

AP Physics 1 Investigation 7:

Rotational Motion

What physical characteristics of an object affect the translational speed of the object after it has rolled to the bottom of an incline?

Central Challenge

This investigation introduces students to concepts of rotational motion as they analyze how characteristics of objects such as mass, radius, and shape affect the linear speeds of those objects at the bottom of a ramp. This lab provides instructions for both qualitative and quantitative investigations in rotational motion, giving you the option of choosing which type of investigation is best for your students. If time permits, you might choose to have them complete both investigations.

Background

Without friction, an object at the top of an incline would slide down the incline without rolling, resulting in only linear (or translational) motion. A friction force exerts a torque on the object, allowing it to roll down the incline. Basic kinematic equations already familiar to students can describe the linear (or translational) motion of the center of mass of the object as it changes position, but rotational motion equations must be incorporated to describe the rotational motion of each object as it rolls without slipping down the ramp. Additionally, the way in which an object rotates depends upon the rotational inertia of the object. Although students will not calculate rotational inertia in this course, they will use the concept of rotational inertia in calculations of quantities such as torque and rotational kinetic energy. This lab helps to provide a conceptual understanding of the physics properties of an object that define the object's rotational inertia.

Real-World Application

It is not difficult for students to visualize numerous everyday objects that rotate. Understanding how an object's properties impact rotational motion allows students to critically examine designs used for rotating objects. For example, bicycle racers will choose wheel designs that have properties that can enhance their racing performance. Wheels that are fairly uniform from hub to rim with light rims have low rotational inertia, so they start quickly for a short race. However, bicycle wheels with light spokes and heavier rims have higher rotational inertia, which make the bicycle more difficult to start, but once these wheels are turning they are less influenced by other forces and require more torque to stop — better for a long race or for stability on a rough terrain.

Another common example is a spinning skater. The skater can exert a torque by pushing on the ice with an extended toe. Once the skater starts rotating, bringing legs and arms in close to the spin axis causes a faster spin. Extending the arms or a leg slows the spinner down to a stop. With arms and legs spinning close to the body (and close to the spin axis), the skater has a lower effective radius of spin and lower rotational inertia. Since angular momentum is the product of rotational inertia and angular speed, angular momentum is conserved when that product remains constant. If no external torque is exerted on the skater, reducing the rotational inertia results in a faster angular speed (and faster spin), and extending to increase the rotational inertia results in lower angular speed.

Inquiry Overview

Students are provided with materials to setup a ramp and objects of various shapes, sizes, and masses to design an experiment to test how objects rotate as they roll down a ramp. If students are provided with a large assortment of objects and options to create the inclined plane, they are given more opportunity for guided inquiry that approaches open inquiry, which is recommended. Students should be given latitude to make decisions about which objects to use, how many trials are adequate, how to make measurements to determine the speed of the object at the bottom of the ramp, and how to analyze their results. Students should be provided with the opportunity prior to actual lab time to meet in groups to design their lab procedure (even though some directions are provided). It adds to the inquiry process for students to report out their procedural plans to the other groups in order to gain feedback about oversights or gain suggestions prior to actually conducting the experiment. This can also happen postlab, giving students the opportunity to engage in critical discussions with the other groups.

Initially, student groups will make qualitative predictions about how object shape, size, and mass will affect the speed of the object as it reaches the bottom of the ramp. These predictions will be discussed and compared in small student groups and recorded. Then students will run the trials and make qualitative observations. Finally, students will design methods to make measurements of the speeds of the objects at the bottom of the ramp to compare to their predictions and observations.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.A All forces share certain common characteristics when considered by observers in inertial reference frames.	3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (Science Practices 1.5, 2.1, and 2.2) 3.A.1.2 The student is able to design an experimental investigation of the motion of an object. (Science Practice 4.2) 3.A.1.3 The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding	Learning Objectives
4.C Interactions with other objects or systems can change the total energy of a system.	4.C.1.1 The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. (Science Practices 1.4, 2.1, and 2.2)

