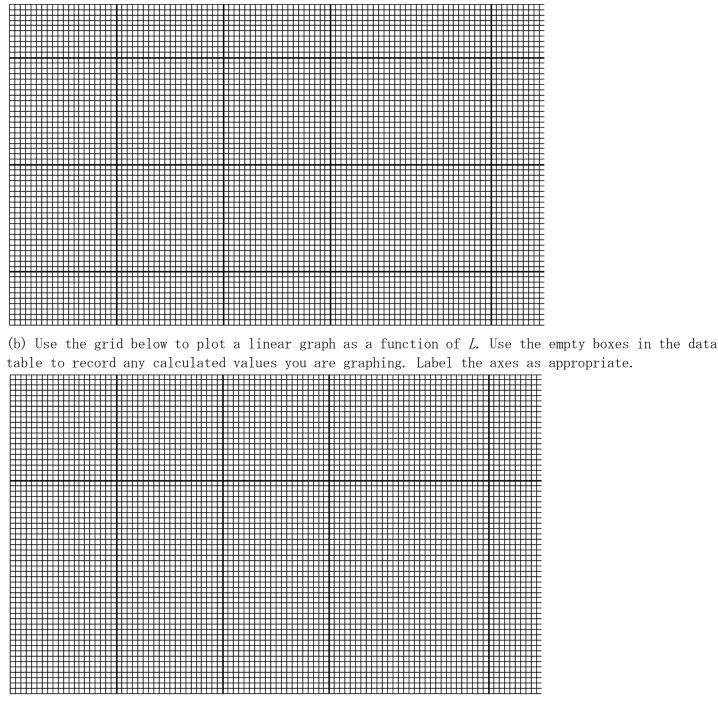




5. Students are to conduct an experiment to investigate the relationship between the length of a pendulum and its period. Their procedure involves hanging a 0.5 kg mass on a string of varying length, L, setting it into oscillation, and measuring the period. The students conduct the experiment and obtain the following data.

Trial	1	2	3	4	5	6
Length (m)	0.25	0.50	0.75	1.0	1.5	2.0
Period (s)	1.0	1.4	1.7	2.0	2.5	2.8
			5	-		

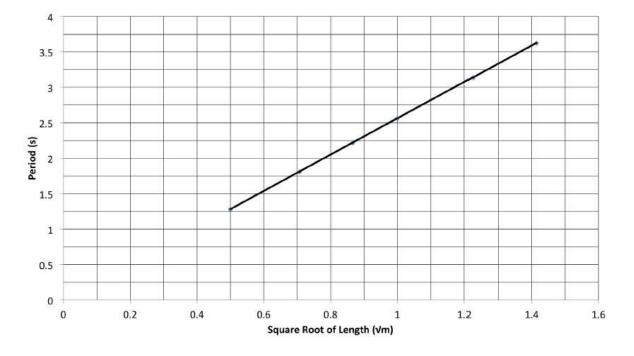
(a) On the grid below, plot the period of the pendulum T as a function of the length of the string, L, and draw a best-fit curve. Label the axes as appropriate.



Students are then given strings of various lengths, a hanging mass, a meter stick, a stopwatch, appropriate survival equipment, and are transported to the surface of Planet X. There, they are asked to determine the acceleration due to gravity on the surface of Planet X using just this equipment.

(c) Describe an experimental procedure that the student could use to collect the necessary data as accurately as possible.

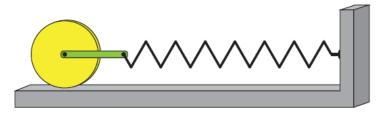
In order to determine the acceleration due to gravity, the students then create a plot of T vs. the square root of L, as shown below.



(d) Using the graph, calculate the acceleration due to gravity on Planet X.

(e) Following further analysis and experiments, a comparison of the students' experimental value for the acceleration due to gravity on Planet X is greater than the actual acceleration due to gravity on Planet X. Offer a reasonable explanation for this difference.

6. A disk with a mass M, a radius R, and a rotational inertia of $I = \frac{1}{2} MR2$ is attached to a horizontal spring which has a spring constant of k as shown in the diagram. When the spring is stretched by a distance x and then released from rest, the disk rolls without slipping while the spring is attached to the frictionless axle within the center of the disk.



(a) Calculate the maximum translational velocity of the disk in terms of M, R, x, k.

(b) What would happen to the period of this motion if the spring constant of the spring increased? Justify your answer.

(c) What would happen to the period of this motion if the surface was now frictionless and the disk was not allowed to roll? Justify your answer.

7. A 5-kg mass attached to a linear spring undergoes simple harmonic motion along a frictionless tabletop with amplitude of 0.35 m and frequency of 0.67 Hz.

(a) Explain what the two criteria are for simple harmonic motion.

(b) Calculate the value of the spring constant.

(c) What is the ratio of the mass' s acceleration when it is at half its amplitude to its acceleration when it is at full amplitude?

(d) Sketch a graph of the mass' s acceleration as a function of its velocity for half of a cycle. Start when the mass is at its greatest positive amplitude. Mark the initial acceleration as a_0 .

8. On a flat surface in Toledo, Ohio, a simple pendulum of length 0.58 m and mass 0.34 kg is pulled back an angle of 45° and released.

(a) Determine the theoretical period of the pendulum.

(b) When the period is measured (with a photogate, for 20 oscillations) it is found that the experimental period is 5 percent higher than expected. Repeated measurements consistently give similar values. What is the most likely explanation for this systematic error?

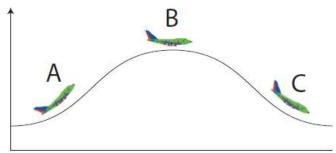
(c) What would be the period of this pendulum if it was in a free-fall environment? Explain.

9. A group of students want to design an experiment where they test the period of simple pendulum as a function of the acceleration due to gravity, g. Part of their motivation is to verify the equation of the period of a simple pendulum.

 $T_{p} = 2\pi \sqrt{\frac{l}{\sigma}}$

They propose to bring their apparatus on NASAs Zero-g airplane that can create a range of values of g for short periods of time. Their proposal calls for three trials each for two situations where $g < 9.8 \text{ m/s}_2$ (where everything feels lighter than normal), $g = 9.8 \text{ m/s}_2$ and two situations where $g > 9.8 \text{ m/s}_2$ (where everything feels heavier than normal).

(a) If the Zero-g airplane moves in a continuous series of up-and-down parabolas, as shown below, identify at which position(s) in its trajectory someone would feel heavier than normal.



(b) If the simple pendulum the students bring consists of a 1.0 kg sphere with a string attached, discuss two other important factors that must be controlled throughout the experiment and how they will be controlled.

(c) Since the students have a limited number of trials for each value of g, they need a method of minimizing uncertainty in the measurement of the period of the pendulum. Discuss a method for doing so. Be sure to list necessary equipment and how it will be applied.

(d) Assuming that the students carry out the experiment and gather valid data, they want to graph their data. If they plot the variable T_p^2 on the y-axis, what variable should they plot on the x-axis so they get a straight line best fit?

(e) When they draw the line of best fit, the students calculate a slope of 9.8 m. What is the length of their pendulum?