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.E The angular momentum of a system is conserved.	5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. (Science Practice 2.2)

[NOTE: In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.4 The student can <i>use representations and models</i> to analyze situations or solve problems qualitatively and quantitatively.	Students include diagrams of objects and experimental setups in order to describe procedures, and they provide qualitative explanations and/or mathematical calculations as part of their analysis.
1.5 The student can <i>re-express key elements of natural phenomena across multiple representations</i> in the domain.	Students support work with written observations of the objects' motion as part of the analysis, and they include diagrams as part of background and analysis. If the quantitative method is used, students also express the motion with equations and calculations.
2.1 The student can justify <i>the selection of a mathematical routine</i> to solve problems.	If the quantitative method is selected, students use equations and calculations to support predictions about which objects move with greater translational speed at the bottom of the ramp.
2.2 The student can <i>apply mathematical routines</i> to quantities that describe natural phenomena.	Students apply selected mathematical routines to the calculations of speed if the qualitative method is selected.
4.2 The student can <i>design a plan</i> for collecting data to answer a particular scientific question.	Students make decisions about which objects to test, how to design ramps, how to measure translational speed at the bottom of the ramp, and how to appropriately analyze the data.
4.3 The student can <i>collect data</i> to answer a particular scientific question.	Students use observations in the qualitative method or numerical measurements in the quantitative method.
5.1 The student can <i>analyze data</i> to identify patterns or relationships.	Students decide what methods will be used to analyze the data, such as graphing speed at the bottom of the ramp as a function of object radius for objects of the same mass and shape.

[NOTE: Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- ▶ Objects of different shapes, masses, and diameters (that can roll down an incline)
- ▶ Inclined plane or inclined grooved track (with sufficient coefficient of friction that chosen objects only roll and do not slide)

- ▶ Objects to prop up the inclined plane (books, bricks, pieces of wood, clamps on ring stands, etc.)
- ▶ Metersticks
- ▶ Rulers
- ▶ Stopwatch
- ▶ Mass scale
- ▶ (Optional) Motion sensor or video analysis tools

The number of different shapes of objects to roll down the incline is up to you, but it is recommended to have at least three. Examples of the most common shapes used include a hoop (a PVC or metal pipe cut into thin pieces); spherical cylinder (small sections of pipe or small metal cans); solid cylinder (samples of different metal cylinders available from lab supply density sets, or 1- and 2-inch wooden dowels cut into short sections); hollow sphere (ping pong ball, tennis ball); and solid sphere (ball bearings, marbles, or wooden balls of various diameters found in craft stores). Other shapes can be included depending on time and teacher preference.

[NOTE: On a larger scale, students can take identical small empty food cans and refill them with various types of solid material (wood putty, cement, marshmallow fluff) and then reseal them. To make objects of the same diameter and radius but different mass, pieces of PVC pipe can be cut and materials can be stuffed inside the PVC pieces to create different masses. **NOTE ALSO:** If the distribution of mass inside the PVC is different or if the mass inside the PVC can move, that introduces another factor in rotational inertia. The distribution of material inside the pipe pieces must be uniform.]

For the qualitative investigation you will need the same shapes with varying masses and radii so that students have enough to test their predictions and to come to the appropriate conclusions. Examples include a set of spheres of the same diameter but different masses (such as 1-inch steel or wooden balls) or the small metal cans as described above.

The inclined plane can be any material that allows a smooth path for the objects to roll. However, since this investigation may require timing the objects, it is recommended that the length of the inclined plane be more than 1 meter to minimize timing errors. Examples of materials you can use to construct an inclined plane include boards, ring stands, and wood strips or metersticks (which can be taped to the boards to create channels of different widths for objects to roll through). Aluminum sliding door C-channel track also works (it can be cut with a hacksaw and bent into tracks for balls to roll on).

If the analysis is done correctly, students should realize that the masses and radii of the objects do not affect the final linear speed of the objects, making the mass scale and ruler unnecessary.

Timing and Length of Investigation

- ▶ **Total Time:** 3.5–4.5 hours
- ▶ **Teacher Preparation/Set-up:** 5–10 minutes

Most of this time will be spent gathering the equipment. If you also setup the equipment for students (not highly recommended) or if you need to saw dowels, etc., into sections, more time will be needed.

Qualitative Investigation

- ▶ **Total Student Time:** 135–200 minutes
- ▶ **Part I:** 45–65 minutes
 - Prediction/Setup/Observation Time: 10–15 minutes
 - Data Collection/Calculations: 15–20 minutes
 - Discussion: 20–30 minutes

- ▶ **Parts II and III:** 30–45 minutes each
 - Prediction/Setup/Observation Time: 10–15 minutes
 - Data Collection/Calculations: 15–20 minutes
 - Discussion: 5–10 minutes

- ▶ **Part IV:** 30–45 minutes
 - Prediction/Setup/Observation Time: 5–10 minutes
 - Answer questions: 15–20 minutes
 - Discussion: 10–15 minutes

Quantitative Investigation

- ▶ **Total Student Time:** 45–65 minutes
 - Prediction/Setup/Observation Time: 10–15 minutes
 - Equation Derivation: 15–20 minutes (or more), depending on students' algebraic abilities; includes equations involving energy analysis and kinematic analysis
 - Data Collection/Calculations: 15–20 minutes; includes both energy and kinematic calculations
 - Error Analysis: 5–10 minutes

[**NOTE:** If you need to fit this investigation into a shorter class period (50–55 minutes), have the prediction/observation portion done on one day, assign the equation derivation portion as homework, and complete the rest of the investigation the next day.]

Safety

There are no specific safety concerns for this lab. However, all general lab safety guidelines should always be observed.

Preparation and Prelab

Students should have previously studied and developed proficiency with applications of kinematics equations to solutions of problems on linear motion. They should also have had previous laboratory experience determining speeds of objects in linear motion. The “Ladybug Revolution” interactive simulation on the PhET web site (see Supplemental Resources) has teacher materials available that provide ideas for student assignments and “clicker” questions to assess students’ understanding of the differences between translational and rotational motion.

Students may have previously done a similar lab for an object sliding down a ramp in which they used a rolling ball. In that situation, they may have already noticed that the ball consistently had less linear speed than predicted by energy calculations. This is an opportunity to build on that lab by giving students the opportunity to rethink the uncertainty in that previous experiment in terms of the rotational kinetic energy of the ball.

If students are still learning experimental protocols, it may be necessary to point out that they need to think about the independent and dependent variables and control other variables. For example, if students are examining the effect of mass on speed of the object, they should keep the object’s radius constant.

The Investigation

You have significant leeway here in how to proceed with this investigation. If time is limited, have students proceed with only the qualitative or the quantitative investigation. To make the lab more inquiry based, simply set out a variety of objects (disks, hoops, spheres, etc.) and have students design their own investigation to determine which factors affect the speed of an object after it has rolled to the bottom of a ramp.

QUALITATIVE INVESTIGATION

The qualitative investigation is divided into three parts as follows. Each part may be conducted by each lab group or different parts may be assigned to different lab groups, with the groups sharing their observations in a larger group discussion for final analysis.

Part I:

Students investigate the question, “How does the mass of a rolling object affect its final speed at the bottom of an incline if radius and shape are held constant?”

Have students make a prediction about whether heavy or light objects will reach the bottom with more speed. After their predictions are made, they design an experiment with the provided equipment that can be used to answer the investigation question.

This part of the investigation should lead to two sets of good discussions, within the groups and also in a whole-class debrief. The first set is about control of variables: When comparing heavier versus lighter, did students hold the shape of the objects constant? The second set is about uncertainty: The heavy and light objects will not reach the bottom with exactly the same speed. How do students decide whether the small difference in speed is a “real” difference? Depending on how deep you want to go, this issue could lead to students taking more data, representing the dispersion in their data points in some way, and using those representations to make arguments about whether the differences are real. Ideally, students would choose which representations to create and the all-class discussion would be about which representations most convincingly supported the related arguments. In any case, the class should arrive at a consensus about the (non) effect of mass on final speed before proceeding to Part II. In both the small group and all-class discussions, students should also explain why the mass didn’t matter.

Part II:

Students investigate the question, “How does the radius of a rolling object affect its final speed at the bottom of an incline if mass and shape are held constant?”

Students repeat the same type of procedure as in Part I and come to a conclusion. There will be less discussion about controlled variables and uncertainty, since time may have already been spent on this in Part I. More discussion can be devoted to why the radius doesn’t affect the final speed.

Part III:

Students investigate the question, “How does the shape of a rolling object affect its final speed at the bottom of an incline if radius and mass are held constant?”

Students again repeat the same type of procedure as in Part I and come to a conclusion. However, this time there should be an obvious difference in the final speeds of the different shapes. More discussion can be devoted to why the shape does affect the final speed and this discussion can lead into energy concepts and the difference between translational and rotational kinetic energies.

Part IV:

Students investigate the question, “Would a cart that has four solid disks for wheels have a final speed that is *greater than*, *less than*, or *equal to* the final speed of a single disc that has the same mass as the cart and wheels?”

Students then reason through the question, using observations and conclusions from Part I. In their small groups they work through the following questions to help guide their thinking, and then come together as a class to discuss their results and answers:

1. Suppose a cart with four wheels and a disk whose mass is equal to the total mass of the cart roll down the ramp. Which, if either, has more gravitational potential energy at the top?
2. Which of those objects has more kinetic energy at the bottom? Why?
3. Imagine the disk just spinning in place instead of rolling. Would it have kinetic energy? Why?
4. Why does the cart have more speed at the bottom even though it doesn't have more kinetic energy than the disk? Build upon your answers to questions 1 and 2 to answer.

QUANTITATIVE INVESTIGATION

Give students several objects of different shapes (not necessarily the same mass or radius, but they can be) that are capable of rolling down an incline. Then pose the question: "If each of these objects were rolled down an incline, each starting at the same height, how would their linear speeds compare at the bottom of the incline?" Ask the students to predict the results of the investigation before the investigation is performed.

After the predictions are made, students setup the equipment and allow the different shapes to roll down the incline, finding an appropriate method to measure the speed of each object at the bottom. (For example, students might decide to use a motion sensor to determine speed or might decide to allow each object to roll onto a level section and measure distance and time on that section to calculate linear speed.) After these initial observations are made, students must then take the necessary measurements and complete the required calculations to support their observations.

Students must use the law of conservation of energy to derive equations for the linear speed of each object at the bottom of the incline. To do this, rotational inertia equations (see Equations 1–4 below) for each object and the relationship between linear speed and angular speed will be needed; you can choose to provide these or have students research and find the equations themselves.

Next, ask the students to calculate the linear speed of each shape using kinematics. (For example, students may allow each object to roll off a table onto the floor and measure the range to determine initial speed. Or they may leave a long level section at the bottom of the ramp and measure distance and time for the object after it leaves the slope and rolls along the level section to calculate a velocity.) Necessary measurements need to be made and calculations shown for the kinematic analysis. Students should then do an error analysis since the linear speed should be the same whether it was determined experimentally using kinematics or calculated using measurements and energy conservation. After the investigation is complete, ask students if their predictions were correct. If not, have them explain why the prediction did not match the observations (i.e., resolve inconsistencies).

The following equations should be provided to students for this investigation:

$$I_{\text{cylinder}} = \frac{1}{2}mr^2$$

[Equation 1]

$$I_{\text{solid sphere}} = \frac{2}{5}mr^2$$

[Equation 4]

$$I_{\text{hoop}} = mr^2$$

[Equation 2]

$$v_{\text{cm}} = r\omega$$

[Equation 5]

$$I_{\text{hollow sphere}} = \frac{2}{3}mr^2$$

[Equation 3]

$$E_{\text{initial}} = E_{\text{final}}$$

[Equation 6]

Students should be able to derive these equations for their use in the experiment:

$$\Delta U_{\text{gravitational}} = \Delta K_{\text{translational}} + \Delta K_{\text{rotational}}$$

[Equation 7]

$$mg\Delta h = \frac{1}{2}m(\Delta v)^2 + \frac{1}{2}I(\Delta\omega)^2$$

[Equation 8]

The following equations are the results students should get for the final speed of each shape at the bottom of the incline using conservation of energy. (You might prefer students to derive an equation for an object of $I = kmr^2$ and then substitute k for each of the shapes.)

$$v_{\text{cylinder}} = \sqrt{\frac{4}{3}gh}$$

[Equation 9]

$$v_{\text{hollow sphere}} = \sqrt{\frac{6}{5}gh}$$

[Equation 11]

$$v_{\text{hoop}} = \sqrt{gh}$$

[Equation 10]

$$v_{\text{solid sphere}} = \sqrt{\frac{10}{7}gh}$$

[Equation 12]

Extension

Small student groups select an activity or sport that operates on wheels, such as bicycle racing or skateboarding. Each group researches wheel design and how it relates to the performance in that activity. They should incorporate conclusions from the lab in their final reporting to the class. Each group should also include a diagram of the wheels and how the structure relates to the activity, estimating rotational inertia, if possible.

A more challenging experiment might be for students to repeat part or all of their investigative procedures with objects and ramp heights where the object slides and rolls (rolls with slipping) down the ramp. This would be a qualitative investigation, as the quantitative measurements and calculations are beyond the scope of the course.

Common Student Challenges

First, students may have an incomplete understanding of the role of friction. If each object rolls without sliding, the friction force is necessary to provide the torque to roll the object. Without friction, the object would not roll but would slide down the ramp. Since both investigations presented in this lab introduce a new topic, they are not designed to deal with specific misconceptions or conceptual challenges. However, they will demonstrate that different shapes roll/rotate differently, and that the final speed of the object at the bottom of the incline does not depend on the mass or radius of the object, which may be surprising to many students.

Since they have been taught and have learned that objects of different mass fall at the same rate with no air resistance and that those same objects will slide down an incline at the same rate if there is no friction, students may predict that all the objects will roll down the incline and reach the bottom at the same time and with the same speed. The observations made in either investigation may surprise some students. It is important to bring the discussion around to what causes objects to roll and help students justify that what they have learned in the case of no friction is still valid.

In the quantitative investigation, the biggest challenge students face is the derivation of the equations for the final speeds of the objects using the conservation of energy, because they are usually not yet familiar with the rotational inertia equations and the equation that relates linear speed to angular speed. A possible way to help students is to show them an example of a derivation in class, using the equations from the AP Physics 1 Equations Sheet using a shape not involved in the investigation. Then students may derive the necessary equations for those shapes that will be used. You might want to limit the number of shapes so that students can spend more time on developing an understanding of the underlying concepts rather than getting bogged down in the algebra. It depends on how much class time you have to devote to the investigation and how comfortable your students are with algebraic manipulation.

Analyzing Results

Qualitative Investigation:

One method of having students analyze their results is to compare their observations to the predictions made at the beginning of the investigation. If the prediction does not match the observations, then ask students to explain/resolve the inconsistencies. This means that students need to provide an explanation of WHY they obtained the observed results. This can be done in small groups and then reported to the larger group for discussion and refinement prior to conducting the quantitative investigation.

Quantitative Investigation:

Students should calculate the percent difference between the theoretical energy analysis and the experimental finding using the kinematic method.

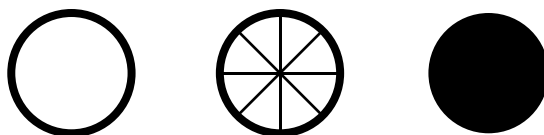
$$\% \text{ difference} = \frac{|\text{result from energy analysis} - \text{result from kinematic method}|}{\text{average of the two results}} \times 100\%$$

You could also ask them to identify sources of uncertainty in measurement, identify the source of the largest uncertainty, and explain what can be done to minimize the uncertainty if the experiment were performed again.

Possible questions you could ask during postlab discussions, or provide for students to consider during their laboratory analysis, include:

1. How well do the final linear speeds, calculated using the theoretical energy analysis and determined by experiment, compare to your predictions? Does one method consistently produce a larger or smaller value? Why?
2. What specific evidence from this investigation supports your answer?
3. How does the rotational inertia of a rolling object affect its final speed at the bottom of an incline?
4. What specific evidence from this investigation supports your answer?
5. Suppose you repeated the experiment with objects of the same radius but larger masses. Would the results of this investigation change? If so, how? If not, why not?
6. Suppose you repeated the experiment with objects of the same mass but larger radii. Would the results of this investigation change? If so, how? If not, why not?
7. If the objects in this investigation were not rolling down an incline, but were each just rotating on their own stationary, fixed axis located through the center of the object, would the mass of the object have an effect on the rotational inertia of the object? Why?
8. If the objects in this investigation were not rolling down an incline, but were each just rotating on their own stationary, fixed axis located through the center of the object, would the *radius* of the object have an effect on the rotational inertia of the object? Why?

9. Based on your observations in this investigation, rank the following objects, which all have the same mass, in terms of rotational inertia, largest to smallest. Explain the reasoning for your ranking.



10. If you were to allow these three objects to roll from rest down an incline simultaneously, in what order would they reach the bottom? Why?

You can either create or find other qualitative and quantitative questions and problems, such as TIPERS (ranking tasks) that would be an effective measure of students' understanding (see Supplemental Resources).

Assessing Student Understanding

Qualitative Investigation:

After completing this investigation, students should be able to:

- ▶ Demonstrate an understanding of what rotational inertia means;
- ▶ Explain how and why different shapes roll/rotate differently using evidence from the investigation;
- ▶ Develop ideas and questions about how and why the location of the mass of a rotating object affects the ease or difficulty of rotating that object and experimental means of verifying this; and
- ▶ Design and analyze an experiment to test the rotational properties of objects of various shapes, masses, and radii.

Quantitative Investigation:

Students will also be able to:

- ▶ Use measurements to calculate the speed of an object after it rolls to the bottom of a ramp based on conservation of energy principles;
- ▶ Use an experimental method to determine the speed of an object after it rolls to the bottom of a ramp based on kinematic principles; and
- ▶ Relate calculations of speed of a rolling object at the bottom of a ramp to a specific aspect of the physical properties of the object, when other factors are held constant.

The quantitative investigation is not intended to get students to derive rotational inertia equations. The equations are given to students or students acquire them for the purpose of using them in the investigation.

Assessing the Science Practices

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Uses diagrams of objects of various shapes to describe the rotational motions of those objects, both verbally and mathematically.
Nearly Proficient	Uses diagrams of objects to determine rotational motions of those objects in most cases, and/or uses equations to describe rotational motions for most shapes.
On the Path to Proficiency	Partially applies diagrams to the analysis of the rotational motion of several shapes or applies equations to the analysis of the motions of several shapes.
An Attempt	Uses only one type of model — either a diagram or kinematic equation — to analyze the motion of a shape.

Science Practice 1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Applies qualitative observations accurately for all the shapes used, and/or correctly makes mathematical derivations for all the shapes provided.
Nearly Proficient	Applies qualitative observations to correct conclusions for most of the shapes provided or to derive most mathematical relationships correctly for the quantitative method.
On the Path to Proficiency	Applies qualitative observations to correct conclusions for several shapes or to derive more than one (but not most) mathematical relationship(s) correctly for the quantitative method.
An Attempt	Applies qualitative observations to correct conclusions for only one shape or to derive only one mathematical relationship correctly for the quantitative method.

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient	Selects all the appropriate equations applying to various shapes and correctly relates variable for linear and rotational motion in a conservation of energy statement.
Nearly Proficient	Selects all the appropriate equations but unable to connect them to all the correct shapes. Possibly addresses conservation of energy with some errors in the derivation.
On the Path to Proficiency	Selects some appropriate equations but unable to connect them to all the correct shapes. Possibly addresses conservation of energy with equations, but not correctly.
An Attempt	Selects some appropriate equations but unable to connect them to the correct shapes. [Applies to quantitative method only.]

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Correctly calculates rotational inertia for all shapes, and correctly applies these calculations to determination of velocity using conservation of energy.
Nearly Proficient	Calculates rotational inertia for most shapes, and calculates velocity from energy conservation with minor errors.
On the Path to Proficiency	Makes some calculations, but they are incomplete; for example, missing some shapes or with a consistent error throughout.
An Attempt	Unable to make complete or correct calculations for any of the shapes, though an attempt is made for each shape. Conservation of energy calculations are missing or incomplete. [Applies to quantitative method only.]

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a plan that is appropriate, clearly thought out, and clearly described. Presents an oral or written laboratory report that has all of the following elements: labeled diagram of the setup, succinctly outlined procedure, multiple trials, clearly shown derivation of mathematical model used (if qualitative treatment is pursued).
Nearly Proficient	Designs a plan that is well thought out with good experimental controls but with a weakness that may affect one set of conclusions or is not clearly described. Offers an oral or written laboratory report that is missing one of the following elements: labeled diagram of the setup, succinctly outlined procedure, multiple trials, clearly shown derivation of mathematical model used (if qualitative treatment is pursued).
On the Path to Proficiency	Designs a plan for the assignment given (quantitative or qualitative or a particular shape) that is generally well thought out but has a flaw (e.g., trying to compare 1" steel balls to 2" wooden balls) that will affect results. Offers an oral or written laboratory report that is missing a significant number of essential elements and contains many errors in labeling, identification, mathematical calculations, and derivations.
An Attempt	Fails to think out the design plan for the assignment given (quantitative or qualitative or a particular shape) well enough to get relevant results from the experiment.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects all relevant data on all rotational shapes, organized in a data table with appropriate units.
Nearly Proficient	Collects all relevant data with some important element missing (e.g., units).
On the Path to Proficiency	Collects data but some relevant data is missing or there are not an appropriate number of trials.
An Attempt	Collects minimal data and presentation is not coherent.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Presents a complete analysis, addressing all aspects of the data, includes analysis of sources of uncertainty, and compares results using the energy method to the kinematic method.
Nearly Proficient	Presents a mostly complete analysis with only some flawed conclusions or final calculations, or doesn't make an attempt at error analysis.
On the Path to Proficiency	Clearly states data and/or observations but analysis methods are somewhat incomplete or contain some flawed conclusions.
An Attempt	Attempts an analysis but the approach is flawed.

Supplemental Resources

Hieggelke, Curtis, J., David P. Maloney, Steve Kanim, and Thomas L. O’Kuma. *TIPERs: Sensemaking Tasks for Introductory Physics*. Boston: Addison-Wesley, 2013.

“Ladybug Revolution.” PhET. University of Colorado Boulder. Accessed September 1, 2014. <http://phet.colorado.edu/en/simulation/rotation>. [*In this simulation students can move a ladybug to different locations on a rotating disk and observe the rotational speed and rotational inertia of the system, as well as several other variables.*]

Rolling Ranking Tasks Solutions. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/ap07_Rolling_Ranking_Tasks_Solutions.pdf.